EUROPEAN CENTRAL BANK

EUROSYSTEM

WORKING PAPER SERIES

NO 797 / AUGUST 2007

JOINT ESTIMATION OF THE NATURAL RATE OF INTEREST, THE NATURAL RATE OF UNEMPLOYMENT, EXPECTED INFLATION, AND POTENTIAL OUTPUT

by Luca Benati

and Giovanni Vitale

EUROPEAN CENTRAL BANK

EUROSYSTEM

WORKING PAPER SERIES

NO 797 / AUGUST 2007

JOINT ESTIMATION OF THE NATURAL RATE OF INTEREST, THE NATURAL RATE OF UNEMPLOYMENT, EXPECTED INFLATION, AND POTENTIAL OUTPUT [1]

by Luca Benati

and Giovanni Vitale [2]

In 2007 all ECB publications feature a motif taken from the €20 banknote.

1 The views expressed in this paper are those of the authors, and do not necessarily reflect those of the European Central Bank.

2 Both authors: European Central Bank, Kaiserstrasse 29, D-60311, Frankfurt am Main, Germany; e-mail: Luca.Benati@ecb.int and Giovanni.Vitale@ecb.int

Address
Kaiserstrasse 29
60311 Frankfurt am Main, Germany

Postal address
Postfach 16 03 19
60066 Frankfurt am Main, Germany

Telephone
+49 69 1344 0

Internet
http://www.ecb.europa.eu

Fax
+49 69 1344 6000

Telex
411 144 ecb d

ISSN 1561-0810 (print)
ISSN 1725-2806 (online)

CONTENTS

Abstract

We jointly estimate the natural rate of interest, the natural rate of unemployment, expected inflation, and potential output for the Euro area, the United States, Sweden, Australia, and the United Kingdom. Particular attention is paid to time-variation in (i) the data-generation process for inflation, which we capture via a time-varying parameters specification for the Phillips curve portion of the model; and (ii) the volatilities of disturbances to inflation and cyclical (log) output, which we capture via break tests.

Time-variation in the natural rate of interest is estimated to have been comparatively large for the United States, and especially for the Euro area, and smaller for Australia and the United Kingdom. Overall, natural rate estimates are characterised by a significant extent of uncertainty.

Keywords: monetary policy; natural rate of interest; time-varying parameters; Monte Carlo integration; median-unbiased estimation; endogenous break tests; bootstrapping.

JEL classification: E31, E32, E52.

Non Technical Summary

The concept of natural rate of interest—first introduced into economics by Wicksell (1898)—has been enjoying in recent years a remarkable revival, with several central banks being today engaged in computing or developing various natural rate (or natural rate gap) measures to inform, either directly or indirectly, the monetary policy process.

Conceptually in line with Laubach and Williams (2003), in this paper we jointly estimate the natural rate of interest, the natural rate of unemployment, expected inflation, and potential output for the Euro area, the United States, Sweden, Australia, and the United Kingdom. Compared with previous contributions—see in particular Laubach and Williams (2003), Clark and Kozicki (2005), and Garnier and Wilhelmsen (2005)—the present work presents, beyond its explicitly international dimension, four main novelties.

First, a time-varying parameters specification for the Phillips curve portion of the model, designed to capture both changes in equilibrium (trend) inflation, and possible changes in inflation's extent of serial correlation and in the impact of the cyclical component of economic activity on inflation. Second, different from previous papers, expected inflation is here generated endogenously within the model, as the one-step-ahead forecast produced by the just-mentioned time-varying parameters Phillips curve. Third, because of the reasons discussed by Stock (2002) in his comment on Cogley and Sargent (2002), the use of time-varying parameters methods in crucial portions of the model, like the Phillips curve, necessarily requires allowing for heteroskedasticity in the relevant shocks. Finally, in order to better filter out the cyclical component of economic activity, we exploit the additional information contained in the unemployment rate, which we introduce into the model *via* Okun's law, linking the cyclical components of unemployment and (log) output.

Time-variation in the natural rate of interest is estimated to have been comparatively large for the United States and especially for the Euro area, and smaller, instead, for Australia and the United Kingdom. Overall, natural rate estimates are characterised by a significant extent of uncertainty.

1 Introduction

The concept of natural rate of interest—first introduced into economics by Wicksell (1898)[1]—has been enjoying in recent years a remarkable revival, with several central banks being today engaged in computing or developing various natural rate (or natural rate gap) measures to inform, either directly or indirectly, the monetary policy process.

The *Federal Reserve Board*—where research on the natural rate of interest originally started, *circa* 1989, following a query from then Chairman Alan Greenspan on what level of the Federal Funds rate would be compatible with unchanging inflation—has pursued two main approaches to the estimation of the natural rate. First, a *structural* approach based on either the MIT-Penn-SSRC (MPS) or the FRB/US models of the U.S. economy,[2] or estimated DSGE models, in which the natural rate is computed essentially *via* 'reverse engineering', by imposing that the output gap be equal to zero after a certain horizon, and then computing the level of the interest rate which, given the current state of the economy, would, if sustained, attain that goal. Second, a *semi-structural* time-series approach pioneered by Laubach and Williams (2003), in which a minimal set of identifying restrictions is imposed on the data within a simple multivariate time-series framework. Conceptually in line with Wicksell (1898), the key intuition behind the Laubach-Williams approach is that the discrepancy between the actual *ex ante* real rate and the natural rate maps into fluctuations in the output gap, which, in turn, cause changes in the rate of inflation, thus allowing to filter out the natural rate *via* standard Kalman filtering methods.

Conceptually in line with Laubach and Williams (2003), in this paper we jointly estimate the natural rate of interest, the natural rate of unemployment, expected inflation, and potential output for the Euro area, the United States, Sweden, Australia, and the United Kingdom. Compared with previous, related contributions—see in particular Laubach and Williams (2003), Clark and Kozicki (2005), and Garnier and Wilhelmsen (2005)—the present work presents, beyond its explicitly international dimension,[3] four main novelties:

- a time-varying parameters specification for the Phillips curve portion of the model, designed to capture both changes in equilibrium (trend) inflation, and possible changes in inflation's extent of serial correlation and in the impact of the cyclical component of economic activity on inflation. Given the crucial role played by expected inflation within the present framework—through its impact on the real interest rate gap, defined as the difference between the natural rate and the *ex ante* real rate—a fixed-coefficients specification for the Phillips

[1] See Jonung (1979).

[2] E.g., Bomfim (1997) used the MPS model, while Bomfim (1998) used the FRB/US model.

[3] With the partial exception of Garnier and Wilhelmsen (2005), who produce natural rate estimates for the Euro area, Germany, and the United States.

curve would present (at least) one crucial drawback, i.e. that of forcing mean-reversion in inflation projections towards an 'equilibrium' reflecting the *average* level of inflation over the sample period. The fact that, for example, Euro area inflation was in double digits around the time of the Great Inflation, and it has instead oscillated around 2 per cent since the start of Stage III of European Monetary Union, in January 1999, clearly highlights the potential relevance of the problem. Within the framework adopted herein, equilibrium inflation is instead by construction time-varying, thus eliminating the problem at the root.[4]

- Different from the three previously mentioned papers, expected inflation is here generated *endogenously* within the model, as the one-step-ahead forecast produced by the just-mentioned time-varying parameters Phillips curve. Laubach and Williams (2003), on the other hand,'[...] proxy inflation expectations with the forecast [...] generated from a univariate AR(3) of inflation estimated over the prior 40 quarters',[5] while Clark and Kozicki (2005) '[...] depart from Laubach and Williams (2003) in using inflation over the past year, instead of a forecast of inflation over the year ahead, to calculate the real interest rate'—i.e., they consider the *ex post*, rather than the *ex ante*, real interest rate. Finally, Garnier and Wilhelmsen (2005) consider, likewise, the *ex post* real interest rate.

- Because of the reasons discussed by Stock (2002) in his comment on Cogley and Sargent (2002), the use of time-varying parameters methods in crucial portions of the model—like the Phillips curve—necessarily requires allowing for heteroskedasticity in the relevant shocks. In a nutshell, Stock's argument is that if reality is characterised by time-variation in *both* the parameters *and* the innovation variances, allowing for variation only in the model's parameters will automatically 'blow up' their estimated extent of time-variation, which will have to compensate for the erroneous imposition of no variation in the innovation variances.

- Finally, in order to better filter out the cyclical component of economic activity, we exploit the additional information contained in the unemployment rate, which we introduce into the model *via* Okun's law, linking the cyclical components of unemployment and (log) output. Different from the Phillips curve portion of the model, we do not employ time-varying parameters specifications for either the 'IS curve' (equation (1) below), or Okun's law (equation (9)).

[4] In response to a referee's comments, we concede that although this point is obviously correct as a matter of principle, its practical importance at the one-steap-ahead horizon which is relevant for the present purposes—see equation (1) below—is entirely an empirical matter. So it might be the case that a fixed-coefficients specification for the Phillips curve portion of the model would still deliver reasonable one-step-ahead inflation forecasts. As it is well known, on the other hand, a drawback of adopting a time-varying parameters specification is an increase in the overall extent of econometric uncertainty.

[5] See Laubach and Williams (2003, page 1064).

The key reason for this is that the model we are using, featuring three (multivariate) random walks, is under this respect already sufficiently complicated, and the addition of further random-walk components would render the estimation even more cumbersome.[6] Further, as for Okun's law, Figure 1, showing the business-cycle components of the unemployment rate and of log real GDP, for the five countries in our sample, points towards a quite remarkable stability in the relationship betwen the two objects over the sample period, thus suggesting no compelling rationale for a time-varying specification.

Time-variation in the natural rate of interest is estimated to have been comparatively large for the United States, Sweden, and especially for the Euro area, and smaller, instead, for Australia and the United Kingdom. Overall, natural rate estimates are characterised by a significant extent of uncertainty.

The paper is organised as follows. The next section describes the structure of the model, while Section 3 outlines in detail the Stock-Watson time-varying parameters median-unbiased estimation method we use to estimate the extent of random-walk time-variation in trend output growth, the Phillips curve parameters, and the natural rate of interest; the procedure we use to deconvolute the probability density functions for the three parameters we estimate *via* the Stock-Watson method; and the Monte Carlo integration procedure we use to compute median estimates and confidence bands for the time-varying objects of interest. Section 4 discusses the results, and Section 5 concludes.

2 The Model

The Phillips curve portion of the model is given by

$$\pi_t = \mu_t + \sum_{j=1}^{J} \beta_{j,t}\pi_{t-j} + \gamma_{1,t}y_{t-1}^C + \gamma_{2,t}y_{t-2}^C + \epsilon_t^\pi = \xi_t' z_t + \epsilon_t^\pi \tag{1}$$

where π_t and y_t^C are inflation and the cyclical component of log output, respectively; μ_t, the β_{jt}'s, and the $\gamma_{i,t}$'s are postulated to evolve according to driftless random walks, in order to capture changes in the equilibrium (trend) component of inflation, in its extent of serial correlation, and in the coefficients capturing the impact of

[6] It is important to remember that *all* the parameters encoding the extents of random-walk time-variation are here estimated *via* the Stock-Watson (1996, 1998) time-varying parameters median-unbiased estimation method, which in the present case is quite remarkably computationally intensive, as we simulate all of the relevant statistics/quantities.

cyclical output on inflation;[7] ϵ_t^π is a heteroskedastic[8] reduced-form shock to inflation, whose properties are discussed below; and ξ_t and z_t are defined as $\xi_t \equiv [\mu_t, \beta_{1,t}, ..., \beta_{J,t}, \gamma_{1,t}, \gamma_{2,t}]'$ and $z_t \equiv [1, \pi_{t-1}, ..., \pi_{t-J}, y_{t-1}^C, y_{t-2}^C]'$.

The logarithm of real GDP, y_t, is postulated to be the sum of two components, a 'natural' one, y_t^N, and a cyclical one, y_t^C:

$$y_t \equiv y_t^N + y_t^C \tag{2}$$

Cyclical output is a lag polynomial of past deviations of the *ex ante* real rate, (r_t-$\pi_{t|t-1}$), from the natural rate, r_t^N, which we model as a driftless random walk:[9]

$$\begin{aligned} y_t^C &= \phi_1 y_{t-1}^C + \phi_2 y_{t-2}^C + \kappa\left[\left(r_{t-1}\text{-}\pi_{t-1|t-2}\text{-}r_{t-1}^N\right) + \left(r_{t-2}\text{-}\pi_{t-2|t-3}\text{-}r_{t-2}^N\right)\right] + \epsilon_t^{yC} && (3)\\ r_t^N &= r_{t-1}^N + \epsilon_t^{rN} && (4)\end{aligned}$$

where ϵ_t^{rN} and ϵ_t^{yC} are a homoskedastic disturbance to the natural rate, and, respectively, a heteroskedastic shock[10] whose properties are discussed below. Finally, the natural level of log output is postulated to evolve according to the I(2) process

$$\begin{aligned} y_t^N &= y_{t-1}^N + \delta_{t-1} + \epsilon_t^{yN} && (5)\\ \delta_t &= \delta_{t-1} + \epsilon_t^\delta && (6)\end{aligned}$$

with ϵ_t^{yN} and ϵ_t^δ being uncorrelated homoskedastic shocks.[11]

The rate of unemployment, U_t, is postulated to be the sum of two components, a 'natural' one, U_t^N, which is assumed to evolve according to a driftless random walk, and a cyclical one, U_t^C:

$$\begin{aligned} U_t &\equiv U_t^N + U_t^C && (7)\\ U_t^N &= U_{t-1}^N + \epsilon_t^{UN} && (8)\end{aligned}$$

[7] Strong evidence on the presence of random-walk time-variation in international inflation rates—admittedly, based on time-varying parameters *univariate* representations for inflation—can be found in Benati (2004). Table 4 contains updated results (to be discussed in detail below) for the five countries in our sample based on the same Stock and Watson (1996, 1998) time-varying parameters median-unbiased estimation methodology used in Benati (2004) and Benati (2007).

[8] The justification for a heteroskedastic specification for ϵ_t^π is provided by the results from break tests in the innovation variance in univariate AR(p) representations for inflation, reported in Table 1, which we briefly discuss below.

[9] The key reason for assuming a pure random-walk specification for the natural rate—in contrast with Laubach and Williams' (2003) original specification, in which part of the variation in the natural rate was automatically linked to changes in the rate of growth of the natural level of output—is to let 'the data speak' as freely as possible.

[10] As in the case of inflation, a justification for assuming heteroskedasticity for ϵ_t^{yC} is provided by the results from break tests in the innovation variance—admittedly, in an AR(p) representation for the output growth—reported in Table 2 below.

