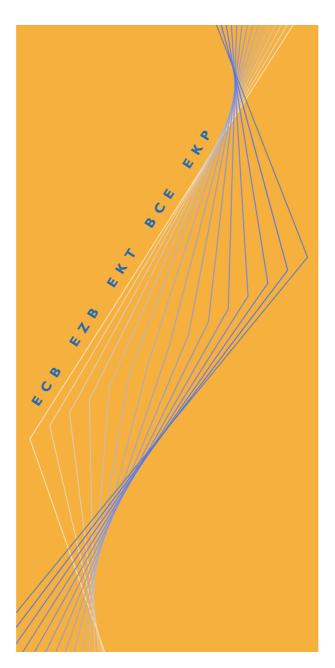
# EUROPEAN CENTRAL BANK

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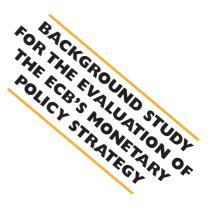


# **WORKING PAPER NO. 255**

IS THE DEMAND FOR EURO AREA M3 STABLE?

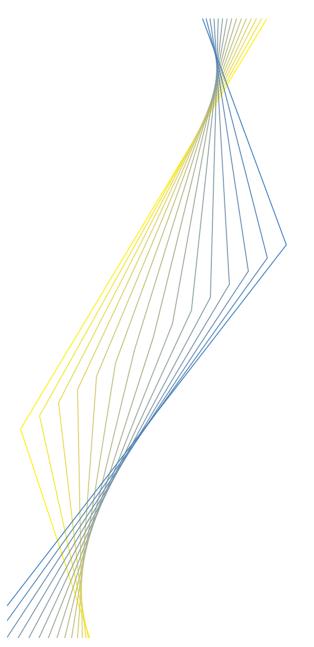
ANNICK BRUGGEMAN, PAOLA DONATI AND ANDERS WARNE

September 2003



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# **IS THE DEMAND FOR EURO AREA M3 STABLE?**

# **ANNICK BRUGGEMAN<sup>2</sup>**, PAOLA DONATI<sup>3</sup> AND **ANDERS WARNE<sup>4</sup>**

# September 2003

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ABSTRACT: This paper re-examines two data issues concerning euro area money demand: aggregation of national data and measurement of the own rate. The main purpose is to study if euro area money demand is subject to parameter non-constancies using formal tests rather than informal diagnostics. As a complement to inference based on asymptotics we perform small-scale bootstraps. The empirical evidence supports the existence of a stable long-run relationship between money and output and that the cointegration space is constant over time. However, the interest rate semi-elasticities of money demand are imprecisely estimated. Conditional on the cointegration relations the remaining parameters of the system appear to be constant. We also examine the relevance of stock prices for money demand and find that our measure does not matter for the long-run relations, but may be useful in forecasting exercises. Finally, the conclusions are robust for the aggregation method and the choice of sample.

KEYWORDS: Aggregation, Bootstrap, Money Demand, Own Rate of Money, Parameter Constancy.

JEL CLASSIFICATION NUMBERS: C22, C32, E41.

For monetary aggregates to be assigned an important role in monetary policy analysis the demand relationship between money, prices, income and interest rates needs to be stable over time and predictable in a statistical sense. Typically such a money demand relation connects real money positively to real income and an own rate of return, and negatively to the rates of return of the alternative assets considered.

This paper addresses three main issues on the demand for the euro area broad monetary aggregate M3:

- (1) The aggregation method for the scale variables and the interest rates;
- (2) The measurement of the own rate of return of M3; and
- (3) The analysis of parameter constancy.

First, national data for M3, the other scale variables and the interest rates need to be consistently aggregated to be appropriately used in a euro area-wide money demand study. In this paper we use two aggregation methods. The primary method sums the national scale variables — M3 and real and nominal GDP — after having converted them into euro at the irrevocably fixed exchange rates. The national interest rates are averaged according to time-varying weights that are determined as the national contributions to euro area M3. The robustness of the empirical results is checked against an alternative aggregation method that adopts the index method suggested by Fagan and Henry (1998). This approach aggregates all national variables on the basis of the share of each country in euro area GDP in 2001 at PPP exchange rates.

Second, a large part of M3 is remunerated at rates that are to a certain extent related to market interest rates and, accordingly, the own rate of return of M3 cannot be ignored or approximated by a constant. This variable is therefore calculated as a weighted average of the rates of return of all M3 components. In particular, it is constructed on the basis of national interest rate series for all components of M3 and for all euro area countries from 1980 onwards. The aggregation across countries is performed using either time-varying M3 weights or constant GDP weights.

Third, the maximum likelihood procedure proposed by Johansen (1996) is applied to a benchmark money demand system consisting of quarterly data on 6 variables: real M3, real income, inflation and a vector of interest rates, composed of the short-term market rate, the long-term bond yield, and the average own rate of return of M3 over the sample 1980–2001. We find that: (i) real money, real income, the short-term and the own rate, and (ii) the 3 interest rates and inflation are "trending" together and form two long-run or steady state relations, where the first is interpreted as long-run money demand.

The main purpose of the paper is to study if the money demand system is subject to parameter non-constancies. Most previous studies on the demand for euro area M3 base their conclusions about parameter constancy on informal diagnostics over a short period of the sample. Conclusions drawn from such a procedure suffer from, at least, two problems: (i) the overall significance level is not taken into account thereby making inference highly subjective; and (ii) a large share of the sample is not studied, thus ignoring information about potential parameter changes outside the examination period. In contrast, the current paper applies formal tests. In additional to having the correct size (at least in the limit), such tests do not require trimming of the sample and, hence, allow us the examine the constancy issue using as much information as possible. As a complement to presenting inferences based on asymptotics, small-scale bootstrap estimates of the small sample distributions are provided.

Five conclusions are drawn from the analysis. First and foremost, there is strong evidence supporting the hypothesis of a stable long-run relationship between real money and real GDP, where — as in previous studies on euro area M3 demand — the estimated income elasticity is greater than unity. Moreover, all freely estimated parameters of the long-run relations seem to be constant over time. Second, the interest rate semi-elasticities of long-run money demand are imprecisely (huge confidence bands) estimated using classical maximum likelihood. Third, once the long-run relations are fixed, the short-run parameters of the money demand system are also found to be constant. Fourth, to investigate if stock market developments matter for the stability of money demand, a measure of real euro area stock prices is added as an endogenous variable to the benchmark money demand system. We find that such a measure of stock prices does not matter for the long-run relations, but may be useful when studying the short-run dynamics, e.g., in forecasting exercises. Finally, stock market volatility can be excluded from the system when it is modelled as a stationary (weakly) exogenous variable.

#### 1. INTRODUCTION

Monetary aggregates have often been assigned an important role in monetary policy analysis by both economists and policy-makers and are believed by many to account for the nature of inflation as a monetary phenomenon in the long run. According to these ideas, periods of sustained inflation cannot occur without monetary accommodation and, ceteris paribus, sustained reductions in money growth will eventually lead to lower inflation or deflation. A close relationship between money and prices in the long run suggests that the analysis of persistent trends in money and money growth may be an important gauge for assessing the outlook for price stability.

For such analyses to be meaningful, the relationship between money, prices and a few other key macroeconomic variables needs to be stable over time and predictable in a statistical sense. One way to assess whether the stability condition is met is to check for parameter constancy in a suitably defined money demand system. Such an analysis — based on formal tests — lies at the core of this paper. The empirical literature on the demand for M3 in the euro area is already considerable. It covers various econometric approaches and sample periods, and it studies single-country and euro area-wide functions. As far as the stability issue is concerned, euro area-wide money demand equations seem, in general, to perform better than many single-country relations, cf. Fagan and Henry (1998) and Golinelli and Pastorello (2002) for extensive literature reviews. The reasons for such results have been subject to much debate, cf. Filosa (1995), Browne, Fagan, and Henry (1997), Dedola, Gaiotti, and Silipo (2001), and Calza and Sousa (2003) for detailed discussions. Some have put forth the possibility that money demand estimated at the euro area level may benefit from the stability property of the German money demand function.<sup>1</sup> Others have argued that euro area functions implicitly average out what could otherwise be the sources of money demand instability at a national level. This, in particular, applies to the case of institutional and regulatory changes, to differences in national historical developments (such as in the speed of financial innovation and deregulation), or to the effect of non-synchronous shocks that hit the euro area countries (Arnold, 1994). Similarly, as highlighted by Kremers and Lane (1990) and Lane and Poloz (1992), the focus on euro area relations may internalize the effects of currency substitution movements.<sup>2</sup> Along the same lines, it has been argued that a money demand relation estimated at the euro area level may help offset misspecification problems due to spillover effects like cross-border trade and capital flows. Confirming the earlier findings of Angeloni, Cottarelli, and Levy (1994), however, Fagan and Henry (1998) conclude that:

...a number of reasons which have been put forward to explain the better performance of the area wide equation such as currency substitution, the operation of the ERM system ... are not strictly necessary to explain the result.

While most other studies base their conclusions concerning stability on informal diagnostics, the present study uses formal tests, which are not only of the correct size (at least in the limit), but also do not require the sample to be trimmed, thereby making it feasible to examine the stability issue using more information.

As emphasized in the literature (e.g. Winder, 1997), national data for M3, other scale variables and interest rates need to be consistently aggregated to be appropriately used in a euro area-wide money

 $<sup>^1</sup>$  Wesche (1997), for example, found that euro area money demand becomes unstable when German data are excluded from a group of countries that also includes France, Italy and the United Kingdom.

 $<sup>^2</sup>$  McKinnon (1982) argues that while movements of liquidity among financially integrated countries may be the source of instability in national money demand functions, these need not affect the stability of the respective aggregate multicountry money demand if well internalized.

demand study. Not only may differences in the availability of national time series be difficult to reconcile, but also the choice of a weighting scheme that suits all variables is not straightforward. The objective of using national aggregates for scale variables, in particular, makes it difficult to take advantage of the contributions of index-number theory. The aggregation adopted by Fagan and Henry (1998) rests on a fixed GDP weight index. Beyer, Doornik, and Hendry (2001), focusing on the distortions that would stem from the simple summation of historical national data due to past exchange rate changes, have constructed an index that also overcomes the difficulties that may arise from possible non-stationarities and structural breaks.

In this paper, two aggregation methods are employed in parallel. The first method uses two aggregation techniques. In the first technique, national M3 and real and nominal GDP are converted into euro at the irrevocably fixed exchange rates and then summed, while in the second national interest rates are averaged according to time-varying weights that depend on the national contributions to euro area M3. The second aggregation method adopts the index method suggested by Fagan and Henry (1998), such that all variables are aggregated according to weights that measure the share of each country in euro area GDP in 2001 at PPP exchange rates. By comparing the results from these two aggregation methods, we not only have a means for checking the robustness of the conclusions with respect to the selected technique, but also a vehicle for evaluating the importance of adopting a consistent aggregation method.

In the present study, it is assumed that there is demand for M3 as a medium of exchange, for precautionary reasons, and as a portfolio asset. Hence, a benchmark long-run money demand relation is considered in which real M3 is related to income and a vector of interest rates, composed of the short-term market rate, the long-term bond yield and the average own rate of return of M3. Furthermore, it is expected that the coefficients for income and the own rate of return are positive (non-negative), while the remaining two coefficients are negative (non-positive).

The measurement and selection of the own rate of return is an important issue addressed carefully by this study. Since a large part of M3 is remunerated, the M3 own rate should be considered as a weighted average of the rates of return of its components. Cassard, Lane, and Masson (1994) estimated the own rate of return of money as a GDP-weighted average of the French and German own rates. Dedola et al. (2001) measured the rate of return of euro area M3 as an average of the interest rates on national M3, weighted by the shares in euro terms of national M3 in euro area M3. Calza, Gerdesmeier, and Levy (2001) constructed a M3 own rate series starting from the euro area interest rates on all M3 components from January 1990 and filled the gap for the previous period using the national interest rates of the five largest euro area countries. The euro area M3 own rate used in the current paper is constructed on the basis of national interest rate series for all components of M3 and for all euro area countries from 1980 onwards. The aggregation across countries is performed on the basis of either M3 or GDP weights.

The selection of the appropriate rates of return of the different assets alternative to M3 depends on the choice made for the own rate of M3. Among the more recent euro area studies, Coenen and Vega (2001), who assume that the own rate of M3 is approximated by the short-term market interest rate, used the spread between long and short-term rates to capture the opportunity cost of holding money. Calza et al. (2001) considered both the spread between the long-term rate and the own rate of M3 and the spread between the short-term rate and the own rate. Brand and Cassola (2000), however, remarked that since the dynamics of the spread between the long-term rate and the own rate are almost fully captured by the dynamics of the long-term rate itself, the latter could be a better proxy for the opportunity cost of M3 than any market interest rate spread. Finally, Cassola and Morana (2002) found a stable long-run money demand relation without any variable representing the opportunity cost of M3. In contrast to these studies, we let all three rates be endogenous in this paper, thus allowing the short and long-term rates (as well as inflation) to represent alternative rates of return for M3. Like Fagan and Henry (1998) and Dedola et al. (2001), however, we found that the interest rate coefficients are imprecisely estimated in all our money demand systems.

The usefulness of analyzing money demand for policy purposes depends also on our ability to separate the developments in M3 that are related to income from those that are due to other factors, as these other factors can generate shifts in the income velocity of money (cf. Dow and Elmendorf, 1998). In particular, this paper addresses the issue of whether euro area stock market developments affect the long-run demand for M3.

The widespread ownership of shares is still a relatively new phenomenon in most euro area countries, growing fairly significantly in the late 1990s, even though it may have reversed recently.<sup>3</sup> While the financial structure of the euro area is still predominantly bank-based, market-oriented instruments (and shares and other equities in particular) have become an increasingly important source of financing for corporations. Furthermore, this trend towards market-oriented financing has resulted in a growing share of these market instruments in the investment portfolios of both non-financial corporations and households. We may therefore expect that market developments affect the "store of value" component of the demand for M3 and that stock prices help to capture the wealth effects behind it.

Cassola and Morana (2002) found evidence that asset prices play an important role in the monetary policy transmission mechanism in the euro area. They study the interactions between nominal interest rates, inflation, real output, real M3 and the euro area real stock price index by means of a structural vector error correction model. Their results broadly support the view that the strong increases in euro area M3 since 2001 can be partly attributed to a temporary liquidity preference shock — that also accounts for the strong declines in stock prices around the world since March 2000 — which made investors increase their holdings of relatively liquid and low risk assets.

Stability of money demand is our main reason for investigating the importance of stock market developments. First, we let the euro area real stock price index be an additional endogenous variable, and, second, we use an estimated stock market volatility series which enters the system as an exogenous variable. In contrast to Kontolemis (2002), who found that the inclusion of a weighted average of the German and French stock price indexes in long-run money demand "produces" parameter constancy for the non-cointegration parameters, our analyses suggest that stock market variables do not seem to be relevant for this issue. Our results show that real stock prices neither seem to matter for the selection of the cointegration rank (i.e. the number of steady states), nor for the estimated parameters of the cointegration parameters appear to be constant for both the system with and the system without real stock prices. If anything, the evidence for non-constancy of the cointegration space is stronger in the real stock price system than in a system without stock prices.

One reason for investigating the effects of stock market volatility on money demand is that volatility may be a proxy for the risk investors are exposed to when holding stocks. Under risky and uncertain conditions, as for example those manifested during financial crises, recessions, structural changes, firms' expectations of future earnings may well worsen. Investors may therefore be

<sup>&</sup>lt;sup>3</sup> For detailed information, see the "Report on Financial Structures", European Central Bank, October 2002.

induced to reallocate their portfolios by increasing the share of short-term money market components, thus increasing the size and possibly influence the developments of a broad monetary aggregate such as M3. In our study we find, however, evidence that stock market volatility does not contain unique information for explaining the endogenous variables. While it is possible that the demand for broad money, especially in more recent years, has been affected by stock market developments, it is conceivable that such a phenomenon is not fully captured by simply looking at the developments in the euro area stock price index or the volatility measure we have used. These issues require further investigation that go beyond the stability analysis we focus on.<sup>4</sup>

The remainder of the paper is organized as follows. Section 2 deals with measurement issues and focuses on the aggregation methodologies adopted in this study and the opportunity costs of holding money. Section 3 introduces the benchmark money demand system and the selection of basic parameters like lag order and cointegration rank. Section 4 is centered around the formal parameters constancy tests, while all results are reexamined in Section 5 using an alternative aggregation method. Section 6 presents the results when stock market variables are included in the model and, finally, a summary and the main conclusions are discussed in Section 7.

## 2. Measurement Issues

This section is concerned with two measurement issues on which the empirical literature on euro area money demand has not reached a consensus. In order to test the robustness of the empirical evidence, one aim of this paper is to investigate these measurement issues in an encompassing framework. The first issue relates to the aggregation methodology for constructing euro area time series from the national data. The need to use a consistent aggregation method for all variables in the statistical model is emphasized in the literature; see, e.g., Winder (1997). While this is straightforward for the scale variables in the model, like money and GDP, it is not obvious how this can be accomplished for the interest rate variables. In this section, we will therefore distinguish between scale variables (denoted in national currencies) on the one hand and interest rates (denoted in percentages per annum) on the other. A second measurement issue involves the selection of both an own rate of return of euro area M3 and one or more alternative rates of return.

### 2.1. The Aggregation Methodology

Most data for the euro area have to be constructed from national data. In this paper, euro area data cover the countries comprising the euro area at each given time, i.e. 11 member states up to December 2000 and 12 member states from January 2001 onwards, i.e. plus Greece.<sup>5</sup>

## 2.1.1. Scale Variables

Since the national data on scale variables, such as M3 and GDP, are denominated in national currencies, they cannot be simply summed to obtain euro area aggregates. The choice of aggregation method, i.e. weighting scheme, is not straightforward, reflecting the problem that it is only from 1999 onwards that a single currency has been in place.

One possible method is to first convert the national data into euro by applying the so called *irrevocably fixed exchange rates*, announced on December 31, 1998 (and determined on June 19,

<sup>&</sup>lt;sup>4</sup> For a recent study on M2 data for the U.S., see Carpenter and Lange (2003)

<sup>&</sup>lt;sup>5</sup> Since our data begins in 1980, it follows that some of the 11 member states were not members of the European Community (EC) in the early 80s. For example, Spain and Portugal became members of the EC in 1986, while Austria and Finland joined the EU in 1995.

2000, in the case of Greece), and then sum these converted series to the euro area aggregate scale variable. This method implies using the following formula:

$$x_{F,t} = \sum_{c} w_{F,c} x_{c,t}$$

where *x* denotes the scale variable, *t* the time period, *c* the individual country, and  $w_{F,c}$  the irrevocably fixed exchange rate for country *c*.

One advantage of using fixed exchange rates instead of current exchange rates is that it avoids that the aggregate series would be affected by nominal exchange rate changes that could give rise to spurious correlations, especially at times of large swings in the exchange rate. A second advantage of this aggregation method is that it is consistent with the method that is used since the start of Stage Three of EMU, i.e. simply summing the national data (already expressed in euro). The main limitation of the method is that it can only be applied to variables that are denominated in national currencies (like stocks and flows of scale variables) and therefore not to the interest rates.

An alternative aggregation method is the so called *index method*, presented in Fagan and Henry (1998). According to this method the log-level index for the euro area scale variable is defined as the weighted sum of the log-levels of the national scale variables, where the weights,  $w_{I,c}$ , are the shares of the countries' GDP in euro area GDP in 2001 measured at PPP exchange rates:

$$\ln x_{I,t} = \sum_{c} w_{I,c} \ln x_{c,t}.$$

This method also uses constant weights which avoids the possible spurious correlations between the euro area series due to changes in the exchange rates. In addition, it implies that the log approximation of the growth rate for the euro area series is a weighted average of the log approximation of the growth rates of the underlying national series. The weights are thus the shares of the countries' GDP in euro area GDP in 2001 (measured at PPP exchange rates). A third advantage is that this method can also be applied to the other variables in the money demand system (albeit without taking the natural logarithm). Finally, the method is consistent with the (very strong) assumption that national money demand relations have the same log-linear specification and similar parameter values across all euro area countries. It therefore facilitates a comparison between area-wide and national money demand models. The main disadvantage of the index method is that it is *not* consistent with the method used since the start of Stage Three of EMU, i.e., simply summing the national data (already expressed in euro). In addition, it does not preserve the balance sheet identities, although this has no direct implication for the money demand models estimated in this paper.

