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OPTIMAL MONETARY POLICY, ASSET PURCHASES, AND CREDIT MARKET FRICTIONS

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Abstract

This paper examines how credit market frictions affect optimal monetary policy and if there is a role for central bank asset purchases. We develop a sticky price model where money serves as the means of payment and ex-ante identical agents borrow/lend among each other. The credit market is distorted as borrowing is constrained by available collateral. We show that the central bank cannot implement the first best allocation and that optimal monetary policy mainly aims at stabilizing prices when only a single instrument is available. The central bank can however mitigate the credit market distortion in a welfare-enhancing way by purchasing loans at a favorable price, which relies on rationing the supply of money.

JEL classification: E4; E5; E32.

Keywords: Optimal monetary policy, borrowing constraints, nominal rigidities, central bank asset purchases, money rationing

Non-technical summary

In response to the recent financial crisis, central banks introduced a variety of policy measures with only little theoretical or empirical guidance available. Since then, researchers have analyzed various channels which are able to rationalize beneficial effects of unconventional monetary policies, like the possibility to ease credit market conditions by direct central bank lending or to influence yields by large scale purchases of assets in specific market segments. Focusing on the asset side of the central bank balances sheet, one aspect that has typically been ignored is the particular role of the central bank's supply of reserves. In this paper, we argue that the fact that a central bank typically supplies money against a set of eligible collateral and not in an unbounded way is crucial for understanding how central bank asset purchases can be effective and lead to welfare enhancements.

This paper presents a normative analysis of monetary policy in a framework that is constructed to account for macroeconomic frictions that have been viewed as essential to understand the central bank trade-offs as well as for a credit market imperfections, providing an efficiency-based justification for policy makers to be aware about credit market developments. The model encompasses the basic New Keynesian model, which rationalizes the paradigm of price stability as the main principle for an optimal monetary policy regime. The analysis in this paper augments the latter model by considering agents that differ with regard to their willingness to spend and thereby to borrow/lend. Despite this property, the model is constructed to ensure the existence of a representative agent, which allows to compare the results with the literature on optimal monetary policy, which mainly builds on models with identical agents. The credit market allocation is distorted by borrowing constraints, which require that agents can only borrow up to the current value of available collateral (i.e., the market price of housing).

In this paper, we first show that credit market imperfections do per se not provide a welfare-based justification for central banks to deviate from the paradigm of price stability. Specifically, we shown that higher inflation is not desirable, even though debt is nominal, as it tends to increase the costs of borrowing (i.e. the nominal lending rate) and thereby to worsen agents' ability to borrow. Thus, a conventionally conducted monetary policy should essentially ignore the distortions induced by credit market frictions. We then extent the analysis by considering central bank asset purchases, i.e. temporary central bank holdings of secured loans. When the central bank offers to purchase loans at the current price market, agents have no incentive to sell them, such that this type of policy is entirely ineffective. If, however, the central bank offers to purchase loans at a more favorable price, it can stimulate the credit market by alleviating the severity of borrowing constraints and can thereby induce a welfare-superior allocation. As a prerequisite, the central bank has to ration the supply of reserves - or equivalently supply reserves at a low price against a bounded set of eligible assets - which implies that the central bank controls both, the price and the amount of reserves in money supply operations. Thus, the ability to stimulate the credit market relies on offering a favorable price for loans, which induces a lower equilibrium lending rate, and on controlling the specific size of the intervention, which endows the central bank with a quantitative instrument that adds to the short-term nominal interest rate that is typically considered as the single instrument of conventional monetary policy regimes.

Overall, the analysis shows that satiating agents' demand for money is in general not recommendable, as rationing of money supply endows the central bank with quantitative instruments that allow manipulating market prices of eligible asset as well as influencing aggregate demand in a welfare-enhancing way.

1 Introduction

How should monetary policy be conducted under credit market frictions? In this paper, we examine how borrowing constraints affect the choices of a central bank that aims at maximizing welfare of a representative agent (see Schmitt-Grohé and Uribe, 2010 for an overview). We develop a macroeconomic model where prices are sticky and money serves as a mean of payment. Private agents can differ with regard to their willingness to spend, giving rise to borrowing/lending between ex-ante identical agents, while borrowing is constrained by available collateral. We analyze how monetary policy affects private borrowing/lending, and how the central bank conducts optimal policy when the tightness of the borrowing constraint is varied. We further show that the central bank can enhance welfare by easing the latter via purchases of secured loans, providing a rationale for central bank purchases of credit market instruments during the recent financial crisis.[3] Specifically, for loan purchases to be welfare enhancing the central bank has to offer a favorable price, implying that money supply will be effectively rationed in equilibrium.

We apply a stylized macroeconomic model where money is essential and private agents borrow/lend among each other. To facilitate aggregation, we consider ex-ante identical agents, as in Shi (1997). In each period, they draw preference shocks from the same time-invariant distribution, i.e. shocks that shift their valuation of the consumption good. Private agents with a high valuation of consumption are willing to consume more, for which they borrow money from other agents. We assume that contract enforcement is limited, such that lending relies on the borrower's ability to pledge collateral, as in Kiyotaki and Moore (1997). Likewise, we assume that the central bank supplies money only against eligible assets, for which we consider treasury securities as collateral in open market operations. We further account for the possibility of central bank purchases of secured loans. To be more precise, the central bank might temporarily hold secured loans under repurchase agreements (which differs from outright purchases as recently conducted by US Federal Reserve, see e.g. Hancock and Passmore, 2014).

Loans are assumed to be intraperiod, as in Jermann and Quadrini (2012), which implies that real debt burden cannot be reduced by higher inflation.[4] In this framework, higher inflation is not beneficial for borrowers, since it tends to increase the nominal lending rate and thereby amplifies the credit market friction. For the analysis of optimal policy, we assume that the central bank acts under full commitment (while we neglect the issue of time inconsistency, as in Schmitt-Grohé and Uribe, 2010). Specifically, it aims at maximizing welfare of a representative agent, taking into account that prices are imperfectly flexible, money is costly, and borrowing is constrained. We

[3] Asset purchases considered in this model are related to the type of policies introduced by the US Federal Reserve during the financial crisis before 2010, which have also been described by the Fed with "credit easing".

[4] This differs from studies on optimal policy under financial market frictions with intertemporal nominal debt (see Monacelli, 2008, or De Fiore et al., 2011).

find that monetary policy cannot implement first best,[5] regardless of price flexibility and of asset purchases, since distortions due to costs of money holdings and due to the borrowing constraint cannot simultaneously be eliminated by the central bank. We first examine a conventional monetary policy regime, where access to central bank money is not effectively constrained by holdings of eligible assets. In this case, central bank asset purchases are neutral and there is a single monetary policy instrument, as usual. Under reasonable degrees of price rigidity, we find that an optimizing central bank mainly aims at stabilizing prices, which accords to the results of related studies (see Schmitt-Grohé and Uribe, 2010, for an overview). If prices were more flexible, the central bank is willing to reduce the inflation rate, which tends to reduce the loan rate but hardly mitigates the credit market distortion.

We then account for additional instruments that might be applied by the central bank, which relies on supplying money in a way that induces asset purchases to be effective. Specifically, purchases of loans are non-neutral if the central bank offers a price that is more favorable than the market price, which is only possible if it simultaneously rations the amount of money supplied. For this, it restrict the set of assets eligible for central bank operations such that money cannot be acquired in an unbounded way (which is typically assumed in macroeconomic theory). By purchasing secured loans at a favorable price, i.e. at a rate below lenders' marginal valuation of money, lenders have an incentive to refinance secured loans and to use the proceeds to extend lending. Central bank loan purchases can thereby induce lenders to charge a lower loan rate, which tends to stimulate private sector borrowing. Compared to the conventional (single instrument) specification of optimal monetary policy where money is supplied in a non-rationed way, we find that the central bank can enhance welfare of the representative agent by mitigating the distortion induced by the borrowing constraint if it purchases secured loans below the market price.[6] We further show that an optimizing monetary policy can even undo the credit market friction by loan purchases in the case where the borrowing constraint is not too tight (i.e. where the liquidation value of collateral is sufficiently large). However, the welfare gains and the scope of effective asset purchases are endogenously limited by the valuation of money and by restrictions on policy instruments (like the zero lower bound on interest rates).[7]

The paper relates to studies on optimal monetary policy in sticky price models (see Kahn et al., 2003, or Schmitt-Grohé and Uribe, 2010) and under financial market frictions, for example, to Monacelli (2008), who examines optimal monetary policy when borrowing households face a

[5] We apply first best as a reference case rather than a constrained efficient allocation, as we will show that the central bank is able to undo the distortion induced by the credit market friction (see Section 4.2.2).

[6] The possibility to enhance welfare by rationing money supply is shown by Schabert (2013) in a framework with frictionless financial markets.

[7] The limits to the effectiveness of balance sheet policies are examined by Hoermann and Schabert (2014) in a related model without collateralized borrowing.

collateral constraint, or to DeFiore et al. (2011), who analyze optimal monetary policy under flexible prices and imperfect monitoring. The analysis of central bank asset purchases relates to studies on unconventional monetary policies like Curdia and Woodford (2011) and Gertler and Karadi (2011), who find that direct central bank lending under costly financial intermediation can be effective if financial market frictions are sufficiently large. The analysis in this paper further relates to Araújo et al. (2013), who show in a model with endogenous collateral constraints and without a special role of currency that central bank purchases of collateral at market prices can potentially improve welfare, though they tend to lower welfare when purchases are sufficiently large. Applying an estimated model with segmented asset markets, Chen et al. (2012) find that large scale asset purchases as recently conducted by the US Federal Reserve can lead to small expansionary effects even at the zero lower bound.

In Section 2 we present the model. In Section 3, we demonstrate how the credit market friction affects the equilibrium allocation and how its severity is altered by monetary policy. In Section 4, we examine optimal monetary policy considering a regime without money rationing and a regime where money supply is effectively rationed and asset purchases are non-neutral. Section 5 concludes.

2 The model

In this Section, we provide an overview of the model, present the details of the private sector behavior and the public sector, and describe the first best allocation.

2.1 Overview

There are three sectors: households, firms, and the public sector. Households consist of members who enter a period with money and government bonds and dispose of a constant time endowment. They can further hold a durable good, i.e. housing, which is supplied at a fixed amount. At the beginning of each period, aggregate productivity shocks are realized and open market operations are conducted, where the central bank sells or purchases assets outright or supplies money via repos against eligible assets at the policy rate R_t^m. Then, idiosyncratic preference shocks are realized.[8] Household members with a high realization of the preference shock (ϵ_b) are willing to consume more than household members with a low realization of the preference shock ($\epsilon_l < \epsilon_b$). Given that purchases of consumption goods rely on money holdings, the former borrow money from the latter at the price $1/R_t^L$. We consider loans being collateralized by the market value of borrowers' housing, which can be justified by limited enforceability of debt contracts. These

[8] The assumption that preference shocks are realized after money is supplied in open market operations against treasuries is made only to facilitate the analysis for the case where money is supplied in a non-rationed way, which is equivalent to the conventional specifiation where money is supplied via lump-sum transfers.

secured loans might be purchased by the central bank, such that the proceeds are available to extend credit supply. After goods are produced, the market for consumption goods opens, where money serves as the means of payment, inducing demand for money and assets eligible for open market operations. In the asset market, borrowing agents repay the secured loans, the government issues new bonds at the price $1/R_t$, and the central bank reinvests payoffs from maturing bonds and leaves money supply unchanged.[9]

The central bank sets the price of money (i.e. the policy rate), decides on the amount of money is supplied against eligible assets in open market operations and by purchases of loans, and it transfers interest earnings to the treasury. The government issues risk-free bonds, which back private sector money holdings, and has access to lump-sum taxes. Firms produce goods employing labor from households, and they set prices in an imperfectly flexible way.

2.2 Details

Households There are infinitely many households of measure one. Each household has a unit measure of members i. Following Shi (1997), we assume that assets of all household members are equally distributed at the beginning of each period. Their utility increases with consumption $c_{i,t}$ of a non-durable good and holdings of a durable good, i.e. housing $h_{i,t}$, and is decreasing in working time $n_{i,t}$. Like in Iacoviello (2005), we assume that the supply of housing is fixed at $h > 0$. Members of each household can differ with regard to their marginal valuation of consumption due to preference shocks $\epsilon_i > 0$, which are i.i.d. across members and time. The instantaneous utility function of ex-ante identical members is given by

$$u(c_{i,t}, h_{i,t}, n_{i,t}, \epsilon_{i,t}) = \epsilon_{i,t}(c_{i,t}^{1-\sigma} - 1)\,(1-\sigma)^{-1} + \gamma(h_{i,t}^{1-\sigma_h} - 1)\,(1-\sigma_h)^{-1} - \chi n_{i,t}^{1+\eta}\,(1+\eta)^{-1}\,, \quad (1)$$

where $\sigma_{(h)} > 0$, $\gamma > 0$, $\chi > 0$, and $\eta \geq 0$ and $h_{i,t}$ denotes the end-of-period stock of housing, which might differ between both types of members. For simplicity, we assume that ϵ_i exhibits two possible realizations, $\epsilon_i \in \{\epsilon_b,\ \epsilon_l\}$, with equal probabilities $\pi_\epsilon = 0.5$, where $\epsilon_l < \epsilon_b$. Household members rely on money for purchases of consumption goods. For this, they hold money $M_{i,t-1}^H$ and can acquire additional money $I_{i,t}$ from the central bank, for which they hold eligible assets, in particular, risk-free government bonds $B_{i,t-1}$. Household members $i \in \{b, l\}$ can further acquire money $I_{i,t}$ from the central bank in open market operations, where money is supplied against treasury securities discounted with the policy rate R_t^m (see Hoermann and Schabert, 2014):

$$I_{i,t} \leq \kappa_t^B B_{i,t-1}/R_t^m. \quad (2)$$

[9] Further details on the flow of funds within each period can be found in Schabert (2013), where a corresponding framework without credit market frictions is applied.

The central bank supplies money against fractions of (randomly selected) bonds $\kappa_t^B \geq 0$ (see 2) under repurchase agreements and outright. When household member i draws the realization ϵ_b (ϵ_l), which materializes after treasuries can be liquidated in open market operations,[10] it is willing to consume more (less) than members who draw ϵ_l (ϵ_b). Hence, ϵ_b-type members tend to borrow an additional amount of money from ϵ_l-type members.

We assume that borrowing and lending among private agents only takes place in form of short-term loans at the price $1/R_t^L$. As Jermann and Quadrini (2012), we assume that loan contracts are signed at the beginning of the period and repaid at the end of each period, which greatly simplifies the analysis. We account for the fact that debt repayment cannot always be guaranteed and that enforcement of debt contracts is limited. We therefore assume that loans are partially secured by borrowers' holdings of housing, serving as collateral. Specifically, ϵ_b-type members can borrow the amount $L_{b,t} < 0$ up to the liquidation value of collateral at maturity

$$-L_{b,t} \leq z_t P_t q_t h_{b,t}, \tag{3}$$

where q_t denotes the real housing price, and z_t a stochastic liquidation value of collateral (see Iacoviello, 2005). In addition to secured loans, we account for the existence of unsecured loans, measured as a share $\upsilon \geq 0$ of secured loans. Thus, private borrowing/lending takes place both in form of unsecured and secured lending (as in He et al., 2013), while we interpret individual debt as being partially collateralized, for convenience.

We allow for the possibility that the central bank purchases loans. After the preference shocks are realized and loan contract are signed, lenders can refinance secured loans up to the amount that is fully collateralized, $L_{l,t} = -L_{b,t}$, at the central bank. Specifically, it purchases a randomly selected fraction $\kappa_t \geq 0$ of loans at the price $1/R_t^m$:

$$I_{l,t}^L \leq \kappa_t L_{l,t}/R_t^m, \tag{4}$$

where $I_{l,t}^L \geq 0$.[11] Money $I_{l,t}^L$ received from loan purchases, $\kappa_t > 0$, can be used to extend lending. Thus, by purchasing loans the central bank can influence the lenders' valuation of secured loans and can increase the amount of money that is available for credit supply. We assume that loan purchases are conducted in form of repurchase agreements, i.e. loans are repurchased by lenders before they mature (such that lending agents earn the interest on loans).

In the goods market, member i can then use money holdings $M_{i,t-1}^H$ as well as new injections $I_{i,t}$ and $I_{i,t}^L$ plus/minus loans for consumption expenditures, where $I_{b,t}^L = 0$. Hence, the goods

[10] Note that this assumption is only relevant for the case, where money is supplied in a non-rationed way, i.e. when the money supply constraint (2) is not binding.

[11] A value of κ_t that exceeds one can in principle be interpreted as purchases at a price that is even more favorable than the price of money in terms of treasuries (see 2).

market constraints for both types b and l read:

$$P_t c_{i,t} \leq I_{i,t} + I_{i,t}^L + M_{i,t-1}^H - \left[(1+\upsilon) L_{i,t} + L_{i,t}^r \right] / R_t^L, \tag{5}$$

where $L_{l,t}^r$ denotes loans funded by the proceeds of central bank purchases, $L_{l,t}^r / R_t^L \leq I_{l,t}^L$ and $L_{l,t}^r = -L_{b,t}^r$. Notably, loans that are refinanced by the central bank L_t^r and the fraction υL_t of original loans are not secured. They both are assumed not to be eligible for central bank operations, which accords to common central bank practice.

Before, the asset market opens, wages, taxes, and profits are paid, and repos are settled, i.e. agents buy back loans and treasuries from the central bank. In the asset market, members repay intraperiod loans and invest in treasuries. Thus, the asset market constraint of both types of members is

$$\begin{aligned} &M_{i,t-1}^H + B_{i,t-1} + (1+\upsilon) L_{i,t} \left(1 - 1/R_t^L\right) + L_{i,t}^r \left(1 - 1/R_t^L\right) + P_t w_t n_{i,t} + P_t \delta_{i,t} + P_t \tau_{i,t} \\ &\geq M_{i,t}^H + (B_{i,t}/R_t) + \left(I_{i,t} + I_{i,t}^L\right)(R_t^m - 1) + P_t c_{i,t} + P_t q_t \left(h_{i,t} - h_{i,t-1}\right), \end{aligned} \tag{6}$$

where $q_t = P_{h,t}/P_t$ and $P_{h,t}$ is the nominal price of housing. Maximizing $E \sum_{t=0}^{\infty} \beta^t u_{i,t}$ subject to (2), (4), (5), (6), and the borrowing constraints (3), $-L_{bt}^r \leq I_{l,t}^L R_t^L$, $M_{i,t}^H \geq 0$, and $B_{i,t} \geq 0$, leads to the following first order conditions for consumption, working time, holdings of treasuries and money, and additional money from open market operations $\forall i \in \{b, l\}$

$$\epsilon_{i,t} c_{i,t}^{-\sigma} = \lambda_{i,t} + \psi_{i,t}, \tag{7}$$

$$\chi n_{i,t}^{\eta} = w_t \lambda_{i,t}, \tag{8}$$

$$\lambda_{i,t} = \beta R_t E_t \left[\left(\lambda_{i,t+1} + \kappa_{t+1}^B \eta_{i,t+1} \right) / \pi_{t+1} \right], \tag{9}$$

$$\lambda_{i,t} = \beta E_t \left[\left(\lambda_{i,t+1} + \psi_{i,t+1} \right) / \pi_{t+1} \right], \tag{10}$$

$$(\pi_\epsilon \psi_{b,t} + \pi_\epsilon \psi_{l,t}) = (R_t^m - 1)(\pi_\epsilon \lambda_{b,t} + \pi_\epsilon \lambda_{l,t}) + R_t^m \eta_{i,t}, \tag{11}$$

where $\lambda_{i,t} \geq 0$ is the multiplier on the asset market constraint (6), $\eta_{i,t} \geq 0$ the multiplier on the money supply constraint (2), and $\psi_{i,t} \geq 0$ the multiplier on the cash-in-advance constraint (5). The cash-constraint implies – for $\psi_{i,t} > 0$ – the usual distortion regarding the optimal choices for consumption and working time (see 7 and 8). Condition (9) indicates that the interest rate on government bonds might be reduced by a liquidity premium, stemming from the possibility to exchange a fraction κ_t^B of bonds in open market operations (see 2).