[11] A strong rationale for allowing for the possibility that the rate of growth of the natural level of output does contain a small random-walk component is provided by results from Stock and Watson's (1996, 1998) time-varying parameters median-unbiased estimation methodology applied to univariate AR(p) representations for output growth, reported in Table 4 and to be discussed below.

with ϵ_t^{UN} a homoskedastic shock. U_t^C, in turn, is postulated to be the sum of a component proportional to the cyclical component of log GDP, y_t^C, and of a homoskedastic disturbance, ϵ_t^{UC}, capturing labor market-specific influences:[12]

$$U_t^C = \alpha y_t^C + \epsilon_t^{UC} \tag{9}$$

The justification for (9) is, of course, Okun's law, of which Figure 1 offers the simplest, and starkest, possible illustration, by plotting the business-cycle components[13] of the rate of unemployment and of the logarithm of real GDP for the five countries in our sample (in order to make the figure easier to read, the series have been standardised). For all countries, a remarkably strong negative correlation between the two components is manifestly apparent even to the naked eye.

Both heteroskedastic shocks—ϵ_t^{π} and ϵ_t^{yC}—are postulated to be zero-mean. As for the specification for their time-varying volatilities, we adopt the following approach.[14] First, we test for multiple structural breaks at unknown points in the sample in the innovation variance in AR(p) representations for π_t and Δy_t,[15] based on the Andrews and Ploberger (1994) *exp*-Wald test statistic, and the Bai (1997) method of estimating multiple breaks sequentially, one at a time, bootstrapping the critical values as in Diebold and Chen (1996). Then, at a second stage, we impose the identified volatility breaks in (1) and (3), and we estimate different volatilities for each sub-sample. Tables 1 and 2 report, for the two variables, the identified break dates, together with 90 per cent confidence intervals based on Bai (2000); the *exp*-Wald test statistics and the bootstrapped p-values; and, for each sub-sample, the estimated standard deviation of the innovation together with a 90 per cent confidence interval. (In order to correctly interpret the numbers reported in the tables, it is important to keep in mind that inflation and output growth are here computed as the simple log-difference of the

[12] This specification for the cyclical component of the unemployment rate is conceptually in line with Kim and Nelson (2000)—see Kim and Nelson (2000, pp. 37-43).

[13] Business-cycle components have been extracted via the Christiano and Fitzgerald (2003) band-pass filter. Following established conventions in business-cycle analysis, business-cycle components have been defined as those with periodicities between 6 quarters and 8 years.

[14] As we discuss below, this specification is conceptually in line with Boivin (2004) and Benati (2007), and is fully consistent with the way the Stock-Watson's (1996, 1998) time-varying parameters median-unbiased estimation method has been derived. (We thank Mark Watson for confirming this to us.)

[15] Given that y_t^C is unobserved, we are, as a matter of fact, compelled to test for breaks in the volatility of ϵ_t^{yC} by testing for breaks in the innovation variance of Δy_t. One possible objection would be: 'Why don't you compute a reasonable proxy for the cyclical component of log output, and then test for breaks in the innovation variance in an AR(p) representation for that proxy?' The problem here is that the only reasonable proxy we can think of is HP-filtered (or band-pass filtered) log output. As it is well known, linear filters distort the stochastic properties of a process' innovation, in the specific sense that, given that the filtered series is a moving-average of either the entire sample, or a portion of it, its 'innovation' is, by the same token, a moving-average of the original series' innovation. As a result, testing for breaks in the 'innovation' of either HP- or band-pass filtered log output is technically incorrect, and we have therefore settled, although grudgingly, for performing tests in the innovation variance of Δy_t, which we regard as the least-worst option.

GDP deflator and of real GDP, respectively.) For all countries, with the exception of Sweden, we identify a single volatility break for both inflation and output growth.[16]

Equations (1)-(9) can be put in state-space form, and the log-likelihood of the data can be computed *via* the Kalman filter by means of the traditional prediction-error decomposition formula, as found in (e.g.) Harvey (1989), Hamilton (1994b), and Kim and Nelson (2000). Given the non-linearity of (1), in implementing the Kalman filter we follow Harvey (1989) and Hamilton (1994a), and we linearise it around $s_{t|t-1}$—with s_t being the state vector within the state-space form, and $s_{t|t-1}$ being its expectation conditional on information at time t-1—as

$$\begin{aligned}\pi_t \;\cong\; & \mu_t - \gamma_{1,t|t-1} y^C_{t-1|t-1} - \gamma_{2,t|t-1} y^C_{t-2|t-1} + \sum_{j=1}^{J} \beta_{j,t} \pi_{t-j} + \\ & + \gamma_{1,t|t-1} y^C_{t-1} + \gamma_{2,t|t-1} y^C_{t-2} + y^C_{t-1|t-1} \gamma_{1,t} + y^C_{t-2|t-1} \gamma_{2,t} + \epsilon^\pi_t \end{aligned} \tag{10}$$

We then perform the updating step of the Kalman filter based on (10), whereas we perform the forecasting step based on (1). All of the remaining details of the traditional Kalman filtering algorithm remain unchanged.

3 Methodology

Three parameters—σ^2_δ, σ^2_{rN}, and λ, the extent of random-walk drift in the Phillips curve (see below)—are estimated *via* the Stock-Watson time-varying parameters median-unbiased (henceforth, TVP-MUB) estimation method,[17] whereas the remaining parameters are estimated *via* maximum likelihood, conditional on the MUB estimates of λ, σ^2_δ, and σ^2_{rN}. Given that joint estimation of all the parameters is, in practice, unfeasible, in the spirit of Laubach and Williams (2003) we proceed sequentially as follows, by (i) first estimating λ and σ^2_δ; (ii) estimating *via* maximum likelihood a version of the model with a constant neutral rate, conditional on the

[16] In terms of comparison with the previous literature, it is interesting to notice, e.g., that the date of the volatility break we identify for U.S. output growth, 1984:2, is only one quarter apart from that identified by both McConnell and Perez-Quiros (2000) and Kim and Nelson (1999) based on previous vintages of data, 1984:1.

[17] See Stock and Watson (1996) and Stock and Watson (1998). The Stock-Watson method has been specifically designed to effectively deal with those cases in which the standard deviations of the innovations to the random-walk components are especially small, so that—because of the 'pile-up' problem discussed (e.g.) by Stock (1994)—pure maximum likelihood methods tend to estimate them equal to zero. Given that this is very likely to be the case for the extent of random-walk drift in either the Phillips curve, the drift in potential output, or the neutral rate, we have therefore decided to resort to median-unbiased estimation for all of these parameters. As for σ^2_{yN}, on the other hand, the work of (e.g.) Watson (1986) has clearly shown that the 'pile-up' problem is not there for the post-WWII U.S., so that—given the well-known larger extent of time-variation in potential output growth in the Eurozone, compared with the United States—it can be safely regarded as even less of a problem within the present context.

TVP-MUB estimates of λ and σ_δ^2; (iii) using the parameter estimates obtained in (ii) to simulate the model conditional on a grid of values for σ_{rN}^2, thus obtaining a TVP-MUB estimate of the extent of random-walk drift in the neutral rate; and (iv) finally, re-estimating the entire model via maximum likelihood conditional on the TVP-MUB estimates of λ, σ_δ^2, and σ_{rN}^2.

The next three sub-sections describe in detail median-unbiased estimation of λ, σ_δ^2, and σ_{rN}^2, and maximum likelihood estimation of the model's remaining parameters; the procedure we use to deconvolute the probability density functions for the MUB estimates of λ, σ_δ^2, and σ_{rN}^2; and the Monte Carlo integration procedure we use in order to compute median estimates and confidence bands for the time-varying objects of interest, taking into account of both parameter and filter uncertainty.

3.1 Median-unbiased estimation of $\boldsymbol{\lambda}$, σ_δ^2, and σ_{rN}^2

3.1.1 Median-unbiased estimation of $\boldsymbol{\lambda}$

By proxying y_t^C in (1) with HP-filtered log output,[18] we estimate the extent of random-walk drift in the Phillips curve conceptually in line with Stock and Watson (1996). We have

$$\xi_t = \xi_{t-1} + \eta_t \tag{11}$$

with η_t iid $N(0_{J+3},\, \lambda^2\sigma^2 Q)$, with 0_{J+3} being a $(J{+}3)$-dimensional vector of zeros; σ_π^2 being the variance of ϵ_t^π in the homoskedastic version of (1);[19] Q being a covariance matrix; and $E[\eta\epsilon_t^\pi]{=}0$. Following Nyblom (1989) and Stock and Watson (1996, 1998), we set $Q{=}[E(z_t z_t')]^{-1}$.[20]

Our point of departure is the OLS estimate of ξ in the time-invariant version of (1). Conditional on $\hat{\xi}_{OLS}$ we compute the residuals, we estimate of the innovation variance, $\hat{\sigma}_\pi^2$, and we perform an *exp*-Wald joint test for a single break at an unknown point the sample in ξ, using the Andrews (1991) HAC covariance matrix estimator to control for possible autocorrelation and/or heteroskedasticity in the residuals. We

[18] As we will see in Section 4.2 below, for *all* countries our estimate of cyclical log output is very strongly correlated to HP-filtered log output, which justifies *ex post* our use of HP-filtered log output within the present context. On the other hand, an anonymous referee pointed out that our estimates of cyclical log output are in general more volatile than HP-filtered log output, so that proxying y_t^C in (1) with HP-filtered log output might cause a slight upward bias in the estimated extent of random-walk time-variation.

[19] Following Boivin (2004) and Benati (2007), heteroskedasticity is introduced at a later stage.

[20] Under such a normalisation, the coefficients on the transformed regressors, $[E(z_t z_t')]^{-1/2}z_t$, evolve according to a multivariate standard random walk, with λ^2 being the ratio between the variance of each 'transformed innovation' and the variance of u_t. (To be precise, given that the Stock-Watson methodology is based on local-to-unity asymptotics, λ is actually equal to the ratio between τ, a small number which is fixed in each sample, and T, the sample length.)

estimate the matrix Q as in Stock and Watson (1996) as

$$\hat{Q} = \left[T^{-1} \sum_{t=1}^{T} z_t z_t' \right]^{-1}. \tag{12}$$

We consider a 30-point grid of values for λ over the interval [0, 0.2], which we call Λ. For each $\lambda_j \in \Lambda$ we compute the corresponding estimate of the covariance matrix of η_t as $\hat{Q}_j = \lambda_j^2 \hat{\sigma}^2 \hat{Q}$, and conditional on $\hat{Q}_j$ we simulate model (1)-(11) 2,000 times as in Stock and Watson (1996, section 2.4), drawing the pseudo innovations from pseudo random *iid* $N(0, \hat{\sigma}^2)$. For each simulation, we compute an *exp*-Wald test—without however applying the Andrews (1991) correction—thus building up its empirical distribution conditional on λ_j. Based on the empirical distributions of the test statistic we then compute the median-unbiased estimate of λ as that particular value of λ_j such that the median of the distribution conditional on λ_j is closest to the statistic we previously computed based on the actual data. Finally, we compute the p-value based on the empirical distribution of the test conditional on λ_j=0.

Results are reported in the first and fourth columns of Table 3. The p-values, equal or close to zero for all countries, strongly points towards rejection of the null hypothesis of time-invariance against the alternative of random-walk time-variation, whereas the estimates of the extent of random-walk drift, ranging between 0.04828 for the United States, to 0.08966 for Sweden, are quite substantial indeed. The first two columns of Table 4, reporting results from the Stock and Watson (1996, 1998) TVP-MUB methodology applied to univariate AR(p) representations for inflation,[21] provide an informal check of the reliability of the results for the Philips curve reported in Table 3. As the table shows, the p-values are equal to zero, or very low, for all countries except the Euro area, while the MUB estimates of λ—although, quite obviously, not numerically identical to those reported in Table 3, are still comparatively large, ranging from 0.03448 for the Euro area to 0.09655 for Sweden.

3.1.2 Median-unbiased estimation of σ_δ^2

Since, from a conceptual point of view, the methodology we use to estimate σ_δ^2 is identical to the one we just discussed, in this sub-section we proceed faster. We start by performing an *exp*-Wald joint test for a single break at an unknown point the sample in the intercept and the sum of the AR coefficients in an AR(p) representation for Δy_t,[22] using the Andrews (1991) HAC covariance matrix estimator to control for possible autocorrelation and/or heteroskedasticity in the residuals. We then estimate

[21] The methodology is exactly the same as that used in Benati (2007), to which the reader is referred to for further details.

[22] We select the lag order based on the AIC.

via maximum likelihood[23] a model given by (2), (5), and

$$y_t^C = \phi_1 y_{t-1}^C + \phi_2 y_{t-2}^C + \epsilon_t^{yC} \qquad (13)$$

—in other words, we eliminate the term $\kappa\,[\cdot]$ from (3), thus subsuming its impact in the error term—with δ_t=δ, imposing the volatility break identified for the innovation variance of Δy_t in Table 1, and thus estimating a different volatility for each subsample. Conditional on the MLE estimates of ϕ_1, ϕ_2, σ_{yN}^2, and δ, we then simulate the model given by (2), (5), (13) and (6) conditional on a 30-point grid[24] of values for σ_δ^2 [25] over the interval $[0,\, 0.1^2 \times \hat{\sigma}_{yN,MLE}^2]$,[26] drawing the pseudo innovations from pseudo random *iid* $N(0,\, \hat{\sigma}_{yC,MLE}^2)$, where $\hat{\sigma}_{yC,MLE}^2$ is the MLE estimate for the volatility of ϵ_t^{yC} which we obtain by estimating a homoskedastic version of the model (in other words, by not imposing the volatility breaks in estimation).[27] For each simulation, we perform the same *exp*-Wald test we performed based on the actual data—without however applying the Andrews (1991) correction, obviously ...—thus building up its empirical distribution conditional on $\sigma_{\delta,j}^2$. Based on the empirical distributions of the test statistic we then compute the median-unbiased estimate of σ_δ^2 and the p-value.

Results are reported in the second and fifth columns of Table 3. Different from the previous section, both the p-values and the MUB estimates of σ_δ point towards some heterogeneity across countries, with, on the one hand, p-values equal to zero and comparatively large estimates of σ_δ for both Australia and the United Kingdom,[28] and, at the other extreme, a p-value equal to 0.902, and a MUB estimate of σ_δ equal to

[23] We implement maximum likelihood estimation by numerically maximising the log-likelihood of the data *via* simulated annealing. Following Goffe, Ferrier, and Rogers (1994), we implement simulated annealing *via* the algorithm proposed by Corana, Marchesi, Martini, and Ridella (1987), setting the key parameters to T_0=100,000, r_T=0.9, N_t=5, N_s=20, ϵ=10^{-6}, N_ϵ=4, where T_0 is the initial temperature, r_T is the temperature reduction factor, N_t is the number of times the algorithm goes through the N_s loops before the temperature starts being reduced, N_s is the number of times the algorithm goes through the function before adjusting the stepsize, ϵ is the convergence (tolerance) criterion, and N_ϵ is number of times convergence is achieved before the algorithm stops. Initial conditions were chosen stochastically by the algorithm itself.