Since there are no strong arguments to prefer one aggregation method over the other, both methods will be used in this paper. The first as the primary aggregation method, and the second to check the robustness of the results.<sup>6</sup>

For the construction of the data on euro area M3, non-seasonally adjusted data on the national contributions to euro area M3 are used. The adjustment for seasonal and calendar effects is performed at the euro area level. The quarterly data on euro area M3 are averages of seasonally adjusted end-of-month "notional stocks" data, calculated on the basis of flow data. From October 1997 these flow data are computed by adjusting the difference between the end-of-month stocks

<sup>&</sup>lt;sup>6</sup> See, e.g., Beyer et al. (2001), who suggest using the average national growth rates as weights, for yet another alternative aggregation method.

for the effects of non-transaction related factors, i.e. for reclassification, foreign exchange revaluations and other revaluations.<sup>7</sup> As can be seen in Figure 1, the use of irrevocably fixed exchange rates results in somewhat lower annual growth rates of nominal M3 than the use of the fixed 2001 GDP weights, in particular in the first half of the 80s.

The quarterly data on euro area nominal and real GDP are based on seasonally adjusted national accounts data (ESA 95) up to 1998:Q4 and on Eurostat series from 1999:Q1 onwards. For both aggregation methods the GDP deflator for the euro area is derived as the ratio of euro area nominal GDP to euro area real GDP. This implies that the national data for the GDP deflator are not taken into account. The annual growth rates of euro area real GDP seem to hardly depend on the aggregation method used; cf. Figure 2.

The same is, however, not true for nominal GDP. The use of the irrevocably fixed exchange rates results in somewhat lower annual growth rates for both nominal GDP and the GDP deflator than the use of the fixed 2001 GDP weights. In general, the annual inflation rate — defined as the annual percentage change in the GDP deflator — declined during most of the sample period. It is therefore not surprising that conventional unit root tests do not the reject the hypothesis of a unit root in the inflation series for the sample 1981–2001. Since the second half of the 90s the annual inflation rate seems to fluctuate around 2 percent, which could be seen as an indication that the inflation series has "become stationary". The sub-sample period is, however, too short for any formal tests to provide any meaningful evidence on this.<sup>8</sup> A similar pattern can be found in all nominal variables, i.e. the annual growth rate of nominal M3 (cf. Figure 1) and the various interest rates (cf. Figure 3).

#### 2.1.2. Interest Rates

Euro area interest rates are constructed as weighted averages of the national interest rate series.<sup>9</sup> Two alternative weighting schemes are also considered for interest rate variables in this paper. The main consideration in this respect is to use an aggregation method that is as consistent as possible with the method used for the scale variables.

A first (time-varying) weighting scheme uses the national contributions to euro area M3:

$$i_{\mathrm{M3},t} = \sum_{c} \frac{M_{c,t}}{M_{F,t}} i_{c,t},$$

where  $M_{c,t}$  is country *c*'s national contribution to euro area M3 (converted into euro via the irrevocably fixed exchange rates), and  $M_{F,t}$  is the measure of euro area M3.

A second (constant) weighting scheme relies on the shares of the countries' GDP in euro area GDP in 2001:

$$i_{I,t} = \sum_{c} w_{I,c} i_{c,t}.$$

The quarterly data on interest rates are averages of monthly data. The use of the national contributions to M3 as a weighting scheme results (especially in the 80s) in somewhat lower nominal interest rates than the use of the constant 2001 GDP weights; cf. Figure 3. However, the general pattern of a downward trend in the 80s and the first half of the 90s, followed by a more stable development, remains.

<sup>&</sup>lt;sup>7</sup> For a detailed discussion of the statistical procedure and the conceptual background to these adjustments, see the box on "The Derivation and Use of Flow Data in Monetary Statistics" in the February 2001 issue of the ECB Monthly Bulletin.

<sup>&</sup>lt;sup>8</sup> The same argument could, of course, be made for the whole sample 1981-2001.

<sup>&</sup>lt;sup>9</sup> There is only one exception: since January 1999 the 3-month EURIBOR rate is taken as the (average) euro area 3-month market interest rate.

The opportunity costs of holding money can be defined as the rate of return that economic agents forego by holding money instead of some other (financial or real) assets. From a conceptual point of view the opportunity costs should thus be calculated as the difference between the rate of return of the alternative assets and the own rate of return of M3.

For a very narrow monetary aggregate — like the monetary base or M1 — the choice of both rates of return is quite straightforward. The own rate of return of the monetary aggregate can be taken to be zero or at least fairly constant at a low level. The alternative rate of return is then usually approximated by a short-term market interest rate. This would then result in a long-run money demand equation of the form:

$$m_1 = \beta_0 + \beta_y y - \beta_i i_s,$$

where  $m_1$  denotes the log of real M1, y the log of real GDP, and  $i_s$  the short-term market interest rate as a proxy for the opportunity costs of holding money.

However, when considering a broad monetary aggregate, like M3, the appropriate definition of the opportunity costs of holding money is less obvious. The own rate of return of M3 may no longer be well approximated by a constant, because a major part of M3 is remunerated at rates that are to a certain extent determined by the market interest rates. Ideally, one would therefore use an own rate of return of M3 that is a weighted average of the rates of return of the individual components of M3. Although there are some problems related to the unavailability of high quality data for some of these national interest rate series, a number of studies have attempted to construct such an average own rate of return of M3. For example, Cassard et al. (1994) used a "GDP-weighted average of the French and German own rate". Dedola et al. (2001) first calculated national own rates of return of M3 for all euro area countries and then computed a weighted average own rate of return of euro area M3, using the national contributions to M3 as weights. Due to data limitations, however, no distinction could be made between the different categories of deposits for some countries. Calza et al. (2001) constructed an own rate series for euro area M3 based on euro area interest rate series for all components of M3 from January 1990 onwards and extended this series backwards on the basis of national interest rate data for the 5 largest euro area countries. Most other studies either use the 3-month market interest rate as the own rate of M3 or assume that the own rate is constant over time, implying that it should not be considered as a separate variable in the system. In this paper two new time series of the own rate of return of M3 are used. Unlike the series used in Calza et al. (2001) or Dedola et al. (2001), these new series are constructed on the basis of national interest rate series for *all* components of M3 and for *all* euro area countries from January 1980 onwards.<sup>10</sup> Only in this way can the robustness of the results of the money demand models with respect to the different aggregation methodologies be tested properly.

For the alternative rate of return, ideally one would like to include a whole range of longer-term financial assets (like bonds or equities) and real assets (possibly approximated by the inflation rate). However, the inclusion of too many interest rate variables in the system may complicate the analysis of how these various interest rates are related among one another and with money and income. It is therefore not surprising that the issue of the appropriate definition of the alternative rates of return has not been settled in the empirical literature on money demand.

The variables that are used most often are the short-term market interest rate, the long-term bond yield, and the inflation rate. The selection of the appropriate alternative rate of return is

<sup>&</sup>lt;sup>10</sup> For details on the construction of the own rate series, see Appendix A.

strongly related to the choice made for the own rate of return of M3. For example, Coenen and Vega (2001) assume that the own rate can be approximated by the short-term market interest rate and include the spread between the long-term and the short-term rates as the opportunity cost of holding money. Brand and Cassola (2000), however, state that:

...the dynamics of the spread of the long-term interest rate against the own rate of M3 is almost fully captured by the dynamics of the long-term interest rate. This suggests that the long-term interest rate may be a better measure of opportunity costs than the market spread (long-term minus short-term market interest rates).

Their main argument for not including a variable capturing the average own rate of return of euro area M3 is that this reduces the complexity of the model. Calza et al. (2001), who did use an average own rate of return of M3, included both the spread between the long-term market interest rate and their own rate measure, and the spread between the short-term market interest rate and the own rate as possible opportunity cost variables. To reduce the complexity of the model, they opted for directly including the spreads instead of all three variables separately. From their empirical analysis they concluded, however, that the preferred money demand model only includes the spread between the short-term market interest rate and the own rate as the measure of the opportunity costs. Finally, Cassola and Morana (2002) find a stable money demand model that does not have an opportunity cost variable in the long-run money demand relation.

In this study we have opted for the inclusion of the average own rate of return of M3 as a separate variable in the system. In addition, we included the short-term market interest rate, the long-term bond yield, and the inflation rate as possible alternative rates of return.<sup>11</sup> This also has the advantage of making several of the previous euro area M3 demand systems special cases of our system. However, we shall not perform any formal tests of encompassing in this paper since the data are not identical.<sup>12</sup>

## 3. The Benchmark Money Demand System

In this section we shall present and discuss the statistical properties of our benchmark money demand system for the euro area. We focus on the determination of lag order, cointegration rank, and the estimation of cointegration relations in a vector error correction model. Based on this empirical model we shall then turn to the crucial issue of whether or not the parameters of the model are constant over time in Section 4. Alternative specifications and their implications on the parameter constancy issue are considered in Sections 5 and  $6^{13}$ 

The benchmark money demand model consists of the 6 variables: real M3,  $m_t$ , inflation measured by annualized quarterly changes of the GDP deflator,  $\Delta p_t$ , real GDP,  $y_t$ , the short-term market interest rate,  $i_{s,t}$ , the long-term market interest rate,  $i_{l,t}$ , and the own rate of return of M3,  $i_{o,t}$ . The interest rates are all measured in annual percentage rates (divided by 100) while the remaining variables are measured in natural logarithms of the seasonally adjusted data. The money stock,

 $<sup>^{11}</sup>$  The inclusion of the inflation rate in the system can be motivated on several grounds. First, it can be seen as the alternative rate of return of investments in real assets. Second, it allows us to test for the relevance of real interest rates in the system. Third, it allows for some degree of short-run price non-homogeneity in the money demand model. For a more detailed discussion on this issue, see, e.g., Coenen and Vega (2001).

<sup>&</sup>lt;sup>12</sup> Several studies use, e.g., irrevocably fixed exchange rates aggregation for the scale variables and GDP weights for the interest rates, while we use M3 weights for the interest rates under that aggregation method for the scale variables.

<sup>&</sup>lt;sup>13</sup> All computations, including simulations of the limiting distributions of the Nyblom tests (using a step length of 500 and 100,000 replications) as well as bootstraps, have been carried out in Structural VAR, version 0.19, which can be downloaded from http://www.econometrics.texlips.org/. When possible cross-checking has been performed with CATS in RATS, Eviews, and PcFiml.

GDP, and the GDP deflator have been aggregated using the irrevocably fixed exchange rates, while M3 weights have been used for the aggregation of the interest rates.

Following the notation in Johansen (1996) we can express the vector error correction model as:

$$\Delta X_t = \Phi D_t + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \alpha \beta' X_{t-1} + \epsilon_t, \quad t = 1, \dots, T,$$
(1)

where  $X_t = (m_t, \Delta p_t, y_t, i_{s,t}, i_{l,t}, i_{o,t})$  and  $D_t$  is a deterministic vector. The cointegration rank, r, is given by the rank of  $\alpha\beta'$ . In addition, all roots to the system are either unity or greater than one and the number of unit roots is equal to 6 - r, i.e. we restrict the system to be at most integrated of order 1. Finally, the residuals  $\epsilon_t \sim N(0, \Omega)$  and the initial values  $(X_0, \ldots, X_{1-k})$  are assumed to be fixed. In all the systems studied in this paper we let  $D_t = 1$ .

As a complement to presenting inferences based on asymptotics we shall also conduct simple bootstrap simulations for all tests concerning rank determination, restrictions on the cointegration space, as well as parameter constancy. The bootstrap procedure we employ belongs to a family of bootstraps known as parametric bootstrapping (see, e.g., Berkowitz and Kilian, 2000, and Horowitz, 2001). Specifically, we generate pseudo-samples  $\Delta \tilde{X}_t(b)$ ,  $b \in \{1, \ldots, B\}$ , of the same length as the original data (*T*) by drawing standard normal errors, converting them to  $\tilde{\epsilon}_t(b)$  through  $\hat{\Omega}^{1/2}$ , the Choleski decomposition of the ML estimate of  $\Omega$  under the null hypothesis, and using equation (1) with the original initial values, deterministic variables, and parameters evaluated at their estimated values under the null hypothesis. For simplicity we limit the number of pseudo-samples to 1000.<sup>14</sup>

As noted by, e.g., Horowitz (2001), at present there are no theoretical results on the ability of the bootstrap to provide asymptotic refinements for tests or confidence intervals when the data are integrated or cointegrated. The consistency of the bootstrap estimator of the distribution of the slope coefficient or Studentized slope coefficient in a simple AR(1) model has been studied by, e.g., Basawa, Mallik, McCormick, and Taylor (1991), while some more recent developments for a few specific cases are presented by Chang, Sickles, and Song (2001), Davidson (2001), Paparoditis and Politis (2001), and Inoue and Kilian (2002). The results of Monte Carlo experiments (see Li and Maddala, 1996, 1997, and Gredenhoff and Jacobson, 2001) suggest that the differences between the true and the nominal rejection probabilities of tests of hypotheses about integrated and cointegrated data are smaller with bootstrap based than with asymptotic critical values.

#### 3.1. Lag Order and Cointegration Rank

Since the full sample only covers 87 observations and inference on lag order determination is based on classical asymptotic theory, one criterion we use is parsimony. In Table 1 we report a number of specification tests, covering serial correlation and the normality of the residuals, for models based on 2 lags (k = 2).

In Panel A we consider a model without imposing any unit root restrictions. When we test the null of k = 2 lags against 3 and 4 lags we find that the null cannot be rejected at conventional levels of marginal significance; in the case of a model of 1 lag against 2 lags, we find that the 1 lag model is strongly rejected. Furthermore, multivariate tests of serially uncorrelated residuals for the k = 2 model indicate that the null cannot be rejected against the alternative hypothesis of first order correlation and correlation at the 4th lag, respectively. Hence, 2 lags seem to be sufficient for describing the dynamics of the system.

<sup>&</sup>lt;sup>14</sup> Procedures for choosing the number of bootstrap replications are discussed by, e.g., Andrews and Buchinsky (2000) and Davidson and MacKinnon (2000a).

Turning to the issue of normality we consider the multivariate Omnibus statistic, suggested by Doornik and Hansen (1994), which looks at the 3rd and 4th moments of normalized residuals. For the null hypothesis that these two moments are equal to those for a multivariate normal distribution we find that the *p*-value is roughly 5 percent when compared to its approximate asymptotic  $\chi^2(12)$  distribution. Hence, whether or not normality of the residuals is supported by the data is for the unrestricted vector error correction model an open issue.

We have also examined a number of additional specification tests. In particular, tests for ARCH of order 1 and of order 4 for each residual. With the exception of the residuals from the output equation, we do not find any strong signs of conditional heteroskedasticity. However, adding more lags to the model does not alleviate this potential source of misspecification. In what follows we will therefore consider the model with 2 lags.

The tests for the cointegration rank are given in Table 2. When the conventional trace tests,  $LR_{tr}$ , are compared with the relevant limiting distribution we find that the data suggest using 4 cointegration relations at the 10 percent level, 3 at the 5 percent level, and 2 at the 1 percent level.<sup>15</sup> However, several studies have concluded that the trace test tends to be over-sized in small samples; see, e.g. Jacobson, Jansson, Vredin, and Warne (2001) and Toda (1995). For that reason we report Bartlett corrected (mean corrected) trace tests,  $LR_{tr}^c$ , using the correction formulas presented in Johansen (2002b, Theorem 1).<sup>16</sup>

As can be seen in Table 2 the correction factors differ somewhat for the possible choices of rank and the smallest correction factor is roughly 15 percent greater than unity. Hence, the direction of the correction and the magnitudes are consistent with our prior expectations. Applying these factors to the trace statistics and comparing the corrected statistics with the same asymptotic distributions as the uncorrected tests we find that the data suggest using 2 cointegration relations at the 10 percent (to the 2.5 percent) level of marginal significance and 1 relation at the 1 percent level. Moreover, the specification tests in Panel B of Table 1 indicate that the rank restrictions do not alter the whiteness properties of the residuals.

Turning to the bootstrapped *p*-values for the tests we find that the uncorrected and Bartlett corrected tests agree with essentially equal *p*-values. Moreover, the empirical *p*-values are almost equal to the *p*-values from the Bartlett corrected tests when the asymptotic distribution is used for inference. As an illustration we have plotted the empirical null distributions for the Bartlett corrected trace tests against the asymptotic distributions in Figure 4. It is quite surprising how well these distributions match for the current data. Based on all these results we conclude that a cointegration rank of 2 appears, at this stage, to be an appropriate choice.

#### 3.2. The Cointegration Space

The next step in the analysis is to examine the cointegration space. In Table 3 we report *LR* tests of a few interesting hypotheses. The first three examine the null that various interest rate spreads are stationary. When comparing the test values to the asymptotic  $\chi^2(4)$  distribution we find that they are all rejected at conventional levels of marginal significance. Moreover, when we test the hypotheses that the real long-term rate and inflation are stationary the conclusions are the same.

As in the trace test case, several studies have reported that the *LR* test of linear restrictions on the cointegration space is over-sized (see, e.g., Jacobson, Vredin, and Warne, 1997, and Gredenhoff and Jacobson, 2001). In fact, the deviation of, e.g., a bootstrapped empirical distribution from

 $<sup>^{15}</sup>$  The *p*-values have been computed using the simulated distributions in MacKinnon, Haug, and Michelis (1999).

<sup>&</sup>lt;sup>16</sup> In particular, we have employed the approximations given in Corollary 2 of this article.

the  $\chi^2$  is often so large that the asymptotic distribution seems close to being a useless reference distribution for an uncorrected *LR* statistic. For the top 5 hypotheses in Table 3 we could apply the Bartlett correction factor derived in Johansen (2000, Corollary 5), while the Bartlett factor has not been derived for the hypotheses underlying the remaining 7 models in the Table.<sup>17</sup> Alternatively, we can use bootstrapping.

In Table 3 we report *p*-values of all tests from bootstrapped empirical distributions in the last (10th) column and bootstrap estimates of the Bartlett correction factors in the 9th column. The bootstrapped Bartlett factors are given by the average of the *LR*-tests from the bootstrap divided by the mean of the  $\chi^2(q)$  distribution. The latter is equal to *q*, the number of restrictions. In addition, we provide plots of the densities of the asymptotic, the empirical bootstrap, and a Bartlett corrected empirical bootstrap for all 12 models in Figure 5. As can be seen from these plots Bartlett correction can potentially work well if the correction factors are not too big, i.e., for models  $\mathcal{M}_6-\mathcal{M}_9$ ,  $\mathcal{M}_{11}$ , and  $\mathcal{M}_{12}$ .

For the empirical *p*-values we find that models  $\mathcal{M}_1-\mathcal{M}_5$  all lie somewhere between 5 and 10 percent, suggesting that at least one of these models may be consistent with the data. At the same time the bootstrapped Bartlett factors are quite big, ranging from 2.35 to 3.50, indicating that Bartlett correction based on Johansen (2000) may not work well here. In view of the results in Omtzigt and Fachin (2002) it may also be the case that the bootstrap is over-sized. If so, then the *p*-values are too small also for the bootstrap. Henceforth, we let all the models  $\mathcal{M}_1-\mathcal{M}_5$  in Table 3 be special cases of the space spanned by one of the cointegration vectors.

In the 6th row of Table 3, model  $\mathcal{M}_6$ , we report the results from testing the null that a linear combination of (i) real money, output, the short and the own rate, and (ii) the three interest rates and inflation are jointly stationary. With a test value of 2.12 we find that this null cannot be rejected at standard significance levels. Bartlett correction will not change this conclusion since, given an estimate of 1.70, we expect the correction factor to be greater than unity. The estimated parameters of these 2 relations and their *conditional* standard errors are given below.

$$\hat{\beta}' X_t = \begin{bmatrix} 1 & 0 & -1.38 & 0.81 & 0 & -1.31 \\ 0 & -0.63 & 0 & 1 & 0.41 & -1.96 \\ 0 & -0.63 & 0 & 1 & 0.41 & -1.96 \\ 0 & 0.06) & & 0 & 0.41 & 0.11 \end{bmatrix} \begin{bmatrix} m_t \\ \Delta p_t \\ y_t \\ i_{s,t} \\ i_{l,t} \\ i_{o,t} \end{bmatrix}.$$
(2)

The behavior of the system under these restrictions is summarized in Panel C of Table 1. Again we find that the restrictions do not appear to change the properties of the money demand system radically.

