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In 2010 all ECB publications feature a motif taken from the €500 banknote.

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CONTENTS

Abstract

This paper develops a small-scale DSGE model which embeds a demographic structure within a monetary policy framework. We extend the tractable, though non-monetary overlapping-generations model of Gertler (1999) and present a small synthesis model which combines the set-up of Gertler with a New-Keynesian structure, implying that the short-run dynamics related to monetary policy are similar to the paradigm summarized in Woodford (2003). In sum, the model offers a New-Keynesian platform which can be used to investigate in a closed economy set-up the response of macroeconomic variables to demographic shocks, similar to technology, government spending or monetary policy shocks. Empirically, we use a calibrated version of the model to discuss a number of macroeconomic scenarios for the euro area with a horizon of around 20 years. The main finding is that demographic changes, while contributing slowly over time to a decline in the equilibrium interest rate, are not visible enough within the time horizon relevant for monetary policy-making to require monetary policy reactions.

Keywords: Demographic change, Monetary policy, DSGE modelling.
JEL classification numbers: D58, E21, E50, E63.

Non-technical summary

This paper starts out from the observation that most industrialized countries are subject to long-lasting demographic changes. Two key features of these changes, which are particularly pronounced in various European countries, are a secular slowdown in population growth and a substantial increase in longevity. As stressed by Bean (2004), these developments are of relevance for monetary policymakers from a normative perspective since the optimal monetary policy may depend on the age structure of an economy, reflecting that different age cohorts tend to have different inflation preferences because of cohort-specific portfolio compositions. Moreover, they may also be of importance from a positive perspective. In particular, it is well known from economic growth theory that demographic variables are a key determinant of the equilibrium real interest rate, a variable which is important for judging the stance of monetary policy for any given inflation target. Yet, despite these insights, monetary policy is typically addressed in frameworks in which demographic changes are not explicitly modelled. In particular, going back to Clarida et al. (1999) and Woodford (2003), the canonical New-Keynesian DSGE framework which is widely used for monetary policy analysis is based on the assumption of an infinitely lived representative household, thereby abstracting from realistic population dynamics, heterogeneity among agents, and individual life-cycle effects.

Against this background, this paper has the goal to develop a closed economy framework for monetary policy analysis which embeds a tractable demographic structure within an otherwise standard New-Keynesian DSGE model. To this end, we build on the non-monetary overlapping-generations model of Gertler (1999) which introduces life-cycle behaviour by allowing for two subsequently reached states of life of new-born agents, working age and retirement. This structure gives rise to two additional demographic variables besides the growth rate of newborn agents, namely the exit probabilities associated with the two states which can be calibrated to match the average lengths of working age and retirement. Similar to Blanchard (1985) and Weil (1989), these probabilities are assumed to be age-independent. This feature is key to keep the state space of the model small such that there exist closed-form aggregate consumption and savings relations despite the heterogeneity of agents at the micro-level. To extend this set-up into a monetary policy framework, we propose a tractable 'money-in-the-utility-function'-approach and modify the non-expected utility specification, which is a key characteristic of Gertler's model, to include real balances as an additional argument of private sector wealth. Moreover, we combine this structure with New-Keynesian supply-side features, characterized by capital accumulation, imperfect competition in the intermediate goods sector and nominal rigidities along the lines of Calvo (1983). These features give rise to a New-Keynesian Phillips-curve, implying that the short-run dynamics related to monetary policy are similar to the standard framework. Indeed, for the special case in which workers are assumed to be infinitely-lived the proposed framework becomes identical with the standard model. Monetary and fiscal policies follow feedback rules in the

spirit of Leeper (1991), thereby anchoring the economy over time around target levels for the inflation rate and the government debt ratio. Reflecting the underlying overlapping generations structure, the dynamics of the model are critically affected by fiscal policy (which is, by construction, non-neutral) and, in particular, by the design of the pension system which facilitates intergenerational transfers between workers and retirees. In sum, we offer an enlarged New-Keynesian platform which can be used to investigate various macroeconomic questions.

In this paper, we use our model to examine, from a positive perspective, selected long-run macroeconomic implications of demographic changes. Our projection horizon stretches until 2030 and we focus, in particular, on the determinants of the equilibrium real interest rate. The model specifies the demographic processes which drive population growth and life expectancy as time-dependent. This assumption allows us to calibrate the model's demographic parameters according to recent demographic projections for the euro area, as reported in European Economy (2009). Specifically, we take the annual demographic projections for the two series as a deterministic input and verify that the model matches the old-age dependency ratio projected until 2030. We then solve the model numerically under perfect foresight. To carry out such analysis we are forced to make assumptions concerning the future course of the assumed PAYGO pension system. We distinguish between two main types of scenarios in which the rising old-age dependency ratio does or does not lead to changes in the replacement rate (defined as the ratio between individual pension benefits and wages). For the first scenario type, the replacement rate decreases endogenously such that the aggregate benefits-output ratio remains unchanged. This assumption amounts to a strengthening of privately funded elements since it introduces a ceiling on the tax-financed redistribution between workers and retirees. For the second scenario type, the replacement rate remains constant, leading to a rise in the aggregate benefits-output ratio. This assumption models in a simple way a `no reform' scenario which extrapolates the existing pension system into the future, leading to a higher tax burden on workers. To distinguish between such two deliberately ‘extreme’ scenarios is instructive because the distinctly different incentives for individual savings generate plausible lower and upper bounds for the projected path of the equilibrium real interest rate.