Given that household members are ex-ante identical, their expected valuation of payoffs in the subsequent period are identical, implying that $\lambda_{b,t} = \lambda_{l,t} = \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]$ (see 10) and that both types supply the same amount of working time, $n_{b,t}^{\eta} = n_{l,t}^{\eta} = (w_t/\chi)\, \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]$ (see 8). Condition (11) for money supplied against treasuries, which indicates that idiosyncratic shocks are not revealed before open market operations are conducted, can then – by using (7) – be simplified to $0.5(\epsilon_b c_{b,t}^{-\sigma} + \epsilon_l c_{l,t}^{-\sigma})/R_t^m = \lambda_{i,t} + \eta_{i,t}$. Thus, the money supply constraint in (2) is binding if the multiplier $\eta_{i,t}$ satisfies $\eta_{i,t} = 0.5(\epsilon_b c_{b,t}^{-\sigma} + \epsilon_l c_{l,t}^{-\sigma})/R_t^m - \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}] > 0$. For this, the policy rate has to be lower than the average member's marginal (nominal) rate of intertemporal substitution, $R_t^m < [0.5(\epsilon_b c_{b,t}^{-\sigma} + \epsilon_l c_{l,t}^{-\sigma})]/[\beta E_t 0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]$.[12]

Further, the following type-specific first order conditions for loans, housing, money from loan purchases $I_{l,t}^L$, and refinanced loans $L_{b,t}^r$ have to be satisfied

$$\psi_{l,t} = \lambda_{l,t}\left(R_t^L - 1\right) + R_t^L \kappa_t \varsigma_{l,t}/\left(1+\upsilon\right), \quad \text{and} \quad \psi_{b,t} = \lambda_{b,t}\left(R_t^L - 1\right) + \zeta_{b,t} R_t^L/\left(1+\upsilon\right), \tag{12}$$

$$q_t \lambda_{b,t} = \gamma h_{b,t}^{-\sigma_h} + \zeta_{b,t} z_t q_t + \beta E_t q_{t+1} \lambda_{i,t+1}, \text{ and } q_t \lambda_{l,t} = \gamma h_{l,t}^{-\sigma_h} + \beta E_t q_{t+1} \lambda_{i,t+1}, \tag{13}$$

$$\varsigma_{l,t} = \lambda_{l,t}\left(R_t^L - R_t^m\right)/R_t^m, \quad \text{and} \quad \varkappa_{b,t} R_t^L = \psi_{b,t} - \lambda_{b,t}\left(R_t^L - 1\right), \tag{14}$$

where $\varsigma_{l,t}$ denotes the multiplier on the money supply constraint (4), $\zeta_{i,t}$ and $\varkappa_{b,t}$ the multiplier on the borrowing constraints (3) and $-L_{b,t}^r \le I_{l,t}^L R_t^L$. Further, the associated complementary slackness conditions and the transversality conditions hold. The conditions for loan demand and supply in (12) reveal that the credit market allocation can be affected by central bank loan purchases (for $\varsigma_{l,t} > 0$) and by the borrowing constraint (for $\zeta_{b,t} > 0$). The borrower demands loans according to $\left(\psi_{b,t} + \lambda_{b,t}\right)/R_t^L = \lambda_{b,t} + \zeta_{b,t}/(1+\upsilon)$ (see 12), which can – by using (7) and (10) – be rewritten as

$$\frac{1}{R_t^L} = \beta E_t \frac{0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})}{\epsilon_b c_{b,t}^{-\sigma} \pi_{t+1}} + \frac{\zeta_{b,t}}{\epsilon_b c_{b,t}^{-\sigma}(1+\upsilon)}. \tag{15}$$

Hence, a positive multiplier $\zeta_{b,t}$ tends – for a given R_t^L – to raise the RHS of (15), implying that current consumption tends to fall, which can be mitigated by a lower loan rate. Put differently, a binding borrowing constraint (3) tends to reduce the loan rate below the borrowers' marginal rate of intertemporal substitution $1/\beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/(\epsilon_b c_{b,t}^{-\sigma} \pi_{t+1})]$. The distortion due to the borrowing constraint (3) is obviously less pronounced for a higher share υ of unsecured loans. The lender supplies loans according to $\lambda_{l,t} + \varsigma_{l,t}\kappa_t/\left(1+\upsilon\right) = \left(\psi_{l,t} + \lambda_{l,t}\right)/R_t^L$ (see 12), or – using (7) and (10) – to $\beta E_t \frac{0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})}{\pi_{t+1}} + \varsigma_{l,t}\kappa_t/\left(1+\upsilon\right) = \epsilon_l c_{l,t}^{-\sigma}/R_t^L$. Eliminating the multiplier $\varsigma_{l,t}$ with (14), then leads to

$$\frac{1}{R_t^L} = \beta E_t \frac{0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})}{\epsilon_l c_{l,t}^{-\sigma} \pi_{t+1}} \left[1 + \frac{\kappa_t}{1+\upsilon}\left(\frac{R_t^L}{R_t^m} - 1\right)\right]. \tag{16}$$

Condition (16) implies that the loan rate is affected by the lender's marginal rate of intertemporal

[12] It should be noted that the average member's marginal rate of intertemporal substitution $0.5(\epsilon_{b,t} c_{b,t}^{-\sigma} + \epsilon_{l,t} c_{l,t}^{-\sigma})]/[\beta E_t 0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]$ is typically larger than the lender's marginal rate of intertemporal substitution $\epsilon_{l,t} c_{l,t}^{-\sigma}/[\beta E_t 0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}$ when the borrowing constraint is binding.

substitution in nominal terms $1/\beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/(\epsilon_l c_{l,t}^{-\sigma}\pi_{t+1})]$ as well as by the policy rate R_t^m, if the central bank purchases loans, $\kappa_t > 0$.

The borrowing constraint (3) further distorts the borrower's demand for housing and thus the housing price q_t (see 13). Combining the first order conditions for housing (13) gives $\gamma(h_{l,t}^{-\sigma_h} - h_{b,t}^{-\sigma_h}) = \zeta_{b,t} q_t z_t$. Hence, if the collateral constraint is binding $\zeta_{b,t} > 0$, investment in housing differ between both types of members, i.e. $h_{b,t} > h_{l,t}$. Combining the conditions in (12) and substituting out $\lambda_{i,t} + \psi_{i,t}$ with (7), further leads to

$$\epsilon_b c_{b,t}^{-\sigma} - \epsilon_l c_{l,t}^{-\sigma} = R_t^L \left(\zeta_{b,t} - R_t^L \kappa_t \varsigma_{l,t}\right) / \left(1 + \upsilon\right), \tag{17}$$

which implies that the consumption choice (that would ideally satisfy $\epsilon_l c_{l,t}^{-\sigma} - \epsilon_b c_{b,t}^{-\sigma} = 0$, see Proposition 1) is distorted by the borrowing constraint ($\zeta_{b,t} > 0$) and by the possibility that loans can be liquidated at the central bank ($\varsigma_{l,t} > 0$). Condition (17) further implies that the central bank can in principle undo the effects of the borrowing constraint by purchasing loans, $\kappa_t > 0$.

Conditions (12) and (14) imply $\varkappa_{b,t} = \zeta_{b,t}/(1+\upsilon)$, which shows that borrowers demand the maximum amount of refinanced loans ($\varkappa_{b,t} > 0 \Rightarrow -L_{b,t}^r = I_{l,t}^L R_t^L$) when the borrowing constraint is binding ($\zeta_{b,t} > 0$). The money supply constraint (4) will further be binding, $\varsigma_{l,t} > 0$, implying that lenders are willing to refinance loans at the central bank when this allows to extract further rents, i.e. if the policy rate is lower than the loan rate (see 14). Lenders will then refinance the maximum amount of available loans and use these funds to supply further loans, $L_{l,t}^r/R_t^L = I_{l,t}^L$. If, however, the policy rate equals the loan rate, $R_t^m = R_t^L$, lenders have no incentive to refinance loans at the central bank, and lenders do not engage in further lending, $L_{l,t}^r = 0$. Thus, only if $R_t^m < R_t^L$ (and thus $\varsigma_{l,t} > 0$) the central bank can directly influence the loan rate by purchasing loans, $\kappa_t > 0$. It should finally be noted that $R_t^m < R_t^L$ implies that the policy rate is also lower than the lender's marginal rate of intertemporal substitution $\epsilon_l c_{l,t}^{-\sigma}/[\beta E_t 0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}$ (see 16), which is – under a binding borrowing constraint – sufficient for the money supply constraint (2) to be binding, $\eta_{i,t} > 0$. Given that money supply is then effectively constrained by the available amount of eligible assets, i.e. bonds and secured loans, this type of monetary policy implies money rationing.

Firms There is a continuum of identical intermediate goods producing firms indexed with $j \in [0,1]$. They exist for one period, are perfectly competitive, and are owned by the households. A firm j distributes profits to the owners and hires the aggregate labor input $n_{j,t}$ at a common rate rate w_t. It then produces the intermediate good $x_{j,t}$ according to $x_{j,t} = a_t n_{j,t}^{\alpha}$, where $\alpha \in (0,1)$ and a_t is stochastic with an unconditional mean equal to one, and sells it to retailers. Following related studies (see e.g. Schmitt-Grohé and Uribe, 2010), we allow for a constant subsidy τ^p to eliminate long-run distortions due to imperfect competition, such that the problem of a profitmaximizing firm j is given by $\max(1+\tau^p)P_{J,t}a_t n_{j,t}^{\alpha} - P_t w_t n_{j,t}$, where $P_{J,t}$ denotes the price for the intermediate good. The first order conditions are given by $(1+\tau^p)\left(P_{J,t}/P_t\right)\alpha n_{j,t}^{1-\alpha} = w_t$ or $\left(P_{J,t}/P_t\right)a_t\alpha n_{j,t}^{\alpha-1} = (1-\tau^n)$, where we defined $\tau^n = \tau^p/(1+\tau^p)$ as the production (or wage) subsidy rate. The firms transfer profits to the owners in a lump-sum way.

To introduce sticky prices, we assume that there are monopolistically competitive retailers who re-package intermediate goods $x_t = \int_0^1 x_{j,t}dj$. A retailer $k \in [0,1]$ produces one unit of a distinct good $y_{k,t}$ with one unit of the intermediate good (purchased at the common price $P_{J,t}$) and sells it at the price $P_{k,t}$ to perfectly competitive bundlers. They bundle the distinct goods $y_{k,t}$ to a final good $y_t = (\int_0^1 y_{k,t}^{\frac{\varepsilon-1}{\varepsilon}} dk)^{\frac{\varepsilon}{\varepsilon-1}}$ which is sold at the price P_t. The cost minimizing demand for $y_{k,t}$ is then given by $y_{k,t} = \left(P_{k,t}/P_t\right)^{-\varepsilon} y_t$. We assume that each period a measure $1-\phi$ of randomly selected retailers may reset their prices independently of the time elapsed since the last price setting, while a fraction $\phi \in [0,1)$ of retailers do not adjust their prices. A fraction $1-\phi$ of retailers sets their price to maximize the expected sum of discounted future profits. For $\phi > 0$, the first order condition for their price $\widetilde{P}_t$ can be written as $Z_{1,t}/Z_{2,t} = \tilde{Z}_t\left(\varepsilon-1\right)/\varepsilon$, where $Z_{1,t} = \left(1-\tau^n\right)\left(\chi/\alpha\right)0.5^{\eta} n_t^{1+\eta} s_t^{-1} + \phi\beta E_t \pi_{t+1}^{\varepsilon} Z_{1,t+1}$, $Z_{2,t} = \left(1-\tau^n\right)\left(\chi/\alpha\right)0.5^{\eta} n_t^{1+\eta}\left(mc_t s_t\right)^{-1} + \phi\beta E_t \pi_{t+1}^{\varepsilon-1} Z_{2,t+1}$, $\tilde{Z}_t = \widetilde{P}_t/P_t$, and $mc_t = P_{J,t}/P_t$ denotes retailers' real marginal cost. With perfectly competitive bundlers, the price index P_t for the final good satisfies $P_t^{1-\varepsilon} = \int_0^1 P_{k,t}^{1-\varepsilon}dk$. Using that $\int_0^1 P_{k,t}^{1-\varepsilon}dk = (1-\phi)\sum_{s=0}^{\infty}\phi^s \widetilde{P}_{t-s}^{1-\varepsilon}$ holds, and taking differences, leads to $1 = (1-\phi)(\tilde{Z}_t)^{1-\varepsilon} + \phi\pi_t^{\varepsilon-1}$.

Public sector The government issues one-period nominally risk-free bonds at the price $1/R_t$, pays lump-sum transfers τ_t, and a wage subsidy at a constant rate, while we abstract from government spending, distortionary taxation, and issuance of long-term debt, for simplicity. The supply of government bonds, which are either held by households or the central bank, is further assumed to be exogenous to the state of the economy, like in Shi (2013). Specifically, we assume that the total amount of short-term government bonds B_t^T grows at the rate $\Gamma > 0$,

$$B_t^T = \Gamma B_{t-1}^T, \tag{18}$$

given $B_{-1}^T > 0$. Due to the existence of lump-sum transfers/taxes, which balance the budget, we will be able to abstract from fiscal policy, except for the supply of treasuries (18). Note that the growth rate might affect the long-run inflation rate if the money supply constraint (2) is binding. In Appendix A.4, we show how the central bank can nevertheless implement a desired inflation target by long-run adjustments of its instruments. The government further pays a constant wage subsidy τ^p, which is solely introduced to eliminate average distortions from imperfect competitive (as usual in related studies, see Schmitt-Grohé and Uribe, 2010) and receives seigniorage revenues τ_t^m from the central bank, such that its budget constraint reads $(B_t^T/R_t) + P_t\tau_t^m = B_{t-1}^T + P_t\tau_t + P_t\tau^p$.

The central bank supplies money in open market operations either outright or temporarily via repos against treasuries, $M_t^H = \int_0^1 M_{i,t}^H di$ and $M_t^R = \int_0^1 M_{i,t}^R di$. It can further increase the supply of money by purchasing secured loans from lenders, I_t^L, i.e. it conducts repos where secured loans serve as collateral. At the beginning of each period, its holdings of treasuries equals B_{t-1}^c and the stock of outstanding money equals M_{t-1}^H. It then receives treasuries and loans in exchange for money. Before the asset market opens, where the central bank rolls over maturing assets, repos in terms of treasuries and secured loans are settled. Hence, its budget constraint reads

$$(B_t^c/R_t) - B_{t-1}^c + P_t\tau_t^m = R_t^m \left(M_t^H - M_{t-1}^H\right) + (R_t^m - 1)\left(I_t^L + M_t^R\right), \tag{19}$$

while it earns interest from holding bonds and by supplying money at the price $1/R_t^m$. We assume that the central bank transfers its interest earnings from asset holdings and from open market operations to the treasury, $P_t\tau_t^m = (1 - 1/R_t) B_t^c + R_t^m \left(M_t^H - M_{t-1}^H\right) + (R_t^m - 1)\left(I_t^L + M_t^R\right)$. Thus, its budget constraint (19) implies that central bank asset holdings evolve according to $B_t^c - B_{t-1}^c = M_t^H - M_{t-1}^H$. Further assuming that initial values for its assets and liabilities satisfy $B_{-1}^c = M_{-1}^H$, leads to the central bank balance sheet

$$B_t^c = M_t^H. \tag{20}$$

The central bank has four instruments. It sets the policy rate $R_t^m \geq 1$ and can decide how much money to supply against a randomly selected fraction of treasuries, for which it can adjust $\kappa_t^B \in (0, 1]$ (see 2) in a state contingent way. The central bank can further decide whether it supplies money in exchange for treasuries either outright or temporarily via repos. Specifically, it can control the ratio of treasury repos to outright sales of government bonds $\Omega_t > 0 : M_t^R = \Omega_t M_t^H$, where a sufficiently large value for Ω_t ensures that injections are always positive, $I_{i,t} > 0$. Finally, the central bank can decide to purchase loans. In each period, it decides on a randomly selected share of secured loans $\kappa_t \in [0, 1]$ that is offered to be exchanged for money under repos.

Equilibrium A definition of a competitive equilibrium, for which we simplify the notation using $L_t = L_{l,t} = -L_{b,t}$, $L_t^r = L_{l,t}^r = -L_{b,t}^r$, and $I_t^L = I_{l,t}^L$, is given in Appendix A.1. Whether money supply is effectively rationed or not depends, in particular, on policy choices. For the analysis of optimal monetary policy, we will therefore distinguish between the two cases where money supply is either effectively rationed or not rationed, where the latter is equivalent to the case where the central bank supplies money in a lump-sum way (as typically assumed in the literature). In this case, the loan rate is identical to the policy rate $R_t^L = R_t^m$ (see 11-12). Before we examine the policy problem of the central bank, we describe the first best allocation, which serves as a benchmark for the subsequent analysis. The following proposition describes the first best allocation.[13]

Proposition 1 *The first best allocation* $\{c^*_{b,t}, c^*_{l,t}, n^*_{b,t}, n^*_{l,t}, h^*_{b,t}, h^*_{l,t}\}_{t=0}^{\infty}$ *satisfies*

$$\epsilon_b(c^*_{b,t})^{-\sigma} = \epsilon_l(c^*_{l,t})^{-\sigma},\ h^*_{b,t} = h^*_{l,t},\ n^*_{b,t} = n^*_{l,t}, \tag{21}$$

$\epsilon_b(c^*_{b,t})^{-\sigma} = [\chi/(a_t\alpha)]0.5^{\eta}(n^*_{b,t} + n^*_{l,t})^{1+\eta-\alpha}$, $h^*_{b,t} + h^*_{l,t} = h$ *and* $c^*_{l,t} + c^*_{b,t} = a_t(n^*_{b,t} + n^*_{l,t})^{\alpha}$.

Proof. See Appendix A.1. ■

Under the first best allocation, the marginal utilities of consumption are identical for borrowers and lenders, and their end-of-period stock of housing is the same (see 21). This will typically not be the case in a competitive equilibrium where the borrowing constraint is binding. In Section 4.2.2, we examine how the central bank can relax the borrowing constraint by purchases of loans. For this policy to be non-neutral, money has to be supplied at a favorable price which implies that access to money is effectively constrained by the available amount of assets eligible for central bank operations. Specifically, the central bank has to set the policy rate below the lender's marginal rate of intertemporal substitution, implying $R^m_t < R^L_t$ (see 16).

3 Constrained borrowing and monetary policy

In this Section, we examine the impact of the existence of the borrowing constraint (3) on the allocation and on prices. We demonstrate how the long-run equilibrium is affected by this credit market friction and how the tightness of the borrowing constraint is altered by monetary policy. The parameter values applied for this analysis and in the subsequent Sections are given in Table A1 in Appendix A.7. We set most parameter equal to values that are standard in the literature, i.e. $\beta = 0.99$, $\sigma_{(h)} = 2$, $\eta = 1$, $\phi = 0.7$, $\alpha = 0.66$, and $\chi = 98$, the latter implying a first best working time share of roughly one third. Given that the model is evidently too stylized to match empirical measures related to the housing market and to private agents' heterogeneity, we apply values for the remaining parameters that turned out out to be particularly useful for the solution to the optimal monetary policy problem. Specifically, the utility weight on housing of $\gamma = 0.1$ is taken from Iacoviello (2005), the steady state housing share qh/y is set at 0.18, the realizations of the idiosyncratic shock are $\epsilon_l = 0.5$ and $\epsilon_b = 1.5$ with equal probabilities (0.5), the share of unsecured loans υ equals 1/2, and the unconditional mean of the liquidation share is set at z at 0.8. For the stochastic processes z_t and a_t, we assume that the autocorrelation of aggregate shocks equals 0.9 and their standard deviation equals 0.005.