Examining the first cointegration relation, which resembles a long-run money demand relation, we find that the estimated coefficient on output is greater than unity and is of the same magnitude as earlier studies on euro area money demand have found; see, e.g., Brand and Cassola (2000), Golinelli and Pastorello (2002), and Calza et al. (2001).<sup>18</sup> Furthermore, the coefficients on the short

 $<sup>^{17}</sup>$  The analyses in Johansen (2002a, 2000) do not consider the types of linear restrictions on  $\beta$  that these hypotheses imply.

<sup>&</sup>lt;sup>18</sup> See also Brand, Gerdesmeier, and Roffia (2002) for a review of the existing money demand models published by the ECB.

rate and on the own rate have the correct signs when the former is interpreted as the alternative rate of return of holding money. If we were to compute a t-test for the null hypothesis that the coefficient on the short rate and the own rate, respectively, is zero, then both hypotheses would be soundly rejected at the 5 percent level when the Gaussian (or t) distribution is used as a reference distribution.

However, it is hazardous to make use of these conditional, or local, standard errors in this way. Instead, we can impose 1 additional restriction on the first cointegration relation and then reestimate the vector error correction model. In row 7 (model  $\mathcal{M}_7$ ) and 8 (model  $\mathcal{M}_8$ ) of Table 3, respectively, we give the test results from the joint hypotheses of the 2 (over-identifying) restrictions in model  $\mathcal{M}_6$  and the additional restriction that the coefficient on the own and on the short rate, respectively, is zero. The difference between the LR test in row 7 (8) and row 6, gives the appropriate *LR* test value of the hypothesis that  $\beta_{16}$  ( $\beta_{14}$ ) is zero. Since the joint hypotheses both result in small numerical values for the *LR* test, we conclude that both these coefficients may be zero. In addition, in row 9 (model  $\mathcal{M}_9$ ) we report the *LR* test value of the hypothesis that all these 4 (overidentifying) restrictions on  $\beta$  are satisfied and again the null is not rejected. While this serves to illustrate the limited usefulness of the conditional standard errors for the identified  $\beta$  parameters, it also suggests that the semi-elasticities on the interest rates in the money demand relation are imprecisely estimated using classical ML. Moreover, the estimated coefficient on output changes only marginally when these additional restrictions are imposed on the first cointegration relation. Also, when we compare the estimated parameters of the second cointegration relation between models  $\mathcal{M}_6$ - $\mathcal{M}_{12}$ , the point estimates change very little; see Section 4 for discussions on  $\mathcal{M}_9$ ,  $\mathcal{M}_{11}$ , and  $\mathcal{M}_{12}$ .

As a final check on the first cointegration relation we have also tested the hypothesis that the spread between the short and the own rate enters this relation, i.e. model  $\mathcal{M}_{10}$ . Again, neither the joint nor the conditional null hypothesis is rejected at conventional levels of marginal significance.

In Figure 6 we have graphed the values of the log-likelihood function when the income and interest rate parameters in long-run money demand take on certain values. All other parameters are reestimated in these experiments. The horizontal line shows the value of the log-likelihood function (at the 95 percent quantile of the  $\chi^2(1)$  distribution) where the *LR* test signals rejection of the null that the parameter is equal to that value when compared to the case when it is estimated freely, i.e., the maximum point for the likelihood function. From Figure 6 it can thus be seen that based on this measure the income elasticity is well determined; the 95 percent "confidence interval" is between 1.30 and 1.45. The interest rate semi-elasticities, on the other hand, have very wide confidence bands, where the own rate semi-elasticity is between 3.7 and -1.2 with 95 percent confidence and the short rate semi-elasticity is between 0.7 and -2.2.

To illustrate this issue further, the upper plot in Figure 7 displays the values of the log-likelihood function for fixed values of the short rate and the own rate semi-elasticities. The plane reflects the value of the log-likelihood when a joint *LR* test of ( $\beta_{14}$ ,  $\beta_{16}$ ) is exactly equal to the 95 percent critical value of the  $\chi^2(2)$  distribution. The region above this plane reflects all combinations of pairs of values that these parameters can take on that are not rejected at the 5 percent level of marginal significance. In the lower plot we present confidence regions at the 80, 90, 95, 97.5, and 99 percent levels. That is, we have sliced the log-likelihood function from the upper graph at these levels of marginal significance. As can be seen from the latter graph, these regions are huge, especially in the own rate parameter direction, even at the 80 percent level. At the same time, the parameters seem to be negatively correlated.

The second cointegration relation relates the own rate to the short and the long-term market interest rates, i.e. a long-run equilibrium relation for the interest rates. In addition, inflation enters the relation with a coefficient almost equal to the coefficient on the long rate, but with the opposite sign. Potentially, one may interpret this as a long-run pricing relation for the own rate.

In the next section we shall study the parameter constancy properties of model  $\mathcal{M}_0$ , where the cointegration space is unrestricted given a rank of 2, and some of its "siblings". In particular, we shall examine the constancy of the cointegration space as well as most of the remaining parameters.

### 4. CONSTANCY ANALYSIS

In contrast to most previous studies of euro area money demand we shall apply formal tests to investigate the parameter constancy issue; the main exception is Fagan and Henry (1998) who use some of the tests suggested by Hansen (1992).<sup>19</sup> Typically, recursive estimates over a limited time period and ocular inspection of recursive Chow forecast, break-point, or predictive failure tests have been used to examine this problem. While such diagnostics may be useful for preliminary analyses, any inferences drawn from these exercises neglect a large fraction of the sample period and do not take into account that, e.g., the Chow test is a formal test only for a single point in time.

Ideally, one would like to have estimates of all parameters in the statistical model for each period in time and compare these with, e.g., the full sample estimates using a simple statistic with good power properties against a wide range of non-constancies. Until such a statistic exists, we may still consider using formal tests for parameter constancy which do compare estimated parameters across time, are relatively easy to compute, and take as much of the sample as possible into account. Moreover, there are test statistics satisfying these criteria whose limiting distributions are known and free from nuisance parameters.

In this section we shall examine the parameter constancy of three sets of parameters in equation (1). First we shall study the non-zero eigenvalues used in the cointegration rank analysis. The main tool here is the fluctuation test suggested by Hansen and Johansen (1999). Second, we examine the constancy of  $\beta$  using the Nyblom (1989) tests studied by Hansen and Johansen (1999). We also propose versions of these tests which are likely to be more reliable. Third, we take a look at the constancy of the  $\Phi$ ,  $\Gamma_1$ , and  $\alpha$  parameters using the fluctuation test due to Ploberger, Krämer, and Kontrus (1989), and finally we compare some of these results to a sample that begins in 1983:Q1. It may be noted that all the formal tests do *not* require trimming of the sample. For computational reasons, however, we will use about 30 percent of the sample as a base period and examine constancy over the remainder.

#### 4.1. Preliminary Considerations

When studying subsets of parameters, one issue to consider for the parameter constancy analysis is how to treat the remaining parameters. One approach is to fix the latter parameters at the full sample estimates, and the alternative is to update them along with the parameters of interest. Below we shall focus on the former approach, but at times also discuss the results when the latter approach is taken. In principle, the second approach should be preferred since fixing parameters that do indeed vary over time can lead to the wrong conclusion about those that are updated. However, given the short sample and given that the tests are based on asymptotic theory, the

<sup>&</sup>lt;sup>19</sup> Coenen and Vega (2001) also apply some of Hansen's (1992) tests for parameter constancy, but only for the money equation. The relationship between these tests and some of those applied in this paper is discussed in Hansen and Johansen (1999).

more parameters we update the more likely it is that the asymptotic distributions provide poor approximations of the unknown small sample distributions. We therefore extend the analyses with bootstrapped empirical distributions of all constancy tests.

## 4.2. The Non-Zero Eigenvalues

The evidence from applying the Hansen and Johansen fluctuation tests to our data, when we condition on the full sample estimates of the parameters on the constant and the first lag, are given in Panel A of Table 4.

As can be seen from the Table at least one of the two non-zero eigenvalues,  $\lambda_1$  and  $\lambda_2$ , appears to be non-constant over the examination period, 1987:Q2–2001:Q4. In particular, the largest eigenvalue may be time varying. It should be emphasized here that the small sample properties of the fluctuation test when applied to non-zero eigenvalues are unchartered territory. Given what we know about, e.g., the *LR* test for linear restrictions on  $\beta$  and the trace test one may suspect that the fluctuation test is over-sized as well. Indeed, the bootstrapped *p*-values in the 4th column are always higher than the asymptotic, but at the 5 percent level do not change the results that  $\lambda_1$  seems to be non-constant over the experimentation period. Still, the bootstrap is only based on 1000 replications, we only consider residuals drawn from a normal distribution, and do not make use of double or fast double bootstrap; see, e.g., Davidson and MacKinnon (2000b). Hence the empirical distribution may not be very accurate, especially in the tails, and our results should therefore be interpreted with great care.

If we instead also update the  $\Phi$  and the  $\Gamma_1$  parameters over the 1987:Q2–2001:Q4 period, then as can be seen from Panel B of Table 4, all null hypotheses are firmly rejected when the reference distribution is the asymptotic. The empirical bootstrap distribution, however, gives a different picture, suggesting that the observed test statistics may not be so unlikely, especially the sum of the transformed eigenvalues.

To summarize, there are some indications that the non-zero eigenvalues may not be constant over the experimentation period. If one of these parameters is indeed time-varying, it may be due to either time-varying  $\alpha$  or  $\beta$  parameters. Alternatively, the tests may indicate time-variation of these parameters when the selected cointegration rank is incorrect.<sup>20</sup> The fluctuation tests seem to be over-sized regardless of whether the  $\Phi$  and the  $\Gamma_1$  parameters are updated or not. To evaluate the small-sample properties of the fluctuation tests more thoroughly, however, is left for future research.

### 4.3. The Cointegration Space

To examine the constancy of the cointegration space we shall consider 2 types of Nyblom tests. The first (supremum) test is based on the maximum value of a weighted *LM*-type statistic over the experimentation period and the second (mean) test on the average of this statistic. In addition, the *LM*-type statistic is calculated using 2 different methods. The first method was suggested by Hansen and Johansen (1999) and involves a first order Taylor expansion of the score function, while the second method is new for the purpose of examining the constancy of  $\beta$  and it uses the scores directly.

 $<sup>^{20}</sup>$  See, e.g., Quintos (1997, Theorem 4) for the behavior of a related fluctuation test when the cointegration rank is over and under-specified.

All Nyblom tests are computed for a model with  $\Phi$  being unrestricted and  $D_t = 1$ . Using the notation from Hansen and Johansen (1999) this means that

$$c = \hat{eta}^{(T)}, \quad c_{\perp} = \hat{eta}^{(T)}_{\perp},$$

and the normalization matrix  $\bar{c} = c(c'c)^{-1}$  such that  $\hat{\beta}_c^{(t)} = \hat{\beta}^{(t)}(\bar{c}'\hat{\beta}^{(t)})^{-1}$  and  $\hat{\alpha}_c^{(t)} = \hat{\alpha}^{(t)}\hat{\beta}^{(t)'}\bar{c}$ . Moreover, defining

$$q^{(t)} = T \bar{c}'_{\perp} (\hat{\beta}^{(t)}_c - \hat{\beta}^{(T)}_c)$$
$$V^{(T)} = \hat{\alpha}^{(T)'}_c \hat{\Omega}^{(T)^{-1}} \hat{\alpha}^{(T)}_c,$$
$$M^{(t)} = T^{-1} c'_{\perp} S^{\tau(t)}_{11} c_{\perp},$$

Hansen and Johansen (1999) shows that a first order Taylor expansion of the score function in the Nyblom statistic for constant  $\beta$  yields the statistic

$$Q_T^{(t)}(HJ) = \left(\frac{t}{T}\right)^2 \operatorname{tr}\left[V^{(T)} q^{(t)'} M^{(t)} M^{(T)^{-1}} M^{(t)} q^{(t)}\right], \quad t = 1, \dots, T.$$
(3)

The matrices  $S_{ij}^{\tau(t)} = (1/t) \sum_{s=1}^{t} R_{i,s}^{(\tau)} R_{j,s}^{(\tau)'}$  for  $i, j \in \{0, 1\}$ . The time index  $\tau = T$  when  $(\Phi, \Gamma_1)$  are fixed at the full sample estimates, while  $\tau = t$  when these parameters are updated. The residuals  $R_{i,s}^{(\tau)} = Z_{i,s} - M_{i2}^{(\tau)} M_{22}^{(\tau)^{-1}} Z_{2,s}$ , where  $Z_{0,s} = \Delta X_s$ ,  $Z_{1,s} = X_{s-1}$ ,  $Z_{2,s} = (1, \Delta X_{s-1})$  and  $M_{ij}^{(\tau)} = \sum_{s=1}^{\tau} Z_{i,s} Z_{j,s}'$ .

The limiting distribution of  $Q_T^{(t)}(HJ)$  is independent of which estimate of  $S_{11}^{\tau(t)}$  is selected. Moreover, it can be shown that Theorem 4 in Hansen and Johansen (1999) is still valid, but with J(s)and S(s) given by:

$$J(s) = \int_0^s FF' du, \quad S(s) = \int_0^s F(dB_2)', \quad F(s) = \begin{bmatrix} B_1(s) - \int_0^1 B_1(u) du \\ s - (1/2) \end{bmatrix}, \tag{4}$$

where  $B_1$  and  $B_2$  are independent standard Brownian motions of dimension (n - r - 1) and r, respectively, and n is the number of endogenous variables.

Instead of using a first order Taylor expansion of the score function, we may consider using the score function directly. For that formulation we obtain the following version of the statistic:

$$Q_T^{(t)}(S) = \left(\frac{t}{T}\right)^2 \operatorname{tr}\left[V^{(T)}S^{(t)'}M^{(T)^{-1}}S^{(t)}\right], \quad t = 1, \dots, T,$$
(5)

where  $S^{(t)} = c'_{\perp} [S^{\tau(t)}_{01} - \hat{\alpha}^{(T)} \hat{\beta}^{(T)'} S^{\tau(t)}_{11}]' \hat{\Omega}^{(T)^{-1}} \hat{\alpha}^{(T)}$ . By construction,  $\hat{\alpha}^{(T)}_{c} = \hat{\alpha}^{(T)}$  and  $\hat{\beta}^{(T)}_{c} = \hat{\beta}^{(T)}$ , thus simplifying these expressions further. It may be noted that  $M^{(t)}q^{(t)}$  is a representation of the first order Taylor expansion of  $S^{(t)}$  in the direction of the appropriately defined free parameters of  $\beta$ . The expression for  $Q^{(t)}_T(S)$  in (5) is a weighted *LM* statistic, while  $Q^{(t)}_T(HJ)$  in (3) is a first order approximation. The test statistic suggested by Nyblom (1989) corresponds to the average of  $Q^{(t)}_T(S)$  (the mean statistic), but like in Hansen and Johansen (1999) we shall also consider its supremum.

Based on the arguments in the proof of Theorem 4 in Hansen and Johansen (1999) it can be shown that  $Q_T^{(t)}(HJ)$  and  $Q_T^{(t)}(S)$  are asymptotically equivalent.<sup>21</sup> In small samples, however, they differ since the remainder term from the first order Taylor expansion is non-zero for t < T. Moreover, this remainder term can be quite large if the log-likelihood function is flat in some direction of the

<sup>&</sup>lt;sup>21</sup> See also the discussion on page 314–316 of Hansen and Johansen (1999) for, e.g., relations to statistics previously suggested in the literature on testing parameter constancy.

unique parameters of the cointegration space. If that is the case, one may suspect that the statistic  $Q_T^{(t)}(HJ)$  will not be well behaved. As we shall see below, this is indeed the case here.<sup>22</sup>

In Panel A of Table 5 we present the tests for the constancy of  $\beta$  when we condition on the full sample estimates of  $\Phi$  and  $\Gamma_1$ . In row 1 we find the Nyblom supremum test,  $\sup Q_T^{(t)}(HJ)$ , and the Nyblom mean test, mean  $Q_T^{(t)}(HJ)$ , for the model  $\mathcal{M}_0$  with 2 unrestricted cointegration relations. It can be seen from this Table that the Hansen-Johansen version of the Nyblom tests generate extremely large values, while the score versions in row 2 behave less "suspect". For the latter version both the supremum and the mean tests are far below their asymptotic 95 percent critical values of 4.16 and 1.87, respectively. Turning to the statistics in Panel B where  $\Phi$  and  $\Gamma_1$  are updated we find similar results.

Furthermore, when we attempt to bootstrap the distributions for the Nyblom tests we find, not surprisingly, that the *HJ* versions do not have meaningful empirical distributions. Hence, we do not report any bootstrap *p*-values for these tests. For the *S* versions of the Nyblom tests the distributions are well behaved by comparison. When we condition on the full sample estimates of  $\Phi$  and  $\Gamma_1$  the empirical *p*-values are lower than the asymptotic, but not sufficiently low to suggest that the null of constancy should be rejected. Still, this indicates that the tests are under-sized in this situation. When we instead update  $\Phi$  and  $\Gamma_1$  the empirical *p*-values are quite close to the asymptotic. For the supremum test we find that it is slightly under-sized, while the mean test is somewhat over-sized.

We have already noted in the previous section that for model  $\mathcal{M}_6$  the interest rate semi-elasticities are very imprecisely estimated since the log-likelihood function is flat over a large section of the parameter space in those 2 directions. This may explain why the HJ versions of the Nyblom tests have such extreme values. To investigate this further we have calculated the HJ version using the restrictions on  $\beta$  in models  $\mathcal{M}_6$  and  $\mathcal{M}_9$ . When the full sample estimates of  $\Phi$  and  $\Gamma_1$  are used we find that the former model gives a supremum value of 304516.01 and a mean value of 5837.62, while the latter where  $\beta_{14} = \beta_{16} = 0$  provides us with 2.55 and 1.13, respectively. Values similar to those for model  $\mathcal{M}_9$  are obtained under models  $\mathcal{M}_{11}$  and  $\mathcal{M}_{12}$ , while model  $\mathcal{M}_{10}$ , where the spread parameter can vary freely, again yields extreme values. Since the log-likelihood function is also very flat in the direction of the spread parameter these numerical results suggest that the HJ versions may be "numerically unreliable". Further research on this issue is, however, necessary before any definite conclusion can be drawn.

To sum up, based on the suggested score version of the Nyblom tests we conclude tentatively that the cointegration space is constant for the irrevocably fixed exchange rate data. Moreover, the first order Taylor expansion version of the Nyblom tests provides numerically unreliable results. The reason for this unreliability seems to be that the log-likelihood function is flat over a large region of the cointegration space, represented by, e.g., the interest rate semi-elasticities of long-run money demand in model  $\mathcal{M}_6$ . Finally, the constancy of  $\beta$  should be treated with caution since the tests rely on the constancy of all other parameters.

<sup>&</sup>lt;sup>22</sup> It may also be noted that  $Q_T^{(t)}(S)$  can be computed faster than  $Q_T^{(t)}(HJ)$  since updated estimates of the cointegration space are not required.

#### 4.4. The Short-Run Dynamics

The estimated cointegration relations for models  $\mathcal{M}_6$ ,  $\mathcal{M}_9$ ,  $\mathcal{M}_{11}$  and  $\mathcal{M}_{12}$  are depicted in Figure 8. The estimated  $\beta_{ij}$  parameters for  $\mathcal{M}_9$  and  $\mathcal{M}_{11}$  are:

$$\hat{\beta}'_{\mathcal{M}_9} = \begin{bmatrix} 1 & 0 & -1.40 & 0 & 0 & 0 \\ & & (0.01) & & \\ 0 & -0.66 & 0 & 1 & 0.44 & -1.98 \\ & & (0.07) & & 0 & 1 & 0.44 & -1.98 \\ & & (0.08) & (0.12) \end{bmatrix}, \quad \hat{\beta}'_{\mathcal{M}_{11}} = \begin{bmatrix} 1 & 0 & -1.38 & 0.8 & 0 & -1.3 \\ & & (0.01) & & \\ 0 & -0.63 & 0 & 1 & 0.41 & -1.96 \\ & & (0.06) & & 0 & 1 & 0.41 & -1.96 \\ & & (0.06) & & 0 & 1 & 0.41 & -1.96 \\ \end{bmatrix}$$

Given the small differences in most of the parameter estimates, it is perhaps not surprising that these relations are so similar for the two models. The first relation, "long-run money demand", primarily has a different mean for the models, while the second relation, which only involves the 3 interest rates and inflation, is virtually identical across models.<sup>23</sup>

The estimated  $\alpha$  parameters for models  $\mathcal{M}_9$ ,  $\mathcal{M}_{11}$  and  $\mathcal{M}_{12}$  are presented in Table 6.<sup>24</sup> From the point estimates and the standard errors we find that long-run money demand enters the money, the short rate, and the own rate equations significantly at the 5 percent level in all models. Moreover, the signs of these parameters are all negative. Turning to the interest rate relation we find that it primarily matters for explaining the behavior in inflation, but also seems to be important for explaining the behavior of the own rate.