The main finding is that under either scenario the decrease in population growth and the increase in life expectancy are two independent forces which contribute over the entire projection horizon of about 20 years to a smooth decline in the equilibrium interest rate. This decline, while being more pronounced for first scenario type, does not exceed 50 basis points. Such decline is not visible enough within the shorter time horizon relevant for monetary policy-making to require monetary policy reactions. This finding supports the reasoning of Bean (2004) that because of the `glacial nature of demographic change' the implications for monetary policy, at least from a positive perspective, should be modest.

1 Introduction

This paper starts out from the observation that most industrialized countries are subject to long-lasting demographic changes. Two key features of these changes, which are particularly pronounced in various European countries, are a secular slowdown in population growth and a substantial increase in longevity. As stressed by Bean (2004), these developments are of relevance for monetary policymakers from a normative perspective since the optimal monetary policy may depend on the age structure of an economy, reflecting that different age cohorts tend to have different inflation preferences because of cohort-specific portfolio compositions. Moreover, they may also be of importance from a positive perspective. In particular, it is well known from economic growth theory that demographic variables are a key determinant of the equilibrium real interest rate, a variable which is important for judging the stance of monetary policy for any given inflation target. Yet, despite these insights, monetary policy is typically addressed in frameworks in which demographic changes are not explicitly modelled. In particular, going back to Clarida et al. (1999) and Woodford (2003), the canonical New-Keynesian DSGE framework which is widely used for monetary policy analysis is based on the assumption of an infinitely lived representative household, thereby abstracting from realistic population dynamics, heterogeneity among agents, and individual life-cycle effects.

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[1] Given the Cobb-Douglas assumption for the composite flow utility of agents in Gertler (1999), real balances can be included as an additional variable without creating an extra analytical burden. As we derive below, the solutions for the value functions of the monetary economy can be conjectured and verified in a straightforward manner, similar to the non-monetary model by Gertler.

Monetary and fiscal policies follow feedback rules in the spirit of Leeper (1991), thereby anchoring the economy over time around target levels for the inflation rate and the government debt ratio. Reflecting the underlying overlapping generations structure, the dynamics of the model are critically affected by fiscal policy (which is, by construction, non-neutral) and, in particular, by the design of the pension system which facilitates intergenerational transfers between workers and retirees. In sum, we offer an enlarged New-Keynesian platform which can be used to investigate various macroeconomic questions.

In this paper, we use our model to examine, from a positive perspective, selected long-run macroeconomic implications of demographic changes. Our projection horizon stretches until 2030 and we focus, in particular, on the determinants of the equilibrium real interest rate. The model specifies the demographic processes which drive population growth and life expectancy as time-dependent. This assumption allows us to calibrate the model's demographic parameters according to recent demographic projections for the euro area, as reported in European Economy (2009). Specifically, we take the annual demographic projections for the two series as a deterministic input and verify that the model matches the old-age dependency ratio projected until 2030. We then solve the model numerically under perfect foresight. To carry out such analysis we are forced to make assumptions concerning the future course of the assumed PAYGO pension system. We distinguish between two main types of scenarios in which the rising old-age dependency ratio does or does not lead to changes in the replacement rate (defined as the ratio between individual pension benefits and wages).[2] For the first scenario type, the replacement rate decreases endogenously such that the aggregate benefits-output ratio remains unchanged. This assumption amounts to a strengthening of privately funded elements since it introduces a ceiling on the tax-financed redistribution between workers and retirees. For the second scenario type, the replacement rate remains constant, leading to a rise in the aggregate benefits-output ratio. This assumption models in a simple way a 'no reform' scenario which extrapolates the existing pension system into the future, leading to a higher tax burden on workers. To distinguish between such two deliberately 'extreme' scenarios is instructive because the distinctly different incentives for individual savings generate plausible lower and upper bounds for the projected path of the equilibrium real interest rate.

The main finding is that under either scenario the decrease in population growth and the increase in life expectancy are two independent forces which contribute over the entire projection horizon of about 20 years to a smooth decline in the equilibrium interest rate. This decline, while being more pronounced for first scenario type, does not exceed 50 basis points. Such decline is not visible enough within the shorter time horizon relevant for monetary policy-making to require monetary policy reactions. This finding supports the reasoning of Bean (2004) that because of the 'glacial nature of demographic change' the implications for monetary policy, at least from a positive perspective, should be modest.