Suppose that monetary policy acts in a non-optimizing way and that money supply is not rationed. Under the parameter values described above, the borrowing constraint will be binding in a long-run

[13] According to the conditions in proposition 1, the solution for $c^*_{b,t}$ and $h^*_{b,t}$ are given by $c^*_{b,t} = a_t^{\frac{1+\eta}{1-\alpha+\eta+\alpha\sigma}}[\alpha\epsilon_{b,t}/(\chi 0.5^{\eta})]^{\frac{\alpha}{1-\alpha+\eta+\alpha\sigma}}[1+(\epsilon_{l,t}/\epsilon_{b,t})^{\frac{1}{\sigma}}]^{-\frac{1-\alpha+\eta}{1-\alpha+\eta+\alpha\sigma}}$ and $h^*_{b,t} = 0.5h$.

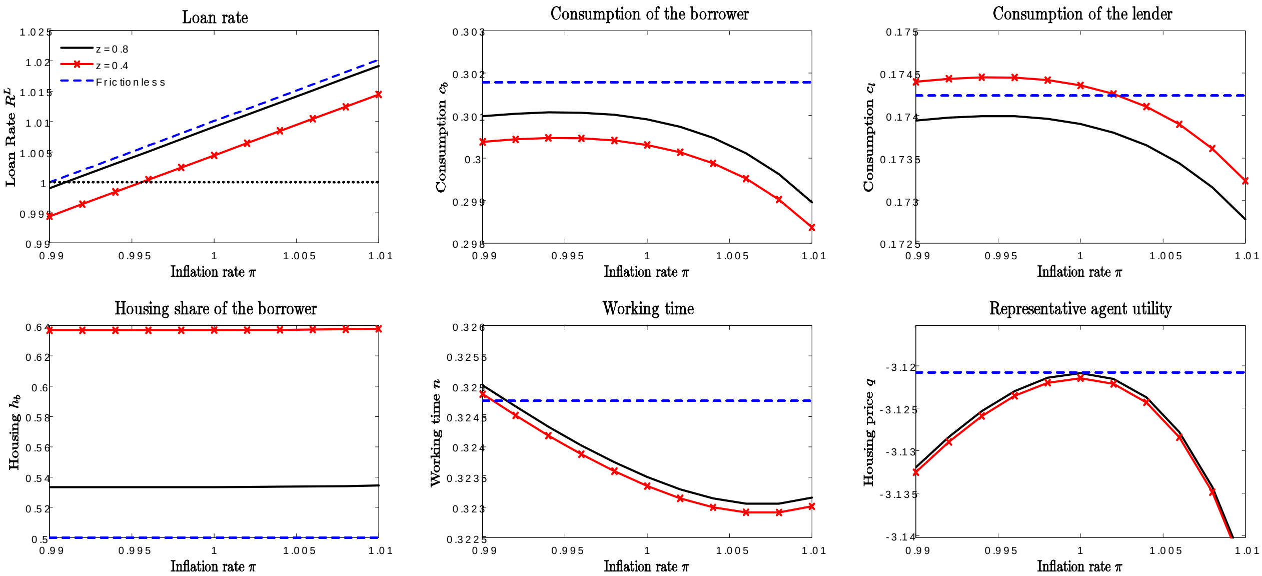

Figure 1: Steady state values for different inflation rates

equilibrium. In the steady state, the central bank is then endowed with a single choice variable, which is assumed to be the inflation rate (or the inflation target). The loan rate is then determined by the demand and the supply of loans in the private credit market as summarized in (15) and (16). Given that money is assumed not to be rationed, the loan rate equals the lender's marginal rate of intertemporal substitution in nominal terms (see 16)

$$R^L = (\pi/\beta) \cdot \left(\epsilon_l c_l^{-\sigma} / \overline{c}^{-\sigma}\right), \tag{22}$$

where $\overline{c} = [0.5\epsilon_l c_l^{-\sigma} + 0.5\epsilon_b c_b^{-\sigma}]^{-1/\sigma}$ and variables without a time index denote steady state values. If borrowing were unconstrained or the borrowing constraint were slack $\zeta_{b,t} = 0$, the borrower's and the lender's marginal utility of consumption would be identical $\epsilon_l c_l^{-\sigma} = \epsilon_b c_b^{-\sigma}$. Thus, consumption of the lender satisfies (see 17 for $\varsigma_{l,t} = 0$):

$$c_l = (\epsilon_l/\epsilon_b)^{1/\sigma} c_b \;\text{ if } \zeta_b = 0 \qquad \text{and} \qquad c_l > (\epsilon_l/\epsilon_b)^{1/\sigma} c_b \;\text{ if } \zeta_b > 0. \tag{23}$$

Thus, constrained borrowing ($\zeta_b > 0$) increases relative consumption of the lender, which tends to reduce the loan rate (see 22). This effect is more pronounced, the tighter the borrowing constraint is, e.g. when the liquidation value of housing z is lower (see Figure 1). When the central bank raises the inflation rate, the loan rate also increases (see 22). The higher inflation rate further tends to reduce overall consumption, due to the inflation tax on consumption as a cash good, $\chi 0.5 n^\eta = w\beta\overline{c}^{-\sigma}/\pi$ (see 8, 11, and 10). The impact of a tighter borrowing constraint on consumption of both types is intuitive: A lower liquidation value z leads to a larger reduction in the borrower's consumption, while the lender's consumption can even exceed first best (see 23). The impact of the borrowing constraint on housing is most pronounced. Borrowers are willing to increase investment in housing in order to raise the stock of collateral and, thereby, to relax the borrowing constraint. Thus, the borrowing constraint distorts the allocation of resources (goods and housing), while this distortion is amplified by a higher inflation rate (and thus by a higher loan rate).

Based on this line of arguments, the central bank should choose a low inflation rate to mitigate the distortions due to the inflation tax and the borrowing constraint. Put differently, there is no gain from higher inflation, which would reduce the real value of nominal debt if it were issued intertemporally (as, for example, in DeFiore et al., 2011). Given that prices are set in an imperfectly flexible way, the price level should however be stable in the long-run to avoid welfare losses from an inefficient allocation of resources (working time) due to price dispersion (see Section 4.2). Thus, a welfare-maximizing central bank should set the inflation rate close to one, as indicated by the steady state utility of the representative agent (which is always strictly smaller than under first best). If, however, it were able to control the loan rate independently from the inflation rate, it might be able to increase welfare. This is in principle possible under money rationing where the long-run loan rate is not given by (22), but instead by $\frac{1}{R^L} = \frac{\beta}{\pi} \frac{\bar{c}^{-\sigma}}{\epsilon_l c_l^{-\sigma}} [1 + \frac{\kappa}{1+\upsilon} (\frac{R^L}{R^m} - 1)]$, which shows that the central bank can influence the loan rate not only via the inflation rate. Specifically, it can control the inflation rate via money supply by adjusting κ_t^B and can further manipulate the loan rate by adjusting the policy rate R^m and the share of purchased loans κ (see Section 4.2.2).

4 Optimal monetary policy

In this Section, we examine the policy plan of a central bank that aims at maximizing welfare of a representative agent (i.e. of a representative household member), for which we assume that it is able to perfectly commit to future policies. We restrict our attention to time-invariant policies plans, neglecting the issue of time inconsistency that typically prevails in such a framework (see e.g. Schmitt-Grohé and Uribe, 2010). We consider the entire set of conditions that describe the competitive equilibrium (see Definition 2 in Appendix A.1) as constraints to the optimization problem of the central bank. Given that fiscal policy is assumed to have access to lump-sum taxation, we can neglect fiscal policy except for the supply of treasuries, which serve as eligible assets for open market operations. In the first part of this Section, we briefly assess the case of flexible prices and perfect competition, and we show that first best cannot be implemented (regardless whether money supply is rationed or not). In the second part of this Section, we consider sticky prices and examine first optimal monetary policy under the assumption that money is supplied in a non-rationed way. We then show that once the central bank rations money supply it can enhance welfare by purchasing loans at a favorable price.

4.1 A flexible price version

Before we turn to the empirically relevant case of imperfectly set prices, we briefly examine how the monetary policy decision is affected by the existence of the borrowing constraint under flexible prices. For this, we examine a reduced set of equilibrium sequences. Details can be found in Appendix A.4, where we further show how an inflation target can be implemented in a competitive equilibrium under money rationing regime. For the case where prices are perfectly flexible and competition is perfect, an equilibrium can be defined as follows.

Definition 1 *A competitive equilibrium under perfectly flexible prices and perfect competition is given by a set of sequences* $\{c_{b,t},\, c_{l,t},\, n_t,\, R_t^L,\, h_{b,t},\, q_t,\, l_t^r,\, \pi_t\}_{t=0}^{\infty}$ *satisfying*

$$0 = n_t^{1+\eta-\alpha} - \omega a_t \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}], \tag{24}$$

$$1/R_t^L = \left(c_{b,t}^{\sigma}/\epsilon_b\right)\{\beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}] + \gamma((1+\upsilon)q_t z_t)^{-1}[(h-h_{b,t})^{-\sigma_h} - h_{b,t}^{-\sigma_h}]\}, \tag{25}$$

$$0 = -[q_t n_t^{\eta+1-\alpha}/a_t] + \beta E_t[q_{t+1} n_{t+1}^{\eta+1-\alpha}/a_{t+1}] + \gamma\omega(h-h_{b,t})^{-\sigma_h}, \tag{26}$$

$$a_t n_t^{\alpha} = c_{l,t} + c_{b,t}, \tag{27}$$

$$c_{b,t} = c_{l,t} + \left[z_t q_t h_{b,t} 2\,(1+\upsilon) + l_t^r\right]/R_t^L,\ \text{if } \zeta_{b,t} = \gamma(q_t z_t)^{-1}(h_{l,t}^{-\sigma_h} - h_{b,t}^{-\sigma_h}) > 0, \tag{28}$$

$$\text{or}\ \ c_{b,t} \leq c_{l,t} + \left[z_t q_t h_{b,t} 2\,(1+\upsilon) + l_t^r\right]/R_t^L,\ \text{if } \zeta_{b,t} = 0,$$

and if $\varsigma_{l,t} = \chi n_{l,t}^{\eta}/w_t\left(R_t^L - R_t^m\right)/R_t^m > 0:$

$$1/R_t^L = \beta\left(c_{l,t}^{\sigma}/\epsilon_l\right)E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\{1 + [\kappa_t/(1+\upsilon)][(R_t^L/R_t^m) - 1]\}, \tag{29}$$

$$c_{l,t} = 0.5(1+\Omega_t)m_t^H - (1+\upsilon)z_t q_t h_{b,t}/R_t^L, \tag{30}$$

$$\text{where } (1+\Omega_t)m_t^H = \kappa_t^B b_{t-1}\pi_t^{-1}/R_t^m + m_{t-1}^H\pi_t^{-1},\ \text{and } b_t + m_t^H = \Gamma\left(b_{t-1} + m_{t-1}^H\right)/\pi_t,$$

$$l_t^r = \kappa_t z_t q_t h_{b,t} R_t^L/R_t^m, \tag{31}$$

or if $\varsigma_{l,t} = 0:$

$$1/R_t^L = \beta\left(c_{l,t}^{\sigma}/\epsilon_l\right)E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}], \tag{32}$$

$$R_t^L = R_t^m, \tag{33}$$

$$l_t^r = 0, \tag{34}$$

where $\omega = \frac{\alpha}{(1-\tau^n)\chi(0.5)^{\eta}}$ *and* $\tau^n = 0$*, and the transversality conditions, for a monetary policy* $\{\kappa_t,\, R_t^m \geq 1\}_{t=0}^{\infty}$ *and exogenous sequences* $\{a_t, z_t\}_{t=0}^{\infty}$*, given* $h > 0$*, and* $m_{-1}^H > 0$*, and* $b_{-1} > 0$ *if* $\varsigma_{l,t} > 0$*.*

As revealed by the conditions in Definition 1, there are more instruments available for the central bank if it supplies money in a rationed way (see 29-31). Notably, the fraction of bonds eligible for open market operations κ_t^B can be adjusted by the central bank to support a particular competitive equilibrium (see Appendix A.4), such that the cash constraint (30) is not a binding restriction for implementable allocations from the point of view of the central bank. Under money rationing, the central bank can then manipulate the loan rate not only via the inflation rate but also by setting the policy rate R_t^m and the share of purchased loans κ_t (see 29). To effectively ration money supply, it has to set the policy rate below the marginal rate of intertemporal substitution of the lender, such that the multipliers on the money supply constraints (2) and (4) are strictly positive, $\eta_{i,t} > 0$ and $\varsigma_{l,t} > 0$. Otherwise, the money supply constraint is slack and the loan rate equals the lender's marginal rate of intertemporal substitution. Thus, rationing money supply endows the central bank with additional instruments, which can be used to address welfare-reducing distortions in a more effective way than under a single instrument regime. According to this simple principle, the central bank is able to enhance welfare by simultaneously controlling money supply and the policy rate. However, the central bank is – even under flexible prices and perfect competition – not able to implement the long-run efficient allocation (as described in Proposition 1) regardless of whether money supply is rationed or not. This property is summarized in the following proposition.

Proposition 2 *Consider a competitive equilibrium as given in Definition 1. The first best allocation can, in general, neither be implemented under rationed money supply nor under non-rationed money supply.*

Proof. See Appendix A.1 ■

The implementation of the long-run efficient allocation would in principle require the central bank to set the inflation rate according to the Friedman rule to undo the distortion induced by the costs of money holdings (see 24). Efficiency further requires holdings of housing and marginal utilities of consumption to be identical for all members (see 21), which implies the loan rate to be equal to one (see 25) and the policy rate to be identical to the loan rate (see 29). Hence, a central bank cannot implement the long-run efficient allocation under a money rationing regime (which relies on setting the policy rate at $R_t^m < R_t^L$). Moreover, the credit market is distorted by the borrowing constraint, which will only be slack in equilibrium at borrowing costs implying a particular loan rate that is in general different from one.

Even though money rationing does not matter for the impossibility to implement first best, it can affect the allocation under second best. To demonstrate this, we compare the steady state under non-optimizing policy regimes with money rationing (see Appendix A.4) to the steady state under optimal monetary policy without money rationing (see last part of Appendix A.5). Specifically, we compute the steady state values for under the first best allocation and for different monetary policy regimes for two liquidation values of collateral ($z = 0.8$ and $z = 0.4$) (see Table A2 in Appendix A.7). It should be noted that these results are presented for demonstration purposes only, given that the values are computed while ignoring the zero lower bound on interest rates and the restriction $\kappa_t \leq 1$ (values that violate of these constraints are marked with a star). The results for the optimal policy regime (without money rationing) reveal that the central bank will not apply the Friedman rule (i.e. $\pi = \beta = 0.99$) when it faces distortions due to the credit market

Table 1: Steady state values under optimal monetary policy w/o money rationing

	First best	Benchmark parameter values	More flexible prices	Tighter borrowing constraint
Consumption of the borrower	0.3018	0.3009	0.3010	0.3003
Consumption of the lender	0.1742	0.1739	0.1739	0.1744
Borrower's housing share	0.5	0.5334	0.5333	0.6369
Working time	0.3248	0.3235	0.3237	0.3233
Loan rate	–	1.0091	1.0007	1.0044
Inflation rate	–	1	0.9982	1
Representative agent utility	–3.12078	–3.12086	–3.12085	–3.12145

friction. In fact, it sets the inflation rate at an even lower value $\pi < \beta$ to ease the borrowing constraint by reducing the loan rate ($R^L < 1$).

Under a more severe credit market friction, $z = 0.4$, deviations to the first best allocation and from the Friedman rule are more pronounced under an optimal policy regime without money rationing. A monetary policy regime that rations money supply can then reduce the deviations from the first best allocation and increase steady state utility of a representative agent, for example, by setting the inflation rate at the Friedman rule (see Appendix A.4 on details how the central bank implements the long-run inflation rate), which eliminates the inflation tax, and by purchasing loans at a low policy rate to address the credit friction, which is demonstrated for $z = 0.8$ with $R^m = 0.99$ and $\kappa = 0.3$ and for $z = 0.4$ with $R^m = 0.98$ and $\kappa = 1.2$. Thus, the central bank can in principle implement a more favorable outcome by purchasing loans, which will subsequently be shown for a version with a plausible degree of price rigidity and without violating constraints on policy instruments.

4.2 Optimal monetary policy under sticky prices

For the flexible price version of the model, it has already been established that monetary policy cannot implement first best (see Proposition 2). Here, we examine monetary policy for the empirically more relevant case of sticky prices and compare optimal monetary policy with and without money rationing. Throughout the analysis, all relevant constraints on the policy instruments are – in contrast to the analysis in the previous Section – taken into account.

4.2.1 Non-rationed money supply

In this Section, we examine optimal monetary policy for the case where the central bank faces three frictions: the borrowing constraint, the cash-credit good distortion, and sticky prices. Notably, we assume that the distortion due to the average price mark-up is eliminated by a subsidy, $\tau^n = 1/\varepsilon$,

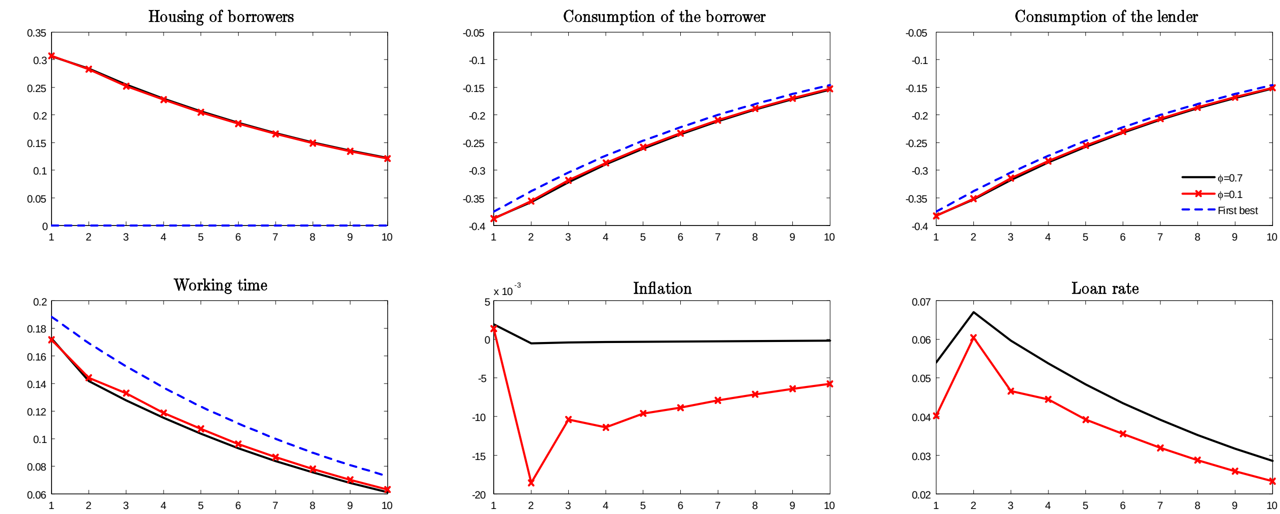

Figure 2: Responses to a contractionary productivity shock under optimal policy w/o money rationing [Note: Steady states are not identical.]

as typically assumed in the related literature (see Schmitt-Grohé and Uribe, 2010). The policy problem for the case where money is supplied in a non-rationed way is described in Appendix A.5. As discussed in Section 2.2, asset purchases are then irrelevant for the equilibrium allocation.