From Table 6 a number of possible restrictions on  $\alpha$  emerge. Below, we shall consider 2 sets of restrictions. For both sets we let output and the long rate be weakly exogenous with respect to ( $\alpha$ ,  $\beta$ ) and impose a zero restriction on the long-run money demand relation in the inflation equation.<sup>25</sup> For the first set of restrictions ( $\mathcal{M}_{s,1}$ , where s = 9, 11, 12) we also let the  $\alpha_{i1} = \alpha_{i2}$  in the money and the short-term rate equations, while the  $\alpha$  parameters in the own rate equation are equal with opposite signs; a total of 8 restrictions. For the second set of restrictions ( $\mathcal{M}_{s,2}$ , where s = 9, 11, 12) we let the  $\alpha$  coefficients on the interest rate relation be equal to zero in the money and the short-term rate equations; adding up to 7 restrictions. The estimated parameters, standard errors, and *LR* tests of these restrictions are also reported in Table 6.

In all cases but  $\mathcal{M}_{9,2}$  we find that the restrictions cannot be rejected at conventional levels of marginal significance. Hence, there does not seem to be any information in the output and the long rate beyond the information contained in the other 4 equations about the two cointegration relations. Moreover, the *p*-values are higher and the test statistics are lower for the first set of restrictions, indicating that the data may be more "comfortable" with the equality restrictions than with the pure zero restrictions. Still, for the parameters which are allowed to be different from zero the differences are generally small when comparing across the two sets of restrictions.

$$\hat{\beta}'_{\mathcal{M}_{12}} = \begin{vmatrix} 1 & 0 & -1.37 & 0.4 & 0 & -0.4 \\ & & & & & \\ 0 & -0.66 & 0 & 1 & 0.42 & -1.95 \\ & & & & & & & \\ 0.06) & & & & & & & & \\ 0.000 & & & & & & & & \\ 0.000 & & & & & & & & \\ 0.000 & & & & & & & & \\ 0.000 & & & & & & & & \\ 0.000 & & & & & & & & \\ 0.000 & & & & & & & & \\ 0.000 & & & & & & & & \\ 0.000 & & & & & & & & \\ 0.000 & & & & & & & & \\ 0.000 & & & & & & & & \\ 0.000 & & & & & & \\ 0.000 & & & & & & \\ 0.00$$

<sup>&</sup>lt;sup>23</sup> The estimated cointegration matrix for the restricted spread model is given by:

As can be seen in Figure 8 the cointegration relations formed using this  $\beta$  matrix are not very different from the series for the other models.

<sup>&</sup>lt;sup>24</sup> Model  $\mathcal{M}_6$  ( $\mathcal{M}_{10}$ ) is not considered since it is basically identical to model  $\mathcal{M}_{11}$  ( $\mathcal{M}_{12}$ ).

<sup>&</sup>lt;sup>25</sup> The restriction that long-run money demand does not enter the inflation equation should *not* be interpreted as evidence that money does not matter for forecasting inflation, even if it were the case that all first difference terms in the inflation equation were equal to zero. The reason is, of course, that money seems to matter for forecasting the short rate and the own rate one period ahead, and these variables seem to be important for forecasting inflation one period ahead. Hence, it may very well be that money incorporates unique information for improving the forecasts of inflation, e.g., two periods ahead; see, e.g., Vega and Trecroci (2002) for a study of the information content in M3 for future inflation and Nicoletti Altimari (2001).

The issue of whether or not to include the interest rate relation in the money equation cannot be resolved from the tests and the difference in point estimates is minor. The log-likelihood value is somewhat larger when the interest rate relation is included, as reflected through the lower test statistics for that case. Given the strong trends in real money and output it is perhaps not so surprising that it is difficult to obtain precise information about the relevance of the interest rates for long-run money demand. Yet, 5 of these 6 models do give a levels role for the interest rates to play in the money equation. Moreover, all these 6 models are consistent with the existence of something resembling a long-run "money supply" relation.<sup>26</sup>

In Table 7 we have formed linear combinations of the two cointegration relations for models  $\mathcal{M}_9$ ,  $\mathcal{M}_{11}$ , and  $\mathcal{M}_{12}$ , respectively, under the two sets of  $\alpha$  restrictions for 3 of the equations. We find that only in the case of model  $\mathcal{M}_{9,2}$  do the interest rates not enter the money equation in levels. And this is precisely the model where the  $\alpha$  restrictions may be rejected at the 5 percent level. For the other models, the signs of the short rate and the own rate are consistent with the interpretation of the linear combination being a long-run money demand relation. Notice that the long rate and inflation are also included in those linear combinations.

For the short rate equation we find that a linear combination, consistent with a long-run money demand relation, enters in all 6 models. In the own rate equation, however, the signs of all coefficients on the interest rates and inflation have been reversed. This is consistent with the interpretation of a long-run money supply relation being important for explaining the changes in the own rate.

At this point it is worthwhile to emphasize that whenever there are 2 or more cointegration relations in the system, we are faced with an economic identification problem concerning the long-run relations. While cointegration analysis may help us identify stationary linear combinations of potentially non-stationary time series, it generally cannot clarify what these cointegration relations mean economically. The reason is, of course, that any linear combination of two or more cointegration relations is also a cointegration relation. Hence, exactly which linear combination of our two statistically identified relations (if any) is the economically identified long-run money demand relation cannot be determined by the data.

In Table 8 we report fluctuation tests (cf. Ploberger et al., 1989) for the constancy of the (unrestricted)  $\Phi$ ,  $\Gamma_1$ , and  $\alpha$  parameters in the 6 equations. There are signs of non-constancy in several of the equations when we rely on the asymptotics for a reference distribution. Generally, models  $\mathcal{M}_{11}$  and  $\mathcal{M}_{12}$  display fewer signs of non-constancy than does model  $\mathcal{M}_9$ . However, at the 5 percent level the parameters in the money equation are non-constant for models  $\mathcal{M}_{11}$  and  $\mathcal{M}_{12}$ , but not for  $\mathcal{M}_9$ .

Still, these results are based on the asymptotic distribution and it may not be a good approximation of the unknown small sample distribution. Consequently we have also bootstrapped these fluctuation tests and the *p*-values from the empirical distributions are also given in Table 8. This time all *p*-values are greater than 10 percent and always greater than the *p*-values based on the asymptotic distribution. Hence, it seems as if these fluctuation tests are quite severely over-sized.

<sup>&</sup>lt;sup>26</sup> For money demand systems of narrow monetary aggregates, like M1, the quantity of money is probably best thought of as being demand determined, i.e. the central bank supplies money as it is demanded by agents of the economy. For broad monetary aggregates, like M3, the picture may be somewhat different. An important component of M3 includes, e.g., savings and time deposits and these instruments typically yield a time-varying rate of return. While banks who supply access to such accounts may be expected to accept an increase (due to, e.g., portfolio shifts from equities) in such deposits, they are likely to react by lowering the rate of return on such accounts when increases are sufficiently big. A similar argument can be made for marketable instruments included in M3. Hence, from this perspective it can be argued that there is a supply side to M3.

Moreover, the empirical *p*-values suggest that the parameters in the individual equations are constant over the experimentation period.

### 4.5. Excluding Data Prior to 1983

As a robustness check we will reexamine the 6 variable model when we exclude the first 3 years of data. While the choice of sub-sample is always to some extent arbitrary, it makes sense to exclude the first years of our sample since the countries making up the euro area were most likely less integrated, especially the financial markets, in the early 80s than at some other point of our sample.

In Table 9 we report the cointegration rank tests for the sample 1983:Q3-2001:Q4 for a model with 2 lags.<sup>27</sup> Compared with the full sample, we now find that the uncorrected trace tests suggest using a cointegration rank of 2 at the 5 percent level, while the corrected trace tests indicate that we should choose only one cointegration relation even at the 10 percent level. Since the bootstrapped empirical distributions yield *p*-values comparable with those from the Bartlett corrected test using the asymptotic distribution we will proceed the analysis here with 1 cointegration relation.

For the model with cointegration rank equal to 1 we find that the S version of the Nyblom supremum test is 1.52 while the mean test is 0.51; yielding asymptotic (bootstrap) *p*-values of 57 (57) percent and 54 (63) percent, respectively, when  $\Phi$  and  $\Gamma_1$  are updated.<sup>28</sup> Hence, without restricting  $\beta$  further we may conclude that the cointegration relation:

$$\hat{\beta}' = \begin{bmatrix} 1 & 3.90 & -1.32 & -5.88 & -3.03 & 13.07 \\ _{(0.67)} & _{(0.06)} & _{(1.11)} & _{(0.67)} & _{(2.25)} \end{bmatrix}$$

is indeed seem to be constant over the sample in question.

If we restrict the parameters of  $\beta$  according to the long-run money demand relation in model  $\mathcal{M}_6$  we obtain:

$$\bar{\beta}' = \begin{bmatrix} 1 & 0 & -1.37 & 0.43 & 0 & -0.48 \\ 0.02) & 0.50) & 0 & 0.98 \end{bmatrix}.$$

The *LR* test value is 13.15 with a *p*-value equal to 0 when compared with the  $\chi^2(2)$  distribution. However, if we were to compute a Bartlett corrected *LR* test for this case we would need a correction factor of 2.20 to obtain a test value equal to the 95 percent critical value from the asymptotic distribution. The *p*-value from the bootstrapped distribution of the test statistic is 7 percent, while the bootstrap estimate of the Bartlett factor is 2.57, thus suggesting that the null hypothesis may be consistent with the data. Moreover, the point estimates suggest that the spread between the short-term rate and the own rate appear in the money demand relation. Again, however, the interest rate semi-elasticities are imprecisely estimated with huge confidence bands.

Turning to the Ploberger et al. (1989) fluctuation tests for the non-cointegration parameters in the individual equations, inference based on the asymptotic distribution suggests that the parameters in the output, short-term and long-term rates as well as the money equation need not be constant when the cointegration space is unrestricted. However, the bootstrapped distributions again suggest that these tests are over-sized in small samples and the empirical *p*-values are always greater than 10 percent. When we impose the cointegration vector above the results from the fluctuation tests are broadly in line with those from the unrestricted  $\beta$  case. Hence, when we condition on 1

<sup>&</sup>lt;sup>27</sup> The first 2 quarters of 1983 are used as initial values.

 $<sup>^{28}</sup>$  When these parameters are fixed at their full sample estimates we obtain a supremum value of 1.45 and a mean value of 0.50. This corresponds to bootstrapped (asymptotic) *p*-values of 42 (61) and 53 (55) percent.

cointegration relation it seems that the parameters of the model are constant over the experimentation period. Moreover, when testing restrictions on  $\alpha$  we find that output and the long-term rate seem to be weakly exogenous for the cointegration space since the *LR* test is equal to 0.12 when we impose the long-run money demand relation above.

In summary, shortening the sample to begin in 1983:Q1 leads to a reduction in the preferred cointegration rank. Still, the main conclusions from the full sample analysis survive. That is, there is strong evidence in the data of constant parameters for the cointegration relation, and the interest rate semi-elasticities are difficult to estimate precisely. Moreover, the  $\Phi$ ,  $\Gamma_1$ , and  $\alpha$  parameters appear to be constant over the experimentation period.

#### 5. AN ALTERNATIVE AGGREGATION METHOD

In this section we shall reexamine the 6 variable model using an alternative aggregation method. Namely, when money, prices, output, and all interest rates have been aggregated using the 2001 GDP weights at PPP exchange rates. Moreover, the sample begins in 1981:Q3 due to data limitations.

For the irrevocably fixed exchange rate aggregated data we found that 2 lags was sufficient for capturing the serial correlation in the data. The GDP weights aggregated data is also consistent with this choice of lag order. Turning to the selection of cointegration rank, however, the issue is now somewhat trickier. The *LR* trace tests along with the Bartlett corrected tests are presented in Table 10. For the uncorrected tests we now find that 4 cointegration relations are supported by the data at the 20 percent (to the 5 percent) level while 3 are supported at the 1 percent level. When we use the corrected tests for rank selection we prefer 3 at the 20 percent (to the 5 percent) level and 2 at the 1 percent level. Moreover, when inference is based on bootstrapped empirical distributions we find that they confirm the evidence for the Bartlett corrected trace tests using the asymptotic distributions. In what follows we shall therefore examine the case of 2 and 3 cointegration relations separately and thereafter compare the main results.

Before we turn to these issues, note that the Nyblom supremum tests in Table 11 indicate possible non-constancy of the cointegration space under the empirical distributions for the score versions with *p*-values generally between 1 and 10 percent. The mean tests are somewhat more in line with the constancy hypotheses. When inference is based on the asymptotic distribution the score versions suggest that the null of constancy cannot be rejected at the 5 percent level for any one of these tests. As noted above, the Taylor expansion version of the scores yield extreme values for the tests that are most likely unreliable.

#### 5.1. Two Cointegration Relations

To save space we shall focus on one set of restricted cointegration relations which is supported by the data. The estimated parameters are given by:

$$\hat{\beta}'_{\mathcal{M}^{\mathrm{GDP}}_{2,1}} = \begin{bmatrix} 1 & 0 & -1.25 & 1 & 0 & -1 \\ & & & & \\ 0 & -0.62 & 0 & 1 & 1 & -2.62 \\ & & & & & \\ 0.03) & & & & 1 & 1 & -2.62 \end{bmatrix}.$$

The *LR* test of the 6 (over-identifying) restrictions imposed on the cointegration vectors is 4.83, with a *p*-value of 57 percent according to the  $\chi^2(6)$  distribution. One aspect that deserves some comment is the coefficient on output in the first cointegration relation. If we interpret this parameter as the income elasticity of long-run money demand, then the point estimate is lower here than what we

found for the irrevocably fixed exchange rates data. Given what has been found in previous studies of euro area money demand, this result is not surprising (see, e.g., Brand et al., 2002).

Since the constancy of the unrestricted cointegration space may be questioned we have also calculated Nyblom tests for the restricted  $\beta$  above. The test statistic is based on *LM* statistics as in equation (5). For the restricted  $\beta$  case the score function is given by the first derivative of the log-likelihood function in the direction of the free parameters of  $\beta$ . The second derivatives are calculated in the same direction and all parameters are evaluated at their full sample estimates for each *t* over the experimentation period. As in the case of (5) the *LM* statistic is weighted by  $(t/T)^2$ . The asymptotic distributions of the supremum and mean statistics for these sequences are unknown but will most likely have critical values that are smaller than those from the limiting distribution of the statistics based on (5), i.e., when the cointegration space is unrestricted. When  $\Phi$  and  $\Gamma_1$  are fixed at their full sample estimates, the values for the supremum and mean statistics are here 0.56 and 0.12, respectively, with bootstrapped *p*-values equal to 0.60 and 0.73. Hence, it seems that the restricted  $\beta$  is not non-constant over the experimentation period.

Like in the case of the irrevocably fixed exchange rates data, we have also considered two sets of restrictions on  $\alpha$  for the GDP weights data. The first set involves 8 restrictions and the second 7 restrictions and they are similar to those used in models  $\mathcal{M}_{9.s}$ ,  $\mathcal{M}_{11.s}$  and  $\mathcal{M}_{12.s}$ .<sup>29</sup> Denoting the models by  $\mathcal{M}_{2.1.1}^{\text{GDP}}$  and  $\mathcal{M}_{2.1.2}^{\text{GDP}}$ , respectively, we obtain the following restricted  $\alpha$  parameters:

$$\hat{\alpha}_{\mathcal{M}_{2,1,1}^{\text{GDP}}} = \begin{bmatrix} -0.084 & -0.084 \\ (0.020) & (0.020) \\ 0 & 0.732 \\ (0.141) \\ 0 & 0 \\ -0.135 & 0 \\ (0.025) & \\ 0 & 0 \\ -0.039 & 0.039 \\ (0.008) & (0.008) \end{bmatrix}, \quad \hat{\alpha}_{\mathcal{M}_{2,1,2}^{\text{GDP}}} = \begin{bmatrix} -0.095 & 0 \\ (0.023) & \\ 0 & 0.591 \\ (0.134) \\ 0 & 0 \\ -0.107 & 0 \\ (0.027) & \\ 0 & 0 \\ -0.028 & 0.065 \\ (0.009) & (0.014) \end{bmatrix}.$$

The *LR* tests for these two models are 13.12 and 9.22, respectively, with *p*-values equal to 11 and 24 percent. Hence, at the 5 percent level we cannot reject either of these models conditional on the selected cointegration relations. Furthermore, when we examine the parameter constancy properties of the model with an unrestricted  $\alpha$ , the results (cf. Table 12) are broadly in line with those obtained in Section 4.4 (cf. Table 8). Hence, it seems as if the non-cointegration parameters are constant over the experimentation period for the case of two cointegration relations.

#### 5.2. Three Cointegration Relations

If we instead select 3 cointegration relations, a set of interesting restrictions on the cointegration space emerges. In particular, the following restricted estimate of  $\beta$  yields a *LR* statistic of 2.51

 $<sup>^{29}</sup>$  In fact, the only difference is found in the short rate equation, where the coefficient on the interest rates relation is set to 0 for the GDP weights data, and equal to the coefficient on the long-run money demand relation for the irrevocably fixed exchange rates data.

which, when compared to the  $\chi^2(6)$  distribution, has a *p*-value of 87 percent:

$$\hat{\boldsymbol{\beta}}'_{\mathcal{M}^{\text{GDP}}_{3,1}} = \begin{bmatrix} 1 & 0 & -1.26 & 1 & 0 & -1 \\ 0 & -0.77 & 0 & 1 & 1.77 & -3.54 \\ 0 & -1.30 & 1 & -1 & 0 \end{bmatrix}$$

The first two cointegration vectors resemble those found in the two vector case above. The third cointegration relation is, however, at first sight a bit more perplexing. It is almost identical to the first with the exception that the long rate enters the relation instead of the own rate. Potentially, one may be inclined to interpret this as a long-run money demand relation, but the question is then how to interpret the first. Still, if we take the linear combination of the first relation minus the third we obtain a long-run relation between output and the spread between the long rate and the own rate. Hence, the third relation is perhaps best interpreted as a linear combination between long-run money demand and aggregate demand.

Concerning the constancy of the restricted  $\beta$  we have computed the same type of Nyblom statistics as those discussed in Section 5.1. We now find that the supremum and mean statistics are equal to 1.30 and 0.46, respectively, when  $\Phi$  and  $\Gamma_1$  are fixed at their full sample estimates. Comparing these with bootstrapped empirical distributions we find that the *p*-values are equal to 15 and 18 percent, thus suggesting that the cointegration space may indeed by constant over time.

Moving on to the fluctuation tests for the  $\Phi$ ,  $\Gamma_1$ , and  $\alpha$  parameters, however, there are indications especially in the output and the short-term rate equations (cf. Table 12) that some of these parameters may not be constant. However, these tendencies are probably due to the fluctuation tests being over-sized and the bootstrapped *p*-values are always greater than 25 percent. Hence, the empirical evidence suggests that model  $\mathcal{M}_{3,1}^{\text{GDP}}$  is not subject to parameter non-constancy.