In related literature, Ferrero (2005), Roeger (2005) and Kilponen et al. (2005) consider non-monetary versions of the Gertler set-up which, similar to ours, allow for time-dependent demographic processes. Yet, our paper differs from these studies in that we consider a closed economy set-up in which the equilibrium interest rate is endogenously

[2] As will become clear below, we also allow for variations in the retirement age of workers.

determined. Fujiwara and Teranishi (2008) offer a New-Keynesian Gertler-type economy which is similar to ours in a number of respects. Yet, the focus is distinctly different in that Fujiwara ad Teranishi (2008) compare the effects of technology and monetary policy shocks in economies characterized by different steady-state age structures, while implications of time-varying demographic changes are not addressed. Moreover, in the absence of ageing-related fiscal policy and social security aspects, the study does not explore links between demographic developments, pension systems and monetary policy, as also pointed out by Ripatti (2008).[3] It is worth stressing that, despite our modelling decision in favour of a tractable small-scale structure, our predictions are in line with those obtained in large-scale settings. In particular, Miles (1998, 2002) uses a rich overlapping generations framework of a closed economy in the spirit of Auerbach and Kotlikoff (1987). Miles considers various specifications for pension systems and he reports in simulations for the European economy qualitatively and quantitatively predictions similar to ours. As stressed by Batini et al. (2006), Boersch-Supan et al. (2006), and Krueger and Ludwig (2007), additional open-economy channels matter in multi-country or global settings. In particular, to the extent that the euro area ages more rapidly than most OECD countries, closed-economy predictions for the decline in the interest rate tend to be overstated, i.e. capital mobility tends to moderate the pressure on factor price adjustments.[4]

As already stressed, our framework can be used to address a variety of macroeconomic questions. In this particular paper, because of its predominantly long-run focus, the monetary margin plays a limited role. However, the set-up is sufficiently generic to use it for the analysis of questions in which the monetary margin naturally does play a much more significant role (like questions of optimal monetary and fiscal policymaking or a comparison of short-run features of New-Keynesian models with and without life-cycle effects). We plan to address questions of this type in future work.

This paper is structured as follows. Section 2 presents the model. Section 3 summarizes the general equilibrium conditions. Section 4 discusses the numerical assumptions that are used to calibrate the benchmark steady state to stylised features of the euro area. Moreover, it summarizes major demographic trends facing the euro area. Section 5 takes a comparative statics perspective and explains the logic of the model by reporting long-run predictions under different policy assumptions concerning future pension systems. Section 6 uses annual demographic projections for the euro area as a deterministic input for the model and discusses two alternative scenarios lasting until 2030. Section 7 offers conclusions. Technical issues are delegated to three Appendices at the end of the paper.

[3] Another core modelling difference concerns labour supply specifications. Differently from us, the labour supply of retirees in Fujiwara ad Teranishi (2008) is not restricted to be zero. This feature leads to qualitatively different long-run predictions for the equilibrium interest rate, in the sense that a 'greyer' society may well be characterized by a higher equilibrium interest rate.

[4] The models of Boersch-Supan et al. (2006) and Krueger and Ludwig (2007) are in the tradition of Auerbach and Kotlikoff (1987), while Batini et al. (2006) uses a large-scale extension of Blanchard (1985), assuming that agents face a constant probability of death.

2 The Model

The model includes a number of features which are essential to analyze macroeconomic effects of demographic changes. The general modelling approach is to add tractable life-cycle features to an otherwise canonical New-Keynesian DSGE model with monopolistic competition, price rigidities and capital accumulation, as familiar from the monetary policy literature. The exposition below aims to outline the basic building blocks of the model, addressing in turn the demographic structure of the economy as well as the behaviour of households, firms, and monetary and fiscal policymakers.

2.1 Demographic structure

In the spirit of Gertler (1999), the population consists of two distinct groups of agents, workers (N^w) and retirees (N^r). Newborn agents enter directly the working age population which grows at rate n^w. Workers face a probability ω to remain a worker, while they retire with probability $(1-\omega)$. Similarly, retirees stay alive with probability γ, while $(1-\gamma)$ denotes the probability of death of retirees. Hence, the total lifespan of agents between birth and death is made up of two distinct states, working age and retirement age. These two states are subsequently reached by agents, giving rise to life-cycle patterns which are different from a standard representative agent economy. For tractability ω and γ are assumed to be independent of the age of agents, similar to Blanchard (1985) and Weil (1989). However, we assume that the three demographic variables of interest, namely n_t^w, ω_t, and γ_t, are time-dependent, similar to Ferrero (2005), Roeger (2005) and Kilponen et al. (2005). The laws of motion for workers and retirees are given by

$$\begin{aligned} N_{t+1}^w &= (1-\omega_t+n_t^w)N_t^w+\omega_t N_t^w=(1+n_t^w)N_t^w \\ N_{t+1}^r &= (1-\omega_t)N_t^w+\gamma_t N_t^r \end{aligned}$$

Let $\psi_t = N_t^r/N_t^w$ denote the ratio between retirees and workers, the so-called 'old-age dependency ratio'. Then, the growth rate of retirees (n_t^r) satisfies the equation

$$N_{t+1}^r/N_t^r=(1+n_t^r)=\frac{1-\omega_t}{\psi_t}+\gamma_t,$$

while the law of motion for the dependency ratio can be calculated as

$$\psi_{t+1}=\frac{1-\omega_t}{1+n_t^w}+\frac{\gamma_t}{1+n_t^w}\psi_t.$$

Hence, any given specification of $\{n_t^w,\omega_t,\gamma_t\}$ implies laws of motion for n_t^r and ψ_t. A demographic balanced growth path is characterized by $n_t^w=n^w$, $\omega_t=\omega$, and $\gamma_t=\gamma$, implying

$$\psi=\frac{1-\omega}{1+n^w-\gamma}, \tag{1}$$

i.e. the old-age dependency ratio (ψ) increases in the survival probability of retirees (γ) and in the retirement probability of workers $(1-\omega)$, while it decreases in the growth rate of newborn agents (n^w). Finally, along a balanced growth path

$$n^r = n^w = n,$$

i.e. the growth rate of the two groups coincides with the population growth rate.