Table 1 presents steady state values for optimal monetary policy without money rationing for the benchmark parameterization (specifically, for $\phi = 0.7$ and $z = 0.8$), for the case where prices are more flexible ($\phi = 0.1$), and for the case where the borrowing constraint is tighter ($z = 0.4$, see last column). As indicated by the borrower's share of housing ($h_b/h > 0.5$), the borrowing constraint (3) is binding in all cases. For the benchmark case, the steady state inflation rate turns out to equal one – implying long-run price stability – for an empirically plausible degree of price rigidity ($\phi = 0.7$), while the long-run loan rate is then given by $R^L = 1.0091$. When the degree of price rigidity is smaller ($\phi = 0.1$), the central bank implements a mean inflation rate below one and a mean loan rate that is lower than under more rigid prices (see Table 1). The reduction in the inflation rate and in the loan rate tend to stimulate consumption due to the reduced inflation tax and the lower borrowing costs. The (almost unchanged) borrower's share of housing, however, indicates that the credit market friction plays a minor role for the optimal monetary policy choice.

This pattern can also be observed in the impulse responses to aggregate shocks presented in the Figures 2 and 3, where the responses are shown as deviations from steady state values that differ between both versions (with higher and lower degree of price rigidity). All impulse responses in the paper are given in percentage deviations from the steady state. The responses to a contractionary productivity shock are very similar for both cases (see Figure 2). Substantial differences can only be observed for the responses of the inflation rate and the loan rate. The latter increases to a larger extent under more rigid prices, which tends to amplify the adverse borrowing conditions. Hence,

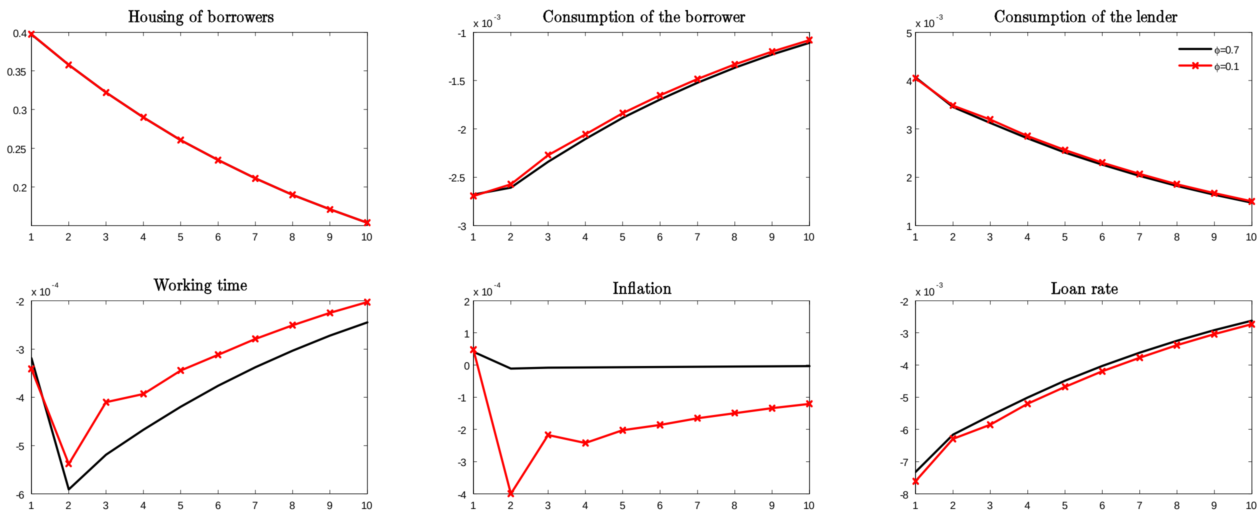

Figure 3: Responses to a lower liquidation value under optimal policy w/o money rationing [Note: Steady states are not identical.]

in order to stabilize inflation, optimal policy accepts a more pronounced loan contraction than under less rigid prices. The responses of consumption and working time are virtually identical for both versions, while it should be noted that they are presented as deviations from different steady states. Thus, monetary policy hardly mitigates the distortion due to the credit market friction even when prices are more flexible. Figure 3 shows responses to a fall in the liquidation value of housing. Again, the inflation response reveals that under a reasonable degree of price stickiness ($\phi = 0.7$), an optimizing central bank mainly aims at stabilizing prices. Under more flexible prices, the central bank strongly reduces the inflation rate. This is associated with a more pronounced reduction in the loan rate, which mitigates the credit market distortion in a negligible way (as indicated by virtually identical responses of the borrower's housing share).

The last column of Table 1 shows results under an optimal monetary policy for a smaller liquidation value of collateral, $z = 0.4$. Intuitively, the distortion induced by the borrowing constraint is then more pronounced, which leads to larger differences from the first best allocation compared to the case with the benchmark parameter values ($z = 0.8$). The exception is the lender's consumption value which is now slightly larger, given that borrower's consumption is more restricted. Overall, the central bank is not willing to deviate from fully stabilizing prices in favor of reducing distortions due to financial frictions (see the impulse response functions in Appendix A.8).

4.2.2 Rationed money supply and loan purchases

When the central bank sets the policy rate R_t^m below the lender's marginal rate of intertemporal substitution, which does not exceed the borrower's marginal rate of intertemporal substitution, it effectively rations money supply. Specifically, the borrower's and the lender's marginal valuation

Table 2: Steady state values with and w/o money rationing for z=0.4

	Optimal policy w/o m. rationing	Policy regime I with m. rationing	Policy regime II with m. rationing	First best
Consumption of the borrower	0.3003	0.3004	0.3005	0.3018
Consumption of the lender	0.1744	0.1743	0.1742	0.1742
Borrower's housing share	0.6369	0.6150	0.5954	0.5
Working time	0.3233	0.3234	0.3234	0.3248
Loan rate	1.0044	1.0049	1.0052	–
Inflation rate	1	1	1	–
Policy rate	–	1.0040	1.0040	–
Share of purchased loans	–	0.5	1	–
Representative agent utility	–3.12145	–3.12126	–3.12112	–3.12078

of money are then larger than its price, such that the money supply constraints (2) and (4) are binding, $\eta_{i,t} > 0$ and $\varsigma_{l,t} > 0$. The money supply instruments κ_t^B and κ_t are then non-neutral in the sense that the central bank can affect the private sector behavior by changing the amount of money supplied in exchange for eligible assets, i.e. treasuries and secured loans. Specifically, the loan rate can be manipulated not only via the lender's marginal rate of intertemporal substitution but also via central bank purchases of loans (see 16).

Non-optimizing policy with money rationing We first consider the case of a severe credit market friction, i.e. a particularly low average liquidation value for collateral ($z = 0.4$). For this case and for the other parameter values applied in this paper (see also Table A1 in Appendix A.7) the borrowing constraint will be binding even when the central bank conducts loan purchases. We therefore examine the steady state under two non-optimizing monetary policy regimes acting under money rationing. The steady state values of selected variables for the two regimes with money rationing are given in Table 2. These policy regimes are both characterized by an inflation rate equal to one and a policy rate set at 1.004. They only differ with regard to the share of purchased loans κ, which equals 50% (regime I) and 100% (regime II). The results presented in Table 2 show that these two non-optimizing policies outperform the optimal policy without money rationing. Specifically, the deviations of the allocation of consumption, housing, and working time from the first best allocation are reduced under the money rationing regimes and when more loans are purchased (see regime II). The superiority of these regimes is confirmed by the steady state utility values of the representative agent.

Optimizing policy under money rationing We finally consider the case where the credit market friction is less severe, $z = 0.8$. For this case, we find that the central bank it actually able to undo the distortions stemming from the borrowing constraint by purchasing loans. Under a money rationing regime, it can be shown in a straightforward way that the policy problem can be greatly simplified by using that the central bank is equipped with additional instruments (see Appendix A.6). In particular, we use that the central bank can set the fraction of eligible bonds κ_t^B to adjust the amount of money available for household members in a way that is consistent with the optimally chosen allocation, and that the policy rate R_t^m together with the share of purchased loans κ_t can be set to implement a favorable loan rate and to ease the borrowing constraint. In fact, the central bank instruments R_t^m and κ_t can be used to slacken the borrowing constraint by setting them according to (28) and (29) for $\zeta_{b,t} = 0$. This property is summarized in the following proposition.

Proposition 3 *Let $\{\widetilde{c}_b, \widetilde{c}_l\}$ be a long-run allocation of borrowers' and lenders' consumption that is not constrained by (3) and satisfies $(\widetilde{c}_b - \widetilde{c}_l)\,\epsilon_b \widetilde{c}_b^{-\sigma} > 2(1+\upsilon)z\mathsf{v}(h)/(1-\beta)$, where $\mathsf{v}(h) = \gamma(0.5h)^{1-\sigma_h}$. Then, this consumption allocation $\{\widetilde{c}_b, \widetilde{c}_l\}$ can only be implemented by the central bank if it purchases loans under money rationing such that the pair $\{R^m, \kappa\}$ satisfies $\kappa \in (0,1]$, $R^m \in [1, \widetilde{R}^L)$, where $\widetilde{R}^L = \epsilon_b \widetilde{c}_b^{-\sigma} \widetilde{\widetilde{c}}^{\sigma} \widetilde{\pi}/\beta$, as well as $\kappa/R^m \geq [(\widetilde{c}_b - \widetilde{c}_l)(1-\beta)\,\epsilon_b \widetilde{c}_b^{-\sigma} - 2(1+\upsilon)z\mathsf{v}(h)]/[z\mathsf{v}(h)\widetilde{R}^L]$, and $R^m(1 + [(\epsilon_l \widetilde{c}_l^{-\sigma} - \epsilon_b \widetilde{c}_b^{-\sigma})\epsilon_b^{-1}\widetilde{c}_b^{\sigma}]\,(1+\upsilon)/\kappa) = \widetilde{R}^L$.*

Proof. See Appendix A.6 ■

When the central bank is able to implement a long-run allocation that is not distorted by the borrowing constraint (see Proposition 3), there exist pairs of sequences $\{R_t^m \in [1, R_t^L), \kappa_t \in (0,1]\}_{t=0}^{\infty}$ that can also undo the distortion due to the borrowing constraint in a sufficiently small neighborhood of this steady state, which we verify numerically for the optimal policy plan under a liquidation value z equal to 0.8 (see Table 3). Given that there are many pairs of sequences for the policy instruments that can implement the optimal plan (see proof of Proposition 3), the policy rate R_t^m, which is below the lender's marginal rate of intertemporal substitution, and the share of liquidated loans κ_t are identified by assuming that the borrowing constraint is just not binding. Optimal policy under money rationing, which – for the applied parameter values – implies a positive amount of loan purchases, then enhances welfare compared to the case where monetary policy without money rationing is conducted in an optimal way (see Section 4.2.1). We compute welfare of the representative agent using

$$V = E_0 \sum_{t=0}^{\infty} \beta^t 0.5\,(u_{b,t} + u_{l,t}),$$

for different policy regimes and assume that the initial values are identical with the corresponding steady state values. Deviations from welfare under the first best allocation ($*$) are then measured as permanent consumption values that compensate for the welfare loss under alternative policy regimes, $(c_{perm} - c^*_{perm})/c^*_{perm}$, where $c_{perm} = ((1-\beta)(1-\sigma)V + 1)^{1/(1-\sigma)}$. The computed welfare gain of money rationing (combined with loan purchases) is considerably small, while the loss under

Table 3: Steady state values with and w/o money rationing for z=0.8

	Optimal policy w/o money rationing	Optimal policy with money rationing	First best
Consumption of the borrower	0.3009	0.3012	0.3018
Consumption of the lender	0.1739	0.1737	0.1742
Borrower's housing share	0.5334	0.5	0.5
Working time	0.3235	0.3236	0.3248
Loan rate	1.0091	1.0086	–
Inflation rate	1	1	–
Policy rate	–	1.0026	–
Fraction of purchased loans	–	0.6860	–
Representative agent utility	–3.12086	–3.12083	–3.12078

a non-rationing regime compared to welfare under the first best allocation is almost twice as large, 0.0021, as under the rationing regime, 0.0012 (where computations are based on second order approximations).

The steady state values given in Table 3 reveal that the differences between the two types of optimal policy regimes are relatively small (except for the allocation of housing), since the credit market friction is less severe. Nonetheless, they show that an optimal policy under money rationing is able to reduce the differences between the first best allocation and the allocation in a competitive equilibrium. The only exception refers to the lender's consumption, which is lower under both optimal policy regimes than under first best. In the case of non-rationed money supply, the value is slightly larger than under money rationing, given that the borrower's consumption is effectively constrained by its collateral value. The allocation under non-rationed money supply exhibits the largest difference to first best for the borrower's housing. This, however, does not have a strong impact on welfare, due to the small utility weight assigned to housing ($\gamma = 0.1$ compared to $\chi = 98$ for the disutility on working time).

The Figures 4 and 5 further show impulse responses to a contractionary productivity shock and to a reduction in the liquidation value of loans. The responses to the productivity shock (see Figures 4) correspond to the results for the steady state values (see Table 3), i.e. that the allocation hardly differs between both types of optimal policy regimes, except for the distribution of housing. Under money rationing, the central bank is nevertheless able to undo the distortion due to the borrowing constraint by purchasing loans, such that the consumption gap is reduced and housing is equally held by borrowers and lenders. In contrast to productivity shocks, the responses to a reduction in the liquidation value of loans can be associated with substantial differences between both types of policies. As long as the reduction is not too pronounced, the central bank can off-

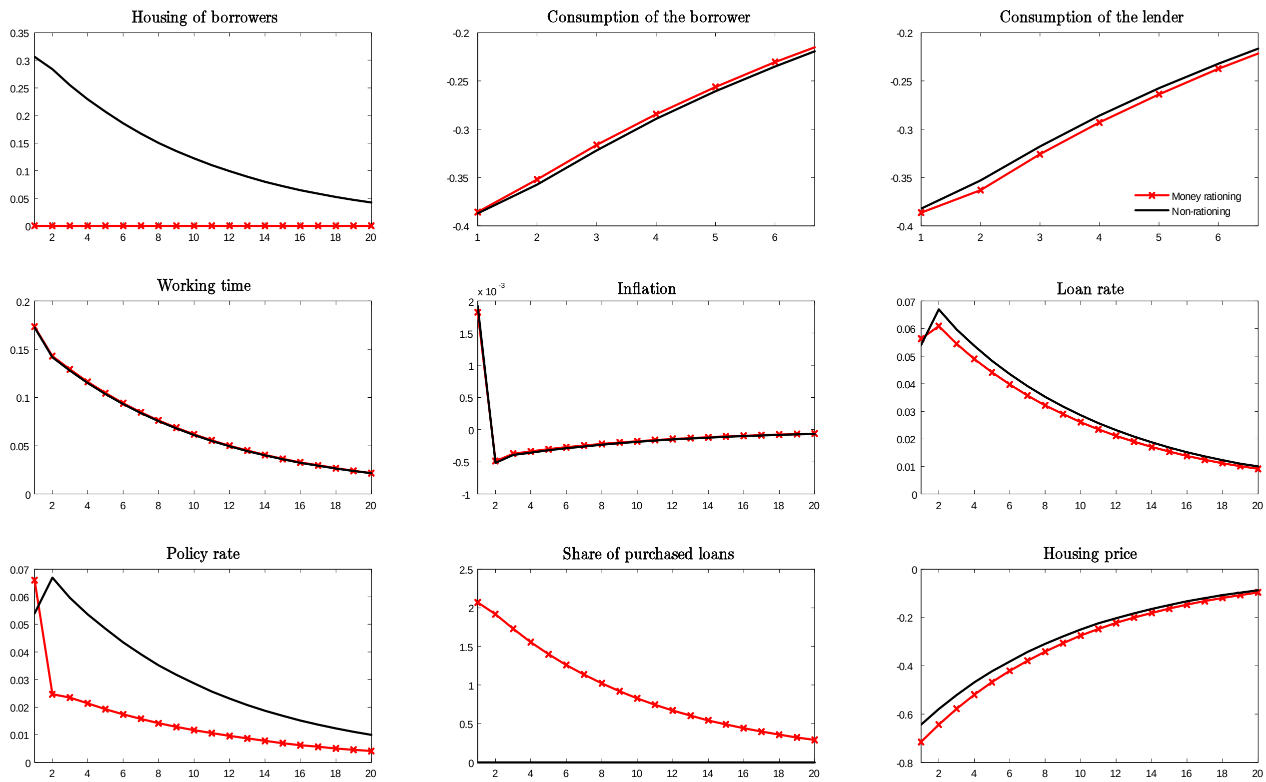

Figure 4: Responses to a contractionary productivity shock under optimizing policies [Note: Steady states are not identical.]

set this shock under a money rationing regime by purchasing loans at a below-market rate. The allocation is then unaffected by the decline in the liquidation value and prices are fully stabilized (see Figure 5). For this, the increase in the share of purchased loans, which exerts an expansionary and thus inflationary effect, is accompanied with an increases the policy rate to avoid an upward shift in prices. Overall, the impulse responses indicate that loan purchases are particularly effective to mitigate exogenous shifts in credit market conditions.

5 Conclusion

In this paper, we examined optimal monetary policy in a sticky price model where money is essential and borrowing between private agents is constrained by available collateral. While the credit market friction could be eased by a low nominal interest rate, a welfare-maximizing central bank predominantly aims at minimizing distortions due to imperfectly set prices, such that the paradigm of price stability prevails. As a consequence, optimal policy largely ignores the credit friction, when monetary policy is conducted in a conventional way, in the sense that only one instruments is available. If, however, the central bank supplies money at a low price against a bounded set of eligible assets, access to money is effectively rationed, which allows the central bank to simultaneously control the price and the amount of money. In this case, the central bank

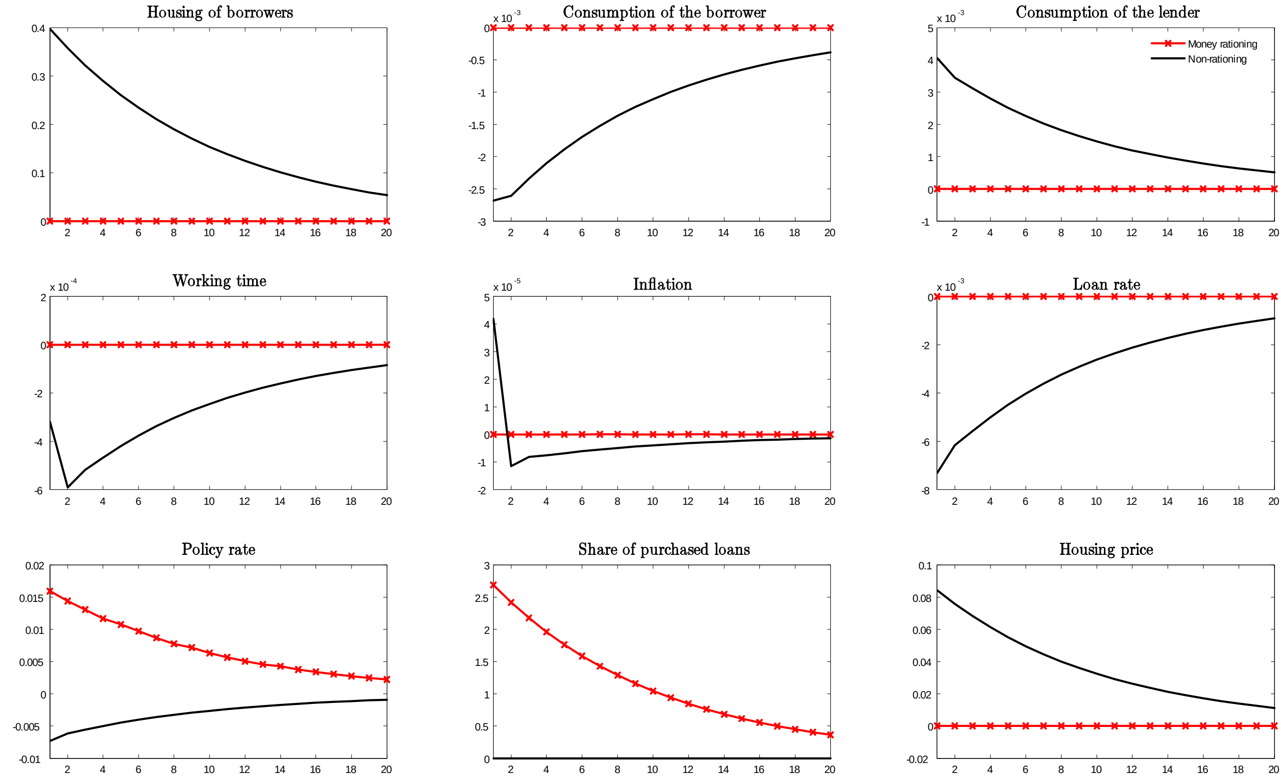

Figure 5: Responses to a lower liquidation value under optimizing policies [Note: Steady states are not identical.]

can – in addition to the price rigidity – address the credit market friction by purchasing secured loans. Such a policy tends to reduce the lending rate and can be welfare enhancing compared to a conventionally conducted optimal policy monetary regime (without money rationing). The analysis further suggests that satiating agents' demand for money is in general not recommendable, as rationing of money supply endows the central bank with quantitative instruments that allow manipulating market prices of eligible asset as well as influencing aggregate demand in a welfare-enhancing way.