Already from the unrestricted  $\alpha$  parameters (not reported) an interesting pattern of possible restrictions emerges. In the spirit of the restrictions for the 2 cointegration relations case and for the irrevocably fixed exchange rates data, we shall discuss the following two models:

$$\hat{\alpha}_{\mathcal{M}_{3,1,1}^{\text{CDP}}} = \begin{bmatrix} -0.067 & -0.067 & 0 \\ (0.017) & (0.017) & \\ -1.146 & 0.930 & 1.146 \\ (0.206) & (0.132) & (0.206) \\ 0 & 0 & 0 \\ 0 & 0 & -0.139 \\ (0.024) \\ 0 & 0 & 0 \\ 0 & 0.038 & -0.038 \\ (0.006) & (0.006) \end{bmatrix}, \quad \hat{\alpha}_{\mathcal{M}_{3,1,2}^{\text{CDP}}} = \begin{bmatrix} -0.075 & 0 & 0 \\ (0.022) & -1.138 & 0.828 & 1.138 \\ (0.207) & (0.131) & (0.207) \\ 0 & 0 & 0 \\ 0 & 0 & -0.135 \\ (0.027) \\ 0 & 0 & 0 \\ 0 & 0 & -0.135 \\ (0.027) \\ 0 & 0 & 0 \\ 0 & 0 & -0.135 \\ (0.007) \\ 0 & 0 & 0 \\ 0 & 0 & -0.135 \\ (0.007) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.040 & -0.036 \\ (0.009) & (0.009) \end{bmatrix}.$$

The  $\alpha$  matrices are subject to 13 and 12 restrictions, respectively. The *LR* tests are in these cases 15.27 and 18.05, respectively, with *p*-values equal to 29 and 11 percent. Hence, data seems to be quite comfortable with either set of restrictions. As before, the only differences between these two models are found in the money and the own rate equations. For model  $\mathcal{M}_{3,1.1}^{GDP}$  we let the coefficient on the interest rate relation be equal to the coefficient on the money demand relation, while for model  $\mathcal{M}_{3,1.2}^{GDP}$  the coefficient on the interest rate relation is set to 0 in the money equation. Notice that these two sets of restrictions are not very different from those used in the 2 cointegration relations case above. The main difference can, in fact, be found in the inflation equation. In the

3 cointegration relations case the long-run money relations enter, while long-run money demand did not enter when we examined 2 cointegration relations. However, the fact that the first and the third relations enter with equal coefficients with opposite signs, means that it is the difference between these two relations that matters for inflation, i.e. the aggregate demand relation. Since the coefficient on output is so small relative to the interest rate coefficients, an approximation of that difference is the spread between the long rate and the own rate.

## 5.3. Comparing the Results from the Two Aggregation Methods

In Table 13 we list linear combinations of the cointegration relations that appear in the money, short rate, and the own rate equations. For all 4 models we find that changes in real money react to something that looks like a long-run money demand relation; we have a positive income elasticity, a negative semi-elasticity on the short rate, a positive semi-elasticity on the own rate, and a negative (positive) or zero semi-elasticity on the long-term rate (inflation). This is basically the same picture we obtained for the irrevocably fixed exchange rates aggregated data; cf. Table 7. Moreover, the change in the short rate also reacts to something resembling a long-run money demand relation and an aggregate demand relation. Finally, the first difference of the own rate depends on what may be a money supply relation. Again, this is consistent with what we found for the irrevocably fixed exchange rates aggregated data.

Still, it is worth emphasizing that the two aggregation methods share a fundamental identification problem. It has been suggested by Davidson (1998) that the principle of irreducible cointegration relations be applied to such cases, i.e. that a set of non-stationary variables is irreducibly cointegrated if these variables are cointegrated, but the exclusion of any of the variables leaves a set that is not cointegrated. According to Davidson's ideas, structural economic interpretations can only be made for irreducible cointegration relations. However, the principle is based on statistics (mathematics), not economics, and therefore neglects the possibility that, for instance, an economically interpretable transformation of the cointegration space is just as irreducible as a transformation of the space which is not economically meaningful.<sup>30</sup>

Given the strong trending behavior of real money and income, it is perhaps not so surprising that it is difficult to obtain precise information about the relevance of the interest rates for longrun money demand. Moreover, for the purpose of forecasting the level of real money it is unlikely that the exact values for the interest rate semi-elasticities in money demand matter. The income elasticity in the irrevocably fixed exchange rate data is, as a point estimate, somewhat larger than the estimates of the same parameter in the GDP weights data. But the difference is by no means huge and it is difficult to assess the uncertainty of the point estimates using asymptotic results.

We have computed bootstrapped 95 percent confidence intervals for the income elasticity in 3 models;  $\mathcal{M}_6$  for the irrevocably fixed exchange rate data and  $\mathcal{M}_{2,1}^{\text{GDP}}$  and  $\mathcal{M}_{3,1}^{\text{GDP}}$  for the GDP weights data. For  $\mathcal{M}_6$  we find that this interval is given by [1.26, 1.52] while  $\mathcal{M}_{2,1}^{\text{GDP}}$  gives us [1.20, 1.29] and  $\mathcal{M}_{3,1}^{\text{GDP}}$  yields [1.21, 1.29]. Since these intervals are overlapping the income elasticities for the

<sup>&</sup>lt;sup>30</sup> It should be pointed out that Davidson (1998) does not claim that all irreducible cointegration relations are structural in an economic (Cowles Commission) sense. For example, if we replace the third cointegration relation in Section 5.2 by the aggregate demand relation (the first minus the third), the first two relations are structural according to Davidson (1998, Theorem 5), while aggregate demand need not be structural since it does not contain a variable which does not appear in another cointegration relation. On the other hand, if we replace the first cointegration relations are structural according to Davidson' Theorem 5, while the third, then the second and the third cointegration relations are structural according to Davidson' Theorem 5, while the first need not be. Hence, the principle may lead to increased confusion rather than increased understanding about what is meant by structural relations or structural parameters. The case of verifying "generic" identification is beautifully treated by Johansen (1995, Theorem 3), while economic interpretations of cointegration relations are still best handled by referring to economic theory.

two aggregation methods may not be different. However, as noted by, e.g., Horowitz (2001) one should keep in mind that bootstrapping is best suited for (asymptotically) pivotal statistics, i.e., a statistic whose (limiting) distribution is free from nuisance parameters, and hence these confidence intervals may not be very accurate. It would be interesting to study the parameter uncertainty issue using Bayesian methods (see, e.g., Villani, 2001), but this is left for future work.

#### 6. The Importance of Asset Markets

The usefulness of analyzing money demand systems in monetary policy analyses depends on our ability to separate changes in its behavior that are due to income from changes that come about from other factors, since these may generate shifts in the income velocity of money (see, e.g., Dow and Elmendorf (1998)). For example, euro area households and firms increased their equity holdings significantly in the late 1990s, even though they may have reduced them somewhat afterwards.<sup>31</sup> We may then expect that in the last few years the behavior of, e.g., M3 has been affected by the stock market.<sup>32</sup>

Friedman (1988) put forth the hypothesis that stock price developments affect money demand in a direction that depends on whether the substitution effect or the wealth effect prevails. While the substitution effect predicts a fall in the demand for money when stock prices rise, the wealth effect would lead to a higher demand for liquidity. Along these lines and under the assumption that stock market variables may help capture the store of value and portfolio reallocation motives behind the demand for M3, we first allow for a measure of euro area real stock prices as an additional endogenous variable. Second, a proxy for euro area stock market volatility is added to the system as a stationary weakly exogenous variable. Stock market volatility can be seen as a measure of the risk investors face when holding stock market portfolios. It often appears to move countercyclically and tends to exhibit spikes during recessions, financial crises, structural change, and periods of uncertainty; see, e.g., Campbell, Lettau, Malkiel, and Xu (2001). Under such conditions expectations of firms' future earnings may well worsen and investors may be inclined to reallocate their portfolios in favor of instruments that are included in M3. This suggests that the effects of stock market volatility on money demand may primarily be a short-term phenomenon. Moreover, if we view volatility as a stationary random variable it cannot affect the long run money demand relation.<sup>33</sup> Below we shall first examine the effects of real stock prices and then turn to the volatility issue.

### 6.1. Adding Real Stock Prices to the Statistical Model

The stock price index for the euro area that we use in this paper is taken from Datastream. The quarterly observations are constructed as averages of the daily data. The currency basis for this index is in euro and to construct a real stock market price index we have taken the natural logarithm of the index minus the natural logarithm of the GDP deflator series. The resulting real stock market index is displayed on the left hand side in Figure 9. The remaining 6 variables are taken from the irrevocably fixed exchange rate data.

<sup>&</sup>lt;sup>31</sup> See, e.g., the box on "Financial investment of the non-financial sectors in the euro area up to the third quarter of 2002" in the ECB Monthly Bulletin (2003, March).

<sup>&</sup>lt;sup>32</sup> See, for example, Cassola and Morana (2002) and Kontolemis (2002) for related studies.

 $<sup>^{33}</sup>$  In contrast, volatility may matter for all non-stationary endogenous variables in the long run for the same reason that some shock can have permanent effects on such variables; see, e.g., Jacobson, Vredin, and Warne (1998) for an *LR* test of such effects.

The cointegration rank results are presented in Table 14. We have chosen to use 2 lags in the VAR model, i.e. k = 2 in equation (1). With real stock prices added to the vector of endogenous variables, we find that the uncorrected trace tests suggest that we should select 2 cointegration relations at the 1 percent level, 3 relations at the 5 percent level, and 4 at the 10 percent level. In contrast, the corrected tests indicate that 1 cointegration relation is suitable at the 1 percent level, 2 relations at the 5 percent level and 3 at the 10 percent level. Thus, while the selection of an appropriate cointegration rank has become somewhat more problematic for the 7 variable model than for the 6 variable model, below we shall discuss the case of 2 cointegration relations. This facilitates comparisons to the 6 variable model, but is also the preferred number of such relations when we follow the results for the Bartlett corrected *LR* trace tests at the 5 percent level as well as for the bootstrapped empirical distributions of the uncorrected and Bartlett corrected trace tests.

Conditional on this choice of cointegration rank, it turns out that the results for the 7 variable model are similar to those for the 6 variable model. In particular, the hypothesis that the real stock market variable can be excluded from the cointegration space cannot be rejected at conventional levels of marginal significance. When we impose exactly the same restrictions on  $\beta$  as in equation (2), i.e. model  $\mathcal{M}_6$ , we obtain the following:

$$\hat{\beta}' X_{t} = \begin{bmatrix} 1 & 0 & -1.39 & 0.62 & 0 & -1.04 & 0.001 \\ & & & (0.03) & (0.30) & & (0.65) & (0.011) \\ 0 & -0.62 & 0 & 1 & 0.28 & -1.92 & -0.004 \\ & & & & (0.09) & (0.10) & (0.003) \end{bmatrix} \begin{bmatrix} m_{t} \\ \Delta p_{t} \\ i_{s,t} \\ i_{l,t} \\ i_{o,t} \\ p_{t}^{s} \end{bmatrix},$$
(6)

where  $p_t^s$  is the real stock price index. Hence, the estimated coefficients on the real stock market price index in these cointegration relations are approximately zero.<sup>34</sup>

In the 6 variable model we settled for 3 alternative sets of restrictions on  $\beta$ , namely models  $\mathcal{M}_9$ ,  $\mathcal{M}_{11}$  and  $\mathcal{M}_{12}$ . Given the lack of importance of the real stock price in the present case, it is not surprising that the same type of restrictions emerge here as well. Moreover, the estimated  $\beta$  parameters are only marginally affected. For example, what would correspond to model  $\mathcal{M}_9$  yields a *LR* test value of 3.45, which when compared with the  $\chi^2(6)$  distribution has a *p*-value of 75 percent.

The parameter constancy results for unrestricted  $\beta$  are somewhat more troublesome than for the 6 variable model. Based on the score version and fixing  $\Phi$  and  $\Gamma_1$  at their full sample estimates, the supremum test is 3.78 while the mean test is 1.91. If we compare these with the limiting distributions we obtain *p*-values of 16 and 8 percent, respectively. The bootstraps again suggest that the tests are under-sized and the empirical *p*-values are 5 percent for both tests. If we instead update  $\Phi$  and  $\Gamma_1$ , the supremum test is 4.46 while the mean test is 2.39. From both the asymptotic and the bootstrapped empirical distributions these values correspond to roughly 6 and 3 percent respectively.

<sup>&</sup>lt;sup>34</sup> The *LR* test for the 2 restrictions on  $\beta$  in equation (6) is 1.48, with a *p*-value of 48 percent. Conditional on these cointegration relations we also find that the *LR* statistic of two zero restrictions on the coefficients on the real stock price index is 0.52, with a *p*-value of 77 percent.

Turning to restricted cointegration spaces, the situation is similar to the unrestricted space. For example, the model with the  $\mathcal{M}_6$  restrictions, i.e., the restrictions on  $\hat{\beta}$  in equation (6) plus 2 zero restrictions on  $p_t^s$ , provides us with a supremum (mean) test of 2.43 (1.29). When these Nyblom statistics for restricted  $\beta$  are bootstrapped, the empirical *p*-values are 10 and 6 percent, respectively. Hence, introducing real stock prices to the system seems to provide somewhat stronger evidence of parameter non-constancy for the cointegration space. Regarding the constancy of the  $\Phi$ ,  $\Gamma_1$ , and  $\alpha$  parameters we obtain similar results to those for the 6 variable system, i.e., we cannot reject constancy of these parameters in any equation when inference is based on bootstrapping.

The  $\alpha$  parameters also obey the same sets of restrictions as in the 6 variable models. Moreover, in the real stock price equation we find that the hypothesis of the sum of the  $\alpha$  parameters being equal to 0 is supported by the data, i.e. the same type of restriction as in the own rate equation. If we test the hypothesis that the real stock price index is weakly exogenous for ( $\alpha$ ,  $\beta$ ), however, the Wald test strongly rejects the null. Hence, real stock prices seem to contain unique information about the cointegration relations. Still, neither the estimated  $\beta$  nor the estimated  $\alpha$  parameters in the other equations are much affected by the inclusion of the real stock price index.

From all these results it seems natural to ask: Do real stock prices really matter? If we examine the Granger non-causality tests (cf. Table 15, where we report the results when  $\beta$  has been restricted as in model  $\mathcal{M}_9$ ) it can be seen that real stock prices primarily contain unique information for predicting the next period change in the short-term rate. In addition, they may be useful for predicting changes in real money, in inflation, in the long rate, and in the own rate.<sup>35</sup> Hence, from a forecasting perspective it makes sense to include real stock prices in the data set. Let us therefore consider the case when the impact of stock markets on money demand is represented by stock market volatility.

#### 6.2. Does Volatility Matter?

While it seems plausible that stock market volatility can be influenced by the behavior of at least some of the variables in the 6 variable model, we shall only consider the case when volatility is weakly exogenous for the parameters of interest. The time series observations on the volatility variable are displayed in Figure 9. It is interesting to note that measured volatility has been fairly stable until 1998, with a few peaks in the late 80s and early 90s. From 1998, however, it seems to drift upward. In what follows we shall assume that the volatility series is stationary since it seems plausible given its behavior and, moreover, the underlying estimation procedure relies on such an assumption.<sup>36</sup>

When weakly exogenous stationary variables are added to the VEC model in equation (1), it has been shown by Rahbek and Mosconi (1999) that the trace test for the cointegration rank depends on nuisance parameters.<sup>37</sup> The solution suggested by Rahbek and Mosconi is to accumulate the stationary regressors and allow these variables to enter the cointegration relations. Once the rank has been determined, it is possible to estimate the cointegration space under the restrictions that the accumulated stationary variables have zero coefficients.

<sup>&</sup>lt;sup>35</sup> The real stock price index is, in fact, the only variable in the 7 variable model which seems to Granger cause the long rate.

 $<sup>^{36}</sup>$  The volatility series is the conditional standard deviation of an estimated leverage GARCH model for weekly data. The leverage term (an interactive dummy variable for negative stock price changes) is significant and positive, meaning that volatility tends to be higher when stock prices decline.

<sup>&</sup>lt;sup>37</sup> The nuisance parameters are characterized as canonical correlations between the common trends, which include the accumulated stationary weakly exogenous variables, and the accumulated residuals.

The limiting distribution of the cointegration rank test in the case with I(1) weakly exogenous variables has been treated by Harbo, Johansen, Nielsen, and Rahbek (1998). One important result in their article is that a model with an unrestricted constant and a zero restriction on a linear trend leads to a nuisance parameter in the limiting distribution. If we relax the restriction on the linear trend such that its coefficients span the same space as  $\alpha$ ,<sup>38</sup> then the nuisance parameter drops out.<sup>39</sup>

From this discussion it follows that a natural model to consider when volatility is viewed as a stationary weakly exogenous variable is the following:

$$\Delta X_{t} = \Phi_{0} + \alpha \Phi_{1} t + \sum_{i=1}^{k-1} \Gamma_{i} \Delta X_{t-i} + \alpha \beta' X_{t-1} + \alpha \beta'_{\nu} \sum_{i=1}^{t-1} \nu_{i} + \sum_{i=0}^{k-1} \Psi_{i} \nu_{t-i} + \epsilon_{t}, \quad t = 1, \dots, T,$$
(7)

where  $v_t$  is the volatility measure. Once the rank has been determined, we can restrict  $\Phi_1$  and  $\beta_v$  to be zero, and then conduct the analysis of the influence of the stationary volatility variable in the restricted VEC model.

The cointegration rank tests, along with the 80, 90, 95, and 97.5 percent quantiles for the trace test are presented in Table 16.<sup>40</sup> If we base our inference on the *LR* trace tests, then at the 5 percent level we would pick 3 cointegration relations, and 4 cointegration relations at the 20 percent level. We have already discussed, in Section 3.1, that the trace test is typically over-sized in small samples. To minimize this size distortion problem we would like to Bartlett correct the trace tests for the model in equation (7) as well. Since this problem has, to our knowledge, not been addressed in the literature yet, we will instead make an approximation based on the correction factors found above. The smallest correction factors we obtained for the 6 variable model was 1.15 and the biggest around 1.5. Typically, the correction factors took on values around 1.15 to 1.2. In Table 16 we therefore list the values of the trace test when they are divided by 1.15 and 1.2; denoted by  $LR_{tr}^{1.15}$  and  $LR_{tr}^{1.2}$ , respectively. When we assume that the Bartlett correction factor is 1.15 for all ranks, the corrected trace tests suggest 3 cointegration relations at the 5 percent level, and 2 relations at the 2.5 percent level. For the case when we correct all the trace tests with 1.2, we instead obtain the results that there are 2 cointegration relations at the 5 percent level and 3 at the 10 percent level.

We have also bootstrapped the distributions for the uncorrected trace test for the models represented by equation (7), i.e., for the null hypotheses r = 0, 1, ..., 5. To construct pseudo-data we condition on the observed time series for volatility. As can be seen from Table 16 the empirical *p*-values agree best with asymptotic inference based on a Bartlett correction factor of 1.20. All in all this leads us to conclude that a choice of 2 cointegration relations is supported by the data.

Given 2 lags and 2 cointegration relations we next restrict the coefficients on the linear trend, i.e.  $\Phi_1$  is equation (7), to be zero. A *LR* test of these 2 restrictions yields a value of 28.28 and is thus strongly rejected by the data. Nevertheless, we exclude the linear trend from the model. Conditional on these choices we can now test if the volatility variable can be excluded from the

<sup>&</sup>lt;sup>38</sup> That is, there are no quadratic trends in the levels of the endogenous variables.

<sup>&</sup>lt;sup>39</sup> The nuisance parameter essentially includes  $\alpha_{\perp}$  and the coefficient on the highest order deterministic variable. If this product is zero, then the nuisance parameter is also zero. For the model where the linear trend is restricted to the cointegration space, i.e. a linear trend is the highest order deterministic variable, it follows that its coefficient spans the column space of  $\alpha$  and is therefore orthogonal to  $\alpha_{\perp}$ . If the coefficient on the linear trend in the VEC model is restricted to zero, but the coefficient on the constant is unrestricted, then there is no guarantee that the nuisance parameter is zero. This follows from the fact that the constant is now the highest order deterministic variable, and its coefficient need not be orthogonal to  $\alpha_{\perp}$ . If it were, then again the limiting distribution of the cointegration rank test would be free of nuisance parameters.

<sup>&</sup>lt;sup>40</sup> The critical values are taken from Harbo et al. (1998, Table 2).

model. Such a test involves 14 restrictions on the parameter space and the *LR* statistic is equal to 16.63. When compared with the  $\chi^2(14)$  distribution we find that the asymptotic *p*-value of the test is roughly 28 percent. If we instead consider an alternative hypothesis where  $\beta_v = 0$  and test the null that  $\Psi_0 = \Psi_1 = 0$ , the *LR* test is equal to 10.82, corresponding to an asymptotic *p*-value of 54 percent. Hence, the data does not object to the volatility variable being dropped from the model altogether.

#### 7. SUMMARY AND CONCLUSIONS

The main purpose of this paper is to study if the demand for euro area M3 is subject to parameter non-constancies. In contrast to most previous studies of euro area money demand, we apply formal tests rather than informal diagnostics. In addition to having the correct size (at least asymptotically), the tests do not require trimming of the sample, thus making it feasible to examine the constancy issue using as much information as possible. As a complement we have also performed small scale bootstrap simulations of the constancy tests, as well as some other statistics of interest, as a means for obtaining better small sample approximations of the unknown distributions.