2.2 Decision problems of retirees and workers

The structure of the preferences of agents follows closely Gertler (1999). To align this structure with a monetary economy, we introduce real balances as an additional element in the utility function, leading to an additional first-order condition. This modifies below the conjectures for the aggregate consumption function and the value functions associated with the two states, respectively. Otherwise, however, the procedure for solving the decision problems of retirees and workers is similar to Gertler. For brevity, technical aspects are delegated to Appendix I.

Let V_t^z denote the value function associated with the two states of working age and retirement, i.e. $z = w, r$. Then,

$$\begin{aligned}
V_t^z &= [[(c_t^z)^{v_1}(m_t^z)^{\nu_2}(1-l_t^z)^{v_3}]^\rho + \beta^z E_t [V_{t+1} \mid z]^\rho]^{\frac{1}{\rho}} \\
\beta^w &= \beta, \beta^r = \beta\gamma_t \\
E_t [V_{t+1} \mid w] &= \omega_t V_{t+1}^w + (1-\omega_t) V_{t+1}^r \\
E_t [V_{t+1} \mid r] &= V_{t+1}^r,
\end{aligned}$$

where c_t, m_t, and $1 - l_t$ denote consumption, real balances and leisure, respectively. The parameter ν_2 denotes the weight of real balances in the Cobb-Douglas flow utility of agents. If $\nu_2 \to 0$ preferences of agents converge against the economy with variable labour supply examined by Gertler (1999). The effective discount rates of the two types of agents differ since retirees face a positive probability of death, while workers, when leaving their state, stay alive and switch to retirement. Going back to Epstein and Zin (1989), such non-expected utility specification can be used to separate risk aversion from intertemporal substitution aspects. For this particular functional form, as discussed in Farmer (1990), agents are risk-neutral with respect to income risk, while $\sigma = 1/(1-\rho)$ denotes the a priori unspecified intertemporal elasticity of substitution. The advantages of this specification become clear when considered together with the idiosyncratic risks faced by individuals and the (un)availability of insurance markets. There are two aspects to this. First, workers face an income risk when entering retirement. To allow for life-cycle behaviour, there exists no insurance market against this risk, and the assumption of risk-neutrality acts like a cushion to dampen the effects of this risk at the individual level. Second, retirees face the risk of death. To eliminate the uncertainty about the remaining lifetime horizon of retirees there exists a perfect annuities market similar to Blanchard (1985). This market is operated by competitive mutual funds which collect the non-human wealth of retirees and pay in return to surviving retirees a return rate $(1+r)/\gamma$ which is above the pure real interest rate $(1 + r)$.

2.2.1 Decision problem of the representative retiree

The representative retiree (with index j) maximizes in period t the objective

$$V_t^{rj} = \left[\left[\left(c_t^{rj}\right)^{v_1}(m_t^{rj})^{\nu_2}(1-l_t^{rj})^{v_3}\right]^{\rho} + \beta\gamma_t\left[V_{t+1}^{rj}\right]^{\rho}\right]^{\frac{1}{\rho}}$$

subject to the flow budget constraint

$$c_t^{rj} + \frac{i_t}{1+i_t}m_t^{rj} + a_t^{rj} = \frac{1+r_{t-1}}{\gamma_{t-1}}a_{t-1}^{rj} + \xi w_t l_t^{rj} + e_t^j,$$

where a_{t-1}^{rj} denotes his predetermined stock of non-human wealth.[5] The retiree receives benefits e_t^j and faces an effective wage rate ξw_t. The parameter $\xi \in (0,1)$ captures the productivity differential between retirees and workers, and in the equilibrium discussed below ξ will be adjusted such that the labour supply l_t^{rj} is zero. With i_t denoting the nominal interest rate, the term $\frac{i_t}{1+i_t}m_t^{rj}$ describes, in a sense, the 'consumption level of real balances', reflecting that real balances are dominated in return by interest-bearing assets. The decision problem gives rise to three first-order conditions. Consumption follows the intertemporal Euler equation

$$c_{t+1}^{rj} = [\beta\,(1+r_t)\,(\frac{1+i_{t+1}}{i_{t+1}}\frac{i_t}{1+i_t})^{\nu_2\rho}(\frac{w_t}{w_{t+1}})^{v_3\rho}]^{\sigma}c_t^{rj},$$

while upon appropriate substitutions the first-order conditions associated with leisure and real balances have a purely intratemporal representation, i.e.

$$\begin{aligned} 1-l_t^{rj} &= \frac{v_3}{v_1}\frac{c_t^{rj}}{\xi w_t} \\ m_t^{rj} &= \frac{v_2}{v_1}\frac{1+i_t}{i_t}c_t^{rj}. \end{aligned}$$