[24] For each point in the grid we simulate the model 2,000 times.

[25] An anonymous referee pointed out that, by estimating the value of σ_δ^2, rather than its ratio with σ_{yN}^2, we might end up slightly over-estimating the extent of random-walk time-variation in potential output.

[26] The key idea here is that the $I(2)$ component can't be 'too large' compared with the $I(1)$ component, which we make it operational by imposing that the standard deviation of ϵ_t^δ be at most equal to 10% of the standard deviation of ϵ_t^{yN}.

[27] The key reason for using, in simulating the model, the estimates of ϕ_1, ϕ_2, σ_{yN}^2, and δ obtained based on the heteroskedastic version of the model, together with the estimated volatility from the homoskedastic version, is that, on the one hand, we want to use the best possible estimates for ϕ_1, ϕ_2, σ_{yN}^2, and δ—which are, quite obviously, those coming from the heteroskedastic version of the model; on the other hand, however, for reasons of computational intensity, we want to simulate a homoskedastic model, in order to avoid having to perform a time-consuming HAC correction when performing the break tests.

[28] For these two countries, the constraint provided by the upper value of the grid is binding.

zero, for the United States. Concerning the result for the United States, several things ought to be stressed. First, the result is identical to that based on simple time-varying parameters AR(p) representations reported in the third and fourth columns of Table 4, with the same MUB estimate of σ_δ, and a near-identical p-value, 0.901. Second, although the MUB estimate of the extent of random-walk drift is equal to zero, about 8 per cent of the probability mass of the deconvoluted PDF of $\hat{\sigma}^2_\delta$—on deconvoluting the PDFs of the MUB estimates of the extents of random walk drift, see Section 3.2 below—is associated with values of σ_δ greater than zero. This is the reason why, as we will see in Section 4 below, the Monte Carlo integration procedure we use to compute both median estimates and confidence bands for the time-varying objects of interest (described in Section 3.3) will produce a tiny extent of time-variation in U.S. trend output growth. Given that the first step of the procedure involves integrating out parameter uncertainty, and given that some draws are associated with a positive extent of time-variation, the average across draws for the elements of the state vector will indeed be characterised by a tiny non-zero extent of time-variation. Third, such a result is in contrast with that of Laubach and Williams (2003), who identify a quite significant extent of time-variation in U.S. trend output growth. It is important to stress, however, that Laubach and Williams (2003) impose a direct link between trend output growth and the natural rate, so that the finding of time-variation in both objects can in principle be driven by time-variation in just the natural rate.

3.1.3 Median-unbiased estimation of σ^2_{rN}

We start by performing an *exp*-Wald test for a single break at an unknown point the sample in the mean of the *ex post* real rate. Following Bai and Perron (2003) and Benati (2007), we perform the break test by regressing the series on a constant, using the Andrews (1991) HAC covariance matrix estimator to control for possible autocorrelation and/or heteroskedasticity in the residuals.

Conditional on the TVP-MUB estimates of λ and σ^2_δ, we then estimate, *via* maximum likelihood, a version of model (1)-(6) with a constant natural rate, imposing in estimation all the volatility breaks identified in Table 1. (Numerical optimisation of the log-likelihood is implemented *via* the simulated annealing algorithm described in footnote 19.) We then use the MLE parameter estimates, together with the TVP-MUB estimates of λ and σ^2_δ, to simulate the entire model[29] conditional on a 30-point grid of values for σ^2_{rN} over the interval [0, 1.5567×10^{-6}],[30] drawing the pseudo innovations in (1) and (3) from pseudo random *iid* $N(0, \hat{\sigma}^2_{\pi,MLE})$ and, respectively, *iid*

[29] A subtle issue in simulating (1)-(6) is imposing a stationarity condition. We do that by exploiting the fact that the relevant portion of the model has a (time-varying) VAR representation. For each single quarter we compute the associated time-varying companion form of the VAR, and we reject all unstable draws.

[30] The upper limit of the grid, 1.5567×10^{-6}, corresponds to a standard deviation of ϵ^{rN}_t of 50 basis points per quarter on an annualised basis, which we regard as a quite significant extent of time-variation for an object like the natural rate.

$N(0,\, \hat{\sigma}^2_{yC,MLE})$, where $\hat{\sigma}^2_{\pi,MLE}$ and $\hat{\sigma}^2_{yC,MLE}$ are the MLE estimates for the volatilities of ϵ^{π}_t and ϵ^{yC}_t which we obtain by estimating a homoskedastic version of the model (in other words, by not imposing the volatility breaks for inflation and the output gap equations in estimation).[31] Given that model (1)-(6) does not include an equation for the determination of the nominal rate, in performing the simulations we postulate that r_t is determined based on the information the public has at time t-1 according to the Fisherian relationship $r_t{=}r^N_{t-1}{+}\pi_{t|t-1}$.[32] For each simulation we then compute an *exp*-Wald test for a single break at an unknown point the sample in the mean of the *ex post* real rate (without however applying the Andrews (1991) correction), thus building up its empirical distributions. Finally, based on the empirical distributions of the test statistic we compute the median-unbiased estimate of σ^2_{rN} as that particular value of $\sigma^2_{rN,\, j}$ which is closest to the statistic we previously computed based on the actual *ex post* real rate, and we compute the p-value based on the empirical distribution of the test conditional on σ^2_{rN}=0.

Results are reported in Table 3. For all countries except Sweden, and possibly Australia, p-values point towards weak evidence of time-variation in the natural rate. As discussed at length in Benati (2007), however, a key reason for significantly underplaying the informational content of simulated p-values for TVP-MUB estimates of the extent of random-walk drift is that a p-value above 10 per cent should be regarded as significant evidence against time-variation if and only if the researcher had very compelling reasons for believing in time-invariance. It is not clear at all, however, *why* this should be the case—to put it differently, it is not clear why the hypothesis of time-invariance should be granted such a privileged status.

3.2 Deconvoluting the probability density functions of the MUB estimates of λ, σ^2_{δ}, and σ^2_{rN}

In order to compute both median estimates and confidence bands for the time-varying objects of interest *via* the Monte Carlo integration procedure detailed in the next sub-section, a crucial preliminary step involves deconvoluting the probability density functions of the MUB estimates of λ, σ^2_{δ}, and σ^2_{rN}, which, following Benati (2007), we do as follows.

Let x be $x = \lambda,\, \sigma^2_{\delta},\, \sigma^2_{rN}$. To fix ideas, let's start by considering the construction of a (1-α)% confidence interval for $\hat{x}$, $[\hat{x}^L_{(1-\alpha)}, \hat{x}^U_{(1-\alpha)}]$, and let's assume, for the sake of simplicity, that x_j and $\hat{x}$ can take any value over $[0; \infty)$. Given the duality between hypothesis testing and the construction of confidence intervals, the (1-α)% confidence set for $\hat{x}$ comprises all the values of x_j that cannot be rejected based on a two-sided test at the α% level. Given that an increase in x_j automatically shifts the PDF of $\hat{L}_j$

[31] The logic here is exactly the same as that discussed in footnote 24.

[32] Here an obvious, more sophisticated alternative would have been to specify a Taylor rule, to jointly estimate it together with (1)-(6), and to use it in performing the simulations. We have chosen the present, less sophisticated alternative uniquely for reasons of simplicity.

conditional on x_j upwards, $\hat{x}^L_{(1-\alpha)}$ and $\hat{x}^U_{(1-\alpha)}$ are therefore such that

$$P\left(\hat{L}_j > \hat{L} \mid x_j = \hat{x}^L_{(1-\alpha)}\right) = \alpha/2 \tag{14}$$

$$P\left(\hat{L}_j < \hat{L} \mid x_j = \hat{x}^U_{(1-\alpha)}\right) = \alpha/2 \tag{15}$$

Let $\phi_{\hat{x}}(x_j)$ and $\Phi_{\hat{x}}(x_j)$ be the probability density function and, respectively, the cumulative probability density function of $\hat{x}$, defined over the domain of x_j. The fact that $[\hat{x}^L_{(1-\alpha)}, \hat{x}^U_{(1-\alpha)}]$ is a (1-α)% confidence interval automatically implies that (1-α)% of the probability mass of $\phi_{\hat{x}}(x_j)$ lies between $\hat{x}^L_{(1-\alpha)}$ and $\hat{x}^U_{(1-\alpha)}$. This in turn implies that $\Phi_{\hat{x}}(\hat{x}^L_{(1-\alpha)})$=$\alpha/2$ and $\Phi_{\hat{x}}(\hat{x}^U_{(1-\alpha)})$=1-$\alpha/2$. Given that this holds for any 0<α<1, we therefore have that

$$\Phi_{\hat{x}}(x_j) = P\left(\hat{L}_j > \hat{L} \mid x_j\right) \tag{16}$$

In this way, based on the *exp*-Wald test statistic, $\hat{L}$, and on the simulated distributions of the $\hat{L}_j$'s conditional on the x_j's in Λ, we obtain an estimate of the cumulative probability density function of $\hat{x}$ over the grid Λ, let's call it $\hat{\Phi}_{\hat{x}}(x_j)$. Finally, we fit a logistic function to $\hat{\Phi}_{\hat{x}}(x_j)$ via non-linear least squares and we compute the implied estimate of $\phi_{\hat{x}}(x_j)$—call it $\hat{\phi}_{\hat{x}}(x_j)$—scaling its elements so that they sum to one.

3.3 Computing median estimates and confidence bands for the time-varying objects of interest

Conditional on the median-unbiased estimates of of $\boldsymbol{\lambda}$, σ^2_δ, and σ^2_{rN}, we then re-estimate the entire model—i.e., all of the remaining parameters—*via* maximum likelihood (once again, numerical optimisation of the log-likelihood is performed *via* the simulated annealing algorithm described in footnote 19). We then compute both median estimates and confidence bands for the time-varying objects of interest *via* the following Monte Carlo integration procedure, taking into account of both parameter and filter uncertainty. The procedure is an adaptation to the case at hand of the Monte Carlo integration procedure proposed by Hamilton (1985) and Hamilton (1986).

The first step consists in integrating out *parameter* uncertainty, i.e. uncertainty pertaining to the true values of both the parameters we estimate *via* the Stock-Watson TVP-MUB methodology (λ, σ^2_δ, and σ^2_{rN}), and the remaining parameters which we estimate *via* maximum likelihood conditional on the MUB estimates of λ, σ^2_δ, and σ^2_{rN} (κ, σ^2_{yN}, ϕ_1, ϕ_2, $\sigma^2_{yC,1}$, ..., σ^2_{yC,N_1}, $\sigma^2_{\pi,1}$, ..., σ^2_{π,N_2}). Let x be $x = \lambda$, σ^2_δ, σ^2_{rN}, and let X be $X = [\kappa, \sigma^2_{yN}, \phi_1, \phi_2, \sigma^2_{yC,1}, ..., \sigma^2_{yC,N_1}, \sigma^2_{\pi,1}, ..., \sigma^2_{\pi,N_2}]'$. Further, let $\hat{x}$ and $\hat{\phi}_{\hat{x}}(x_j)$ be the median-unbiased estimate of x and its estimated deconvoluted discretised probability density function, respectively, and let $\hat{X}$ and $\hat{\Sigma}_{\hat{X}}$ be the maximum likelihood estimate of X and its estimated Hessian, respectively.[33] Let $MN(h, H)$ be a multivariate normal distribution with mean h and covariance matrix H. For j = 1, 2, 3, ..., 10000, (1) we draw from $\hat{\phi}_{\hat{x}}(\cdot)$—let's define the j-th draw as $\tilde{x}_j$—for $x = \lambda$, σ^2_δ, σ^2_{rN}, and (2) we draw from $MN(\hat{X},\ \hat{\Sigma}_{\hat{X}})$—let's define the j-th draw as $\tilde{X}_j$—and conditional on these draws we run the Kalman filter[34] and smoother, thus getting estimates of the state vector and of its precision matrix at each t, $\theta^i_{t|\tau}$ and $P^i_{t|\tau}$, respectively, with τ=t for one sided estimates, and τ=T for two-sided ones. Finally, for each t we take the mean across the 10,000 draws for both $\theta^i_{t|\tau}$ and $P^i_{t|\tau}$, τ=t, T—let's define them as $\bar{\theta}_{t|\tau}$ and $\bar{P}_{t|\tau}$, respectively—thus integrating out uncertainty about x and X.

The second step then consists in quantifying the extent of *filter* uncertainty, which we do by repeating the following 10,000 times. For each t from J+1 to T draw from $MN(\bar{\theta}_{t|\tau}, \bar{P}_{t|\tau})$, τ=t, T. Call this draw $\theta^k_{t|\tau}$. Based on $\theta^k_{t|\tau}$, compute the time-varying objects of interest—the natural rate, trend inflation, etc. ...—thus building up their distributions. Finally, based on the distributions of the time-varying objects of interest, we compute both median estimates and 16th and 84th percentiles—i.e., the percentiles corresponding to one standard deviation.

4 Empirical Evidence

Table 5 reports the maximum likelihood estimates of the model's remaining parameters conditional on the MUB estimates of of λ, σ^2_δ, and σ^2_{rN}, while Figures 2-11

[33] We compute $\hat{\Sigma}_{\hat{X}}$ numerically as in (e.g.) An and Schorfheide (2006).

[34] We initialise the Kalman filter's state vector and its precision matrix as follows. The neutral rate is initialised at the mean of the *ex-post* real rate computed over the entire sample. As for the trend and cyclical components of log output, we set them equal to the initial values of the HP-filtered trend and cyclical components of log output, respectively. The random-walk drift in trend log output is initialised at a value equal to the difference between the second and the first value taken by HP-filtered trend log output. Finally, the portion of the state vector pertaining to the Phillips curve is initialised at a value equal to the OLS estimate of the time-invariant model computed over the entire sample.

We postulate that the initial value of the precision matrix has a block-diagonal structure (this assumption is made partly for reasons of convenience, and partly because it is not clear at all how we could reasonably specify the off-diagonal elements), and we set the relevant elements as follows. As for the block corresponding to the Phillips curve, we set it equal to 4 times the Andrews (1991) HAC estimate of the covariance matrix of the OLS estimate of the time-invariant model computed over the entire sample. As for the remaining elements of the state vector, we set the corresponding block of the precision matrix to $0.01^2 \cdot I_k$, where I_k is the identity matrix. Although it is not immediately apparent, it is important to stress that, given the scale of the variables we are working with, a standard deviation equal to 0.01 for the initial extent of uncertainty pertaining the neutral rate, natural and cyclical log output, and the drift in natural output is substantial indeed. Given that inflation is computed as the simple log-difference of the GDP deflator, and that all other variables are (re-)scaled accordingly, a standard deviation equal to 0.01 on a quarter-on-quarter non-annualised basis translates into 4.06 percentage points on an annual basis. This implies that, for example, a 90% confidence interval for the initial value of the neutral rate stretches from -5.78 to 10.13 per cent.

show median estimates for the time-varying objects of interest—the natural rate, the natural rate gap, cyclical log output, output growth's time-varying trend, trend inflation, and the natural and cyclical components of the unemployment rate—together with the 16th and 84th percentiles of the simulated distributions (i.e., the percentiles corresponding to one standard deviation).