The constancy analysis is divided into three sub-sets of parameters. First, we look at the nonzero eigenvalues from the cointegration analysis. As shown by Hansen and Johansen (1999), these eigenvalues are asymptotically Gaussian, meaning that we can use the fluctuation test (of Ploberger et al., 1989) to investigate if these parameters are constant over the experimentation period or not. Should these parameters be non-constant, the cointegration space, the coefficients on the cointegration relations or the covariance matrix for the residuals must vary over time. Second, we test for the constancy of the cointegration space using the Nyblom (1989) statistics studied by Hansen and Johansen. Finally, conditional on the cointegration relations, we apply the Ploberger et al. fluctuation tests directly to the parameters of the individual equations of the vector error correction model.

In addition, the paper addresses a number of issues concerning euro area money demand, specifically the need for a consistent aggregation methodology for scale variables and interest rates and the measurement of the own rate of return on M3. The primary aggregation method used in this paper is based on the irrevocably fixed exchange rates for the scale variables and M3 weights for the interest rates. Hence, we are not using a consistent aggregation technique for scale variables and interest rates. For this reason (as well as for others), we also study data aggregated using the 2001 GDP weights measured at PPP exchange rates for all variables, thus enabling us to compare the results from the primary euro area dataset with those obtained with a method using a consistent aggregation scheme for scale variables and interest rates. The own rate of return on M3 is constructed as a weighted average of national interest rate series for all components of M3 and for all euro area countries.

First and foremost, there is strong evidence favoring the hypothesis that there is a stable long-run relationship between real money and real GDP. The estimated coefficient on real GDP is in all cases greater than unity and is thus consistent with the findings of previous euro area money demand studies (cf. Brand et al., 2002). Moreover, the point estimate is somewhat larger for the irrevocably fixed exchange rate data (roughly 1.4) than for the GDP weights data (about 1.25). This difference is, however, not large and 95 percent confidence bands, constructed from bootstraps, overlap. Thus, the income elasticity estimates from the two data sets need not be different.

The Nyblom statistics suggested by Hansen and Johansen (1999) yield extreme values whenever the cointegration space is unrestricted. As an alternative we suggested Nyblom statistics which do

not rely on a first order Taylor expansion of the score vector (as Hansen and Johansen do), but instead use the score directly. Such statistics are thus functions of the *LM* statistic, whereas the Hansen and Johansen versions are functions of an approximate *LM* statistic. When the constancy tests for the unrestricted cointegration space are calculated using the score form they no longer yield extreme values. Moreover, the empirical evidence based on both asymptotics and bootstraps generally suggests that the cointegration space is not subject to non-constancy.

Second, the interest rate semi-elasticities of long-run money demand are imprecisely estimated using classical maximum likelihood. This has also been pointed out by Fagan and Henry (1998) and Dedola et al.  $(2001)^{41}$  and it would be interesting to discover the extent to which this depends on the use of a classical rather than a Bayesian estimator.<sup>42</sup> For the primary dataset, these elasticities can range from at least -2.2 to 0.7 (with 95 percent asymptotic confidence) for the short-term rate and from -1.2 to 3.7 for the own rate. Standard *LR* tests of such hypotheses result in high p-values and, not surprisingly, the long-run relations are virtually unaffected.

Third, once the coefficient matrix of the cointegration relations in the vector error correction system is fixed, the remaining parameters of the money demand system are typically also found to be constant when inference relies on empirical bootstrapped distributions. It has been suggested by Kontolemis (2002) that the stock market index should be included in the money demand system for the non-cointegration coefficients to be constant over time. Unlike Kontolemis, we include the own rate in the system and do not find that the non-cointegration parameters are non-constant. If we drop the own rate from e.g. the primary dataset, we find that one of the cointegration relations "disappears".<sup>43</sup> Parameter constancy in other respects is, however, preserved. Whereas our methodology is based on formal tests applied to recursively estimated parameters for a large proportion of the sample, Kontolemis uses informal, period-by-period Chow tests for the short sub-sample 1999–2001. Hence, the non-constancy conclusion by Kontolemis may very well be the result of not taking the overall significance level for the (correlated) Chow tests into account.

Once we add a measure of real euro area stock prices as an endogenous variable to our basic six-variable system, we find that stock prices do not matter for the selection of cointegration rank or for the estimated parameters of the cointegration space. It is only when we test if stock prices are Granger non-causal for any of the six other variables that we find a role for stock prices in the money demand system. In particular, real stock prices seem to help predict the next period's change in the short-term interest rate, but may also be useful for predicting the other interest rates as well as real money and inflation changes.

As a second check on the relevance of stock market developments for the stability of the money demand system, we included the estimated volatility for the euro area stock price index as a weakly exogenous stationary regressor. In this case, the coefficients on volatility are not significantly different from zero. One explanation for this result is that the signal contained in our stock market volatility measure about the effects of financial crises, structural change and increased uncertainty (e.g. during the second half of 2001) may be too weak in the selected sample. The effects of stock market variables of money demand are important issues that warrant further research.

Finally, when we shorten the sample by excluding the early years 1980–1982, there is evidence of only one cointegration relation. Nevertheless, the main conclusions regarding the stability of

 $<sup>^{41}</sup>$  For example, Dedola et al. (2001) note that national differences in interest rate elasticity may explain the difficulty of accurately estimating the euro area elasticity.

 $<sup>^{\</sup>rm 42}$  This issue, as well as that of the uncertainty of income elasticity, are left for future research.

<sup>&</sup>lt;sup>43</sup> The results have not been reported in the paper, but are available on request.

the parameters in the long-run money demand equation and the difficulty in precisely estimating the interest rate semi-elasticities remain true for this sample. Additionally, inference based on bootstrapping suggests that the non-cointegration parameters of the system are not subject to non-constancy. The own rate of return of euro area M3 used in this paper is constructed as a weighted average of the national own rates of return of M3, where the latter are calculated as a weighted average of the rates of return of the different instruments included in M3. More formally,

$$i_{o} = \sum_{c} w_{c} i_{c}$$

$$= \sum_{c} w_{c} \left( \sum_{k} w_{c,k} i_{k,c} \right)$$

$$= \sum_{c} w_{c} \left( \sum_{k} \frac{M_{c,k}}{M_{c}} i_{k,c} \right),$$
(A.1)

where  $i_o$  denotes the own rate of return of euro area M3,  $w_c$  the weight of country c in the euro area interest rate,  $i_c$  the national own rate of return of M3 for country c,  $w_{c,k}$  the share of instrument kin M3 for country c ( $M_c$ ),  $M_{c,k}$  instrument k included in M3 for country c, and  $i_{k,c}$  the rate of return of instrument k in country c.

The instruments included in M3 have been grouped as follows: currency in circulation (*cc*); overnight deposits (*of*); deposits redeemable at notice up to three months or short-term savings deposits (*sd*); deposits with an agreed maturity of up to two years or short-term time deposits (*td*); and marketable instruments (*mi*). The rate of return of currency in circulation is assumed to be zero. For the rates of return of the various categories of deposits use has been made of retail bank deposit rates. Finally, it is assumed that the rate of return of marketable instruments can be approximated by the 3-month market interest rate (*i<sub>s</sub>*).

For each country, an own rate of return of M3 is then calculated according to the following equation:

$$i_{c} = \frac{M_{cc,c}}{M_{c}} \cdot 0 + \frac{M_{od,c}}{M_{c}} \cdot i_{od,c} + \frac{M_{sd,c}}{M_{c}} \cdot i_{sd,c} + \frac{M_{td,c}}{M_{c}} \cdot i_{td,c} + \frac{M_{mi,c}}{M_{c}} \cdot i_{s,c}.$$
 (A.2)

Data for the national interest rate series are taken from various sources, namely, ECB, BIS, IMF, and OECD; see Table A.1. In most cases data from several sources had to be combined to obtain a series for the period from January 1980 onwards. Quarterly data refer to averages of monthly data.

To transform the national interest rate series into a national own rate of return of M3 they are weighted by the share of each instrument in M3, according to equation (A.2). To this end, series for the 'notional stocks' of each of the instruments included in M3 were constructed. A time series of notional stocks corrects the series of outstanding amounts for reclassification, foreign exchange revaluations, and other revaluations to give a better indication of the actual transactions that have taken place.<sup>44</sup> When there are major differences between the changes in the end-of-month stocks and the changes in the notional stocks, using the latter results in much smoother series for the own rates of return.

Finally, to combine the national own rates of return of M3 into a series for the own rate of return of euro area M3, two different weighting schemes have been used, in line with the procedure for the other interest rate variables; cf. Section 2.1.2.

<sup>&</sup>lt;sup>44</sup> For a detailed discussion of the statistical procedure, see the box on "The Derivation and the Use Of Flow Data in Monetary Statistics", in the February 2001 issue of the ECB Monthly Bulletin.

TABLE 1: Specification tests and asymptotic p-values for models with 2 lags.

|        |                 |        |                 |              | )                 |   |                         |                 |                 |             |                 |
|--------|-----------------|--------|-----------------|--------------|-------------------|---|-------------------------|-----------------|-----------------|-------------|-----------------|
| LM(36) | <i>p</i> -value | LM(72) | <i>p</i> -value | W(36)        | <i>p</i> -value   | LR(36)  | <i>p</i> -value         | LR(36)          | <i>p</i> -value | $E_{6}(12)$ | <i>p</i> -value |
| s.     | 2 vs. 3 lags    | 2 vs   | 2 vs. 4 lags    | l vs.        | l vs. 2 lags      | lag l   | 5 1                     | lag             | lag 4           |             |                 |
| 34.92  | 0.52            | 85.11  | 0.14            | 174.31       | 0.00              | 8.85  | 1.00                    | 42.38           | 0.21            | 20.93       | 0.05            |
|        |                 |        | (B) (           | Cointegratic | n Rank = 2        | (B) Cointegration Rank = 2, Unrestricted Model $\mathcal{M}_0$  | ted Model J             | $\mathcal{M}_0$ |                 |             |                 |
| 33.47  | 0.59            | 72.07  | 0.48            | 254.84       | 0.00              | 8.48  | 1.00                    | 50.82           | 0.05            | 14.92       | 0.25            |
|        |                 |        | (C)             | Cointegrat   | <i>ion Rank =</i> | (C) Cointegration Rank = 2, Restricted Model $\mathcal{M}_6$    | ed Model M              | _9.             |                 |             |                 |
| 35.90  | 0.47            | 77.98  | 0.29            | 259.89       | 0.00              | 8.67  | 1.00                    | 49.03           | 0.07            | 14.34       | 0.28            |
|        |                 |        | (D)             | Cointegrat   | <i>ion Rank =</i> | (D) Cointegration Rank = 2, Restricted Model $\mathcal{M}_9$    | bd Model M              | _6;             |                 |             |                 |
| 35.89  | 0.47            | 77.57  | 0.31            | 249.00       | 0.00              | 8.03  | 1.00                    | 48.65           | 0.08            | 15.07       | 0.24            |
|        |                 |        | (E)             | Cointegrati  | on Rank = .       | (E) Cointegration Rank = 2, Restricted Model $\mathcal{M}_{11}$ | d Model $\mathcal{M}_1$ |                 |                 |             |                 |
| 36.17  | 0.46            | 77.09  | 0.32            | 255.70       | 0.00              | 8.88  | 1.00                    | 49.03           | 0.07            | 14.00       | 0.30            |
|        |                 |        | (F)             | Cointegrati  | on Rank = 2       | (F) Cointegration Rank = 2, Restricted Model $\mathcal{M}_{12}$ | d Model $\mathcal{M}_1$ | 12              |                 |             |                 |
| 35.77  | 0.48            | 78.36  | 0.28            | 259.46       | 0.00              | 8.20  | 1.00                    | 48.94           | 0.07            | 14.86       | 0.25            |

(A) Cointegration Rank = 6

TABLE 2: Cointegration rank tests with asymptotic and bootstrapped *p*-values for models with 2lags over the sample 1980:Q4-2001:Q4.

| Rank | Eigenvalue | LR <sub>tr</sub> | asymp<br><i>p</i> -value | boot<br><i>p</i> -value | BF    | $LR_{\rm tr}^{\rm c}$ | asymp<br><i>p</i> -value | boot<br><i>p-</i> value |
|------|------------|------------------|--------------------------|-------------------------|-------|-----------------------|--------------------------|-------------------------|
| 0    | 0.47       | 140.46           | 0.00                     | 0.00                    | 1.176 | 119.48                | 0.00                     | 0.00                    |
| 1    | 0.34       | 86.92            | 0.00                     | 0.02                    | 1.178 | 73.81                 | 0.02                     | 0.02                    |
| 2    | 0.24       | 50.85            | 0.03                     | 0.11                    | 1.183 | 42.98                 | 0.13                     | 0.12                    |
| 3    | 0.23       | 27.00            | 0.10                     | 0.26                    | 1.225 | 22.03                 | 0.30                     | 0.28                    |
| 4    | 0.04       | 4.33             | 0.88                     | 0.94                    | 1.147 | 3.77                  | 0.92                     | 0.94                    |
| 5    | 0.00       | 0.35             | 0.56                     | 0.60                    | 1.508 | 0.23                  | 0.63                     | 0.62                    |

TABLE 3: Tests of various hypotheses about the cointegration space for models with 2 lags and 2 cointegration relations.

| Model              | $oldsymbol{eta}_i' X_t$ is I(0)   | $\beta_{13}$ | $eta_{14}$ | $eta_{16}$ | LR    | df | asymp<br><i>p</i> -value | boot<br>BF | boot<br><i>p</i> -value |
|--------------------|---|--------------|------------|------------|-------|----|--------------------------|------------|-------------------------|
| $\mathcal{M}_1$    | $i_{s,t} - i_{l,t}$   |              |            |            | 16.33 | 4  | 0.00                     | 2.35       | 0.12                    |
| $\mathcal{M}_2$    | $i_{l,t} - i_{o,t}$   |              |            |            | 26.11 | 4  | 0.00                     | 3.50       | 0.05                    |
| $\mathcal{M}_3$    | $i_{s,t} - i_{o,t}$   |              |            |            | 21.60 | 4  | 0.00                     | 3.12       | 0.11                    |
| $\mathcal{M}_4$    | $i_{l,t} - \Delta p_t$  |              |            |            | 21.81 | 4  | 0.00                     | 2.80       | 0.06                    |
| $\mathcal{M}_5$    | $\Delta p_t$  |              |            |            | 23.79 | 4  | 0.00                     | 3.25       | 0.08                    |
| $\mathcal{M}_6$    | $m_t+\beta_{13}y_t+\beta_{14}i_{s,t}+\beta_{16}i_{o,t}$                     | -1.38        | 0.81       | -1.31      | 2.12  | 2  | 0.35                     | 1.70       | 0.56                    |
|                    | $i_{s,t} + \beta_{22}\Delta p_t + \beta_{25}i_{l,t} + \beta_{26}i_{o,t}$    |              |            |            |       |    |                          |            |                         |
| $\mathcal{M}_7$    | $m_t + \beta_{13} y_t + \beta_{14} i_{s,t}$                                 | -1.38        | 0.11       | 0          | 3.46  | 3  | 0.33                     | 1.78       | 0.58                    |
|                    | $i_{s,t} + \beta_{22} \Delta p_t + \beta_{25} i_{l,t} + \beta_{26} i_{o,t}$ |              |            |            |       |    |                          |            |                         |
| $\mathcal{M}_8$    | $m_t + \beta_{13} y_t + \beta_{16} i_{o,t}$                                 | -1.39        | 0          | 0.07       | 3.71  | 3  | 0.29                     | 1.75       | 0.56                    |
|                    | $i_{s,t} + \beta_{22} \Delta p_t + \beta_{25} i_{l,t} + \beta_{26} i_{o,t}$ |              |            |            |       |    |                          |            |                         |
| $\mathcal{M}_9$    | $m_t + \beta_{13} y_t$  | -1.40        | 0          | 0          | 3.75  | 4  | 0.44                     | 1.73       | 0.70                    |
|                    | $i_{s,t} + \beta_{22} \Delta p_t + \beta_{25} i_{l,t} + \beta_{26} i_{o,t}$ |              |            |            |       |    |                          |            |                         |
| $\mathcal{M}_{10}$ | $m_t+\beta_{13}y_t+\beta_{14}(i_{s,t}-i_{o,t})$                             | -1.37        | 0.36       | -0.36      | 2.96  | 3  | 0.40                     | 2.74       | 0.66                    |
|                    | $i_{s,t} + \beta_{22} \Delta p_t + \beta_{25} i_{l,t} + \beta_{26} i_{o,t}$ |              |            |            |       |    |                          |            |                         |
| $\mathcal{M}_{11}$ | $m_t + \beta_{13} y_t + 0.8 i_{s,t} - 1.3 i_{o,t}$                          | -1.38        | 0.8        | -1.3       | 2.12  | 4  | 0.71                     | 1.71       | 0.87                    |
|                    | $i_{s,t} + \beta_{22} \Delta p_t + \beta_{25} i_{l,t} + \beta_{26} i_{o,t}$ |              |            |            |       |    |                          |            |                         |
| $\mathcal{M}_{12}$ | $m_t + \beta_{13} y_t + 0.4 (i_{s,t} - i_{o,t})$                            | -1.37        | 0.4        | -0.4       | 2.97  | 4  | 0.56                     | 1.75       | 0.80                    |
|                    | $i_{s,t}+\beta_{22}\Delta p_t+\beta_{25}i_{l,t}+\beta_{26}i_{o,t}$          |              |            |            |       |    |                          |            |                         |

TABLE 4: Fluctuation tests of the constancy of the non-zero eigenvalues for the unrestricted model $\mathcal{M}_0$  with 2 lags and 2 cointegration relations over the period 1987:Q2-2001:Q4.

| Eigenvalue | $\sup_{t\in\mathbb{T}}	au_{t T}(\lambda_i)$                                    | asymptotic <i>p</i> -value                            | bootstrap <i>p</i> -value |  |
|------------|--|---|---------------------------|--|
| 1          | 2.67   | 0.00  | 0.00                      |  |
| 2          | 1.18   | 0.13  | 0.33                      |  |
|            |  |   |                           |  |
|            | $\sup_{t\in\mathbb{T}} \tau_{t T}(\sum_{i=1}^2 \log(\lambda_i/1 - \lambda_i))$ | asymptotic <i>p</i> -value                            | bootstrap <i>p</i> -value |  |
|            | 2.83   | 0.00  | 0.01                      |  |
|            |  |   |                           |  |
|            | (B) Updating of $\dot{\diamond}$   | $\hat{\mathbf{b}}^{(t)}$ and $\hat{\mathbf{L}}^{(t)}$ |                           |  |
|            | (b) Optiming of s  |   |                           |  |
| Eigenvalue | $\sup_{t\in\mathbb{T}}	au_{t\mid T}(\lambda_i)$                                | asymptotic <i>p</i> -value                            | bootstrap <i>p</i> -value |  |
| 1          | 2.73   | 0.00  | 0.02                      |  |
| 2          | 2.46   | 0.00  | 0.06                      |  |
|            |  |   |                           |  |
|            | $\sup_{t\in\mathbb{T}} \tau_{t T}(\sum_{i=1}^2 \log(\lambda_i/1 - \lambda_i))$ | asymptotic <i>p</i> -value                            | bootstrap <i>p</i> -value |  |
|            | 3.70   | 0.00  | 0.11                      |  |

(A) Conditional on  $\hat{\Phi}^{(T)}$  and  $\hat{\Gamma}_1^{(T)}$ 

NOTES: The experiment period is given by  $\mathbb{T} = \{1987 : Q2, \dots, 2001 : Q4\}$ . The fluctuation test converges weakly to a Brownian bridge; see Ploberger et al. (1989) for details.