Let $\epsilon_t\pi_t$ denote the marginal propensity of retirees to consume out of wealth, where consumption is meant to include the term $\frac{i_t}{1+i_t}m_t^{rj}$. In other words, $\epsilon_t\pi_t$ corresponds to $c_t^{rj} + \frac{i_t}{1+i_t}m_t^{rj} = c_t^{rj}(1+\frac{v_2}{v_1})$. Moreover, with d_t^{rj} and h_t^{rj} denoting the disposable income of a retiree and his stock of human capital, respectively, consider the following recursive law of motion for human capital

$$\begin{aligned} h_t^{rj} &= d_t^{rj} + \frac{\gamma_t}{1+r_t}h_{t+1}^{rj} \\ d_t^{rj} &= \xi w_t l_t^{rj} + e_t^j, \end{aligned}$$

[5] The budget constraint, if written like this, assumes that the retiree was already in retirement during the previous period $t-1$. For a complete description of the cohort-specific behavior of all agents the decision problem would have to be conditioned on the year of birth and the age at which retirement takes place. However, this is not needed for the derivation of the aggregate behaviour of retirees and workers, anticipating the linear structure of the decision rules derived below.

which captures that the retiree survives with probability γ_t. Then, in combination with the flow budget constraint, one can establish that the consumption function and the law of motion for $\epsilon_t\pi_t$ satisfy the relationships

$$c_t^{rj} + \frac{i_t}{1+i_t} m_t^{rj} = c_t^{rj}(1+\frac{v_2}{v_1}) = \epsilon_t\pi_t \left(\frac{1+r_{t-1}}{\gamma_{t-1}} a_{t-1}^{rj} + h_t^{rj} \right)$$

and

$$\epsilon_t\pi_t = 1 - [(\frac{1+i_{t+1}}{i_{t+1}} \frac{i_t}{1+i_t})^{\nu_2\rho} (\frac{w_t}{w_{t+1}})^{v_3\rho}]^\sigma \beta^\sigma (1+r_t)^{\sigma-1} \gamma_t \frac{\epsilon_t\pi_t}{\epsilon_{t+1}\pi_{t+1}}. \quad (2)$$

These expressions can be used to establish an analytical expression for the value function V_t^{rj} which is a key input for the decision problem of the representative worker. In particular, the proportionality between m_t^{rj} and c_t^{rj} (which leads to the 'gross 'consumption term $c_t^{rj}(1+\frac{v_2}{v_1})$) ensures that in Appendix I the conjectured solutions for the value functions of the monetary economy can be verified similarly to the non-monetary model by Gertler.

2.2.2 Decision problem of the representative worker

Similarly, the representative worker maximizes in period t the objective

$$V_t^{wj} = \left[\left[\left(c_t^{wj} \right)^{v_1} (m_t^{wj})^{\nu_2} (1-l_t^{wj})^{v_3} \right]^\rho + \beta \left[\omega_t V_{t+1}^{wj} + (1-\omega_t) V_{t+1}^{rj} \right]^\rho \right]^{\frac{1}{\rho}}$$

subject to the flow budget constraint

$$c_t^{wj} + \frac{i_t}{1+i_t} m_t^{wj} + a_t^{wj} = (1+r_{t-1}) a_{t-1}^{wj} + w_t l_t^{wj} + f_t^j - \tau_t^j,$$

which assumes that the worker was already in the workforce during the period $t-1$.[6] Notice that the return rate associated with a_{t-1}^{wj} is different from the previous section because of the discussed asymmetries of insurance possibilities in working age and retirement age. Moreover, the representative worker faces the full wage rate (w_t), receives profits (f_t^j) of imperfectly competitive firms in the intermediate goods sector and pays lump-sum taxes (τ_t^j).[7] Again, the decision problem gives rise to three first-order conditions. The consumption-Euler equation

$$\omega_t c_{t+1}^{wj} + (1-\omega_t)(\epsilon_{t+1})^{\frac{\sigma}{1-\sigma}} (\frac{1}{\xi})^{v_3} c_{t+1}^{rj} = \left[\beta(1+r_t)\Omega_{t+1} (\frac{1+i_{t+1}}{i_{t+1}} \frac{i_t}{1+i_t})^{\nu_2\rho} (\frac{w_t}{w_{t+1}})^{v_3\rho} \right]^\sigma c_t^{wj}$$

with associated

$$\Omega_{t+1} = \omega_t + (1-\omega_t)\, \epsilon_{t+1}^{\frac{1}{1-\sigma}} (\frac{1}{\xi})^{v_3} \quad (3)$$

[6] New born agents are assumed to enter the workforce with zero non-human wealth.

[7] In a richer framework, it would be straightforward to modify the simplifying assumption that all taxes (profits) are paid (received) by workers and all benefits are received by retirees. Since the key results depend only on the net transfers made between the two groups, this simple specification, however, captures the main redistribution effects occurring in a life-cycle framework.

is now more complicated, reflecting the possibility that the worker may switch into retirement in the next period. Specifically, the weighting term Ω_{t+1} (which is specific to the solution of the worker's problem) indicates that a worker, when switching into retirement, reaches a state which is characterized by a different effective wage rate (captured by ξ) and, as will become clear below, by a different marginal propensity to consume (captured by ϵ_{t+1}). By contrast, the first-order conditions with respect to leisure (adjusted for the absence of ξ) and real balances are unchanged, i.e.