4.1 The Euro area

Starting from the Euro area, the top-left panels of Figures 2 and 3 show median estimates of the natural rate and of trend output growth, respectively, together with the 16th and 84th percentiles of the two distributions based on Monte Carlo integration. The natural rate is estimated to have gently declined from 4.0 per cent at the very beginning of the sample to 1.7 per cent at the end of 2006. As the width of the one standard deviation confidence bands clearly shows, however, natural rate estimates have historically been characterised by a significant extent of uncertainty, to the point that (e.g.) a 90 per cent confidence interval for the last quarter of the sample, 2006 Q4, stretches from -4.3 to 8.1 per cent. Interestingly, although—different from, e.g. Laubach and Williams (2003) and Clark and Kozicki (2005)—our model does not impose *any* correlation whatsoever between the natural rate and trend output growth,[35] our results clearly point, nonetheless, towards a remarkably close comovement between the two objects, with trend output growth estimated to have decreased from 3.3 per cent during the first half of the 1970s to 1.9 per cent at the end of the sample.

The top-right and bottom-left panels of Figure 2 show median estimates and one standard deviation percentiles for the real interest rate gap—defined as the difference between the natural rate and the *ex ante* real rate, r_t^N-$\left(r_t\text{-}\pi_{t|t-1}\right)$—and for the output gap, respectively. In order to provide an informal assessment of the ability of our output gap measure to reliably capture fluctuations in the cyclical component of economic activity in the Euro area, the output gap estimate has been plotted together with HP-filtered log output. As the figure shows, the correlation between the two objects, although not perfect, has indeed been very close, having been, in particular, extremely high at the very beginning and at the very end of the sample, while during the period between the first half of the 1980s and the beginning of the new century the correlation was still very high, but our output gap measure was systematically lower than HP-filtered log output. The broad picture emerging from the bottom-left panel of Figure 2 is that of an overheated economy around the time of the Great Inflation episode, up until the very beginning of the 1980s, with an overall large and positive output gap; a significant economic slowdown during the first half of

[35] As it is well known—and it is discussed in (e.g.) Laubach and Williams (2003, Section II)—neoclassical growth theory predicts indeed a close relationship between the two objects, with an increase (decrease) in trend output being associated with a corresponding increase (decrease) in the natural rate of interest.

the 1980s, when (based on median estimates) the output gap is estimated to have decreased from about 4 per cent to between -3 and -4 per cent; an economic recovery around the turn of the decade, and a further slowdown during the second half of the 1990s; and a comparatively greater stability over the most recent period, with the cyclical component of economic activity estimated to have oscillated around zero. A comparison between cyclical output and the real interest rate gap—see in particular the bottom-right panel of Figure 2, where cyclical output is plotted together with the real interest rate gap three years before[36]—clearly suggests the interest rate gap to pre-date[37] comparatively lower-frequency movements in the output gap. As the figure shows, indeed, on the one hand the real interest rate gap has pre-dated very broad swings in cyclical output, from large and positive to negative, and then, slowly and progressively, to positive again. On the other hand, the interest rate gap failed to predict two comparatively higher frequency upswings around end of the 1980s-beginning of the 1990s, and at the very beginning of the new century; and it missed the depth of the downswing around mid-1980s.

The top-right panel of Figure 3 shows median estimates and one standard deviation percentiles for trend inflation,[38] together with actual inflation, while the two bottom panels show the actual and natural unemployment rate, and the cyclical components of unemployment and log output, respectively. Trend inflation is estimated to have fluctuated between 6.5 and 9.5 per cent around the time of the Great Inflation episode; to have progressively declined starting in 1982; and to have been around 2 per cent since the beginning of Stage III of the European Monetary Union (henceforth, EMU), in January 1999.[39]

4.2 The United States

Figures 4 and 5 report the corresponding objects for the United States. Starting from trend inflation, cyclical output, the natural rate of unemployment, and trend output growth—for which we have some reasonable prior information, and whose estimates should therefore be regarded as a sort of 'cross-check' of the model's overall adequacy and reliability—trend inflation exhibits the well-known hump-shaped pattern iden-

[36] In order to make the comparison clearer, both series have been demeaned and standardised, and had all the components with frequencies of oscillation faster than six quarters removed. Filtering was performed *via* the Christiano and Fitzgerald (2003) band-pass filter.

[37] An important point to stress is that, while the leading property of the real interest rate gap for the output gap holds *by construction*—see equation (3)—the specific extent by which the interest rate gap leads the output gap is entirely determined by the data.

[38] Trend inflation is defined as $\mu_t / \left(1 - \sum_{j=1}^{J} \beta_{j,t}\right)$.

[39] In order to correctly assess trend inflation estimates for the post-January 1999 period, it is worth stressing that (i) the *European Central Bank* aims at keeping *HICP* inflation 'below but close' to 2 per cent; and (ii) whereas, over the entire sample period (1970:2-2006:4), GDP deflator inflation has exceeded HICP inflation, on average, by 0.75 percentage points, post-January 1999 HICP inflation has exceeded GDP deflator inflation by an average of 0.12 percentage points.

tified, e.g., by Cogley and Sargent (2002), Cogley and Sargent (2005), Cogley and Sbordone (2005), and Benati and Mumtaz (2007), with a peak (based on median estimates) around 7 per cent in 1981, a marked decrease over the next decade and a half, and a mild increase over the most recent years. The cyclical component of output is, once again, very strongly correlated with HP-filtered log GDP, being, in particular, numerically very close both before the beginning of the 1970s, and over the last decade. The most significant difference between our output gap measure and HP-filtered log output pertains to the 1980s, and especially to the cyclical trough associated with the Volcker recession, when HP-filtered log GDP 'stops' at around minus 4 per cent, while our gap measure falls all the way to about minus 9 per cent. The natural rate of unemployment exhibits a pattern of time-variation in line with that found, e.g., in Kim and Nelson (2000), with a peak of 6 per cent at the end of the 1970s-beginning of the 1980s, and a gentle decline in subsequent years, reaching 5.2 per cent at the end of the sample. Conceptually in line with the weak evidence of time-variation in U.S. trend output growth of Cogley (2005), equilibrium GDP growth is estimated to have experienced a very mild decline over the sample period, from 3.6 per cent around mid-1950s to 3.2 per cent at the end of 2006. Turning, finally, to the natural rate, our estimates point towards a quite significant extent of time-variation, starting at 0.8 per cent around mid-1950s, increasing—albeit non monotonically—up to 2.7 per cent around mid-1980s, and decreasing over the following years, fluctuating around 1.7 per cent since the beginning of the new century. In line with the Euro area, U.S. natural rate estimates are characterised by a significant extent of uncertainty, although lower than in the previous section, due to the longer span of data.

4.3 Sweden, Australia, and the United Kingdom

Finally, turning to Sweden, Australia, and the United Kingdom, cyclical output is, once again, and reassuringly, very strongly correlated with HP-filtered log output, while both trend inflation and output growth, and the natural rate of unemployment, well capture low-frequency movements in inflation, output growth, and the unemployment rate, respectively. The natural rate exhibits a significant extent of time-variation in Sweden, starting at about 3 per cent around the end of the 1970s, increasing up to 3.8 per cent ten years later, and then declining up to 2 per cent at the end of the sample. Australia and the United Kingsdom, on the other hand, exhibit a lower extent of time-variation, with an overall mild decline from 2.1 to 1.5 per cent, and from 2.5 to 1.6 per cent, respectively, over the sample period.

5 Conclusions

In this paper we have jointly estimated the natural rate of interest, the natural rate of unemployment, expected inflation, and potential output for the Euro area, the

United States, Sweden, Australia, and the United Kingdom. Particular attention has been paid to time-variation in (i) the data-generation process for inflation, which we have captured via a time-varying parameters specification for the Phillips curve portion of the model; and (ii) the volatilities of disturbances to inflation and cyclical (log) output, which we capture via break tests. Time-variation in the natural rate of interest is estimated to have been comparatively large for the United States, and especially the Euro area and Sweden, and smaller for Australia and the United Kingdom. Overall, natural rate estimates are characterised by a significant extent of uncertainty.

References

An, S., and F. Schorfheide (2006): "Bayesian Analysis of DSGE Models," *Econometric Reviews*, forthcoming.

Andrews, D. K. (1991): "Heteroskedasticity and Autocorrelation-Consistent Covariance Matrix Estimation," *Econometrica*, 59(3), 817–858.

Andrews, D. K., and W. Ploberger (1994): "Optimal Tests When a Nuisance Parameter is Present Only Under the Alternative," *Econometrica*, 62(6), 1383–1414.

Bai, J. (1997): "Estimating Multiple Breaks One at a Time," *Econometric Theory*, 13(3), 315–352.

——— (2000): "Vector Autoregressive Models with Structural Changes in Regression Coefficients and in Variance-Covariance Matrices," *Annals of Economics and Finance*, 1, 303–339.

Bai, J., and P. Perron (2003): "Computation and Analysis of Multiple Structural Change Models," *Journal of Applied Econometrics*, 18(1), 1–22.

Benati, L. (2004): "International Evidence on the Persistence of Inflation," available at at: http://www.nber.org/ confer/2004/si2004/efwwprg.html.

——— (2007): "Drift and Breaks in Labor Productivity," *Journal of Economic Dynamics and Control*, 31, 2847–2877.

Benati, L., and H. Mumtaz (2007): "U.S. Evolving Macroeconomic Dynamics: A Structural Investigation," *European Central Bank Working Paper N. 746, April 2007.*

Boivin, J. (2004): "Has U.S. Monetary Policy Changed? Evidence from Drifting Coefficients and Real-Time Data," *Journal of Money, Credit and Banking*, 38(5), 1149–1173.

Bomfim, A. N. (1997): "The Equilibrium Fed Funds Rate and the Indicator Properties of Term-Structure Spreads," *Economic Inquiry*, October, 830–846.

——— (1998): "Measuring Equilibrium Real Interest Rates," Manuscript, Federal Reserve Board.

Christiano, L., and T. Fitzgerald (2003): "The Band-Pass Filter," *International Economic Review*, 44(2), 435–465.

Clark, T., and S. Kozicki (2005): "Estimating Equilibrium Real Interest Rates in Real Time," *North American Journal of Economics and Finance*, December 2005.

Cogley, T. (2005): "How Fast Can the New Economy Grow?," *Journal of Macroeconomics*, 27(2), 179–207.

Cogley, T., and T. J. Sargent (2002): "Evolving Post-WWII U.S. Inflation Dynamics," *in B. Bernanke and K. Rogoff, eds. (2002), NBER Macroeconomics Annuals 2001.*

——— (2005): "Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S.," *Review of Economic Dynamics*, 8(April), 262–302.

Cogley, T., and A. M. Sbordone (2005): "A Search for a Structural Phillips Curve," Federal Reserve Bank of New York Staff Report No. 203, March 2005.

Corana, L., M. Marchesi, C. Martini, and S. Ridella (1987): "Minimizing Multimodal Functions of Continuous Variables with the Simulated Annealing Algorithm," *ACM Transactions on Mathematical Software*, 13, 262–80.

Diebold, F., and C. Chen (1996): "Testing Structural Stability with Endogenous Breakpoint: A Size Comparison of Analytic and Bootstrap Procedures," *Journal of Econometrics*, 70(1), 221–241.

Garnier, J., and B.-R. Wilhelmsen (2005): "The Natural Real Interest Rate and the Output Gap in the Euro Area: A Joint Estimation," *ECB Working Paper n. 546, November 2005.*

Goffe, W. L., G. Ferrier, and J. Rogers (1994): "Global Optimization of Statistical Functions with Simulated Annealing," *Journal of Econometrics*, 60,(1/2), 65–99.

Hamilton, J. (1985): "Uncovering Financial Markets Expectations of Inflation," *Journal of Political Economy*, 93(6), 1224–1241.

——— (1986): "A Standard Error for the Estimated State Vector of a State-Space Model," *Journal of Econometrics*, 33(3), 387–397.

——— (1994a): "State-Space Models," *in R. Engle and D. McFadden, eds., Handbook of Econometrics, vol. IV, Amsterdam, North Holland.*

——— (1994b): *Time Series Analysis.* Princeton, NJ, Princeton University Press.

Harvey, A. (1989): *Forecasting, Structural Time Series Models, and the Kalman Filter.* Cambridge University Press.

Jonung, L. (1979): "Knut Wicksell's Norm of Price Stabilisation and Swedish Monetary Policy in the 1930s," *Journal of Monetary Economics*, 5, 459–496.

Kim, C. J., and C. Nelson (1999): "Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov-Switching Model of the Business-Cycle," *Review of Economics and Statistics*, 81, 608–616.

Kim, C. J., and C. Nelson (2000): *State-Space Models with Regime Switching*. Cambridge, Mass., The MIT Press.

Laubach, T., and J. Williams (2003): "Measuring the Natural Rate of Interest," *The Review of Economics and Statistics*, 85(4), 1063–1070.

McConnell, M., and G. Perez-Quiros (2000): "Output Fluctuations in the United States: What Has Changed Since the Early 1980s?," *American Economic Review*, 90, 1464–1476.

Nyblom, J. (1989): "Testing for the Constancy of Parameters Over Time," *Journal of the American Statistical Association*, 84(405), 223–230.

Stock, J. (2002): "Discussion," *in B. Bernanke and K. Rogoff, eds. (2002), NBER Macroeconomics Annuals 2001.*

Stock, J., and M. Watson (1996): "Evidence of Structural Instability in Macroeconomic Time Series Relations," *Journal of Business and Economic Statistics*, 14(1), 11–30.

——— (1998): "Median-Unbiased Estimation of Coefficient Variance in a Time-Varying Parameter Model," *Journal of the Americal Statistical Association*, 93(441), 349–358.

Stock, J. H. (1994): "Unit Roots, Structural Breaks, and Trends," *in R. Engle and D. MacFadden (eds.), 'Handbook of Econometrics', Vol. 4, Amsterdam, Elsevier*, pp. 2739–2841.

Watson, M. (1986): "Univariate Detrending with Stochastic Trends," *Journal of Monetary Economics*, 18, 49–75.

Wicksell, K. (1898): "The Influence of the Rate of Interest on Commodity Prices," *Lecture delivered to the Swedish Economic Association, April 14, 1898.*

A The Data

Euro area Quarterly seasonally adjusted series for real GDP, the GDP deflator, and the unemployment rate, and a quarterly seasonally unadjusted series for the short rate, are from the *European Central Bank*'s database. The sample period is 1970:1-2006:4.