TABLE 5: Nyblom tests for the constancy of  $\beta$  for the unrestricted model  $\mathcal{M}_0$  with 2 lags and 2 cointegration relations over the period 1987:Q2–2001:Q4.

| i  | $\sup_{t\in\mathbb{T}}Q_T^{(t)}(i)$ | asymp<br><i>p</i> -value | boot<br><i>p</i> -value | $\operatorname{mean}_{t\in\mathbb{T}}Q_T^{(t)}(i)$ | asymp<br><i>p</i> -value | boot<br><i>p</i> -value |
|----|-------------------------------------|--------------------------|-------------------------|--|--------------------------|-------------------------|
| HJ | 4240.36                             | 0.00                     | -                       | 95.17  | 0.00                     | -                       |
| S  | 2.86                                | 0.30                     | 0.15                    | 1.05   | 0.41                     | 0.32                    |

(A) Conditional on  $\hat{\Phi}^{(T)}$  and  $\hat{\Gamma}_1^{(T)}$ 

(B) Updating of  $\hat{\Phi}^{(t)}$  and  $\hat{\Gamma}_{1}^{(t)}$ 

| i  | $\sup_{t\in\mathbb{T}}Q_T^{(t)}(i)$ | asymp<br><i>p</i> -value | boot<br><i>p</i> -value | $\operatorname{mean}_{t\in\mathbb{T}}Q_T^{(t)}(i)$ | asymp<br><i>p</i> -value | boot<br><i>p</i> -value |
|----|-------------------------------------|--------------------------|-------------------------|--|--------------------------|-------------------------|
| HJ | 22315.63                            | 0.00                     | -                       | 457.85   | 0.00                     | -                       |
| S  | 3.45                                | 0.14                     | 0.13                    | 1.50   | 0.14                     | 0.18                    |

| Equation       | ${\mathcal M}$  | 9       | $\mathcal{M}_{2}$ | ).1       | $\mathcal{M}_{9}$    | .2        |  |
|----------------|-----------------|---------|-------------------|-----------|----------------------|-----------|--|
|                |                 |         | LR(8)=9.          | 70 [0.29] | LR(7)=13.            | 93 [0.05] |  |
| т              | -0.118          | -0.088  | -0.075            | -0.075    | -0.073               | 0         |  |
|                | (0.034)         | (0.089) | (0.025)           | (0.025)   | (0.027)              |           |  |
| $\Delta p$     | 0.154           | 1.022   | 0                 | 0.993     | 0                    | 0.900     |  |
|                | (0.076)         | (0.201) |                   | (0.172)   |                      | (0.168)   |  |
| У              | 0.033           | 0.172   | 0                 | 0         | 0                    | 0         |  |
|                | (0.042)         | (0.110) |                   |           |                      |           |  |
| i <sub>s</sub> | -0.132          | -0.152  | -0.128            | -0.128    | -0.121               | 0         |  |
|                | (0.042)         | (0.112) | (0.025)           | (0.025)   | (0.036)              |           |  |
| il             | -0.049          | -0.040  | 0                 | 0         | 0                    | 0         |  |
|                | (0.033)         | (0.087) |                   |           |                      |           |  |
| i <sub>o</sub> | -0.059          | 0.050   | -0.055            | 0.055     | -0.054               | 0.088     |  |
|                | (0.012)         | (0.031) | (0.008)           | (0.008)   | (0.011)              | (0.018)   |  |
|                |                 |         |                   |           |                      |           |  |
| Equation       | $\mathcal{M}_1$ | .1      | $\mathcal{M}_1$   | 1.1       | $\mathcal{M}_{11.2}$ |           |  |
|                |                 |         | LR(8)=5.          | 78 [0.67] | LR(7) = 6.6          | 52 [0.47] |  |
| т              | -0.124          | -0.034  | -0.083            | -0.083    | -0.092               | 0         |  |
|                | (0.037)         | (0.095) | (0.026)           | (0.026)   | (0.028)              |           |  |
| $\Delta p$     | 0.111           | 0.994   | 0                 | 1.063     | 0                    | 0.951     |  |
|                | (0.086)         | (0.216) |                   | (0.178)   |                      | (0.173)   |  |

0

-0.154

(0.025)

0

-0.059

(0.009)

0

-0.154

(0.025)

0.059

(0.009)

0

0

-0.169

(0.027)

-0.062

(0.011)

0

0

0

0

0.102

(0.020)

TABLE 6: Tests of linear restrictions on  $\alpha$  for models  $\mathcal{M}_9$ ,  $\mathcal{M}_{11}$ , and  $\mathcal{M}_{12}$  over the period 1980:Q4-2001:Q4.

0.002

(0.046) -0.176

(0.046)

-0.034

(0.036)

-0.066

(0.013)

y

 $i_s$ 

 $i_l$ 

 $i_o$ 

0.176

(0.116)

-0.079

(0.115)

-0.021

(0.092) 0.080

(0.033)

| Equation       | $\mathcal{M}_1$ | 12      | $\mathcal{M}_{12}$ | 2.1       | $\mathcal{M}_{12}$ | 2.2       |
|----------------|-----------------|---------|--------------------|-----------|--------------------|-----------|
|                |                 |         | LR(8)=6.           | 51 [0.59] | LR(7) = 9.2        | 16 [0.24] |
| т              | -0.152          | -0.059  | -0.085             | -0.085    | -0.091             | 0         |
|                | (0.034)         | (0.090) | (0.024)            | (0.024)   | (0.027)            |           |
| $\Delta p$     | 0.108           | 1.008   | 0                  | 1.040     | 0                  | 0.921     |
|                | (0.079)         | (0.208) |                    | (0.173)   |                    | (0.169)   |
| у              | -0.003          | 0.174   | 0                  | 0         | 0                  | 0         |
|                | (0.043)         | (0.112) |                    |           |                    |           |
| i <sub>s</sub> | -0.159          | -0.116  | -0.142             | -0.142    | -0.144             | 0         |
|                | (0.042)         | (0.112) | (0.025)            | (0.025)   | (0.035)            |           |
| i <sub>l</sub> | -0.046          | -0.027  | 0                  | 0         | 0                  | 0         |
|                | (0.034)         | (0.089) |                    |           |                    |           |
| i <sub>o</sub> | -0.061          | 0.065   | -0.056             | 0.056     | -0.056             | 0.096     |
|                | (0.012)         | (0.032) | (0.009)            | (0.009)   | (0.011)            | (0.019)   |

TABLE 6: Continued.

TABLE 7: Linear combinations of the cointegration relations in the money, short rate, and the own rate equations for models  $\mathcal{M}_{9.s}$ ,  $\mathcal{M}_{11.s}$  and  $\mathcal{M}_{12.s}$ .

| Equation              | $\mathcal{M}_{9.1}$                                     | $\mathcal{M}_{9.2}$                                      |
|-----------------------|---|--|
| m                     | $m - 1.40y + i_s - 1.98i_o + 0.44i_l - 0.66\Delta p$    | <i>m</i> – 1.40 <i>y</i>                                 |
| <i>is</i>             | $m - 1.40y + i_s - 1.98i_o + 0.44i_l - 0.66\Delta p$    | m - 1.40y  |
| i <sub>o</sub>        | $m - 1.40y - i_s + 1.98i_o - 0.44i_l + 0.66\Delta p$    | $m - 1.4y - 1.6i_s + 3.17i_o - 0.7i_l + 1.06\Delta p$    |
|                       |   |  |
| Equation              | $\mathcal{M}_{11.1}$                                    | $\mathcal{M}_{11.2}$                                     |
| т                     | $m - 1.38y + 1.8i_s - 3.26i_o + 0.41i_l - 0.63\Delta p$ | $m - 1.38y + 0.8i_s - 1.3i_o$                            |
| <i>i</i> s            | $m - 1.38y + 1.8i_s - 3.26i_o + 0.41i_l - 0.63\Delta p$ | $m - 1.38y + 0.8i_s - 1.3i_o$                            |
| i <sub>o</sub>        | $m - 1.38y - 0.2i_s + 0.66i_o - 0.41i_l + 0.63\Delta p$ | $m - 1.38y - 0.85i_s + 1.93i_o - 0.68i_l + 1.04\Delta p$ |
|                       |   |  |
| Equation              | $\mathcal{M}_{12.1}$                                    | $\mathcal{M}_{12.2}$                                     |
| т                     | $m - 1.37y + 1.4i_s - 2.34i_o + 0.42i_l - 0.66\Delta p$ | $m - 1.37y + 0.4(i_s - i_o)$                             |
| i <sub>s</sub>        | $m - 1.37y + 1.4i_s - 2.34i_o + 0.42i_l - 0.66\Delta p$ | $m - 1.37y + 0.4(i_s - i_o)$                             |
| <i>i</i> <sub>o</sub> | $m - 1.37y - 0.6i_s + 1.54i_o - 0.42i_l + 0.66\Delta p$ | $m - 1.37y - 1.3i_s + 2.9i_o - 0.71i_i + 1.612\Delta p$  |

TABLE 8: Ploberger-Krämer-Kontrus fluctuation tests for the constancy of  $\Phi$ ,  $\Gamma_1$ ,  $\alpha$  for models with 2 lags and 2 cointegration relations over the period 1987:Q2-2001:Q4.

| Equation       | $\mathcal{M}_9$ | asymp<br><i>p-</i> value | boot<br><i>p-</i> value | $\mathcal{M}_{11}$ | asymp<br><i>p</i> -value | boot<br><i>p</i> -value | $\mathcal{M}_{12}$ | asymp<br><i>p</i> -value | boot<br><i>p-</i> value |
|----------------|-----------------|--------------------------|-------------------------|--------------------|--------------------------|-------------------------|--------------------|--------------------------|-------------------------|
| m              | 1.59            | 0.11                     | 0.49                    | 2.00               | 0.01                     | 0.22                    | 1.83               | 0.02                     | 0.29                    |
| $\Delta p$     | 1.76            | 0.04                     | 0.34                    | 1.63               | 0.08                     | 0.44                    | 1.62               | 0.09                     | 0.43                    |
| у              | 2.19            | 0.00                     | 0.14                    | 2.23               | 0.00                     | 0.13                    | 2.23               | 0.00                     | 0.13                    |
| <i>i</i> s     | 2.03            | 0.00                     | 0.18                    | 1.56               | 0.13                     | 0.51                    | 1.85               | 0.02                     | 0.26                    |
| <i>i</i> 1     | 1.84            | 0.02                     | 0.30                    | 1.32               | 0.43                     | 0.76                    | 1.47               | 0.22                     | 0.61                    |
| i <sub>o</sub> | 1.77            | 0.03                     | 0.32                    | 1.25               | 0.57                     | 0.81                    | 1.41               | 0.28                     | 0.65                    |

TABLE 9: Cointegration rank tests with asymptotic and bootstrapped p-values for the model with 2 lags over the sample 1983:Q3–2001:Q4.

| Rank | Eigenvalue | <i>LR</i> <sub>tr</sub> | asymp<br><i>p</i> -value | boot<br><i>p</i> -value | BF    | <i>LR</i> <sup>c</sup> <sub>tr</sub> | asymp<br><i>p</i> -value | boot<br><i>p</i> -value |
|------|------------|-------------------------|--------------------------|-------------------------|-------|--------------------------------------|--------------------------|-------------------------|
| 0    | 0.43       | 118.90                  | 0.00                     | 0.03                    | 1.191 | 99.82                                | 0.02                     | 0.02                    |
| 1    | 0.35       | 76.57                   | 0.01                     | 0.14                    | 1.207 | 63.42                                | 0.14                     | 0.14                    |
| 2    | 0.22       | 44.67                   | 0.10                     | 0.32                    | 1.189 | 37.56                                | 0.32                     | 0.34                    |
| 3    | 0.19       | 25.48                   | 0.14                     | 0.32                    | 1.354 | 18.81                                | 0.51                     | 0.42                    |
| 4    | 0.12       | 9.70                    | 0.30                     | 0.45                    | 1.144 | 8.49                                 | 0.42                     | 0.42                    |
| 5    | 0.00       | 0.07                    | 0.78                     | 0.84                    | 1.383 | 0.05                                 | 0.82                     | 0.87                    |

TABLE 10: Cointegration rank tests with asymptotic and bootstrapped p-values for the model with<br/>2 lags using the 2001 GDP weights at PPP exchange rates aggregated data over the sample<br/>1982:Q1-2001:Q4.

| Rank | Eigenvalue | <i>LR</i> <sub>tr</sub> | asymp<br><i>p</i> -value | boot<br><i>p</i> -value | BF    | $LR_{\rm tr}^{\rm c}$ | asymp<br><i>p</i> -value | boot<br><i>p</i> -value |
|------|------------|-------------------------|--------------------------|-------------------------|-------|-----------------------|--------------------------|-------------------------|
| 0    | 0.42       | 134.08                  | 0.00                     | 0.00                    | 1.175 | 114.11                | 0.00                     | 0.00                    |
| 1    | 0.34       | 91.12                   | 0.00                     | 0.02                    | 1.185 | 76.89                 | 0.01                     | 0.02                    |
| 2    | 0.28       | 58.22                   | 0.00                     | 0.02                    | 1.190 | 48.94                 | 0.04                     | 0.03                    |
| 3    | 0.23       | 31.58                   | 0.03                     | 0.10                    | 1.342 | 23.53                 | 0.22                     | 0.16                    |
| 4    | 0.12       | 10.69                   | 0.23                     | 0.42                    | 1.177 | 9.08                  | 0.36                     | 0.40                    |
| 5    | 0.00       | 0.06                    | 0.80                     | 0.84                    | 1.310 | 0.05                  | 0.82                     | 0.85                    |

TABLE 11: Nyblom tests for the constancy of  $\beta$  for models with 2 lags using the 2001 GDP weights at PPP exchange rates aggregated data over the period 1988:Q2-2001:Q4.

| r | i  | $\sup_{t\in\mathbb{T}}Q_T^{(t)}(i)$ | asymp<br>p-value | boot<br><i>p</i> -value | $\operatorname{mean}_{t\in\mathbb{T}}Q_T^{(t)}(i)$ | asymp<br>p-value | boot<br><i>p</i> -value |
|---|----|-------------------------------------|------------------|-------------------------|--|------------------|-------------------------|
| 2 | HJ | 1082.46                             | 0.00             | -                       | 357.53   | 0.00             | -                       |
|   | S  | 3.89                                | 0.08             | 0.01                    | 1.41   | 0.17             | 0.10                    |
| 3 | HJ | 987.31                              | 0.00             | _                       | 87.72  | 0.00             | _                       |
|   | S  | 3.23                                | 0.30             | 0.06                    | 1.15   | 0.53             | 0.27                    |

(A) Conditional on  $\hat{\Phi}^{(T)}$  and  $\hat{\Gamma}_1^{(T)}$ 

(B) Updating of  $\hat{\Phi}^{(t)}$  and  $\hat{\Gamma}_1^{(t)}$ 

| r | i  | $\sup_{t\in\mathbb{T}}Q_T^{(t)}(i)$ | asymp<br><i>p</i> -value | boot<br><i>p</i> -value | $\operatorname{mean}_{t\in\mathbb{T}}Q_T^{(t)}(i)$ | asymp<br><i>p</i> -value | boot<br><i>p</i> -value |
|---|----|-------------------------------------|--------------------------|-------------------------|--|--------------------------|-------------------------|
| 2 | HJ | 3854.80                             | 0.00                     | _                       | 568.66   | 0.00                     | -                       |
|   | S  | 4.09                                | 0.06                     | 0.04                    | 1.94   | 0.04                     | 0.04                    |
| 3 | HJ | 36425.92                            | 0.00                     | -                       | 802.27   | 0.00                     | -                       |
|   | S  | 4.24                                | 0.09                     | 0.04                    | 1.90   | 0.11                     | 0.07                    |

TABLE 12: Ploberger-Krämer-Kontrus fluctuation tests for the constancy of  $\Phi$ ,  $\Gamma_1$ ,  $\alpha$  for models with<br/>2 lags using the 2001 GDP weights at PPP exchange rates aggregated data over the period<br/>1988:Q2-2001:Q4.

|                |                                 | <i>r</i> = 2             |                         |                                 | <i>r</i> = 3             |                         |
|----------------|---------------------------------|--------------------------|-------------------------|---------------------------------|--------------------------|-------------------------|
| Equation       | $\mathcal{M}_{2,1}^{	ext{GDP}}$ | asymp<br><i>p-</i> value | boot<br><i>p-</i> value | $\mathcal{M}_{3,1}^{	ext{GDP}}$ | asymp<br><i>p-</i> value | boot<br><i>p</i> -value |
| m              | 1.44                            | 0.25                     | 0.66                    | 1.82                            | 0.03                     | 0.49                    |
| $\Delta p$     | 0.95                            | 0.97                     | 0.98                    | 1.63                            | 0.09                     | 0.61                    |
| У              | 1.69                            | 0.06                     | 0.41                    | 2.18                            | 0.00                     | 0.26                    |
| $i_s$          | 1.94                            | 0.01                     | 0.25                    | 2.02                            | 0.01                     | 0.30                    |
| $i_l$          | 1.64                            | 0.08                     | 0.49                    | 1.56                            | 0.14                     | 0.69                    |
| i <sub>o</sub> | 1.71                            | 0.05                     | 0.41                    | 1.88                            | 0.02                     | 0.42                    |

TABLE 13: Linear combinations of the cointegration relations in the money, short rate, and the own rate equations for models  $\mathcal{M}_{r,1.s}^{\text{GDP}}$ , r = 2, 3 and s = 1, 2.

| Equation       | $\mathcal{M}_{2,1.1}^{	ext{GDP}}$                     | $\mathcal{M}_{2,1.2}^{	ext{GDP}}$                        |
|----------------|---|--|
| m              | $m - 1.25y + 2i_s - 3.62i_o + i_l - 0.62\Delta p$     | $m - 1.25y + i_s - i_o$                                  |
| i <sub>s</sub> | $m - 1.25y + i_s - i_o$                               | $m-1.25y+i_s-i_o$  |
| i <sub>o</sub> | $m - 1.25y + 1.62i_o - i_l + 0.62\Delta p$            | $m - 1.25y + 4.76i_o - 1.32i_s - 2.32i_l + 1.44\Delta p$ |
|                |   |  |
| Equation       | $\mathcal{M}_{3,1.1}^{	ext{GDP}}$                     | $\mathcal{M}^{	ext{GDP}}_{3,1.2}$                        |
| m              | $m - 1.26y + 2i_s - 4.54i_o + 1.77i_l - 0.77\Delta p$ | $m - 1.26y + i_s - i_o$                                  |
| i <sub>s</sub> | $m - 1.30y + i_s - i_l$                               | $m-1.30y+i_s-i_l$  |
| i <sub>o</sub> | $m - 1.30y + 3.54i_o - 2.77i_l + 0.77\Delta p$        | $m - 1.30y + 3.93i_o - 0.11i_s - 2.96i_l + 0.85\Delta p$ |

TABLE 14: Cointegration rank tests with asymptotic and bootstrapped *p*-values for models with 2lags and including the real stock price index over the sample 1980:Q4-2001:Q4.

| Rank | Eigenvalue | <i>LR</i> tr | asymp<br><i>p</i> -value | boot<br><i>p</i> -value | BF    | $LR_{\rm tr}^{\rm c}$ | asymp<br><i>p</i> -value | boot<br><i>p-</i> value |
|------|------------|--------------|--------------------------|-------------------------|-------|-----------------------|--------------------------|-------------------------|
| 0    | 0.48       | 174.51       | 0.00                     | 0.00                    | 1.196 | 145.91                | 0.00                     | 0.00                    |
| 1    | 0.37       | 119.37       | 0.00                     | 0.02                    | 1.204 | 99.16                 | 0.03                     | 0.03                    |
| 2    | 0.33       | 80.23        | 0.01                     | 0.07                    | 1.204 | 66.61                 | 0.09                     | 0.08                    |
| 3    | 0.24       | 46.66        | 0.06                     | 0.26                    | 1.215 | 38.39                 | 0.28                     | 0.27                    |
| 4    | 0.20       | 23.81        | 0.21                     | 0.47                    | 1.186 | 20.07                 | 0.42                     | 0.49                    |
| 5    | 0.05       | 4.76         | 0.83                     | 0.94                    | 1.259 | 3.78                  | 0.92                     | 0.94                    |
| 6    | 0.00       | 0.15         | 0.69                     | 0.77                    | 1.486 | 0.10                  | 0.75                     | 0.78                    |

 TABLE 15: Granger non-causality tests for the 2 lag model with real stock prices and with 2 restricted cointegration relations.

| Hypothesis                 | W     | <i>p</i> -value | F    | <i>p</i> -value |
|----------------------------|-------|-----------------|------|-----------------|
| $p^s \Rightarrow m$        | 4.22  | 0.04            | 3.73 | 0.06            |
| $p^s \Rightarrow \Delta p$ | 4.09  | 0.04            | 3.61 | 0.06            |
| $p^s \neq y$               | 0.55  | 0.46            | 0.48 | 0.49            |
| $p^s \Rightarrow i_s$      | 10.52 | 0.00            | 9.28 | 0.00            |
| $p^s \Rightarrow i_l$      | 3.99  | 0.04            | 3.52 | 0.06            |
| $p^s \Rightarrow i_o$      | 4.46  | 0.03            | 3.94 | 0.05            |