$$\begin{aligned} 1 - l_t^{wj} &= \frac{v_3}{v_1}\frac{c_t^{wj}}{w_t} \\ m_t^{wj} &= \frac{v_2}{v_1}\frac{1+i_t}{i_t}c_t^{wj} \end{aligned}$$

Let π_t denote the marginal propensity of workers to consume out of wealth, again, inclusive of the term $\frac{i_t}{1+i_t}m_t^{wj}$. Moreover, with d_t^{wj} and h_t^{wj} denoting the disposable income of a worker and his stock of human capital, consider the recursive law of motion

$$\begin{aligned} h_t^{wj} &= d_t^{wj} + \frac{\omega_t}{\Omega_{t+1}}\frac{1}{1+r_t}h_{t+1}^{wj} + (1 - \frac{\omega_t}{\Omega_{t+1}})\frac{1}{1+r_t}h_{t+1}^{rj} \\ d_t^{wj} &= w_t l_t^{wj} + f_t^j - \tau_t^j, \end{aligned}$$

with h_t^{rj} following the law of motion defined above. Then, similar to the retiree's problem, one can verify that the worker's consumption function is given by

$$c_t^{wj} + \frac{i_t}{1+i_t}m_t^{wj} = c_t^{wj}(1 + \frac{v_2}{v_1}) = \pi_t\left((1+r_{t-1})\,a_{t-1}^{wj} + h_t^{wj}\right).$$

Finally, these relationships are mutually consistent with each other if the marginal propensity to consume out of wealth π_t evolves according to

$$\pi_t = 1 - \left[(\frac{1+i_{t+1}}{i_{t+1}}\frac{i_t}{1+i_t})^{\nu_2\rho}(\frac{w_t}{w_{t+1}})^{v_3\rho}\right]^{\sigma}\beta^{\sigma}((1+r_t)\,\Omega_{t+1})^{\sigma-1}\frac{\pi_t}{\pi_{t+1}} \tag{4}$$

One can show that the marginal propensity to consume of retirees is higher than of workers ($\epsilon > 1$), implying $\Omega > 1$. This in turn indicates that workers discount future income streams at an effective interest rate $(1+r_t)\,\Omega_{t+1}$ which is higher than the pure interest rate, reflecting the expected finiteness of life.

2.3 Aggregation over retirees and workers

To characterize aggregate variables, we use the notation introduced in the previous subsections but drop the index j. With the total number of retirees and workers in period t being given by N_t^r and N_t^w, respectively, aggregate labour supply schedules satisfy

$$l_t^w = N_t^w l_t^{wj} = N_t^w\left(1 - \frac{v_3}{v_1}\frac{c_t^{wj}}{w_t}\right) = N_t^w - \frac{v_3}{v_1}\frac{c_t^w}{w_t} \tag{5}$$

$$l_t^r = N_t^r l_t^{rj} = N_t^r\left(1 - \frac{v_3}{v_1}\frac{c_t^{rj}}{\xi w_t}\right) = N_t^r - \frac{v_3}{v_1}\frac{c_t^r}{\xi w_t} \tag{6}$$

$$l_t = l_t^w + \xi l_t^r. \tag{7}$$

The aggregate stocks of the human capital of retirees and of workers follow the recursive law of motions

$$h_t^r = d_t^r + \frac{\gamma_t}{(1+n_t^r)(1+r_t)} h_{t+1}^r \quad (8)$$

$$h_t^w = d_t^w + \frac{\omega_t}{\Omega_{t+1}} \frac{1}{(1+n_t^w)(1+r_t)} h_{t+1}^w + (1 - \frac{\omega_t}{\Omega_{t+1}}) \frac{1}{(1+n_t^r)(1+r_t)} \frac{1}{\psi_t} h_{t+1}^r, \quad (9)$$

with the aggregate disposable income terms of the two groups being defined as

$$d_t^r = d_t^{rj} N_t^r = \left(\xi w_t l_t^{rj} + e_t^j\right) N_t^r = \xi w_t l_t^r + e_t \quad (10)$$

$$d_t^w = d_t^{wj} N_t^w = \left(w_t l_t^{wj} + f_t^j - \tau_t^j\right) N_t^w = w_t l_t^w + f_t - \tau_t. \quad (11)$$

Compared with the law of motions at the individual level, the two equations (8) and (9) feature the additional discounting terms $1 + n_t^r$ and $1 + n_t^w$, respectively. These terms ensure that the discounted income streams of currently alive retirees and workers do not incorporate contributions of agents which as of today do not yet belong to these two groups. Let a_{t-1}^r and a_{t-1}^w denote the predetermined levels of aggregate non-human wealth of retirees and workers in period t, resulting from savings decisions in period $t-1$. Then, given the linear structure of individual consumption decisions, aggregate consumption levels of retirees and workers can be written as

$$c_t^r (1 + \frac{v_2}{v_1}) = \epsilon_t \pi_t \left((1 + r_{t-1}) a_{t-1}^r + h_t^r\right) \quad (12)$$

$$c_t^w (1 + \frac{v_2}{v_1}) = \pi_t \left((1 + r_{t-1}) a_{t-1}^w + h_t^w\right), \quad (13)$$