United States Seasonally adjusted quarterly series for real GDP ('GDPC96, Real Gross Domestic Product, 3 Decimal, Quarterly, Seasonally Adjusted Annual Rate, Billions of Chained 2000 Dollars'), the GDP deflator ('GDPDEF, Gross Domestic Product: Implicit Price Deflator, Quarterly, Seasonally Adjusted Annual Rate, Index 2000=100'), are from the U.S. Department of Commerce, *Bureau of Economic Analysis*. A monthly seasonally adjusted series for the rate of unemployment ('UNRATE, Civilian Unemployment Rate, Persons 16 years of age and older, Seasonally Adjusted, Monthly, Percent'), are from the U.S. Department of Labor, *Bureau of Labor Statistics*. A monthly seasonally adjusted series for the Federal Funds rate ('FEDFUNDS, effective Federal Funds rate, Monthly, Percent') is from the Board of Governors of the Federal Reserve System. Quarterly series for the unemployment rate and the Federal Funds rate have been constructed by taking averages within the quarter of the corresponding monthly series. The sample period is 1954:3-2006:4.

Sweden A monthly seasonally adjusted series for the rate of unemployment, from *Statistics Sweden*, has been converted to the quarterly frequency by taking averages within the quarter. Quarterly seasonaly adjusted series for real GDP, the 3-month rate on Treasury discount notes, and the CPI for urban and rural areas (series' codes are 14499BVPZF..., 14460C..ZF..., and 14464...ZF..., respectively) are from the *International Monetary Fund*'s *International Financial Statistics* (henceforth, *IFS*). The sample period is 1976:1-2006:2.

Australia A monthly seasonally adjusted series for the rate of unemployment, from *Global Financial Data*, has been converted to the quarterly frequency by taking averages within the quarter. Quarterly seasonaly adjusted series for real GDP, the GDP deflator, and the average rate on the money market (series' codes are 19399BVRZF..., 19399BIRZF..., and 19360B..ZF..., respectively) are from the *International Monetary Fund*'s *International Financial Statistics* (henceforth, *IFS*). The sample period is 1969:3-2006:3.

United Kingdom Quarterly seasonally adjusted series for real GDP and the GDP deflator, and a monthly seasonally unadjusted series for the rate of unemployment (based on the claimant count) are all from the *Office for National Statistics*. The unemployment series has been seasonally adjusted *via* the ARIMA X-12 procedure as implemented in *EViews*, and it has been converted to the quarterly frequency by taking averages within the quarter. A quarterly seasonally unadjusted series for the Treasury Bill rate (series' code is 11260C..ZF...) is from the *IFS*. The sample period is 1957:1-2006:4.

Table 1 Tests for multiple breaks at unknown points in the sample in the innovation variance of inflation based on Andrews-Ploberger (1994) and Bai (1997)

Break dates and 90% confidence intervals	*exp*-Wald (p-value)	Sub-periods	Standard deviation, and 90% confidence interval
	Euro area		
1976:3 [1971:4; 1981:2]	11.00 (0.001)	1970:2-1976:2	8.40E-3 [7.66E-3; 9.32E-3]
		1976:3-2006:4	2.58E-3 [2.35E-3; 2.86E-3]
	United States		
1985:4 [1977:2; 1994:2]	9.97 (0.004)	1954:4-1985:3	3.51E-3 [3.25E-3; 3.82E-3]
		1985:4-2006:4	1.99E-3 [1.85E-3; 2.17E-3]
	United Kingdom		
1984:4 [1979:1; 1990:3]	21.500 (0.008)	1955:2-1984:3	0.014 [0.013; 0.015]
		1984:4-2006:4	5.7E-3 [5.3E-3; 6.3E-3]

Table 2 Tests for multiple breaks at unknown points in the sample in the innovation variance of output growth based on Andrews-Ploberger (1994) and Bai (1997)

Break dates and 90% confidence intervals	*exp*-Wald (p-value)	Sub-periods	Standard deviation, and 90% confidence interval
	Euro area		
1993:2 [1987:2; 1999:2]	12.414 (0.004)	1970:2-1993:1	6.52E-3 [5.95E-3; 7.22E-3]
		1993:2-2006:4	2.87E-3 [2.62E-3; 3.18E-3]
	United States		
1984:2 [1978:2; 1990:2]	16.832 (0.000)	1954:4-1984:1	0.011 [9.9E-3; 0.012]
		1984:2-2006:4	4.8E-3 [4.4E-3; 5.2E-3]
	Australia		
1984:2 [1978:1; 1990:3]	11.087 (0.008)	1969:4-1984:1	0.015 [0.014; 0.017]
		1984:2-2006:3	6.8E-3 [6.2E-3; 7.5E-3]
	United Kingdom		
1992:3 [1988:1; 1997:1]	32.460 (0.000)	1955:2-1992:2	0.012 [0.011; 0.013]
		1992:3-2006:4	2.7E-3 2.5E-3; 2.9E-3]

Table 3 Results based on the Stock-Watson TVP-MUB methodology: *exp*-Wald test statistics, simulated *p*-values, and median-unbiased estimates of λ, σ_δ, σ_{rW}

	exp-Wald test (*p*-value)			MUB estimates		
	Phillips curve	potential output	natural rate	$\hat{\lambda}$	$\hat{\sigma}_\delta$	$\hat{\sigma}_{rN}$
Euro area	51.16 (0.000)	1.77 (0.095)	15.98 (0.102)	0.08276	2.11E-4	6.24E-4
United States	14.74 (0.002)	0.424 (0.902)	2.13 (0.174)	0.04828	0.000	2.32E-4
Sweden	22.85 (0.000)	2.174 (0.115)	21.57 (5.0E-4)	0.08966	6.07E-4	1.27E-3
Australia	21.67 (0.000)	14.489 (0.000)	9.49 (0.089)	0.06207	8.17E-4	1.09E-3
United Kingdom	22.39 (0.000)	12.056 (0.000)	7.10 (0.190)	0.06207	6.69E-4	7.68E-4

Table 4 Univariate results based on the Stock-Watson TVP-MUB methodology: *exp*-Wald test statistics, simulated *p*-values, and median-unbiased estimates of $\hat{\tau}$

	Inflation		Output growth	
	exp-Wald (*p*-value)	$\hat{\tau}$	*exp*-Wald (*p*-value)	$\hat{\tau}$
Euro area	3.758 (0.253)	0.03448	1.765 (0.238)	0.02069
United States	6.825 (0.009)	0.08276	0.424 (0.901)	0.00000
Sweden	17.780 (0.000)	0.09655	2.174 (0.149)	0.03103
Australia	6.229 (0.026)	0.04828	14.489 (0.000)	0.03103
United Kingdom	26.273 (0.000)	0.04828	12.056 (0.000)	0.03103

Table 5 Maximum likelihood estimates of the remaning parameters and 90% confidence intervals conditional on the median-unbiased estimates of λ, σ_δ, σ_{rW}

Parameter	*Euro area*	*United States*	*Sweden*
α	-0.253 [-0.291; -0.209]	-0.560 [-0.646; -0.476]	-0.291 [-0.339; -0.243]
σ_{UN}	2.0E-3 [1.8E-3; 2.2E-3]	6.6E-4 [2.4E-4; 1.01E-3]	4.1E-3 [3.7E-3; 4.5E-3]
σ_{UC}	8.2E-4 [7.5E-4; 9.0E-4]	1.9E-4 [5.2E-5; 3.2E-4]	0.016 [0.014; 0.017]
κ	-0.114 [-0.131; -0.095]	-0.337 [-0.398; -0.288]	-0.316 [-0.369; -0.264]
σ_{yN}	1.0E-3 [9.3E-4; 1.1E-3]	5.6E-3 [5.2E-3; 5.9E-3]	6.6E-3 [6.1E-3; 7.2E-3]
ϕ_1	1.229 [1.044; 1.436]	1.226 [1.023; 1.419]	1.058 [0.886; 1.229]
ϕ_2	-0.530 [-0.610; -0.440]	-0.254 [-0.293; -0.214]	-0.271 [-0.315; -0.226]
$\sigma_{yC,1}$	6.3E-3 [5.7E-3; 6.9E-3]	5.4E-3 [4.9E-3; 5.8E-3]	7.0E-3 [6.4E-3; 7.5E-3]
$\sigma_{yC,2}$	2.9E-3 [2.2E-3; 3.4E-3]	2.9E-3 [2.5E-3; 3.2E-3]	
$\sigma_{\pi,1}$	8.3E-3 [7.6E-3; 8.9E-3]	2.3E-3 [1.1E-3; 3.0E-3]	5.0E-3 [4.6E-3; 5.4E-3]
$\sigma_{\pi,2}$	1.8E-3 [4.4E-4; 2.9E-3]	1.7E-3 [5.3E-4; 3.0E-3]	
	Australia	*United Kingdom*	
α	-0.224 [-0.255; -0.186]	-0.353 [-0.411; -0.294]	
σ_{UN}	4.2E-3 [3.9E-3; 4.7E-3]	3.2E-3 [2.9E-3; 3.5E-3]	
σ_{UC}	0.016 [0.015; 0.017]	0.012 [0.011; 0.013]	
κ	-0.349 [-0.409; -0.305]	-0.370 [-0.431; -0.310]	
σ_{yN}	5.3E-3 [4.8E-3; 5.7E-3]	5.2E-3 [4.8E-3; 5.7E-3]	
ϕ_1	1.090 [0.897; 1.253]	1.050 [0.876; 1.224]	
ϕ_2	-0.317 [-0.384; -0.277]	-0.397 [-0.463; -0.332]	
$\sigma_{yC,1}$	0.014 [0.013; 0.016]	0.010 [9.5E-3; 0.011]	
$\sigma_{yC,2}$	5.5E-3 [3.1E-3; 7.1E-3]	2.6E-3 [7.8E-4; 4.2E-3]	
$\sigma_{\pi,1}$	9.0E-3 [7.7E-3; 0.010]	0.025 [0.022; 0.028]	
$\sigma_{\pi,2}$	4.8E-3 [3.3E-3; 5.9E-3]	8.2E-3 [2.5E-3; 0.013]	

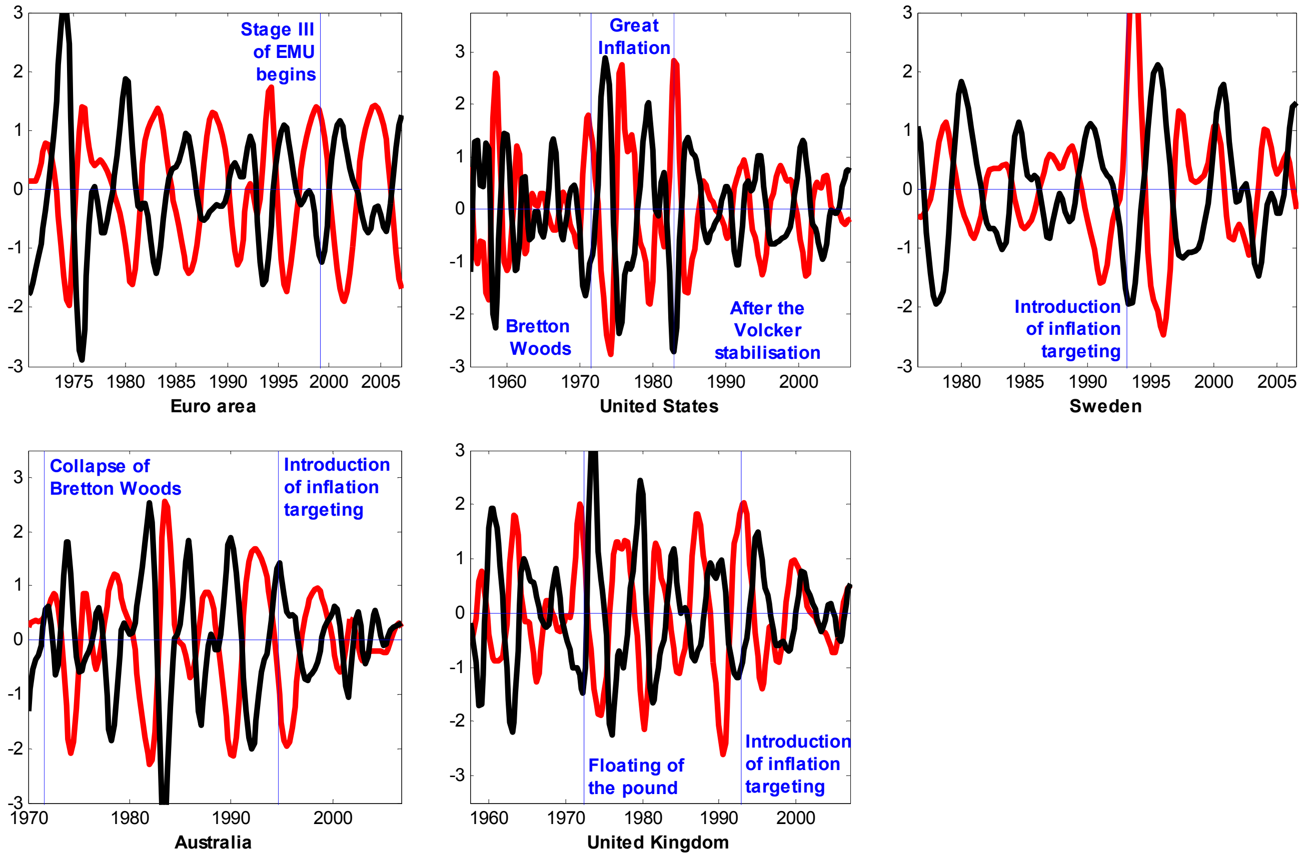

Figure 1 Okun's Law: business-cycle components of the unemployment rate and of log real GDP (both series have been standardised)