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A Appendix

A.1 Competitive equilibrium

Definition 2 *A competitive equilibrium is a set of sequences* $\{c_{b,t},\ c_{l,t},\ n_{b,t},\ n_{l,t},\ n_t,\ l_t,\ l_t^r,\ i_{b,t},\ i_{l,t},\ i_t^L,\ m_{b,t}^H,\ m_{l,t}^H,\ m_t^H,\ b_{b,t},\ b_{l,t},\ b_t,\ b_t^T,\ w_t,\ mc_t,\ \tilde{Z}_t,\ s_t,\ \pi_t,\ R_t^L,\ \zeta_{b,t},\ h_{l,t},\ h_{b,t},\ q_t\ \}_{t=0}^{\infty}$ *satisfying*

$$n_{l,t} = n_{b,t}, \tag{35}$$

$$\chi n_{b,t}^{\eta} = w_t \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}], \tag{36}$$

$$1/R_t^L = \left(c_{b,t}^{\sigma}/\epsilon_b\right)\beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}] + \zeta_{b,t}\left(c_{b,t}^{\sigma}/\epsilon_b\right)/(1+\upsilon), \tag{37}$$

$$1/R_t^L = \beta\left(c_{l,t}^{\sigma}/\epsilon_l\right)E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\{1 + \frac{\kappa_t}{1+\upsilon}[\frac{R_t^L}{R_t^m} - 1]\},\ \textit{if}\ \varsigma_{l,t} > 0, \tag{38}$$

$$\textit{or}\ 1/R_t^L = \beta\left(c_{l,t}^{\sigma}/\epsilon_l\right)E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}],\ \textit{if}\ \varsigma_{l,t} = 0,$$

$$c_{l,t} = i_{l,t} + m_{l,t-1}^H \pi_t^{-1} - (1+\upsilon)\left(l_t/R_t^L\right)\ \textit{if}\ \psi_{l,t} > 0, \tag{39}$$

$$\textit{or}\ c_{l,t} < i_{l,t} + m_{l,t-1}^H \pi_t^{-1} - (1+\upsilon)\left(l_t/R_t^L\right)\ \textit{if}\ \psi_{l,t} = 0,$$

$$c_{b,t} = i_{b,t} + m_{b,t-1}^H \pi_t^{-1} + [(1+\upsilon)l_t + l_t^r]/R_t^L\ \textit{if}\ \psi_{b,t} > 0, \tag{40}$$

$$\textit{or}\ c_{b,t} < i_{b,t} + m_{b,t-1}^H \pi_t^{-1} + [(1+\upsilon)l_t + l_t^r]/R_t^L\ \textit{if}\ \psi_{b,t} = 0,$$

$$R_t^m i_{l,t} = \kappa_t^B b_{l,t-1}\pi_t^{-1}\quad \textit{if}\ \eta_{i,t} > 0,\quad \textit{or}\ R_t^m i_{l,t} < \kappa_t^B b_{l,t-1}\pi_t^{-1}\quad \textit{if}\ \eta_{l,t} = 0, \tag{41}$$

$$R_t^m i_{b,t} = \kappa_t^B b_{b,t-1}\pi_t^{-1}\quad \textit{if}\ \eta_{i,t} > 0,\quad \textit{or}\ R_t^m i_{b,t} < \kappa_t^B b_{b,t-1}\pi_t^{-1}\quad \textit{if}\ \eta_{b,t} = 0, \tag{42}$$

$$l_t = z_t q_t h_{b,t}\quad \textit{if}\ \zeta_{b,t} > 0,\ \textit{or}\ l_t \leq z_t q_t h_{b,t}\quad \textit{if}\ \zeta_{b,t} = 0, \tag{43}$$

$$R_t^m i_t^L = \kappa_t l_t\quad \textit{if}\ \varsigma_{l,t} > 0\ \textit{or}\ i_t^L = 0\quad \textit{if}\ \varsigma_{l,t} = 0, \tag{44}$$

$$l_t^r/R_t^L = i_t^L\ \textit{if}\ \zeta_{b,t} > 0\ \ \textit{or}\ l_t^r/R_t^L \leq i_t^L\ \textit{if}\ \zeta_{b,t} = 0, \tag{45}$$

$$\zeta_{b,t} q_t z_t = \gamma(h_{l,t}^{-\sigma_h} - h_{b,t}^{-\sigma_h}), \tag{46}$$

$$q_t \chi n_{l,t}^{\eta}/w_t = \gamma h_{l,t}^{-\sigma_h} + \beta E_t[q_{t+1}\chi n_{l,t+1}^{\eta}/w_{t+1}], \tag{47}$$

$$h = h_{l,t} + h_{b,t}, \tag{48}$$

$$n_t = n_{l,t} + n_{b,t}, \tag{49}$$

$$m_{b,t}^H = m_{l,t}^H, \tag{50}$$

$$b_t = b_{b,t} + b_{l,t}, \tag{51}$$

$$m_t^H = m_{b,t}^H + m_{l,t}^H, \tag{52}$$

$$i_{b,t} = (1+\Omega_t)m_{b,t}^H - m_{b,t-1}^H \pi_t^{-1}, \tag{53}$$

$$i_{l,t} = (1+\Omega_t)m_{l,t}^H - m_{l,t-1}^H \pi_t^{-1}, \tag{54}$$

$$0 = (1-\tau^n)\,w_t - mc_t a_t \alpha n_t^{\alpha-1}, \tag{55}$$

$$\tilde{Z}_t = [\varepsilon/(\varepsilon-1)][Z_{1,t}/Z_{2,t}], \tag{56}$$

$$1 = (1-\phi)(\tilde{Z}_t)^{1-\varepsilon} + \phi\pi_t^{\varepsilon-1}, \tag{57}$$

$$s_t = (1-\phi)\tilde{Z}_t^{-\varepsilon} + \phi s_{t-1}\pi_t^{\varepsilon}, \tag{58}$$

$$a_t n_t^{\alpha}/s_t = c_{l,t} + c_{b,t}, \tag{59}$$

$$b_t^T = \Gamma b_{t-1}^T/\pi_t, \tag{60}$$

$$b_t^T = b_t + m_t^H, \tag{61}$$

where the multiplier and auxiliary variables $\psi_{l,t}$, $\psi_{b,t}$, $\eta_{i,t}$, $\varsigma_{l,t}$, $Z_{1,t}$, *and* $Z_{2,t}$ *satisfy* $\psi_{l,t} = \left(R_t^L - 1\right)\left(\mu_t \chi n_{l,t}{}^{\eta}/w_t\right) + R_t^L \kappa_t \varsigma_{l,t}/(1+\upsilon) \geq 0$, $\psi_{b,t} = (R_t^L - 1)\left(\mu_t \chi n_{b,t}{}^{\eta}/w_t\right) + \zeta_{b,t} R_t^L/(1+\upsilon) \geq 0$,

$\eta_{i,t} = (\epsilon_b c_{b,t}^{-\sigma} + \epsilon_l c_{l,t}^{-\sigma})/R_t^m - \beta E_t(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1} \geq 0$, $\varsigma_{l,t} = (\chi n_{l,t}^{\eta}/w_t)(R_t^L - R_t^m)/R_t^m \geq 0$, $Z_{1,t} = (\chi n_{b,t}^{\eta}/w_t)(a_t n_t^{\alpha}/s_t)mc_t + \phi\beta E_t \pi_{t+1}^{\varepsilon} Z_{1,t+1}$, $Z_{2,t} = (\chi n_{b,t}^{\eta}/w_t)(a_t n_t^{\alpha}/s_t) + \phi\beta E_t \pi_{t+1}^{\varepsilon-1} Z_{2,t+1}$, *and the transversality conditions, a monetary policy setting* $\{R_t^m \geq 1,\ \kappa_t^B \in (0,1],\ \kappa_t \in [0,1],\ \Omega_t \geq 0\}_{t=0}^{\infty}$, *a subsidy* τ^n, *given* Γ, $\{a_t, z_t\}_{t=0}^{\infty}$, $m_{b,-1}^H = m_{l,-1}^H > 0$, $b_{b,-1} = b_{l,-1} > 0$, $b_{-1} = b_{b,-1} + b_{l,-1} > 0$, $m_{-1}^H = m_{b,-1}^H + m_{l,-1}^H > 0$, *and* $s_{-1} = 1$.

A.2 First best

Proof of proposition 1. Using $n_t = n_{l,t} + n_{b,t}$ and $n_{l,t} = n_{b,t}$, the social planer problem can be summarized as

$$\max_{\{c_{l,t},c_{b,t},h_{l,t},h_{b,t},n_t,n_{j,t},y_{k,t}\}_{t=0}^{\infty}} E\sum_{t=0}^{\infty}\beta^t\Big\{0.5\left[\epsilon_b(c_{b,t}^{1-\sigma}-1)+\epsilon_l(c_{l,t}^{1-\sigma}-1)\right](1-\sigma)^{-1}$$
$$-\chi(0.5n_t)^{1+\eta}(1+\eta)^{-1} + 0.5\gamma[(h_{b,t-1}^{1-\sigma_h}-1)+(h_{l,t-1}^{1-\sigma_h}-1)](1-\sigma_h)^{-1}\Big\}$$
$$\text{s.t.}\ \ a_t\int_0^1 n_{j,t}^{\alpha}dj = \int_0^1 y_{k,t}dk,\ \int_0^1 n_{j,t}dj = n_t,$$
$$h = \int_0^1 h_{b,t}di + \int_0^1 h_{l,t}di,\ \int_0^1 y_{k,t}^{\frac{\varepsilon-1}{\varepsilon}}dk = (\int_0^1 c_{b,t}di + \int_0^1 c_{l,t}di)^{\frac{\varepsilon-1}{\varepsilon}}.$$

The first order conditions can be simplified to $\epsilon_b c_{b,t}^{-\sigma} = \epsilon_l c_{l,t}^{-\sigma}$, $\chi(0.5n_t)^{\eta} = a_t\alpha n_t^{\alpha-1}\epsilon_b c_{b,t}^{-\sigma}$, $h_{b,t}^{-\sigma_h} = h_{l,t}^{-\sigma_h}$, $h_{b,t} + h_{l,t} = h$, and $c_{l,t} + c_{b,t} = a_t n_t^{\alpha}$. These conditions immediately lead to $c_{b,t} = a_t^{\frac{1+\eta}{1-\alpha+\eta+\alpha\sigma}}[\alpha\epsilon_b/(\chi 0.5^{\eta})]^{\frac{\alpha}{1-\alpha+\eta+\alpha\sigma}}[1+(\epsilon_l/\epsilon_b)^{\frac{1}{\sigma}}]^{-\frac{1-\alpha+\eta}{1-\alpha+\eta+\alpha\sigma}}$, $c_{l,t} = (\epsilon_l/\epsilon_b)^{\frac{1}{\sigma}}c_{b,t}$, $h_{b,t} = h_{l,t}$, $n_t = (c_t/a_t)^{1/\alpha}$, which characterize the first best allocation. ■

A.3 Monetary policy under flexible prices

Proof of proposition 2. Consider the competitive equilibrium as given in Definition 1. The long-run equilibrium values $\{c_b,\ c_l,\ n,\ R^L,\ h_b,\ q\}$ then satisfy

$$1/R^L = (c_b^{\sigma}/\epsilon_b)\beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1} + (c_b^{\sigma}/\epsilon_b)\gamma((1+\upsilon)qz)^{-1}\left[(h-h_b)^{-\sigma_h} - h_b^{-\sigma_h}\right] \quad (62)$$

$$1/R^L = [(c_l^{\sigma}/\epsilon_l)\beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1}\{1 + [\kappa/(1+\upsilon)][(R^L/R^m) - 1]\},\ \text{if}\ \varsigma_l > 0, \quad (63)$$
$$\text{or}\ 1/R^L = \beta(c_l^{\sigma}/\epsilon_l)\beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1}\ \text{and}\ R^m = R^L\ \text{if}\ \varsigma_l = 0,$$

$$n^{1+\eta-\alpha}/\omega = \beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1}, \quad (64)$$

$$c_b = c_l + [zqh_b 2(1+\upsilon) + l^r]/R^L,\ \text{if}\ \zeta_b > 0,\ \text{where}\ l^r = 0,\ \text{if}\ \varsigma_l = 0, \quad (65)$$
$$\text{or}\ \ c_b \leq c_l + [zqh_b 2(1+\upsilon) + l^r]/R^L,$$

$q = \gamma\omega(h-h_b)^{-\sigma_h}/\left(n^{\eta+1-\alpha}(1-\beta)\right)$ and $n^{\alpha} = c_l + c_b$. Given that the long-run first best allocation satisfies $\epsilon_b(c_b^*)^{-\sigma} = \epsilon_l(c_l^*)^{-\sigma}$, and $\epsilon_i(c_i^*)^{-\sigma} = (n^*)^{1+\eta-\alpha}/\omega$, (64) implies that the implementation of a long-run efficient allocation would require $\pi = \beta$. Using $h_b^* = h_l^*$ and $\epsilon_b(c_b^*)^{-\sigma} = \epsilon_l(c_l^*)^{-\sigma}$ as well as (62) and (63), shows that long-run efficiency further requires $R^L = R^m = 1$ and thus $\varsigma_l = 0$. Eliminating q in the borrowing constraint (65), and again using $\epsilon_b(c_b^*)^{-\sigma} = \epsilon_l(c_l^*)^{-\sigma}$ and $h_b^* = h_l^*$, then gives $c_b^* = 2z(1+\upsilon)\gamma\omega(0.5h)^{1-\sigma_h}/\{[1-(\epsilon_l/\epsilon_b)^{1/\sigma}]n^{\eta+1-\alpha}(1-\beta)R^L\}$. Further, substituting out n with $n = (2c_b^*)^{1/\alpha}$ and c_b^* with $c_b^* = (\epsilon_b\omega)^{\frac{\alpha}{1-\alpha+\eta+\alpha\sigma}}[1+(\epsilon_l/\epsilon_b)^{\frac{1}{\sigma}}]^{-\frac{1-\alpha+\eta}{1-\alpha+\eta+\alpha\sigma}}$, implies that the implementation of a long-run efficient allocation requires the loan rate R^L to satisfy $R^L = \Lambda$, where

$$\Lambda = \frac{2z(1+\upsilon)}{1-(\epsilon_l/\epsilon_b)^{\frac{1}{\sigma}}}\frac{\gamma\omega(0.5h)^{1-\sigma_h}}{0.5(1-\beta)}\left(2[\epsilon_b\omega]^{\frac{\alpha}{1-\alpha+\eta+\alpha\sigma}}[1+(\epsilon_l/\epsilon_b)^{\frac{1}{\sigma}}]^{-\frac{1-\alpha+\eta}{1-\alpha+\eta+\alpha\sigma}}\right)^{-(\eta+1)/\alpha}. \quad (66)$$

Given that Λ only consists of exogenously given terms (see 66), the two conditions $R^L = \Lambda$ and $R^L = 1$ are in general inconsistent, implying that first best cannot be implemented. ■

A.4 Non-optimizing policy with money rationing

In this Appendix, we describe how a competitive equilibrium can be implemented by a central bank that effectively rations money supply. Consider a competitive equilibrium as given in Definition 2. Under under money rationing a competitive equilibrium can then be reduced to a set of sequences $\{c_{b,t},\, c_{l,t},\, n_t,\, mc_t,\, \tilde{Z}_t,\, s_t,\, \pi_t,\, R_t^L,\, h_{b,t},\, q_t,\, b_t,\, b_t^T,\, m_t^H\}_{t=0}^\infty$ satisfying (57)-(61),

$$n_t^{1+\eta-\alpha}/(\omega mc_t a_t) = \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}], \quad (67)$$

$$\epsilon_b c_{b,t}^{-\sigma}/R_t^L = \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}] + \gamma((h-h_{b,t})^{-\sigma_h} - h_{b,t}^{-\sigma_h})/[q_t z_t(1+\upsilon)] \quad (68)$$

$$\epsilon_l c_{l,t}^{-\sigma}/R_t^L = \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\{1+[\kappa_t/(1+\upsilon)][(R_t^L/R_t^m)-1]\}, \quad (69)$$

$$c_{b,t} - c_{l,t} = z_t q_t h_{b,t}\left((1+\upsilon)\left(2/R_t^L\right) + \kappa_t/R_t^m\right), \quad (70)$$

$$q_t n_t^{1+\eta-\alpha}/(mc_t a_t) = \gamma\omega\left(h-h_{b,t}\right)^{-\sigma_h} + \beta E_t[q_{t+1}n_{t+1}^{1+\eta-\alpha}/(mc_{t+1}a_{t+1})], \quad (71)$$

$$Z_{1,t}/Z_{2,t} = \tilde{Z}_t\left(\varepsilon-1\right)/\varepsilon, \text{ where } Z_{1,t} = n_t^{1+\eta}\left(\omega s_t\right)^{-1} + \phi\beta E_t\pi_{t+1}^{\varepsilon}Z_{1,t+1}, \quad (72)$$

$$\text{and } Z_{2,t} = n_t^{1+\eta}\left(\omega mc_t s_t\right)^{-1} + \phi\beta E_t\pi_{t+1}^{\varepsilon-1}Z_{2,t+1},$$

$$0.5(1+\Omega_t)m_t^H = c_{l,t} + (1+\upsilon)z_t q_t h_{b,t}/R_t^L, \quad (73)$$

$$\kappa_t^B b_{t-1}\pi_t^{-1}/R_t^m = (1+\Omega_t)m_t^H - m_{t-1}^H\pi_t^{-1}, \quad (74)$$

(where $\omega = 1/\left[(1-\tau^n)\left(\chi/\alpha\right)(0.5)^\eta\right]$) the transversality conditions, a monetary policy setting $\{R_t^m \in [1, \epsilon_l c_{l,t}^{-\sigma}/[\beta E_t 0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}),\, \kappa_t \in [0,1],\, \kappa_t^B \in (0,1],\, \Omega_t \geq 0\}_{t=0}^\infty$, a subsidy τ^n, given $\{a_t, z_t\}_{t=0}^\infty$, $\Gamma > 0$, $s_{-1} = 1$, $b_{-1}^T > 0$, $b_{-1} > 0$, and $m_{-1}^H > 0$.