where the absence of the term γ_{t-1} in equation (12) reflects the competitive insurance of death probabilities of retirees. To aggregate these two expressions let $a_t = a_t^r + a_t^w$ and $\lambda_t = a_t^r / a_t$, where λ_t is introduced to summarize compactly the distribution of aggregate non-human wealth between retirees and workers. Using these definitions aggregate consumption (c_t) and aggregate real balances (m_t) can be characterized by the expressions

$$c_t = c_t^r + c_t^w = \frac{1}{1 + \frac{v_2}{v_1}} \pi_t \left[(1 + (\epsilon_t - 1) \lambda_{t-1})(1 + r_{t-1}) a_{t-1} + \epsilon_t h_t^r + h_t^w\right] \quad (14)$$

$$m_t = m_t^r + m_t^w = \frac{1 + i_t}{i_t} \frac{v_2}{v_1} c_t. \quad (15)$$

Finally, to characterize the law of motion for λ_t, notice that the aggregate non-human wealth of retirees evolves according to

$$\lambda_t a_t = \lambda_{t-1} (1 + r_{t-1}) a_{t-1} + d_t^r - c_t^r - \frac{i_t}{1 + i_t} m_t^r + (1 - \omega_t) \left[(1 - \lambda_{t-1})(1 + r_{t-1}) a_{t-1} + d_t^w - c_t^w - \frac{i_t}{1 + i_t} m_t^w\right],$$

while the aggregate non-human wealth of workers follows the law of motion

$$(1-\lambda_t)\,a_t = \omega_t\left[(1-\lambda_{t-1})\,(1+r_{t-1})\,a_{t-1} + d_t^w - c_t^w - \frac{i_t}{1+i_t}m_t^w\right].$$

Combining these two expressions yields

$$\lambda_t a_t = \omega_t\left[(1-\epsilon_t\pi_t)\left(\lambda_{t-1}\left(1+r_{t-1}\right)a_{t-1} + h_t^r\right) - \left(h_t^r - d_t^r\right)\right] + (1-\omega_t)a_t. \qquad (16)$$

2.4 Firms

The supply-side of the economy has a simple New-Keynesian structure, in the spirit of Clarida et al. (1999) and Woodford (2003). Specifically, we combine the assumption of monopolistic competition in the spirit of Dixit and Stiglitz (1977) with Calvo-type nominal rigidities in order to generate short-run dynamics consistent with a New-Keynesian Phillips-curve. Moreover, the production of capital goods is subject to adjustment costs, leading to a persistent reaction of investment dynamics to shocks hitting the economy.

2.4.1 Final goods sector

There is a continuum of intermediate goods, indexed by $z \in [0,1]$, which are transformed into a homogenous final good according to the technology

$$y_t = \left[\int_0^1 y_t(z)^{\frac{\theta-1}{\theta}}dz\right]^{\frac{\theta}{\theta-1}}.$$

The final goods sector is subject to perfect competition, giving rise to a demand function for the representative intermediate good z

$$y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\theta} y_t,$$

where $P_t(z)$ and P_t denote the price of good z and the average price level of intermediate goods, respectively, and $\theta > 1$ denotes the price elasticity of demand. Reflecting the CES-structure of the technology in the final goods sector, P_t is given by

$$P_t = \left[\int_0^1 P_t(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}}.$$

2.4.2 Intermediate goods sector

The representative firm produces the intermediate good z with the technology

$$y_t(z) = \left(X_t l_t(z)\right)^{\alpha} k_t(z)^{1-\alpha},$$

where $l_t(z)$ and $k_t(z)$ denote the input levels of labour and capital and X_t denotes the exogenously determined level of labour augmenting technical progress. For simplicity, we assume that X_t grows at a constant rate, i.e. $X_t = (1+x)X_{t-1}$, with $x > 0$. Markets for the two inputs are competitive, i.e. the real wage rate w_t and the real rental rate r_t^k are taken as given in the production of good z. Cost minimization implies

$$\frac{w_t l_t(z)}{\alpha y_t(z)} = \frac{r_t^k k_t(z)}{(1-\alpha) y_t(z)} = mc_t,$$

where mc_t denotes real marginal costs, which are identical across firms. Profits of firm z are given by

$$f_t(z) = \left(\frac{P_t(z)}{P_t} - mc_t\right) y_t(z).$$

Each firm has price-setting power in its output market. In line with Calvo (1983), in each period only a fraction $(1-\zeta)$ of firms can reset its price optimally, while for a fraction ζ of firms the price remains unchanged. Let $P_t^*(z)$ denote the optimally reset price in period t by a firm which can change its price. Reflecting the forward-looking dimension of the price-setting decision under the Calvo-constraint, $P_t^*(z)/P_t$ evolves over time according to

$$\frac{P_t^*(z)}{P_t} = \frac{\theta}{(\theta-1)} \frac{E_t \sum_{i=0}^{\infty} (\zeta\beta)^i \left(\frac{1}{P_{t+i}}\right)^{1-\theta} y_{t+i} mc_{t+i} \frac{P_{t+i}}{P_t}}{E_t \sum_{i=0}^{\infty} (\zeta\beta)^i \left(\frac{1}{P_{t+i}}\right)^{1-\theta} y_{t+i}}.$$