TABLE 16: Cointegration rank tests with asymptotic critical values and bootstrapped p-values for2 lag models with volatility as a weakly exogenous variable over the sample 1980:Q4-2001:Q4.

| Rank | Eigenvalue | <i>LR</i> <sub>tr</sub> | $LR_{\mathrm{tr}}^{1.15}$ | $LR_{\rm tr}^{1.2}$ | $Q_{80}$ | $Q_{90}$ | $Q_{95}$ | Q <sub>97.5</sub> | bootstrap <i>p</i> -value |
|------|------------|-------------------------|---------------------------|---------------------|----------|----------|----------|-------------------|---------------------------|
| 0    | 0.54       | 205.18                  | 178.42                    | 170.98              | 116.0    | 123.0    | 127.0    | 132.0             | 0.00                      |
| 1    | 0.48       | 138.26                  | 120.23                    | 115.22              | 88.1     | 93.5     | 98.0     | 102.0             | 0.01                      |
| 2    | 0.33       | 83.38                   | 72.50                     | 69.48               | 63.0     | 67.9     | 71.7     | 75.2              | 0.11                      |
| 3    | 0.27       | 49.28                   | 42.85                     | 41.07               | 41.9     | 45.9     | 49.6     | 52.4              | 0.24                      |
| 4    | 0.16       | 22.78                   | 19.81                     | 18.98               | 24.7     | 27.8     | 30.5     | 33.3              | 0.52                      |
| 5    | 0.09       | 7.70                    | 6.70                      | 6.42                | 11.0     | 13.2     | 15.2     | 17.4              | 0.57                      |

| Country     |            | Overnight Deposits                  |            | Time Deposits   |             | Savings Deposits  | she        | short-term Market Interest Rates                     |
|-------------|------------|-------------------------------------|------------|---|-------------|---|------------|--|
| Belgium     | 80:1-89:12 | CP in 90:1                          | 80:1-89:12 | NRIR  | 80:1-89:12  | BIS (savings book deposits -<br>HPHA.BE.91)   | 80:1-98:12 | BIS (treasury certificate, 3-month -<br>HEPA.BE.01)  |
|             | 90:1-01:12 | CP                                  | 90:1-01:12 | G   | 90:1-01:12  | CP  | 99:1-01:12 | 3-month EURIBOR                                      |
| Germany     | 80:1-89:12 | CP in 90:1                          | 80:1-01:12 | ď   | 80:1-01:12  | Weighted average of BIS (savings<br>deposits at 3 months notice -<br>HPHA.DE.02) and CP (higher yield-<br>ing time deposits | 80:1-98:12 | BIS (money market rate, 3-month -<br>HEEA.DE.02)     |
| _           | 90:1-01:12 | CP                                  |            |   |             |   | 99:1-01:12 | 3-month EURIBOR                                      |
| Spain       | 80:1-01:12 | CP                                  | 80:1-89:12 | NRIR (deposits with agreed maturity between 1-2 years)  | 80:1-89:12  | BIS (savings deposits - HPHA.ES.01)   | 80:1-98:12 | BIS (monthly market rate, 3-month -<br>HEEA.ES.02)   |
|             |            |                                     | 90:1-01:12 | CP  | 90:1-01:12  | CP  | 99:1-01:12 | 3-month EURIBOR                                      |
| France      | 80:1-01:12 | zero                                | 80:1-01:12 | CP  | 80:1-89:12  | BIS (savings deposits - HPHA.FR.01)   | 80:1-98:12 | BIS (money market rate, 3-month -<br>HEEA.FR.92)     |
|             |            |                                     |            |   | 90:1-01:12  | CP  | 99:1-01:12 | 3-month EURIBOR                                      |
| Greece      | 01:1-01:12 | CP                                  | 01:1-01:12 | Ċ   | 01:1-01:12  | CP  | 01:1-01:12 | 3-month EURIBOR                                      |
| Ireland     | 80:1-01:12 | zero                                | 80:1-01:12 | same as short-term market rate                          | 80:1-01:12  | CP  | 80:1-98:12 | BIS (money market rate, 3-month -<br>HEEA.IE.02)     |
| _           |            |                                     |            |   |             |   | 99:1-01:12 | 3-month EURIBOR                                      |
| Italy       | 80:1-86:7  | BIS (bank deposits - HPHA.IT.98)    | 80:1-89:12 | BIS (demand deposits - HPBA.IT.96)                      | 80:1-84:6   | BIS (bank deposits - HPHA.IT.98)  | 80:1-90:1  | BIS (treasury bills, 3-month -<br>HEPA.IT.02)        |
|             | 86:8-89:1  | BIS (savings deposits - HPHA.IT.96) | 90:1-01:12 | CP  | 84:7-94:12  | BIS (average current/savings deposits -<br>HPHA.IT.96)  | 90:2-98:12 | BIS (money market rate, 3-month -<br>HEEA.IT.02)     |
| _           | 89:2-01:12 | CP                                  |            |   | 95:1-01:12  | CP  | 99:1-01:12 | 3-month EURIBOR                                      |
| Luxembourg  | 80:1-94:2  | CP in 94:3                          | 80:1-94:2  | regression using OECD annual series                     | 80:1-01:12  | same as time deposits   | 80:1-98:12 | OECD annual series of short-term in-<br>terest rates |
|             | 94:3-01:12 | CP                                  | 94:3-01:12 | CP  |             |   | 99:1-01:12 | 3-month EURIBOR                                      |
| Netherlands | 80:1-01:12 | CP                                  | 80:1-89:12 | NRIR (time deposits with a fixed term of 2 years)       | 80:1-01:12  | BIS (ordinary savings deposits -<br>HPHA.NL.01)   | 80:1-98:12 | BIS (money market rate, 3-month -<br>HEEA.NL.92)     |
| Ī           |            |                                     | 90:1-01:12 | CP  |             |   | 99:1-01:12 | 3-month EURIBOR                                      |
| Portugal    | 80:1-89:12 | CP in 90:1                          | 80:1-89:12 | IMF 60L (minimum time deposits rate)                    | 80:1-01:12  | same as time deposits   | 80:1-89:1  | OECD annual series of short-term in-<br>terest rates |
|             | 90:1-01:12 | Cb                                  | 90:1-01:12 | CP  |             |   | 89:2-98:12 | BIS (money market rate, 3-month -<br>HEEA.PT.32)     |
| _           |            |                                     |            |   |             |   | 99:1-01:12 | 3-month EURIBOR                                      |
| Austria     | 80:1-95:3  | CP in 95.4                          | 80:1-95:3  | German CP series of time deposits                       | 80:1-93:12  | BIS (savings deposits - HPHA.AT.92)   | 80:1-98:12 | BIS (money market rate, 3-month -<br>HEEA.AT.92)     |
|             | 95:4-01:12 | CP                                  | 95:4-01:12 | CP  | 94:1-00:10  | IMF 60L (deposit rate)  | 99:1-01:12 | 3-month EURIBOR                                      |
| Ī           |            |                                     |            |   | 00:11-01:12 | CP time deposits  |            |  |
| Finland     | 80:1-01:12 | C)                                  | 80:1-90:4  | BIS (bank deposits at 24 months notice<br>- HPHA.FI.93) | 80:1-90:12  | BIS (bank deposits at 24 months notice<br>- HPHA.FI.93)   | 80:1-86:12 | OECD annual series of short-term in-<br>terest rates |
|             |            |                                     | 90:5-01:12 | CP  | 91:1-01:12  | CP  | 87:1-98:12 | BIS (money market rate, 3-month -<br>HEEA.FI.92)     |
|             |            |                                     |            |   |             |   | 99:1-01:12 | 3-month EURIBOR                                      |

TABLE A.1: National monthly interest rate series used in the construction of the own rate of return of euro area M3.

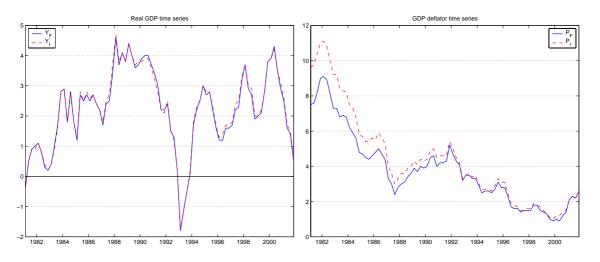
NOTE: CP rates are the national components of the aggregated euro area retail interest rate; NRIR are national retail interest rates. Both sets of variables are taken from the ECB database.



FIGURE 1: Euro area nominal M3 in annual percentage changes

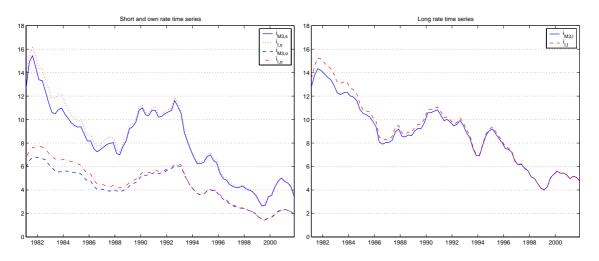
NOTE:  $M3_F$  denotes official euro area M3 aggregated according to the method of irrevocably fixed exchange rates, while  $M3_I$  denotes euro area M3 aggregated according to the index method.

## FIGURE 2: Euro area real GDP and the GDP deflator in annual percentage changes.



NOTE:  $Y_F$  denotes euro area real GDP aggregated according to the method of irrevocably fixed exchange rates, while  $Y_I$  denotes euro area real GDP aggregated according to the index method.

NOTE:  $P_F$  denotes the euro area GDP deflator derived according to the method of irrevocably fixed exchange rates, while  $P_I$  denotes the euro area GDP deflator derived according to the index method.



## FIGURE 3: Euro area short and long-term interest rates in percentages per annum.

NOTE:  $i_{M3,s,t}$  denotes the euro area three-month (short-term) market interest rate using time varying M3 weights,  $i_{I,s,t}$  denotes the euro area three-month market interest rate using constant 2001 GDP weights. Similarly,  $i_{z,o,t}$  for z = M3, I are the (average) own rates of return of M3.

NOTE:  $i_{M3,l,t}$  denotes the euro area ten-year (long-term) government bond yield using time varying M3 weights,  $i_{l,l,t}$  denotes the euro area ten-year government bond yield using constant 2001 GDP weights.

FIGURE 4: Asymptotic and bootstrapped density functions for the Bartlett corrected trace tests along with 80, 90, 95, 98, and 99 percent quantiles from the empirical bootstrap distributions.

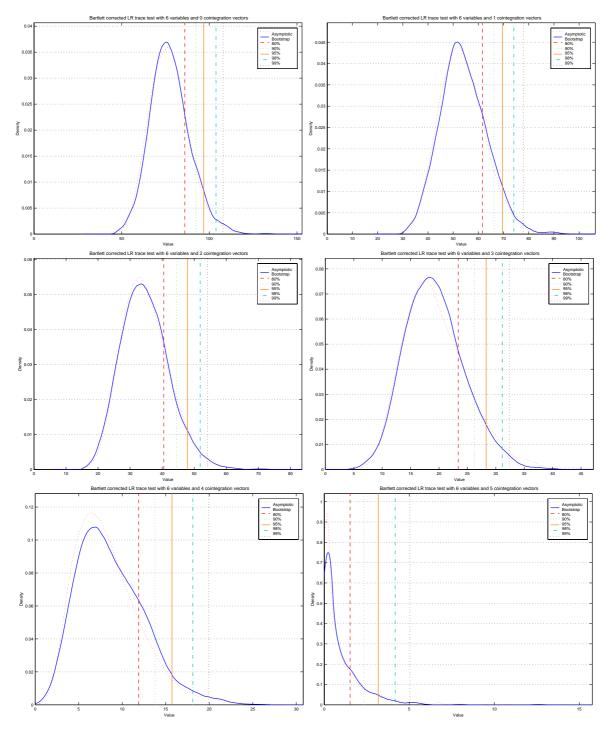


FIGURE 5: Asymptotic and bootstrapped density functions for the *LR* tests of restrictions on  $\beta$  in models  $\mathcal{M}_1$ - $\mathcal{M}_{12}$  with estimated Bartlett corrected empirical distributions along with 80, 90, 95, 98, and 99 percent quantiles from the empirical bootstrap distributions.

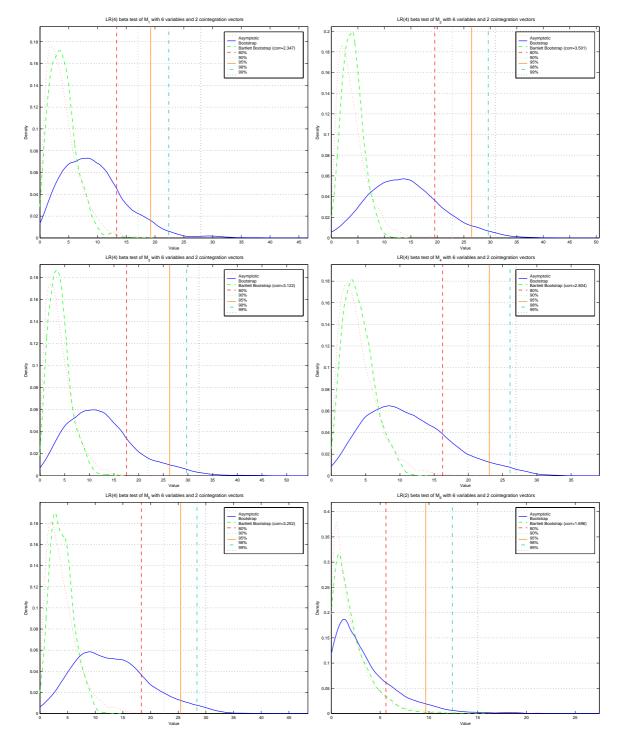
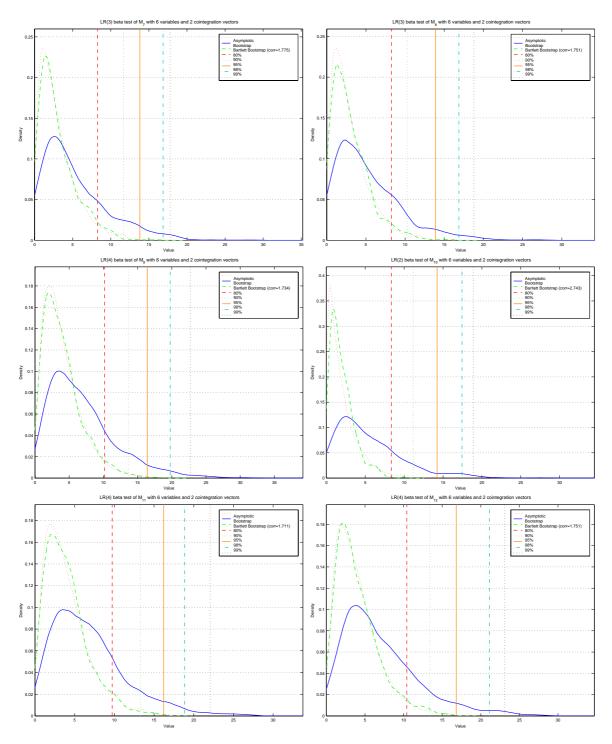


FIGURE 5: Continued.



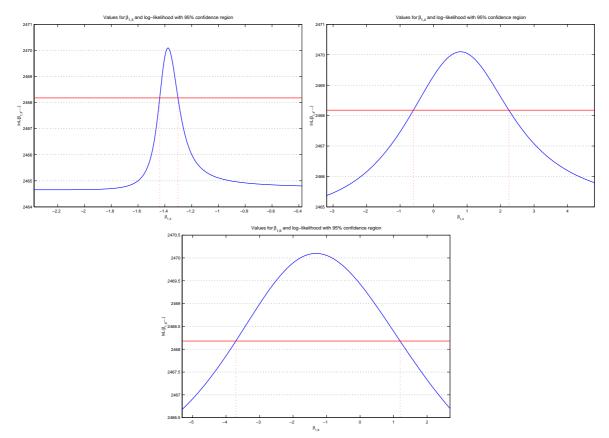
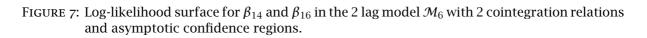
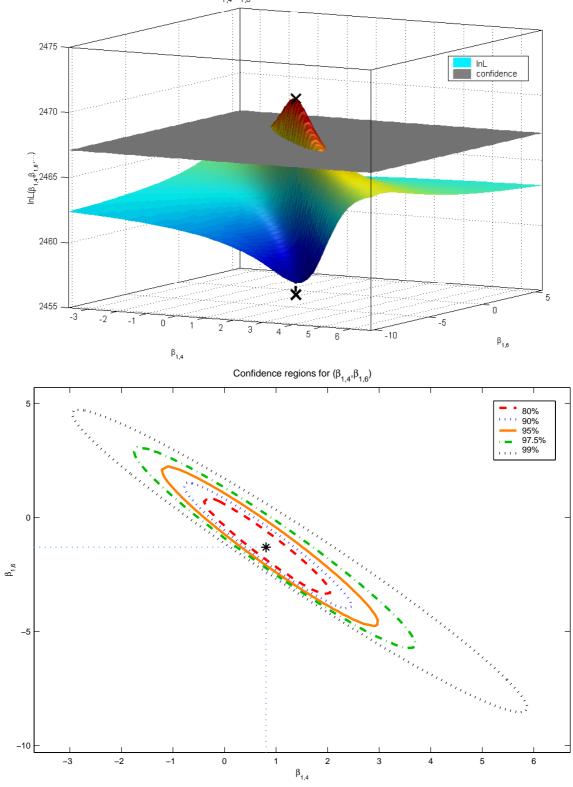


FIGURE 6: Log-likelihood values for  $\beta_{1j}$  (j = 3, 4, 6) in the 2 lag model  $\mathcal{M}_6$  with 2 cointegration relations.





Values for  $(\beta_{1,4},\beta_{1,6})$  and log-likelihood with 95% confidence region

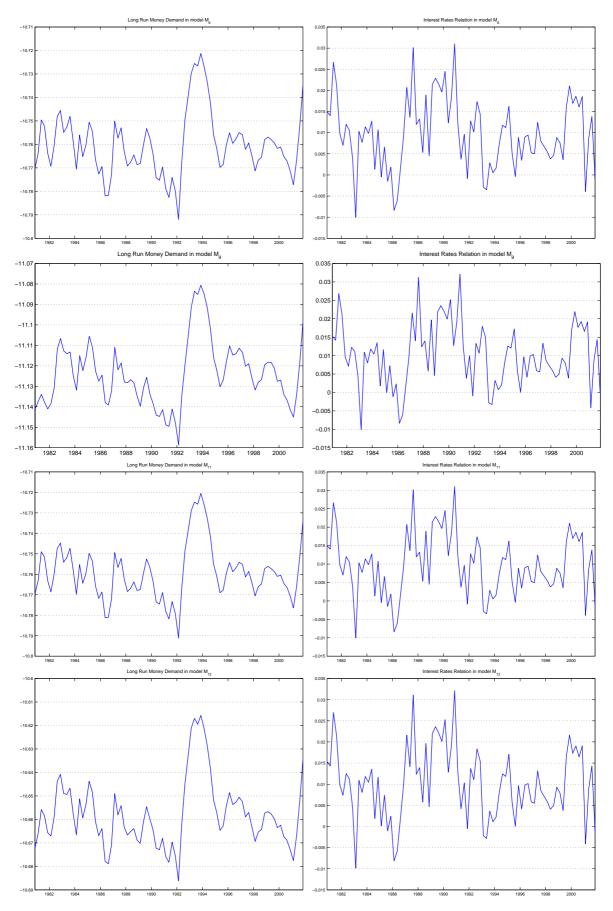
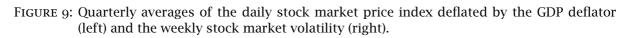
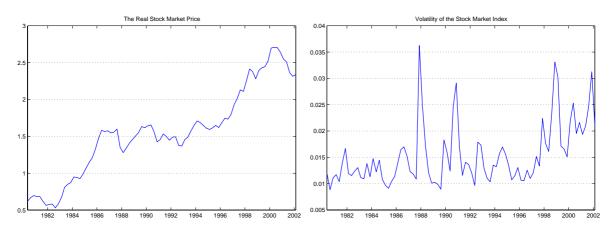


FIGURE 8: The estimated cointegration relations for models  $\mathcal{M}_6$ ,  $\mathcal{M}_9$ ,  $\mathcal{M}_{11}$  and  $\mathcal{M}_{12}$  over the period 1980:Q4-2001:Q4.





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