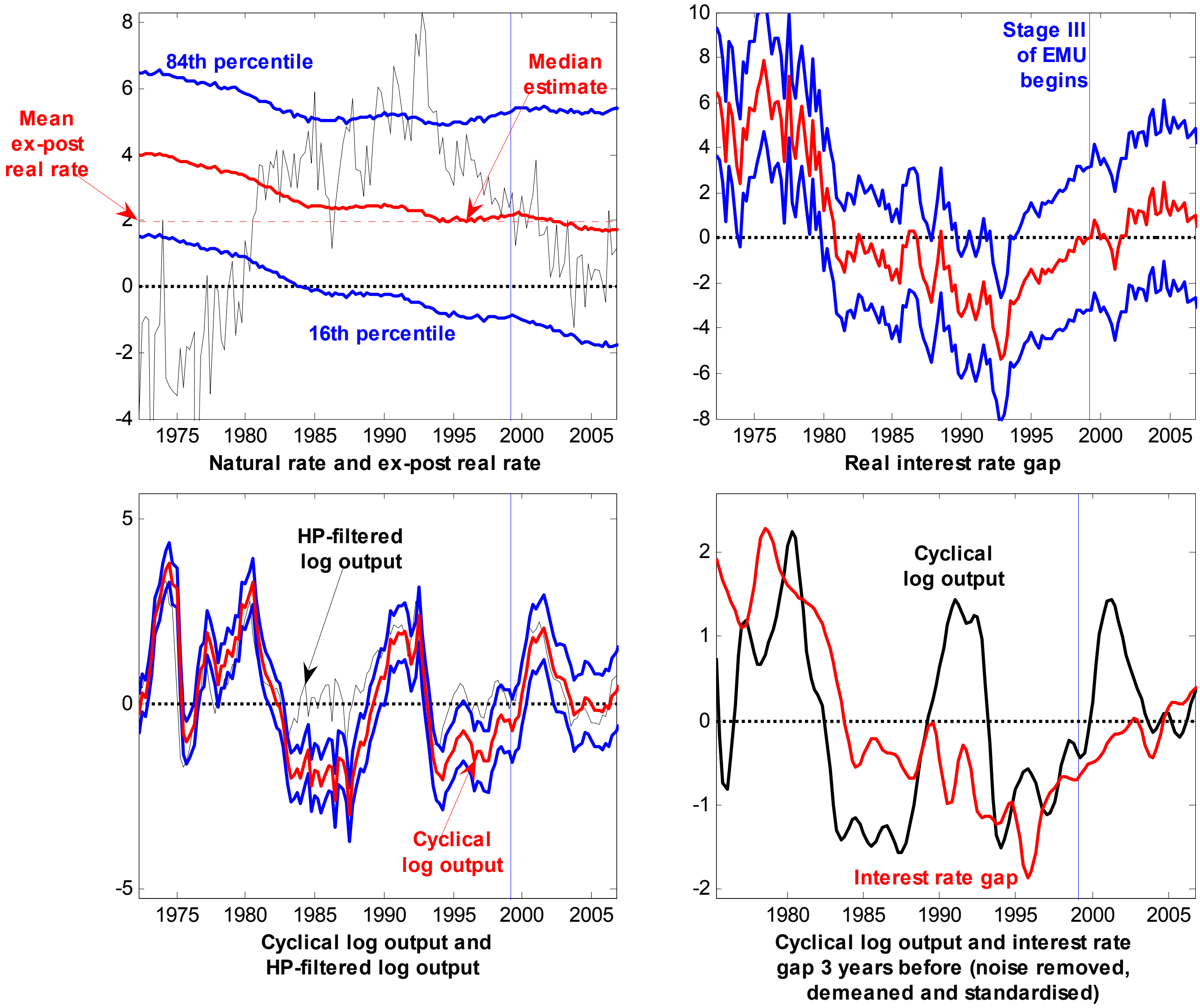

Figure 2 Euro area: the natural rate, the natural rate gap, and cyclical log output (two-sided estimates)

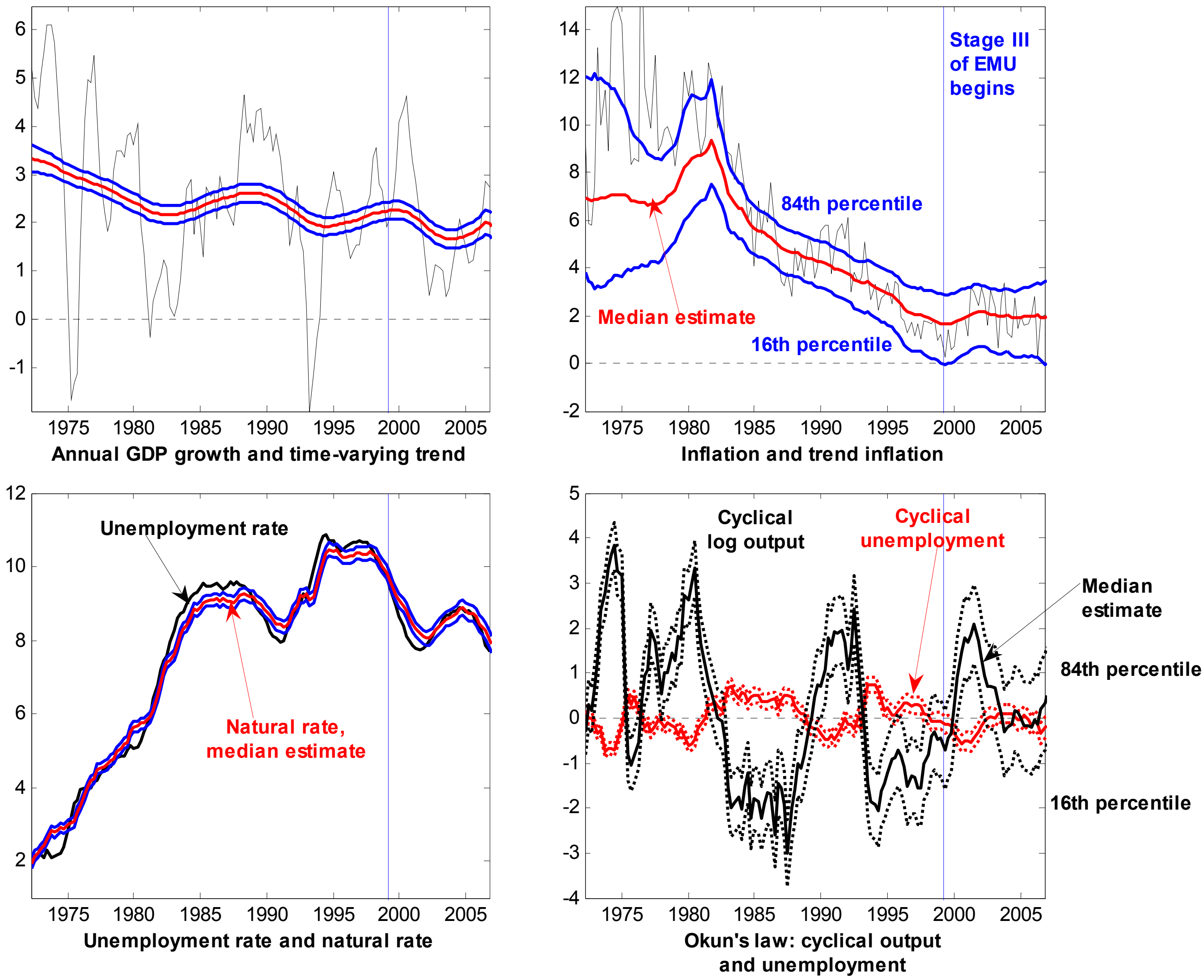

Figure 3 Euro area: annual GDP growth and time-varying trend, inflation and trend inflation,unemployment rate and natural rate of unemployment, and cyclical components of unemployment and log output (two-sided estimates)

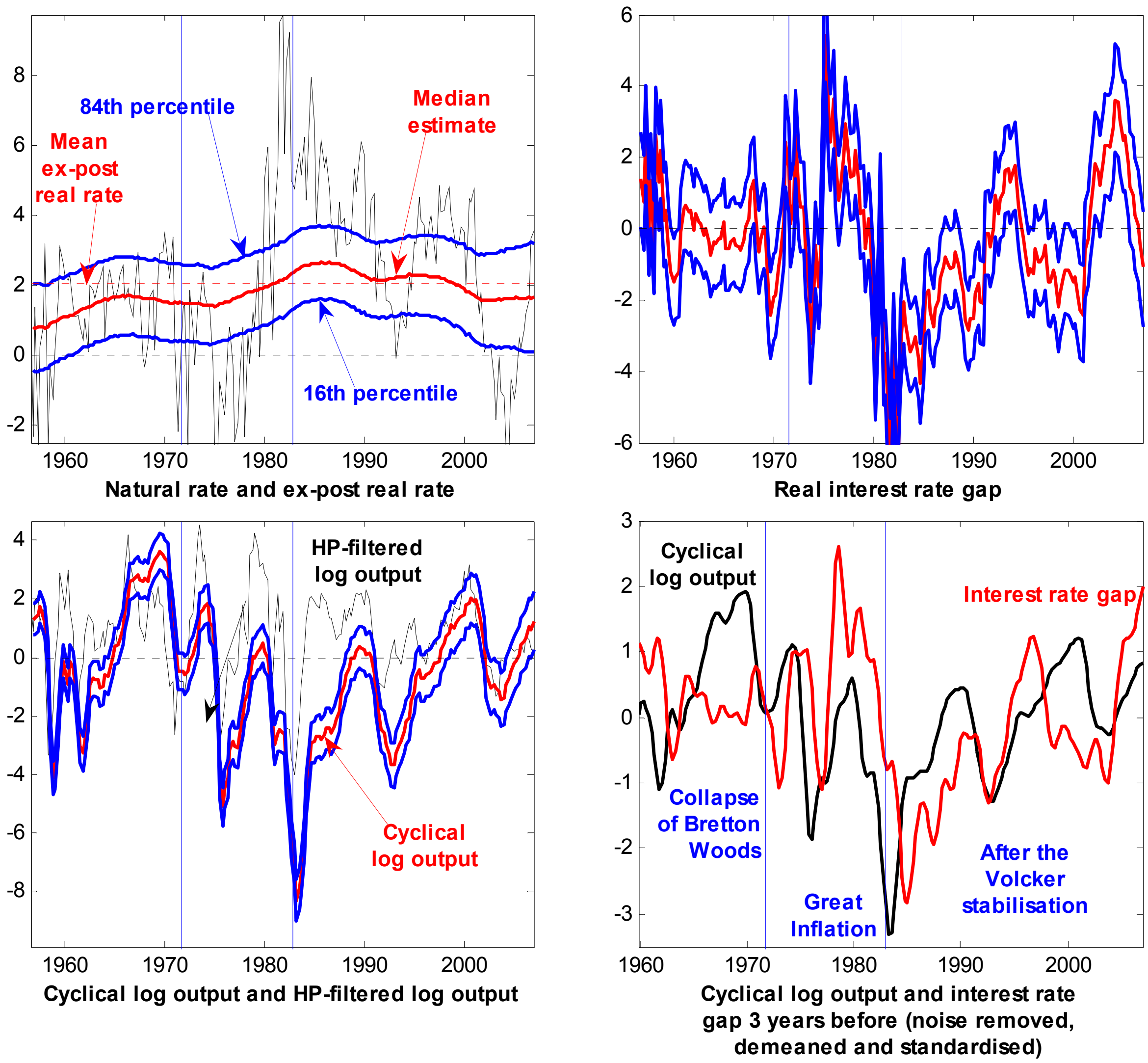

Figure 4 United States: the natural rate, the natural rate gap, and cyclical log output (two-sided estimates)

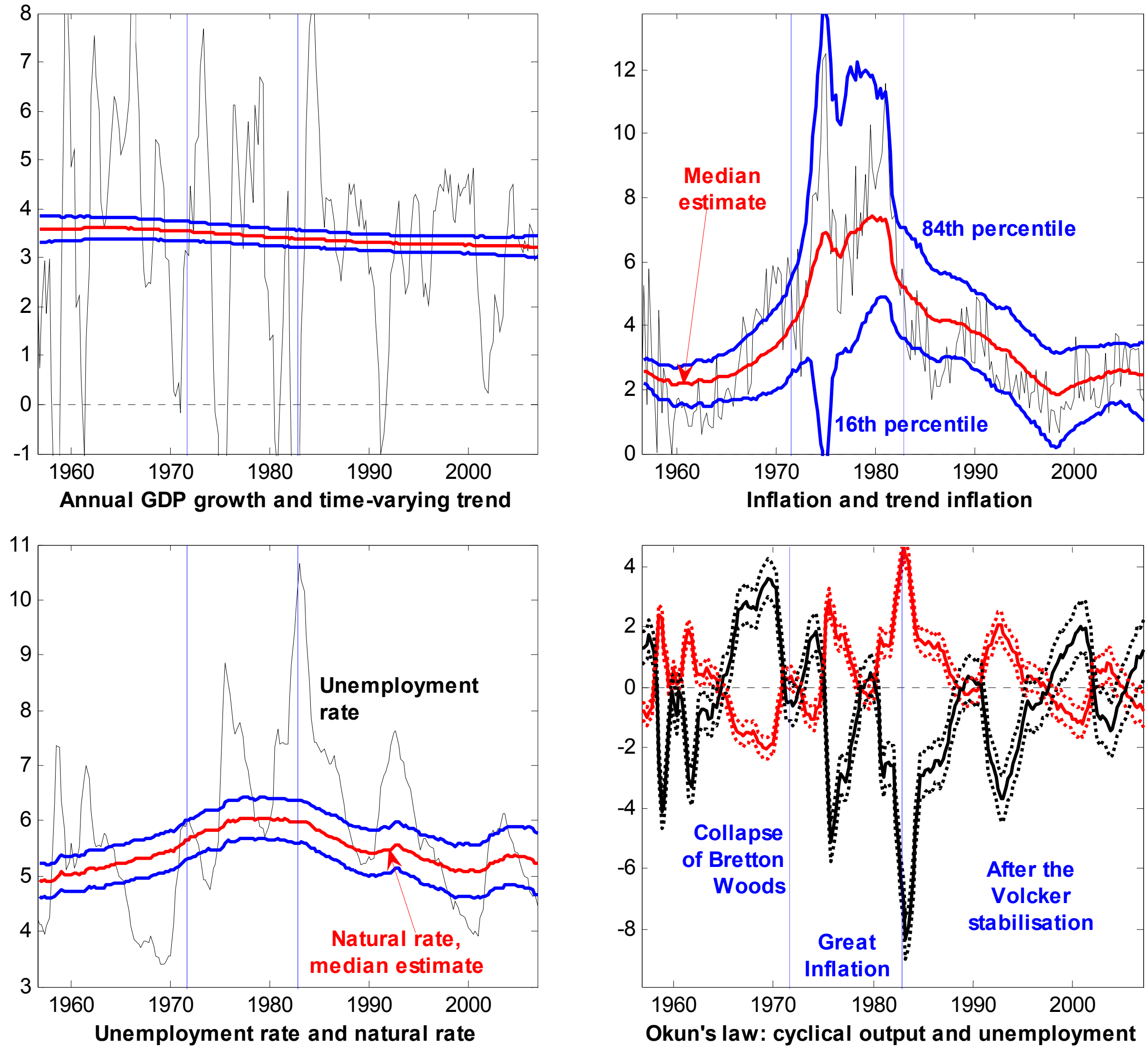

Figure 5 United States: annual GDP growth and time-varying trend, inflation and trend inflation,unemployment rate and natural rate of unemployment, and cyclical components of unemployment and log output (two-sided estimates)