For the characterization of the steady state, where variables describing the allocation are either constant or grow/shrink at a constant rate, we assume that the central bank sets an inflation target π^* that has to be identical with the long-run inflation rate π in a rational expectations equilibrium. To ensure this, we consider two scenarios for the supply of government bonds (60). First, suppose that public debt is issued in a way that is consistent with the inflation target, $\Gamma = \pi^*$, such that (60) is consistent with $\pi^* = \pi$ for constant real debt. Given π, the steady state values for $\tilde{Z}_t$, mc_t, and s_t are given by $\tilde{Z} = [\frac{1-\phi\pi^{\varepsilon-1}}{1-\phi}]^{1/(1-\varepsilon)}$, $mc = \tilde{Z}\frac{\varepsilon-1}{\varepsilon}\frac{1-\phi\beta\pi^\varepsilon}{1-\phi\beta\pi^{\varepsilon-1}}$, and $s = \frac{(1-\phi)\tilde{Z}^{-\varepsilon}}{1-\phi\pi^\varepsilon}$. Then, the steady state values n, c_l, c_b, R^L, h_b, and q, can – for a constant long-run policy rate $R^m \geq 1$ – be determined by solving

$$
\begin{aligned}
n^{\alpha}/s &= c_l + c_b, \\
n^{1+\eta-\alpha}\pi/\beta &= 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\omega mc, \\
1/R^L &= \beta\left(c_l^{\sigma}/\epsilon_l\right)0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1}\left(1 + (\kappa/(1+\upsilon))\left((R^L/R^m) - 1\right)\right), \\
\epsilon_b c_b^{-\sigma} &= R^L\beta 0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})\pi^{-1} + R^L\left(\gamma/qz(1+\upsilon)\right)\left((h-h_b)^{-\sigma_h} - h_b^{-\sigma_h}\right), \\
c_b - c_l &= zqh_b\left((1+\upsilon)\left(2/R^L\right) + \kappa/R^m\right), \\
\gamma\omega(h-h_b)^{-\sigma_h} &= qn^{\eta+1-\alpha}mc^{-1}\left(1-\beta\right),
\end{aligned}
$$

Once c_l, R^L, h_b, and q are known, the steady state values m^H and b are given by $m^H = \frac{c_l+(1+\upsilon)zqh_b/R^L}{0.5(1+\Omega)}$ and $b = \frac{R^m\pi}{\kappa^B}\left(1+\Omega-\pi^{-1}\right)m^H$ given κ^B and Ω.

Now, suppose that government bonds are supplied at a rate that is not identical to the inflation target, $\Gamma \neq \pi^*$. Then, the total stock of bonds b_t^T might grow or shrink in a long-run equilibrium at a constant rate Γ/π (see 60). The money demand condition (73) then requires for constant steady state values c_l, R^L, h_b, q, and z, that the term $\widetilde{m}_t = (1+\Omega_t)m_t^H$ is also constant in the long-run. Using the latter and (61) to eliminate b_t^T and m_t^H in (60) and (61), leads to $\kappa_t^B b_t = R_t^m\pi_t[\widetilde{m}_t - \widetilde{m}_{t-1}(1+\Omega_{t-1})^{-1}\pi_t^{-1}]$ and $[b_t + \widetilde{m}_t/(1+\Omega_t)] = \Gamma\left[b_{t-1} + \widetilde{m}_{t-1}/(1+\Omega_{t-1})\right]/\pi_t$. Further, substituting out b_t, gives

$$
\left[\frac{R_t^m\pi_t}{\kappa_t^B}\left(\widetilde{m}_t - \frac{\widetilde{m}_{t-1}\pi_t^{-1}}{1+\Omega_{t-1}}\right) + \frac{\widetilde{m}_t}{1+\Omega_t}\right] = \frac{\Gamma}{\pi_t}\left[\frac{R_{t-1}^m\pi_{t-1}}{\kappa_{t-1}^B}\left(\widetilde{m}_{t-1} - \frac{\widetilde{m}_{t-2}\pi_{t-1}^{-1}}{1+\Omega_{t-2}}\right) + \frac{\widetilde{m}_{t-1}}{1+\Omega_{t-1}}\right]. \quad (75)
$$

Taking the limit $t \to \infty$ of both sides of (75), we can use that for a constant long-run inflation rate π and a constant policy rate R^m a steady state is characterized by a constant value for $\widetilde{m}_t$. The term in the square brackets in (75) grows/shrinks with the constant rate Γ/π. When the growth rate of bonds exceeds the inflation rate, $\Gamma > \pi$, this can be guaranteed by a permanently shrinking value for κ_t^B. Thus, the central bank can let κ_t^B grow at the rate π/Γ and can let the share of money supplied outright go to zero in the long-run, i.e. it can set κ_t^B and $1/\Omega_t$ according to $\lim_{t\to\infty}\kappa_t^B/\kappa_{t-1}^B = \pi/\Gamma < 1$ and $\lim_{t\to\infty} 1/\Omega_t = 0$ if $\Gamma > \pi$. For $\Gamma < \pi$, the term in the square bracket in (75) permanently shrinks, which can not be supported by a growing value κ_t^B without violating the restriction $\kappa_t^B \leq 1$. In this case, the central bank can let κ_t^B go to zero and can let the share $1/\Omega_t$ of money supplied outright grow in a long-run equilibrium. For $\pi = 1$ and $\Gamma < 1$, it can thus set κ_t^B and $1+1/\Omega_t$ in a steady state according to $\lim_{t\to\infty}\left(1+1/\Omega_t\right)/\left(1+1/\Omega_{t-1}\right) = 1/\Gamma > 1$ and $\lim_{t\to\infty}\kappa_t^B = 0$.

A.5 Optimal monetary policy under non-rationed money supply

In this Appendix, we consider the policy problem of the central bank that neglects the possibility of effectively rationing money supply. Hence, the money supply constraints (2) and (4) are disregarded for the derivation of the optimal policy plan, which can – by accounting for the remaining constraints imposed by a competitive equilibrium (see Definition 2) – be summarized as

$$\max_{\{c_{b,t},c_{l,t},n_t,mc_t,\tilde{Z}_t,Z_{1,t},Z_{2,t},s_t,\pi_t,h_{b,t}\,q_t,R_t^L\}_{t=0}^{\infty}} \min_{\{\lambda_{0,t},\ldots\lambda_{10,t}\}_{t=0}^{\infty}} \tag{76}$$

$$E\sum_{t=0}^{\infty}\beta^t\left[\begin{array}{c}0.5\epsilon_b(c_{b,t}^{1-\sigma}-1)\,(1-\sigma)^{-1}+0.5\epsilon_l(c_{l,t}^{1-\sigma}-1)\,(1-\sigma)^{-1}-\chi\,(0.5n_t)^{1+\eta}\,(1+\eta)^{-1}\\ +0.5\gamma(h_{b,t}^{1-\sigma_h}-1)\,(1-\sigma_h)^{-1}+0.5\gamma((h-h_{b,t})^{1-\sigma_h}-1)\,(1-\sigma_h)^{-1}\end{array}\right],$$

$$+\lambda_{0,t}\left[0.5\epsilon_l c_{l,t}^{-\sigma}-0.5\epsilon_b c_{b,t}^{-\sigma}\qquad+0.5R_t^L\,(\gamma/(1+\upsilon)q_t z_t)\left((h-h_{b,t})^{-\sigma_h}-h_{b,t}^{-\sigma_h}\right)\right]$$

$$+\lambda_{1,t}\left[(1-\tau^n)\,\chi 0.5^\eta n_t^{\eta+1-\alpha}/\,(mc_t a_t\alpha)-\beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma}+\epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\right]$$

$$+\lambda_{2,t}\left[a_t n_t^\alpha/s_t-c_{l,t}-c_{b,t}\right]+\lambda_{3,t}\left[s_t-\phi s_{t-1}\pi_t^\varepsilon-(1-\phi)^{\frac{1}{1-\varepsilon}}\left(1-\phi\pi_t^{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right]$$

$$+\lambda_{4,t}\left[(1-\phi)(\tilde{Z}_t)^{1-\varepsilon}+\phi\pi_t^{\varepsilon-1}-1\right]+\lambda_{5,t}\left[\tilde{Z}_t\,(\varepsilon-1)\,/\varepsilon-Z_{1,t}/Z_{2,t}\right]$$

$$+\lambda_{6,t}\left[Z_{1,t}-(1-\tau^n)\,(\chi/\alpha)\,0.5^\eta n_t^{1+\eta}s_t^{-1}-\phi\beta E_t\pi_{t+1}^\varepsilon Z_{1,t+1}\right]$$

$$+\lambda_{7,t}\left[Z_{2,t}-(1-\tau^n)\,(\chi/\alpha)\,0.5^\eta n_t^{1+\eta}\,(mc_t s_t)^{-1}-\phi\beta E_t\pi_{t+1}^{\varepsilon-1}Z_{2,t+1}\right]$$

$$+\lambda_{8,t}\left[2\left((1+\upsilon)z_t q_t h_{b,t}/R_t^L\right)-c_{b,t}+c_{l,t}\right]+\lambda_{9,t}\left[\beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma}+\epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]-\epsilon_l c_{l,t}^{-\sigma}/R_t^L\right]$$

$$+\lambda_{10,t}\left[\frac{q_t n_t^{\eta+1-\alpha}}{mc_t a_t}-\beta E_t\frac{q_{t+1}n_{t+1}^{\eta+1-\alpha}}{mc_{t+1}a_{t+1}}-\gamma\omega(h-h_{b,t})^{-\sigma_h}\right]$$

Neglecting the conditions for $t=0$, the solution to the policy problem (76) has to satisfy the following first order conditions

$$0=0.5\gamma h_{b,t}^{-\sigma_h}-0.5\gamma(h-h_{b,t})^{-\sigma_h}+\lambda_{0,t}0.5R_t^L\,(\gamma/(1+\upsilon)q_t z_t)\left(\sigma_h(h-h_{b,t})^{-\sigma_h-1}+\sigma_h h_{b,t}^{-\sigma_h-1}\right)$$
$$+\lambda_{8,t}2\left((1+\upsilon)z_t q_t/R_t^L\right)-\lambda_{10,t}\gamma\omega\sigma_h(h-h_{b,t})^{-\sigma_h-1},$$

$$0=\lambda_{0,t}0.5\,(\gamma/(1+\upsilon)q_t z_t)\left((h-h_{b,t})^{-\sigma_h}-h_{b,t}^{-\sigma_h}\right)-\lambda_{8,t}2(1+\upsilon)z_t q_t h_{b,t}\left(R_t^L\right)^{-2}$$
$$+\lambda_{9,t}\epsilon_l c_{l,t}^{-\sigma}\left(R_t^L\right)^{-2},$$

$$0=-\lambda_{0,t}0.5R_t^L\gamma/\left((1+\upsilon)q_t^2 z_t\right)\left((h-h_{b,t})^{-\sigma_h}-h_{b,t}^{-\sigma_h}\right)+\lambda_{8,t}2(1+\upsilon)z_t h_{b,t}/R_t^L$$
$$+\lambda_{10,t}n_t^{\eta+1-\alpha}/(mc_t a_t)-\lambda_{10,t-1}n_t^{\eta+1-\alpha}/(mc_t a_t),$$

$$0=-[\lambda_{1,t}\,(1-\tau^n)\,\chi 0.5^\eta n_t^{\eta+1-\alpha}/\left(mc_t^2\alpha a_t\right)]+\lambda_{7,t}\,(1-\tau^n)\,(\chi/\alpha)\,0.5^\eta n_t^{1+\eta}mc_t^{-2}s_t^{-1}$$
$$-\lambda_{10,t}q_t n_t^{\eta+1-\alpha}/(mc_t^2 a_t)+\lambda_{10,t-1}q_t n_t^{\eta+1-\alpha}/(mc_t^2 a_t),$$

$$0=-(\lambda_{2,t}a_t n_t^\alpha/s_t^2)+\lambda_{3,t}-\beta E_t\lambda_{3,t+1}\phi\pi_{t+1}^\varepsilon+\lambda_{6,t}\,(1-\tau^n)\,(\chi/\alpha)\,0.5^\eta n_t^{1+\eta}s_t^{-2}$$
$$+\lambda_{7,t}\,(1-\tau^n)\,(\chi/\alpha)\,0.5^\eta n_t^{1+\eta}mc_t^{-1}s_t^{-2},$$

$$
\begin{aligned}
0 =& [\lambda_{1,t-1}(0.5\epsilon_b c_{b,t}^{-\sigma} + 0.5\epsilon_l c_{l,t}^{-\sigma})/\pi_t^2] + \lambda_{4,t}(\varepsilon - 1)\phi\pi_t^{\varepsilon-2} - \lambda_{6,t-1}\phi\varepsilon\pi_t^{\varepsilon-1}Z_{1,t} \\
&-\lambda_{7,t-1}\phi(\varepsilon-1)\pi_t^{\varepsilon-2}Z_{2,t} + \lambda_{3,t}[-\phi s_{t-1}\varepsilon\pi_t^{\varepsilon-1} + (1-\phi)^{\frac{1}{1-\varepsilon}}\varepsilon\left(1-\phi\pi_t^{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}-1}\phi\pi_t^{\varepsilon-2}] \\
&-\lambda_{9,t-1}(0.5\epsilon_b c_{b,t}^{-\sigma} + 0.5\epsilon_l c_{l,t}^{-\sigma})/\pi_t^2, \\
0 =& -\chi 0.5^{1+\eta}n_t^{\eta} + [\lambda_{1,t}(\eta+1-\alpha)(1-\tau^n)\chi 0.5^{\eta}n_t^{\eta-\alpha}/(mc_t\alpha a_t)] + (\lambda_{2,t}a_t\alpha n_t^{\alpha-1}/s_t) \\
&-\lambda_{6,t}(1+\eta)(1-\tau^n)(\chi/\alpha)0.5^{\eta}n_t^{\eta}s_t^{-1} - \lambda_{7,t}(1+\eta)(1-\tau^n)(\chi/\alpha)0.5^{\eta}n_t^{\eta}(mc_t s_t)^{-1} \\
&+\lambda_{10,t}[(\eta+1-\alpha)q_t n_t^{\eta-\alpha}/(mc_t a_t)] - \lambda_{10,t-1}[(\eta+1-\alpha)q_t n_t^{\eta-\alpha}/(mc_t a_t)], \\
0 =& 0.5\epsilon_b c_{b,t}^{-\sigma} + 0.5\lambda_{0,t}\epsilon_b\sigma c_{b,t}^{-\sigma-1} + \lambda_{1,t-1}\epsilon_b 0.5\sigma(c_{b,t}^{-\sigma-1}/\pi_t) - \lambda_{2,t} - \lambda_{8,t} \\
&-\lambda_{9,t-1}\epsilon_b 0.5\sigma(c_{b,t}^{-\sigma-1}/\pi_t), \\
0 =& 0.5\epsilon_l c_{l,t}^{-\sigma} - 0.5\lambda_{0,t}\sigma\epsilon_l c_{l,t}^{-\sigma-1} + \lambda_{1,t-1}\epsilon_l 0.5\sigma(c_{l,t}^{-\sigma-1}/\pi_t) - \lambda_{2,t} + \lambda_{8,t} \\
&-\lambda_{9,t-1}\epsilon_l 0.5\sigma(c_{l,t}^{-\sigma-1}/\pi_t) + \lambda_{9,t}\sigma\epsilon_l c_{l,t}^{-\sigma-1}/R_t^L, \\
0 =& -(\lambda_{5,t}/Z_{2,t}) + \lambda_{6,t} - \lambda_{6,t-1}\phi\pi_t^{\varepsilon}, \\
0 =& (\lambda_{5,t}Z_{1,t}/Z_{2,t}^2) + \lambda_{7,t} - \lambda_{7,t-1}\phi\pi_t^{\varepsilon-1}, \\
0 =& \lambda_{4,t}(1-\phi)(1-\varepsilon)(\tilde{Z}_t)^{-\varepsilon} + \lambda_{5,t}(\varepsilon-1)/\varepsilon,
\end{aligned}
$$

as well as the constraints to the policy problem (76) and the transversality conditions, given τ^n, $\{a_t, z_t\}_{t=0}^{\infty}$, $h > 0$, $s_{-1} = 1$, as well as $\lambda_{1,-1} = \lambda_1$, $\lambda_{6,-1} = \lambda_6$, $\lambda_{7,-1} = \lambda_7$, $\lambda_{9,-1} = \lambda_9$ and $\lambda_{10,-1} = \lambda_{10}$. The steady state of the solution, where all variables are constant or grow with a constant rate, is a set $\{c_b, c_l, n, mc, s, \pi, h_b, q, R^L, \lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_8, \lambda_{10}\}$ satisfying

$$
\begin{aligned}
0 =& 0.5\gamma\left[h_b^{-\sigma_h} - (h-h_b)^{-\sigma_h}\right] + \lambda_0 0.5R^L(\gamma/(qz))(\sigma_h(h-h_b)^{-\sigma_h-1} + \sigma_h h_b^{-\sigma_h-1}) \\
&+\lambda_8 2\left(zq/R^L\right) - \lambda_{10}\gamma\omega\sigma_h(h-h_b)^{-\sigma_h-1}, \\
0 =& -\lambda_0 0.5R^L\gamma/(qz)\left((h-h_b)^{-\sigma_h} - h_b^{-\sigma_h}\right) + \lambda_8 2qzh_b/R^L, \\
0 =& -\lambda_2(n^{\alpha}/s^2) + \lambda_3(1-\beta\phi\pi^{\varepsilon}) - \lambda_1(1-\tau^n)(\chi/\alpha)0.5^{\eta}n^{1+\eta-\alpha}(mcs)^{-1}\frac{\pi^{\varepsilon-1}\phi(1-\beta)(\pi-1)}{(1-\phi\beta\pi^{\varepsilon-1})(1-\phi\pi^{\varepsilon})}, \\
0 =& \lambda_1 0.5\left(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma}\right) + \varepsilon\phi s\pi^{\varepsilon}(\pi-1)\left(\lambda_1\frac{(1-\tau^n)(\chi/\alpha)0.5^{\eta}n^{1+\eta-\alpha}(mcs)^{-1}}{(1-\phi\pi^{\varepsilon})(1-\phi\beta\pi^{\varepsilon-1})} - \frac{\lambda_3}{1-\pi^{\varepsilon-1}\phi}\right), \\
0 =& 0.5\epsilon_b c_b^{-\sigma}\left[1 + \lambda_0\sigma c_b^{-1} + \lambda_1\sigma c_b^{-1}/\pi\right] - \lambda_2 - \lambda_8, \\
0 =& 0.5\epsilon_l c_l^{-\sigma}\left[1 - \lambda_0\sigma c_l^{-1} + \lambda_1\sigma c_l^{-1}/\pi\right] - \lambda_2 + \lambda_8, \\
0 =& -\chi 0.5^{1+\eta}n^{\eta} + \lambda_2(\alpha n^{\alpha-1}/s) + \lambda_1(1-\tau^n)(\chi/\alpha)0.5^{\eta}\frac{n^{\eta-\alpha}}{mc}\left(\frac{1-\phi\beta\pi^{\varepsilon}}{1-\phi\beta\pi^{\varepsilon-1}}\frac{1-\phi\pi^{\varepsilon-1}}{1-\phi\pi^{\varepsilon}}(1+\eta) - \alpha\right),
\end{aligned}
$$

as well as the steady state versions of the constraints in (76): $c_l = (\frac{\epsilon_l}{n^{\eta+1-\alpha}}\frac{\omega mc}{R^L})^{1/\sigma}$, $1/R^L = \beta(c_l^{\sigma}/\epsilon_l)0.5(\epsilon_b c_b^{-\sigma} + \epsilon_l c_l^{-\sigma})/\pi$, $\epsilon_b c_b^{-\sigma} - \epsilon_l c_l^{-\sigma} = R^L(\gamma/qz)[(h-h_b)^{-\sigma_h} - h_b^{-\sigma_h}]$, $c_b - c_l = 2zqh_b/R^L$, $\frac{qn^{\eta+1-\alpha}}{mc}(1-\beta) = \gamma\omega(h-h_b)^{-\sigma_h}$, $n^{\alpha}/s = c_l + c_b$, $s = (1-\phi)^{1/(1-\varepsilon)}\frac{(1-\phi\pi^{\varepsilon-1})^{\varepsilon/(\varepsilon-1)}}{1-\phi\pi^{\varepsilon}}$, and $mc = \frac{\varepsilon-1}{\varepsilon}\frac{1-\phi\beta\pi^{\varepsilon}}{1-\phi\beta\pi^{\varepsilon-1}}(\frac{1-\phi}{1-\phi\pi^{\varepsilon-1}})^{1/(\varepsilon-1)}$.