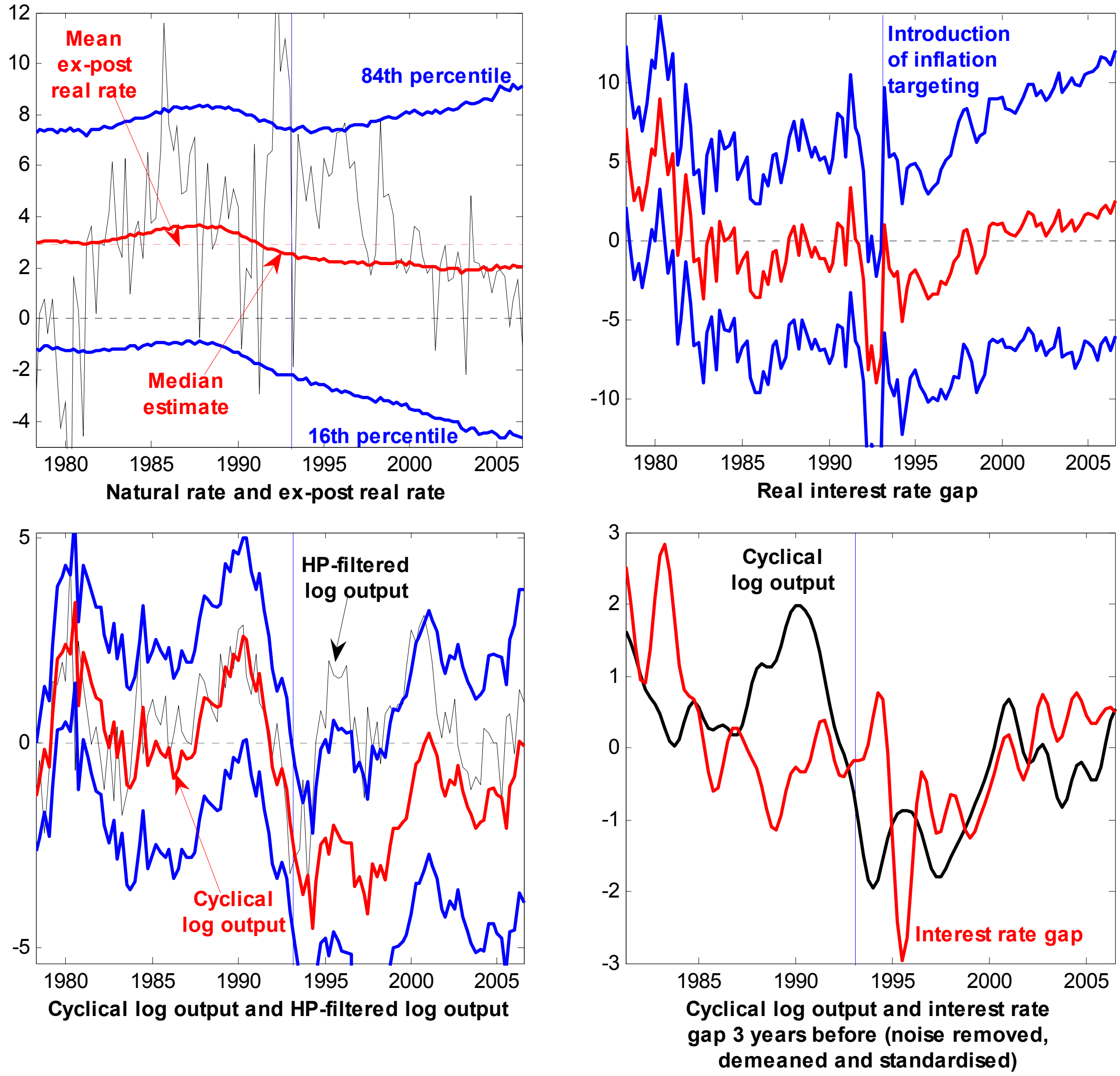

Figure 6 Sweden: the natural rate, the natural rate gap, and cyclical log output (two-sided estimates)

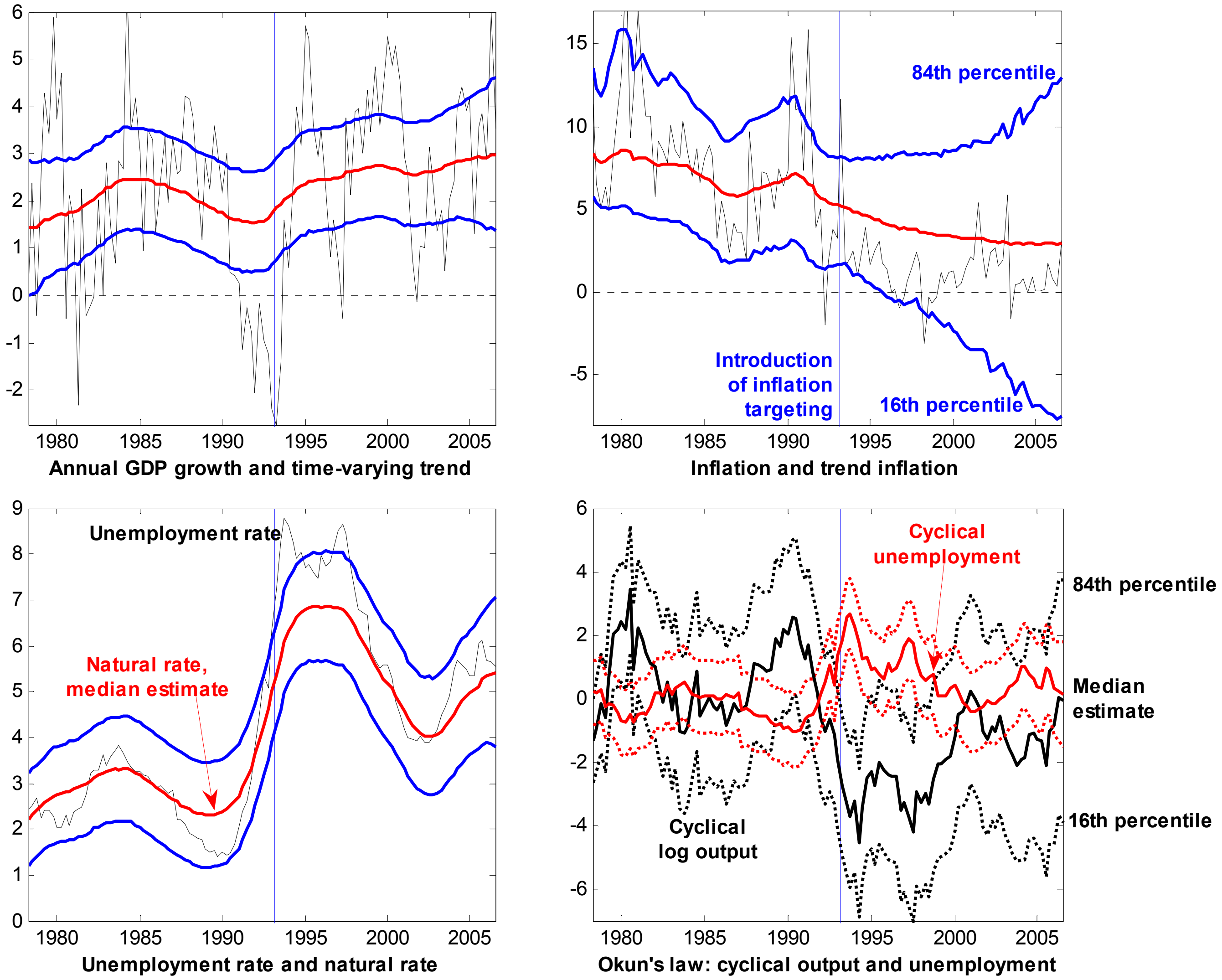

Figure 7 Sweden: annual GDP growth and time-varying trend, inflation and trend inflation,unemployment rate and natural rate of unemployment, and cyclical components of unemployment and log output (two-sided estimates)

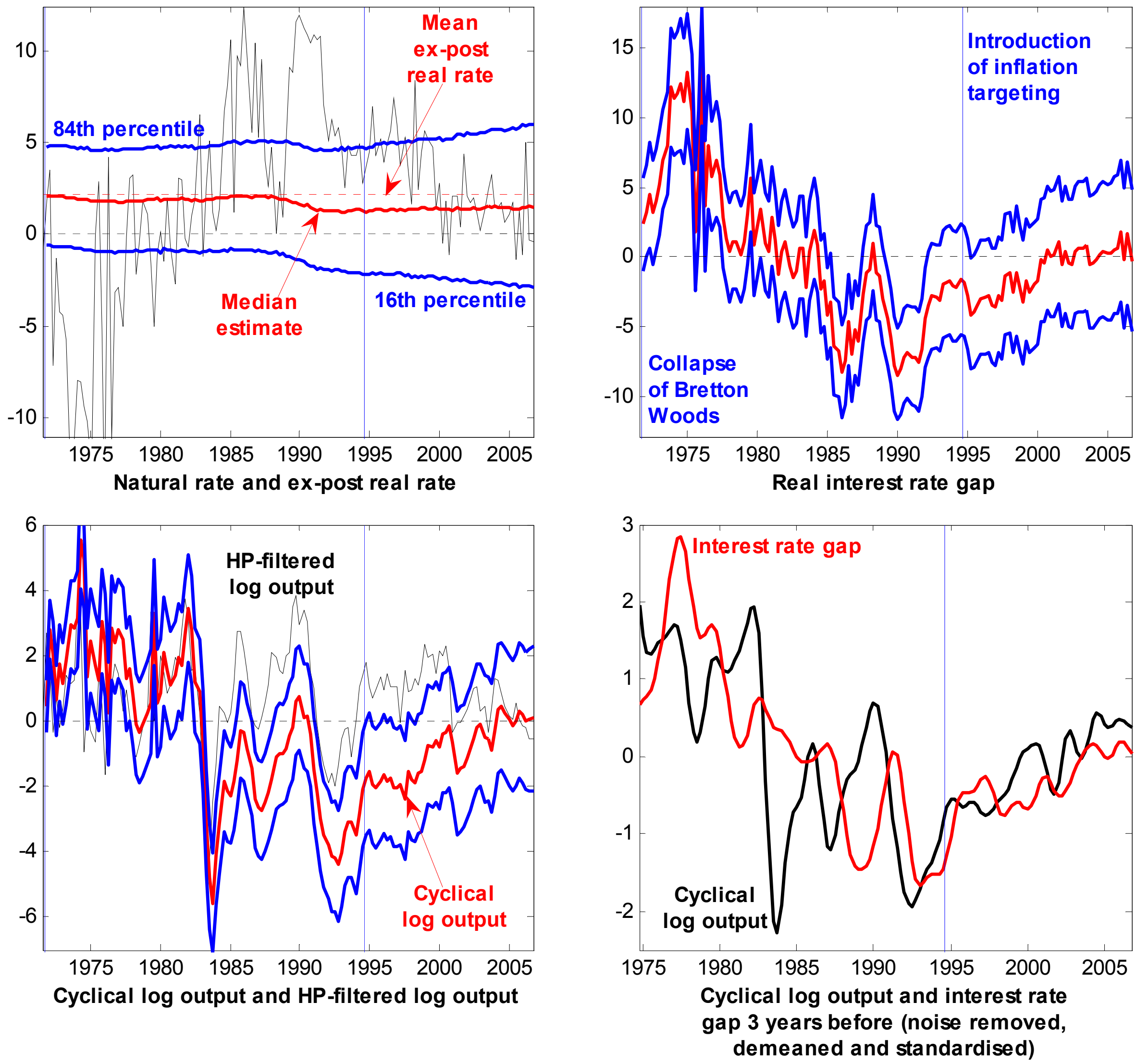

Figure 8 Australia: the natural rate, the natural rate gap, and cyclical log output (two-sided estimates)

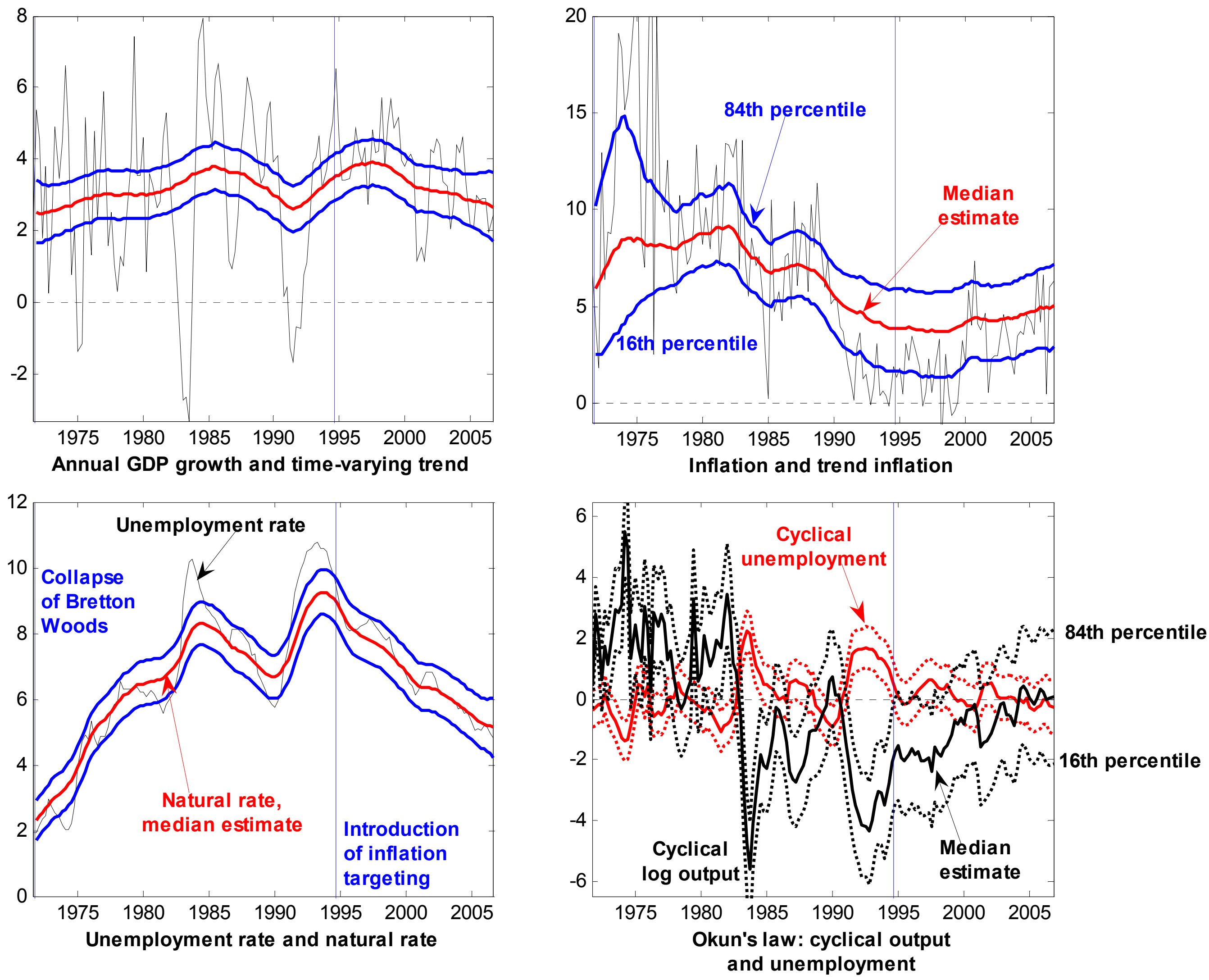

Figure 9 Australia: annual GDP growth and time-varying trend, inflation and trend inflation,unemployment rate and natural rate of unemployment, and cyclical components of unemployment and log output (two-sided estimates)