Under flexible prices and perfect competition the policy problem simplifies to

$$\max_{\{c_{b,t},c_{l,t},n_t,\pi_t,h_{b,t}\,q_t,R_t^L\}_{t=0}^{\infty}} \quad \min_{\{\lambda_{0,t},\lambda_{1,t},\lambda_{2,t},\lambda_{8,t},\lambda_{9,t},\lambda_{10,t}\}_{t=0}^{\infty}} \tag{77}$$

$$E\sum_{t=0}^{\infty}\beta^t\begin{bmatrix}0.5\epsilon_b(c_{b,t}^{1-\sigma}-1)(1-\sigma)^{-1}+0.5\epsilon_l(c_{l,t}^{1-\sigma}-1)(1-\sigma)^{-1}-\chi(0.5n_t)^{1+\eta}(1+\eta)^{-1}\\+0.5\gamma(h_{b,t}^{1-\sigma_h}-1)(1-\sigma_h)^{-1}+0.5\gamma((h-h_{b,t})^{1-\sigma_h}-1)(1-\sigma_h)^{-1}\end{bmatrix},$$
$$+\lambda_{0,t}\left[0.5\epsilon_l c_{l,t}^{-\sigma}-0.5\epsilon_b c_{b,t}^{-\sigma}\qquad+0.5R_t^L(\gamma/(1+\upsilon)q_t z_t)((h-h_{b,t})^{-\sigma_h}-h_{b,t}^{-\sigma_h})\right]$$
$$+\lambda_{1,t}\left[(1-\tau^n)\chi 0.5^{\eta}n_t^{\eta+1-\alpha}/a_t\alpha-\beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma}+\epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\right]+\lambda_{2,t}\left[a_t n_t^{\alpha}{}_t-c_{l,t}-c_{b,t}\right]$$
$$+\lambda_{8,t}\left[2\left((1+\upsilon)z_t q_t h_{b,t}/R_t^L\right)-c_{b,t}+c_{l,t}\right]+\lambda_{9,t}\left[\beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma}+\epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]-\epsilon_l c_{l,t}^{-\sigma}/R_t^L\right]$$
$$+\lambda_{10,t}\left[q_t n_t^{\eta+1-\alpha}a_t^{-1}-\beta E_t(q_{t+1}n_{t+1}^{\eta+1-\alpha}a_{t+1}^{-1})-\gamma\omega(h-h_{b,t})^{-\sigma_h}\right]$$

such that the policy plan is – when neglecting conditions for $t=0$ – characterized by the following first order conditions of the policy problem

$$0=0.5\gamma h_{b,t}^{-\sigma_h}-0.5\gamma(h-h_{b,t})^{-\sigma_h}+\lambda_{0,t}0.5R_t^L(\gamma/(1+\upsilon)q_t z_t)(\sigma_h(h-h_{b,t})^{-\sigma_h-1}+\sigma_h h_{b,t}^{-\sigma_h-1})$$
$$+\lambda_{8,t}2\left((1+\upsilon)z_t q_t/R_t^L\right)-\lambda_{10,t}\gamma\omega\sigma_h(h-h_{b,t})^{-\sigma_h-1},$$
$$0=\lambda_{0,t}0.5(\gamma/(1+\upsilon)q_t z_t)((h-h_{b,t})^{-\sigma_h}-h_{b,t}^{-\sigma_h})-\lambda_{8,t}2(1+\upsilon)z_t q_t h_{b,t}(R_t^L)^{-2}+\lambda_{9,t}\epsilon_l c_{l,t}^{-\sigma}(R_t^L)^{-2},$$
$$0=\lambda_{9,t}\epsilon_l c_{l,t}^{-\sigma}\left(q_t R_t^L\right)^{-1}+n_t^{\eta+1-\alpha}a_t{}^{-1}(\lambda_{10,t}-\lambda_{10,t-1}),\quad 0=\lambda_{1,t}-\lambda_{9,t},$$
$$0=-\chi 0.5^{1+\eta}n_t^{\,\eta}+(\lambda_{2,t}a_t\alpha n_t^{\alpha-1})+((\eta+1-\alpha)\,n_t^{\eta-\alpha}/a_t)[(\chi/\alpha)0.5^{\eta}\lambda_{1,t}+q_t(\lambda_{10,t}-\lambda_{10,t-1})],$$
$$0=0.5\epsilon_b c_{b,t}^{-\sigma}(1+\lambda_{0,t}\sigma c_{b,t}^{-1}+\lambda_{1,t-1}\sigma c_{b,t}^{-1}\pi_t^{-1}-\lambda_{9,t-1}\sigma c_{b,t}^{-1}/\pi_t)-\lambda_{2,t}-\lambda_{8,t},$$
$$0=0.5\epsilon_l c_{l,t}^{-\sigma}(1-\lambda_{0,t}\sigma c_{l,t}^{-1}+\lambda_{1,t-1}\sigma c_{l,t}^{-1}\pi_t^{-1}-\lambda_{9,t-1}\sigma c_{l,t}^{-1}\pi_t^{-1}+\lambda_{9,t}\sigma c_{l,t}^{-1}/R_t^L)-\lambda_{2,t}+\lambda_{8,t},$$

as well as the constraints to the policy problem (77), and the transversality conditions, given $\{a_t,z_t\}_{t=0}^{\infty}$, $h>0$, as well as as well as $\lambda_{1,-1}=\lambda_1$, $\lambda_{9,-1}=\lambda_9$ and $\lambda_{10,-1}=\lambda_{10}$. The steady state of the solution, where all variables are constant or grow with a constant rate, is a set $\{c_b,c_l,h_b,n,R^L,\pi,q,\lambda_0,\lambda_2,\lambda_8,\lambda_{10}\}$ satisfying

$$0=0.5\gamma h_b^{-\sigma_h}-0.5\gamma(h-h_b)^{-\sigma_h}+\lambda_0 0.5R^L(\gamma/(1+\upsilon)qz_t)(\sigma_h(h-h_b)^{-\sigma_h-1}+\sigma_h h_b^{-\sigma_h-1})$$
$$+\lambda_8 2\left((1+\upsilon)zq/R^L\right)-\lambda_{10}\gamma\omega\sigma_h(h-h_b)^{-\sigma_h-1},$$
$$0=-\lambda_0 0.5R^L(\gamma/(1+\upsilon)q^2z)((h-h_b)^{-\sigma_h}-h_b^{-\sigma_h})-\lambda_8 2(1+\upsilon)zh_b/R^L,\ \chi 0.5^{1+\eta}n^{\eta}=\lambda_2\alpha n^{\alpha-1},$$
$$0=0.5\epsilon_b c_b^{-\sigma}\left(1+\lambda_0\sigma c_b^{-1}\right)-\lambda_2-\lambda_8,\ 0=0.5\epsilon_l c_l^{-\sigma}(1-\lambda_0\sigma c_l^{-1})-\lambda_2+\lambda_8$$

as well as the steady state versions of the constraints in (77) $c_l=(\frac{\epsilon_l}{n^{\eta+1-\alpha}}\frac{\omega}{R^L})^{1/\sigma}$, $1/R^L=\beta\,(c_l^{\sigma}/\epsilon_l)\,0.5(\epsilon_b c_b^{-\sigma}+\epsilon_l c_l^{-\sigma})/\pi$, $\epsilon_b c_b^{-\sigma}-\epsilon_l c_l^{-\sigma}=R^L(\gamma/qz)\,[(h-h_b)^{-\sigma_h}-h_b^{-\sigma_h}]$, $c_b-c_l=2zqh_b/R^L$, $qn^{\eta+1-\alpha}(1-\beta)=\gamma\omega(h-h_b)^{-\sigma_h}$, and $n^{\alpha}=c_l+c_b$.

A.6 Optimal monetary policy under rationed money supply

In this Appendix, we consider the policy problem for the case where the central bank takes the possibility of money rationing into account. To examine the optimal policy problem we set-up the policy problem including the money supply constraints (2) and (4), and then examine if the central bank is able to undo several constraints imposed by the private sector equilibrium behavior (see Definition 2) by using its instruments, R_t^m, κ_t, and κ_t^B. For this, we suppose that the restrictions on these instruments, i.e. $R_t^m \geq 1$ and $\kappa^{(B)} \in [0,1]$, are not binding. After solving for the optimal policy plan and the associated sequences for all instruments, we verify numerically that the restrictions on the policy instruments are not violated for the chosen set of parameter values (and in the neighborhood of the long-run equilibrium). The policy problem can be summarized as

$$\max_{\{c_{b,t},c_{l,t},n_t,m_t^H,b_t,b_t^T,l_t,mc_t,\tilde{Z}_t,Z_{1,t},Z_{2,t},s_t,\pi_t,R_t^L,\kappa_t^B,\kappa_t,R_t^m,h_{b,t}q_t\}_{t=0}^{\infty}} \quad \min_{\{\theta_{1,t},\ldots\theta_{16,t}\}_{t=0}^{\infty}} \tag{78}$$

$$E\sum_{t=0}^{\infty}\beta^t\left[\begin{array}{c}0.5\epsilon_b(c_{b,t}^{1-\sigma}-1)(1-\sigma)^{-1}+0.5\epsilon_l(c_{l,t}^{1-\sigma}-1)(1-\sigma)^{-1}-\chi(0.5n_t)^{1+\eta}(1+\eta)^{-1}\\+0.5\gamma(h_{b,t}^{1-\sigma_h}-1)(1-\sigma_h)^{-1}+0.5\gamma((h-h_{b,t})^{1-\sigma_h}-1)(1-\sigma_h)^{-1}\end{array}\right],$$

$$+\theta_{1,t}\left[\epsilon_b c_{b,t}^{-\sigma}/R_t^L-(1-\tau^n)\chi(0.5n_t)^{\eta}/(mc_t a_t\alpha n_t^{\alpha-1})-\gamma((1+\upsilon)q_t z_t)^{-1}((h-h_{b,t})^{-\sigma_h}-h_{b,t}^{-\sigma_h})\right]$$

$$+\theta_{2,t}\left[(1-\tau^n)\chi(0.5n_t)^{\eta}/\left(mc_t a_t\alpha n_t^{\alpha-1}\right)-\beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma}+\epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\right]$$

$$+\theta_{3,t}[(1/R_t^L)-\beta\left(c_{l,t}^{\sigma}/\epsilon_l\right)E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma}+\epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\{1+[\kappa_t/(1+\upsilon)][(R_t^L/R_t^m)-1]\}]$$

$$+\theta_{4,t}\left[0.5(1+\Omega_t)m_t^H+\{(1+\upsilon)l_t/R_t^L\}+(\kappa_t l_t/R_t^m)-c_{b,t}\right]+\theta_{5,t}\left[b_t-b_t^T+m_t^H\right]$$

$$+\theta_{6,t}\left[0.5(1+\Omega_t)m_t^H-(1+\upsilon)\left(l_t/R_t^L\right)-c_{l,t}\right]$$

$$+\theta_{7,t}\left[a_t n_t^{\alpha}/s_t-c_{l,t}-c_{b,t}\right]+\theta_{8,t}\left[\tilde{Z}_t(\varepsilon-1)/\varepsilon-Z_{1,t}/Z_{2,t}\right]+\theta_{9,t}\left[(1-\phi)(\tilde{Z}_t)^{1-\varepsilon}+\phi\pi_t^{\varepsilon-1}-1\right]$$

$$+\theta_{10,t}\left[s_t-(1-\phi)\tilde{Z}_t^{-\varepsilon}-\phi s_{t-1}\pi_t^{\varepsilon}\right]+\theta_{11,t}\left[Z_{1,t}-(1-\tau^n)(\chi/\alpha)0.5^{\eta}n_t^{1+\eta}s_t^{-1}-\phi\beta E_t\pi_{t+1}^{\varepsilon}Z_{1,t+1}\right]$$

$$+\theta_{12,t}\left[Z_{2,t}-(1-\tau^n)(\chi/\alpha)0.5^{\eta}n_t^{1+\eta}(mc_t s_t)^{-1}-\phi\beta E_t\pi_{t+1}^{\varepsilon-1}Z_{2,t+1}\right]+\theta_{13,t}\left[b_t^T-\Gamma b_{t-1}^T/\pi_t\right]$$

$$+\theta_{14,t}\left[\kappa_t^B b_{t-1}/(R_t^m\pi_t)-(1+\Omega_t)m_t^H+m_{t-1}^H\pi_t^{-1}\right]+\theta_{15,t}\left[z_t q_t h_{b,t}-l_t\right]$$

$$+\theta_{16,t}\left[q_t n_t^{\eta+1-\alpha}/(mc_t a_t)-\beta E_t q_{t+1}n_{t+1}^{\eta+1-\alpha}/(mc_{t+1}a_{t+1})-\varrho(h-h_{b,t})^{-\sigma_h}\right]$$

We first examine the optimal choices for policy related variables and, in particular, for the monetary policy instruments. We thereby show that the set of relevant constraints of the original policy problem (78) can be reduced, if the central bank rations money supply. Once we have shown that several constraints in (78) are not binding, we continue with the simplified policy problem. The first order condition for κ_t^B, $\theta_{14,t}b_{t-1}/(R_t^m\pi_t)=0$, immediately leads to $\theta_{14,t}=0$, such that the first order conditions for b_t, $\theta_{5,t}+\beta E_t\theta_{14,t+1}\kappa_{t+1}^B/R_{t+1}^m\pi_{t+1}=0$, and for b_t^T, $\theta_{5,t}=\theta_{13,t}-\Gamma\beta E_t\theta_{13,t+1}/\pi_{t+1}$, imply $\theta_{5,t}=0$ and $\theta_{13,t}=0$. Then, the first order condition for m_t^H, $\theta_{5,t}+\theta_{4,t}0.5(1+\Omega_t)+\theta_{6,t}0.5(1+\Omega_t)-(1+\Omega_t)\theta_{14,t}+\beta E_t\theta_{14,t+1}/\pi_{t+1}=0$, leads to $\theta_{4,t}=-\theta_{6,t}$.

The optimal choices for the policy rate R_t^m and κ_t depend on whether the policy rate is set below the loan rate or not. If $R_t^m = R_t^L$ or if $\kappa_t = 0$, the constraint (16) reduces to $\left(1/R_t^L\right) - \beta(c_{l,t}^{\sigma}/\epsilon_l)E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]$ as in the case of non-rationed money supply (see Appendix A.5). If, however, the policy rate is set such that it is lower than the equilibrium loan rate and loans are purchased, i.e. if

$$R_t^m < R_t^L \text{ and } \kappa_t > 0, \tag{79}$$

(while $1 \leq R_t^m$ and $\kappa_t \leq 1$) the first order condition for R_t^m is given by $\theta_{3,t}\beta(c_{l,t}^{\sigma}/\epsilon_l)E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}][\kappa_t/(1+\upsilon)]R_t^L = \theta_{4,t}\kappa_t l_t + \theta_{14,t}\kappa_t^B b_{t-1}\pi_t^{-1}$, which can by using $\theta_{14,t} = 0$ be further simplified to $\theta_{3,t}\beta(c_{l,t}^{\sigma}/\epsilon_l)\ E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]/(1+\upsilon) = \theta_{4,t} l_t/R_t^L$. Combining the latter with the first order condition for κ_t, $-\theta_{3,t}\beta(c_{l,t}^{\sigma}/\epsilon_l)E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}][1/(1+\upsilon)][(R_t^L/R_t^m) - 1] + \theta_{4,t}(l_t/R_t^m) = 0$, then leads to $\theta_{4,t} l_t/R_t^L = 0$, implying $\theta_{4,t} = 0$. The first order conditions for l_t, $\theta_{4,t}[(1+\upsilon)/R_t^L + (\kappa_t/R_t^m)] - \theta_{6,t}(1+\upsilon)/R_t^L - \theta_{15,t} = 0$, then implies $\theta_{15,t} = 0$. We can therefore conclude that the constraints associated with the multiplier $\theta_{3,t}$, $\theta_{4,t}$, $\theta_{5,t}$, $\theta_{6,t}$, $\theta_{13,t}$, $\theta_{14,t}$, and $\theta_{15,t}$, which are all equal to zero, are not binding for the policy choice. Then, the loan rate can be chosen to ensure that the constraint associated with the multiplier θ_{1t} is satisfied, while the constraint associated with the multiplier $\theta_{16,t}$ can be used to residually determine the sequence of q_t for a given allocation. When (79) is satisfied, the policy problem (78) can therefore be reduced to

$$\begin{aligned}
&\max_{\{c_{b,t},c_{l,t},n_t,mc_t,\tilde{Z}_t,Z_{1,t},Z_{2,t},s_t,\pi_t,h_{b,t}\}_{t=0}^{\infty}} \min_{\{\lambda_{1,t},\ldots\lambda_{7,t}\}_{t=0}^{\infty}} \qquad (80)\\
&E\sum_{t=0}^{\infty}\beta^t\begin{bmatrix} 0.5\epsilon_b(c_{b,t}^{1-\sigma}-1)(1-\sigma)^{-1} + 0.5\epsilon_l(c_{l,t}^{1-\sigma}-1)(1-\sigma)^{-1} - \chi(0.5n_t)^{1+\eta}(1+\eta)^{-1} \\ +0.5\gamma(h_{b,t}^{1-\sigma_h}-1)(1-\sigma_h)^{-1} + 0.5\gamma((h-h_{b,t})^{1-\sigma_h}-1)(1-\sigma_h)^{-1}\end{bmatrix}, \\
&+\lambda_{1,t}\left[(1-\tau^n)\chi 0.5^{\eta} n_t^{\eta+1-\alpha}/(mc_t a_t \alpha) - \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\right] \\
&+\lambda_{2,t}\left[a_t n_t^{\alpha}/s_t - c_{l,t} - c_{b,t}\right] + \lambda_{3,t}\left[s_t - \phi s_{t-1}\pi_t^{\varepsilon} - (1-\phi)^{\frac{1}{1-\varepsilon}}\left(1-\phi\pi_t^{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}\right] \\
&+\lambda_{4,t}\left[(1-\phi)(\tilde{Z}_t)^{1-\varepsilon} + \phi\pi_t^{\varepsilon-1} - 1\right] + \lambda_{5,t}\left[\tilde{Z}_t(\varepsilon-1)/\varepsilon - Z_{1,t}/Z_{2,t}\right] \\
&+\lambda_{6,t}\left[Z_{1,t} - (1-\tau^n)(\chi/\alpha)0.5^{\eta} n_t^{1+\eta} s_t^{-1} - \phi\beta E_t\pi_{t+1}^{\varepsilon} Z_{1,t+1}\right] \\
&+\lambda_{7,t}\left[Z_{2,t} - (1-\tau^n)(\chi/\alpha)0.5^{\eta} n_t^{1+\eta}(mc_t s_t)^{-1} - \phi\beta E_t\pi_{t+1}^{\varepsilon-1} Z_{2,t+1}\right]
\end{aligned}$$

Proof of proposition 3. As shown above, the borrowing constraint $z_t q_t h_{b,t} \geq l_t$ is not a binding constraint for the policy problem (78), i.e. the multiplier $\theta_{15,t}$ equals zero under the optimal choice, if (79) is satisfied. Then, the central bank instruments R_t^m, κ_t, and κ_t^B are non-neutral and can be set to implement a set of sequences $\{\widetilde{c}_{b,t}, \widetilde{c}_{l,t}, \widetilde{n}_t, \widetilde{mc}_t, \widetilde{\tilde{Z}}_t, \widetilde{Z}_{1,t}, \widetilde{Z}_{2,t}, \widetilde{s}_t, \widetilde{\pi}_t, \widetilde{h}_{b,t}\}_{t=0}^{\infty}$ solving (80). Specifically, it can control the overall supply of money and thus aggregate demand by setting κ_t^B, while R_t^m and κ_t can be set such that *i.*) the private sector multiplier $\zeta_{b,t}$ on the borrowing constraint (3) equals zero, which demands $(\epsilon_b c_{b,t}^{-\sigma}/R_t^L) = \beta E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]$ (see 37), and *ii.*) credit supply, which restricts consumption expenditures via (39) and (40), is altered to accommodate desired consumption expenditures of borrowers and lenders via loan purchases (see 44-45). For *i.*), the instruments R_t^m and κ_t, which affect the loan rate by $1/R_t^L = \beta(c_{l,t}^{\sigma}/\epsilon_l)E_t[0.5(\epsilon_b c_{b,t+1}^{-\sigma} + \epsilon_l c_{l,t+1}^{-\sigma})/\pi_{t+1}]\{1 + \frac{\kappa_t}{1+\upsilon}[(R_t^L/R_t^m) - 1]\}$ if (79) holds (see 38), have to satisfy

$$R_t^m \left(1 + \left[\frac{\epsilon_l \widetilde{c}_{l,t}^{-\sigma} - \epsilon_b \widetilde{c}_{b,t}^{-\sigma}}{\epsilon_b \widetilde{c}_{b,t}^{-\sigma}}\right] \frac{1+\upsilon}{\kappa_t}\right) = \frac{\epsilon_b \widetilde{c}_{b,t}^{-\sigma}}{\beta E_t\{0.5(\epsilon_b \widetilde{c}_{b,t+1}^{-\sigma} + \epsilon_l \widetilde{c}_{l,t+1}^{-\sigma})/\widetilde{\pi}_{t+1}^*\}}, \tag{81}$$

where the RHS of (81) equals the associated loan rate $\widetilde{R}_t^L$ and the term in the square brackets is non-negative. For *ii.*), the instruments R_t^m and κ_t further have to ensure $l_t[(\kappa_t/R_t^m) + 2(1+\upsilon)/\widetilde{R}_t^L] \geq \widetilde{c}_{b,t} - \widetilde{c}_{l,t}$, for a feasible amount of loans, $l_t \leq z_t \widetilde{q}_t \widetilde{h}_{b,t}$ and thus

$$z_t \widetilde{q}_t \widetilde{h}_{b,t}[(\kappa_t/R_t^m) + 2(1+\upsilon)/\widetilde{R}_t^L] \geq \widetilde{c}_{b,t} - \widetilde{c}_{l,t}, \tag{82}$$

where $\widetilde{q}_t$ is determined by (35), (36) and (47) for $\{\widetilde{c}_{b,t}, \widetilde{c}_{l,t}, \widetilde{n}_t, \widetilde{mc}_t, \widetilde{\pi}_t, \widetilde{h}_{b,t}\}_{t=0}^{\infty}$. Substituting out $\widetilde{q}$ using the steady state versions of (35), (36) and (47) as well as that $\widetilde{h}_b = \widetilde{h}_l = 0.5h$ holds for $\zeta_b = 0$, the steady state version of condition (82) can be written as

$$\frac{\kappa}{R^m} \geq \frac{(\widetilde{c}_b - \widetilde{c}_l)(1-\beta)\left(\epsilon_b \widetilde{c}_b^{-\sigma}\right) - 2(1+\upsilon)z\mathsf{v}(h)}{z\mathsf{v}(h)\widetilde{R}^L}, \tag{83}$$

where $\mathsf{v}(h) = \gamma(0.5h)^{1-\sigma_h}$. Condition (83) implies that if $(\widetilde{c}_b - \widetilde{c}_l)\,\epsilon_b \widetilde{c}_b^{-\sigma} > 2(1+\upsilon)z\mathsf{v}(h)/(1-\beta)$ holds in the steady state, then the consumption allocation $\{\widetilde{c}_b, \widetilde{c}_l\}$ cannot be implemented without loan purchases. The long-run consumption allocation $\{\widetilde{c}_b, \widetilde{c}_l\}$ can however be implemented if the pair $\{R^m, \kappa\}$ satisfies $R^m < \widetilde{R}^L = \epsilon_b \widetilde{c}_b^{-\sigma} \widetilde{\overline{c}}^{\sigma} \widetilde{\pi}/\beta$, $R^m \geq 1$, $0 < \kappa$, $\kappa \leq 1$, (83), and the steady state version of (81), i.e. $R^m(1 + [(\epsilon_l \widetilde{c}_l^{-\sigma} - \epsilon_b \widetilde{c}_b^{-\sigma})\epsilon_b^{-1}\widetilde{c}_b^{\sigma}]\,(1+\upsilon)/\kappa) = \widetilde{R}^L$. Then, in a sufficiently small neighborhood of this steady state there also exist pairs of sequences $\{R_t^m, \kappa_t\}_{t=0}^{\infty}$ satisfying $R_t^m < \widetilde{R}_t^L$, $R_t^m \geq 1$, $0 < \kappa_t$, $\kappa_t \leq 1$, (81), and (82) which implement $\{\widetilde{c}_{b,t}, \widetilde{c}_{l,t}\}_{t=0}^{\infty}$. ■

Neglecting the conditions for $t = 0$, the solution to the policy problem (80) has to satisfy the following first order conditions:

$$\begin{aligned}
0 &= h_{b,t} - 0.5h \\
0 &= 0.5\epsilon_b c_{b,t}^{-\sigma} + \lambda_{1,t-1}\epsilon_b 0.5\sigma\left(c_{b,t}^{-\sigma-1}/\pi_t\right) - \lambda_{2,t}, \\
0 &= 0.5\epsilon_l c_{l,t}^{-\sigma} + \lambda_{1,t-1}\epsilon_l 0.5\sigma\left(c_{l,t}^{-\sigma-1}/\pi_t\right) - \lambda_{2,t}, \\
0 &= -\chi 0.5^{1+\eta} n_t^{\eta} + [\lambda_{1,t}(\eta+1-\alpha)(1-\tau^n)\chi 0.5^{\eta} n_t^{\eta-\alpha}/(mc_t \alpha a_t)] + (\lambda_{2,t} a_t \alpha n_t^{\alpha-1}/s_t) \\
&\quad - \lambda_{6,t}(1+\eta)(1-\tau^n)(\chi/\alpha)0.5^{\eta} n_t^{\eta} s_t^{-1} - \lambda_{7,t}(1+\eta)(1-\tau^n)(\chi/\alpha)0.5^{\eta} n_t^{\eta}(mc_t s_t)^{-1},
\end{aligned}$$

$$
\begin{aligned}
0 = & [\lambda_{1,t-1}\left(0.5\epsilon_b c_{b,t}^{-\sigma} + 0.5\epsilon_l c_{l,t}^{-\sigma}\right)/\pi_t^2] + \lambda_{4,t}\left(\varepsilon - 1\right)\phi\pi_t^{\varepsilon-2} \\
& -\lambda_{6,t-1}\phi\varepsilon\pi_t^{\varepsilon-1}Z_{1,t} - \lambda_{7,t-1}\phi\left(\varepsilon-1\right)\pi_t^{\varepsilon-2}Z_{2,t} \\
& +\lambda_{3,t}\left[-\phi s_{t-1}\varepsilon\pi_t^{\varepsilon-1} - (1-\phi)^{\frac{1}{1-\varepsilon}}\frac{\varepsilon}{\varepsilon-1}\left(1-\phi\pi_t^{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}-1}\left(-\left(\varepsilon-1\right)\phi\pi_t^{\varepsilon-2}\right)\right], \\
0 = & -(\lambda_{2,t}a_t n_t^{\alpha}/s_t^2) + \lambda_{3,t} - \beta E_t\lambda_{3,t+1}\phi\pi_{t+1}^{\varepsilon} \\
& +\lambda_{6,t}\left(1-\tau^n\right)\left(\chi/\alpha\right)0.5^{\eta}n_t^{1+\eta}s_t^{-2} + \lambda_{7,t}\left(1-\tau^n\right)\left(\chi/\alpha\right)0.5^{\eta}n_t^{1+\eta}mc_t^{-1}s_t^{-2}, \\
& -[\lambda_{1,t}\left(1-\tau^n\right)\chi 0.5^{\eta}n_t^{\eta+1-\alpha}/\left(mc_t^2\alpha a_t\right)] + \lambda_{7,t}\mu_t\left(1-\tau^n\right)\left(\chi/\alpha\right)0.5^{\eta}n_t^{1+\eta}mc_t^{-2}s_t^{-1}, \\
0 = & -(\lambda_{5,t}/Z_{2,t}) + \lambda_{6,t} - \lambda_{6,t-1}\phi\pi_t^{\varepsilon}, \\
0 = & \lambda_{5,t}(Z_{1,t}/Z_{2,t}^2) + \lambda_{7,t} - \lambda_{7,t-1}\phi\pi_t^{\varepsilon-1}, \\
0 = & \lambda_{4,t}(1-\phi)\left(1-\varepsilon\right)(\tilde{Z}_t)^{-\varepsilon} + \lambda_{5,t}\left(\varepsilon-1\right)/\varepsilon,
\end{aligned}
$$

as well as the constraints to the policy problem (80), and the transversality conditions, given τ^n, $\{a_t, z_t\}_{t=0}^{\infty}$, $h > 0$, $s_{-1} = 1$, as well as $\theta_{1,-1} = \theta_1$, $\theta_{6,-1} = \theta_6$, and $\theta_{7,-1} = \theta_7$.

The steady state of the solution, where all exogenous and endogenous variables are constant or grow with a constant rate, can be reduced to a set $\{c_b, c_l, n, \pi, s, \lambda_1, \lambda_3, h_b\}$ satisfying

$$
\begin{aligned}
0 = & h_b - 0.5h, \\
0 = & \epsilon_l c_l^{-\sigma}\left(1+\sigma c_l^{-1}\lambda_1/\pi\right) - \epsilon_b c_b^{-\sigma}\left(1+\sigma c_b^{-1}\lambda_1/\pi\right), \\
0 = & \frac{\pi}{\beta}0.5\left(1+\sigma c_b^{-1}\lambda_1/\pi\right)\left(\alpha n^{\alpha}/s\right) + \lambda_1\frac{0.5\epsilon_b c_b^{-\sigma}+0.5\epsilon_l c_l^{-\sigma}}{\epsilon_b c_b^{-\sigma}}\left(\eta+1-\alpha+\left(1+\eta\right)\Phi(\pi)\right) \\
& -\frac{\chi n^{1+\eta}0.5^{1+\eta}}{\epsilon_b c_b^{-\sigma}}\frac{\pi}{\beta}, \\
0 = & \lambda_1\frac{0.5\epsilon_b c_b^{-\sigma}+0.5\epsilon_l c_l^{-\sigma}}{\epsilon_b c_b^{-\sigma}}\Phi(\pi) + \frac{\pi}{\beta}0.5\left(1+\sigma c_b^{-1}\lambda_1/\pi\right)\left(n^{\alpha}/s\right) - \frac{\pi}{\beta}s\lambda_3\frac{c_b^{\sigma}}{\epsilon_b}\left(1-\beta\phi\pi^{\varepsilon}\right), \\
0 = & -\lambda_1 + \lambda_3\phi\varepsilon\pi^{\varepsilon}\frac{s}{0.5\epsilon_b c_b^{-\sigma}+0.5\epsilon_l c_l^{-\sigma}}\frac{\pi-1}{1-\phi\pi^{\varepsilon-1}} + \lambda_1\beta\frac{\varepsilon\phi\pi^{\varepsilon-1}}{1-\phi\beta\pi^{\varepsilon-1}}\frac{1-\pi}{1-\phi\pi^{\varepsilon}}, \\
0 = & \left(1-\tau^n\right)\frac{\varepsilon}{\varepsilon-1}\frac{\pi}{\beta}\left(\frac{1-\phi\pi^{\varepsilon-1}}{1-\phi}\right)^{\frac{1}{\varepsilon-1}}\frac{\left(1-\phi\beta\pi^{\varepsilon-1}\right)}{\left(1-\phi\beta\pi^{\varepsilon}\right)} - \frac{0.5\epsilon_b c_b^{-\sigma}+0.5\epsilon_l c_l^{-\sigma}}{\left(\chi/\alpha\right)0.5^{\eta}n^{\eta+1-\alpha}}, \\
0 = & (1-\phi)^{\frac{1}{1-\varepsilon}}\frac{\left(1-\phi\pi^{\varepsilon-1}\right)^{\frac{\varepsilon}{\varepsilon-1}}}{\left(1-\phi\pi^{\varepsilon}\right)} - s, \\
0 = & c_l + c_b - n^{\alpha}/s.
\end{aligned}
$$

where $\Phi(\pi) = \frac{1-\phi\beta\pi^{\varepsilon}}{1-\phi\beta\pi^{\varepsilon-1}}\frac{1-\phi\pi^{\varepsilon-1}}{1-\phi\pi^{\varepsilon}} - 1$.

A.7 Additional tables

Table A1: Benchmark parameter values

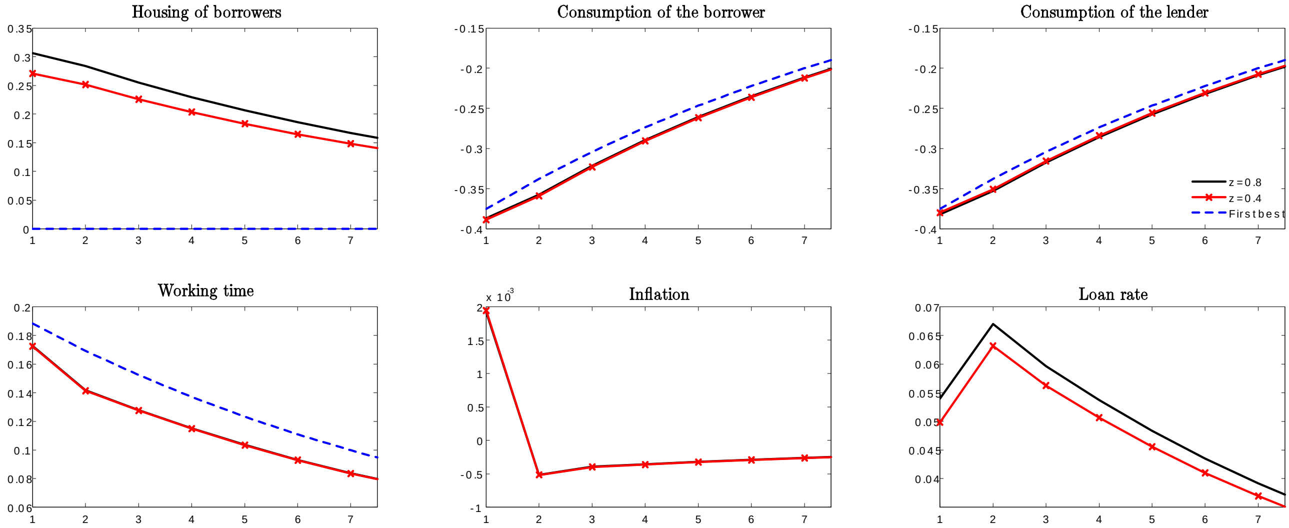
Subjective discount factor	$\beta = 0.99$
Inverse int. elasticity of substitution	$\sigma_{(h)} = 2$
Inverse of Frisch elasticity	$\eta = 1$
Substitution elasticity	$\varepsilon = 10$
Degree of price stickiness	$\phi = 0.7$
Labor income share	$\alpha = 0.66$
Share of unsecured loans	$\upsilon = 0.5$
Utility weight on housing	$\gamma = 0.1$
Utility weight on working time	$\chi = 98$
Housing supply	$h = 28$
Stochastic consumption weights	$\Delta\epsilon = 1$
Mean liquidation share of collateral	$z = 0.8$
Autocorrelation of shocks	$\rho_{a,z} = 0.9$
Standard deviation of shocks	$sd_{a,z} = 0.005$

Table A2: Steady state values under flexible prices

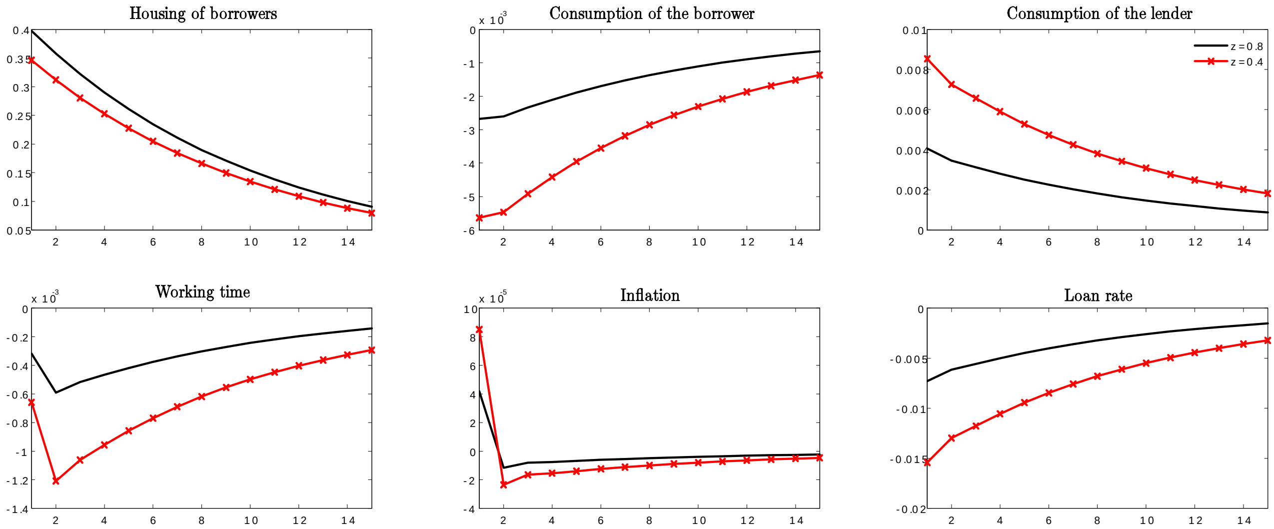
	First best	z=0.8 Optimal policy	z=0.8 Money rationing	z=0.4 Optimal policy	z=0.4 Money rationing
Borrower's consumption	0.3018	0.3017	0.3018	0.3012	0.3018
Lender's consumption	0.1742	0.1743	0.1742	0.1744	0.1742
Borrower's housing share	0.5	0.5323	0.5176	0.63669	0.5879
Loan rate	–	0.9988*	0.9982*	0.9929*	0.9912*
Inflation rate	–	0.9897	0.99	0.9885	0.99
Policy rate	–	–	0.99*	–	0.98*
Share of purchased loans	–	–	0.3	–	1.2*
Representative agent's utility	–3.12078	–3.12081	–3.12079	–3.12138	–3.12101

Note: A star "*" indicates that the lower bound on interest rates or constraints on policy instruments are violated.

A.8 Additional figures



Responses to a contractionary productivity shock under optimal policy w/o money rationing for different liquidation value means [Note: Steady states are not identical.]



Responses to a lower liquidation value under optimal policy w/o money rationing for different liquidation value means [Note: Steady states are not identical.]