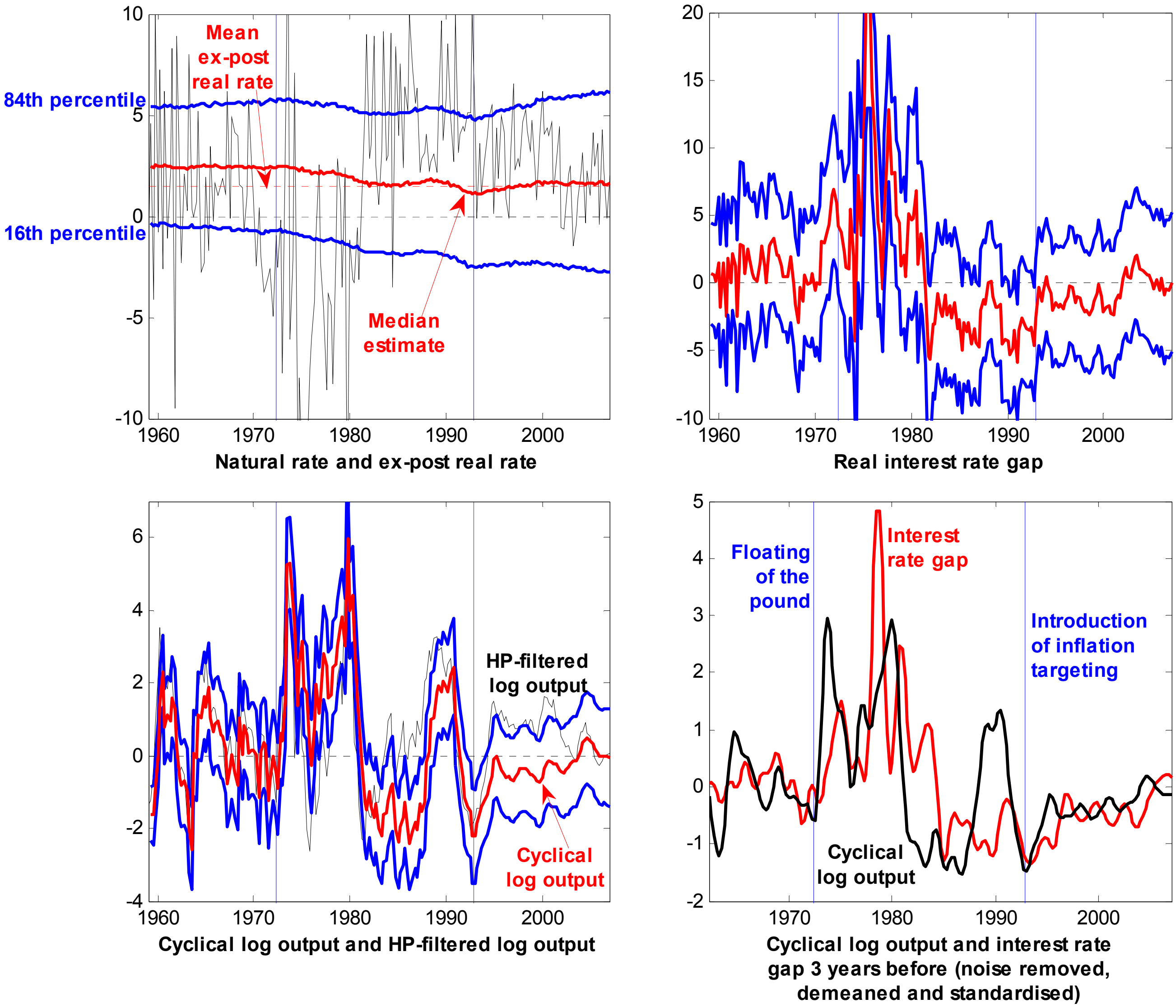

Figure 10 United Kingdom: the natural rate, the natural rate gap, and cyclical log output (two-sided estimates)

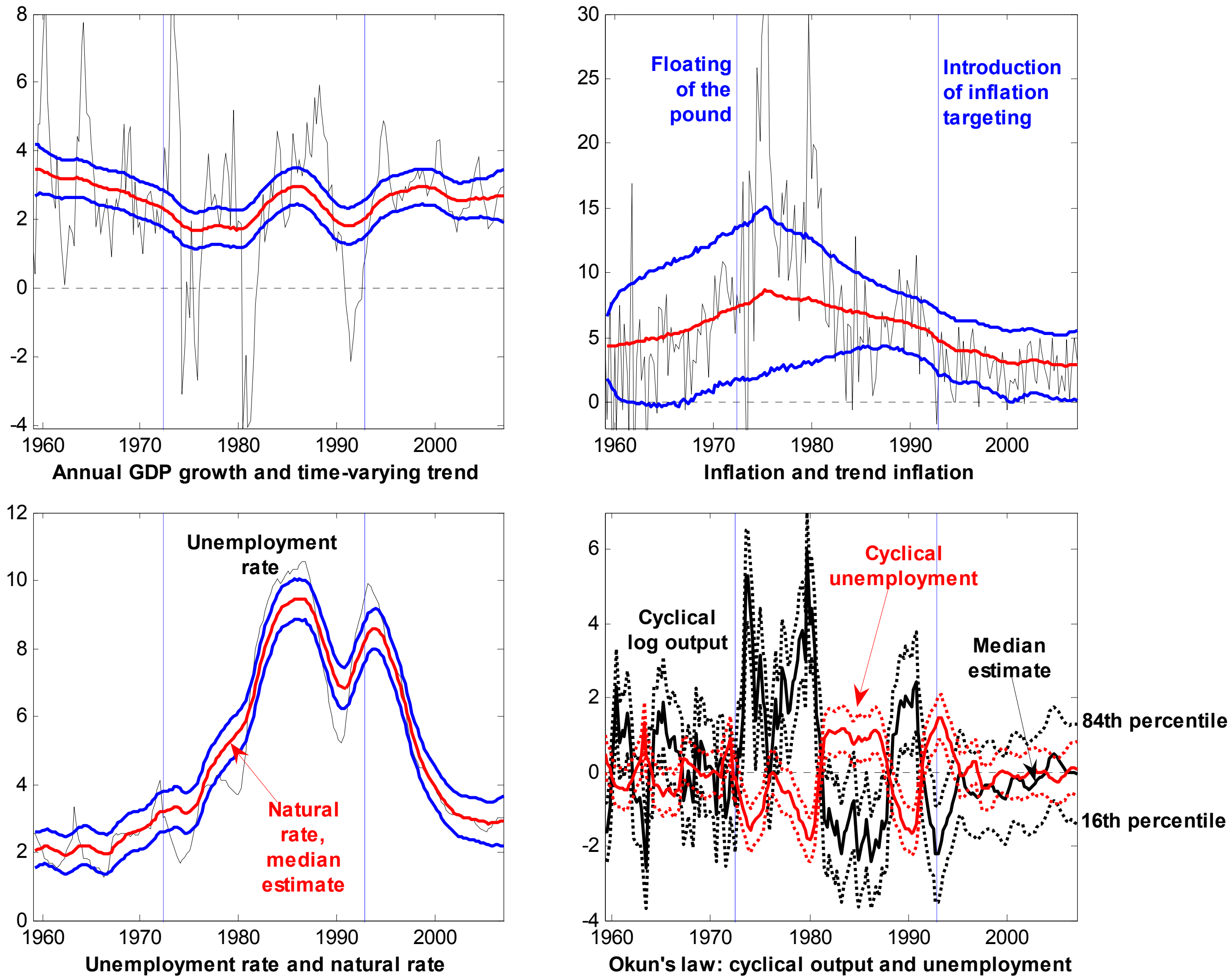

Figure 11 United Kingdom: annual GDP growth and time-varying trend, inflation and trend inflation,unemployment rate and natural rate of unemployment, and cyclical components of unemployment and log output (two-sided estimates)

European Central Bank Working Paper Series

For a complete list of Working Papers published by the ECB, please visit the ECB's website (http://www.ecb.int)

748 "Financial dollarization: the role of banks and interest rates" by H. S. Basso, O. Calvo-Gonzalez and M. Jurgilas, May 2007.

749 "Excess money growth and inflation dynamics" by B. Roffia and A. Zaghini, May 2007.

750 "Long run macroeconomic relations in the global economy" by S. Dées, S. Holly, M. H. Pesaran and L.V. Smith, May 2007.

751 "A look into the factor model black box: publication lags and the role of hard and soft data in forecasting GDP" by M. Bańbura and G. Rünstler, May 2007.

752 "Econometric analyses with backdated data: unified Germany and the euro area" by E. Angelini and M. Marcellino, May 2007.

753 "Trade credit defaults and liquidity provision by firms" by F. Boissay and R. Gropp, May 2007.

754 "Euro area inflation persistence in an estimated nonlinear DSGE model" by G. Amisano and O. Tristani, May 2007.

755 "Durable goods and their effect on household saving ratios in the euro area" by J. Jalava and I. K. Kavonius, May 2007.

756 "Maintaining low inflation: money, interest rates, and policy stance" by S. Reynard, May 2007.

757 "The cyclicality of consumption, wages and employment of the public sector in the euro area" by A. Lamo, J. J. Pérez and L. Schuknecht, May 2007.

758 "Red tape and delayed entry" by A. Ciccone and E. Papaioannou, June 2007.

759 "Linear-quadratic approximation, external habit and targeting rules" by P. Levine, J. Pearlman and R. Pierse, June 2007.

760 "Modelling intra- and extra-area trade substitution and exchange rate pass-through in the euro area" by A. Dieppe and T. Warmedinger, June 2007.

761 "External imbalances and the US current account: how supply-side changes affect an exchange rate adjustment" by P. Engler, M. Fidora and C. Thimann, June 2007.

762 "Patterns of current account adjustment: insights from past experience" by B. Algieri and T. Bracke, June 2007.

763 "Short- and long-run tax elasticities: the case of the Netherlands" by G. Wolswijk, June 2007.

764 "Robust monetary policy with imperfect knowledge" by A. Orphanides and J. C. Williams, June 2007.

765 "Sequential optimization, front-loaded information, and U.S. consumption" by A. Willman, June 2007.

766 "How and when do markets tip? Lessons from the Battle of the Bund" by E. Cantillon and P.-L. Yin, June 2007.

767 "Explaining monetary policy in press conferences" by M. Ehrmann and M. Fratzscher, June 2007.

768 "A new approach to measuring competition in the loan markets of the euro area" by M. van Leuvensteijn, J.A. Bikker, A. van Rixtel and C. Kok Sørensen, June 2007.

769 "The 'Great Moderation' in the United Kingdom" by L. Benati, June 2007.

770 "Welfare implications of Calvo vs. Rotemberg pricing assumptions" by G. Lombardo and D. Vestin, June 2007.

771 "Policy rate decisions and unbiased parameter estimation in typical monetary policy rules" by J. Podpiera, June 2007.

772 "Can adjustment costs explain the variability and counter-cyclicality of the labour share at the firm and aggregate level?" by P. Vermeulen, June 2007.

773 "Exchange rate volatility and growth in small open economies at the EMU periphery" by G. Schnabl, July 2007.

774 "Shocks, structures or monetary policies? The euro area and US after 2001" by L. Christiano, R. Motto and M. Rostagno, July 2007.

775 "The dynamic behaviour of budget components and output" by A. Afonso and P. Claeys, July 2007.

776 "Insights gained from conversations with labor market decision makers" by T. F. Bewley, July 2007.

777 "Downward nominal wage rigidity in the OECD" by S. Holden and F. Wulfsberg, July 2007.

778 "Employment protection legislation and wages" by M. Leonardi and G. Pica, July 2007.

779 "On-the-job search and the cyclical dynamics of the labor market" by M. U. Krause and T. A. Lubik, July 2007.

780 "Dynamics and monetary policy in a fair wage model of the business cycle" by D. de la Croix, G. de Walque and R. Wouters, July 2007.

781 "Wage inequality in Spain: recent developments" by M. Izquierdo and A. Lacuesta, July 2007.

782 "Panel data estimates of the production function and product and labor market imperfections" by S. Dobbelaere and J. Mairesse, July 2007.

783 "The cyclicality of effective wages within employer-employee matches: evidence from German panel data" by S. Anger, July 2007.

784 "Understanding the dynamics of labor shares and inflation" by M. Lawless and K. Whelan, July 2007.

785 "Aggregating Phillips curves" by J. Imbs, E. Jondeau and F. Pelgrin, July 2007.

786 "The economic impact of merger control: what is special about banking?" by E. Carletti, P. Hartmann and S. Ongena, July 2007.

787 "Finance and growth: a macroeconomic assessment of the evidence from a European angle" by E. Papaioannou, July 2007.

788 "Evaluating the real effect of bank branching deregulation: comparing contiguous counties across U.S. state borders" by R. R. Huang, July 2007.

789 "Modeling the impact of external factors on the euro area's HICP and real economy: a focus on pass-through and the trade balance" by L. Landolfo, July 2007.

790 "Asset prices, exchange rates and the current account" by M. Fratzscher, L. Juvenal and L. Sarno, August 2007.

791 "Inquiries on dynamics of transition economy convergence in a two-country model" by J. Brůha and J. Podpiera, August 2007.

792 "Euro area market reactions to the monetary developments press release" by J. Coffinet and S. Gouteron, August 2007.

793 "Structural econometric approach to bidding in the main refinancing operations of the Eurosystem" by N. Cassola, C. Ewerhart and C. Morana, August 2007.

794 "(Un)naturally low? Sequential Monte Carlo tracking of the US natural interest rate" by M. J. Lombardi and S. Sgherri, August 2007.

795 "Assessing the impact of a change in the composition of public spending: a DSGE approach" by R. Straub and I. Tchakarov, August 2007.

796 "The impact of exchange rate shocks on sectoral activity and prices in the euro area" by E. Hahn, August 2007.

797 "Joint estimation of the natural rate of interest, the natural rate of unemployment, expected inflation, and potential output" by L. Benati and G. Vitale, August 2007.

ISSN 1561081-0

9 771561 081005