2.4.3 Capital goods

There exists a continuum of capital goods producing firms, indexed by $u \in [0,1]$, renting out capital to firms in the intermediate goods sector. In each period, after the production of intermediate and final goods is completed, the representative capital goods producing firm combines its existing capital stock $k_t(u)$ with investment goods $i_t^k(u)$ to produce new capital goods $k_{t+1}(u)$ according to the constant returns technology

$$k_{t+1}(u) = \phi(\frac{i_t^k(u)}{k_t(u)}) k_t(u) + (1-\delta) k_t(u),$$

with $\phi'() > 0, \phi''() < 0$. Let $p_t^k = P_t^k/P_t$ denote the relative price of capital goods in terms of final output. Then, the optimal choice of investment levels $i_t^k(u)$ leads to the first-order condition

$$p_t^k \phi'(\frac{i_t^k(u)}{k_t(u)}) = 1.$$

Let $i^k(u)/k(u)$ denote the investment-capital ratio at the firm level along a balanced growth path. It is assumed that the function ϕ satisfies the relations

$$\phi(\frac{i^k(u)}{k(u)}) = \frac{i^k(u)}{k(u)}, \ \phi'(\frac{i^k(u)}{k(u)}) = 1,$$

which are well-known from the q-theory of investment.

2.4.4 Aggregate relationships and resource constraint

At the aggregate level the capital stock is a predetermined variable, leading to

$$k_{t-1} = \int_0^1 k_t(z)dz = \int_0^1 k_t(u)du.$$

Moreover,

$$i_t^k = \int_0^1 i_t^k(u)du$$

and

$$l_t = l_t^w + \xi l_t^r = \int_0^1 l_t(z)dz,$$

while aggregate output and profits are given by

$$y_t = \left[\int_0^1 y_t(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}, \quad \text{with } y_t(z) = \left(X_t l_t(z)\right)^{\alpha} k_t(z)^{1-\alpha} \tag{17}$$

$$f_t = \int_0^1 f_t(z)dz = \int_0^1 \left(\frac{P_t(z)}{P_t} - mc_t\right) y_t(z)dz. \tag{18}$$

The capital-labour ratio in the intermediate goods sector will be identical across firms, implying

$$\frac{k_{t-1}}{l_t} = \frac{w_t}{r_t^k}\frac{1-\alpha}{\alpha}, \tag{19}$$

while real marginal costs can be rewritten as

$$mc_t = \left(\frac{w_t}{\alpha X_t}\right)^{\alpha} \left(\frac{r_t^k}{(1-\alpha)}\right)^{1-\alpha}. \tag{20}$$

In any symmetric equilibrium $P_t^*(z) = P_t^*$ must be identical across all firms which have the chance to adjust their prices, leading to

$$\frac{P_t^*}{P_t} = \frac{\theta}{(\theta-1)} \frac{E_t \sum_{i=0}^{\infty} (\zeta\beta)^i \left(\frac{1}{P_{t+i}}\right)^{1-\theta} y_{t+i} mc_{t+i} \frac{P_{t+i}}{P_t}}{E_t \sum_{i=0}^{\infty} (\zeta\beta)^i \left(\frac{1}{P_{t+i}}\right)^{1-\theta} y_{t+i}}, \tag{21}$$

while the evolution of the price level can be written as

$$P_t = \left(\zeta P_{t-1}^{1-\theta} + (1-\zeta) P_t^{*1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{22}$$

In the capital goods sector the investment-capital ratio is identical across firms, leading to

$$k_t = \phi\left(\frac{i_t^k}{k_{t-1}}\right) k_{t-1} + (1-\delta) k_{t-1} \tag{23}$$

$$1 = p_t^k \phi'\left(\frac{i_t^k}{k_{t-1}}\right). \tag{24}$$

The aggregate resource constraint of the economy is given by

$$y_t = c_t + g_t + i_t^k, \tag{25}$$

where g_t denotes government expenditures in terms of the final output good.

2.5 Government

To discuss the role of the government sector, it is convenient to start with the flow budget constraint of the government in nominal terms (denoted by capital letters)

$$M_t + B_t = M_{t-1} + (1 + i_{t-1})B_{t-1} + G_t + E_t - T_t.$$

With

$$1 + i_t = (1 + r_t)\left(\frac{P_{t+1}}{P_t}\right), \tag{26}$$

the budget constraint can be rewritten in real terms as

$$b_t = (1 + r_{t-1})\left(b_{t-1} + \frac{1}{1 + i_{t-1}} m_{t-1}\right) + g_t + e_t - \tau_t - m_t. \tag{27}$$

Real government expenditures (g_t) are assumed to be exogenously given. The path of aggregate real benefits (e_t) is determined by the replacement rate μ_t between individual benefits and the real wage, i.e.

$$\mu_t = \frac{e_t^j}{w_t} \Rightarrow e_t = e_t^j N_t^r = \mu_t w_t N_t^r. \tag{28}$$

Notice that the budget of the pension system is embedded in the overall budget constraint (27). In present value terms, the pension system is run on a PAYGO-basis, since all benefits received by retirees are backed by taxes (which are entirely paid by workers), and not by proceeds from investments in the economy's capital stock. Real government debt (b_t) and real capital holdings ($p_t^k k_t$) are perceived as perfect substitutes by the private sector. This leads to the definition of total private sector non-human wealth

$$a_t = p_t^k k_t + b_t + \frac{m_t}{1 + i_t}, \tag{29}$$

which is supported by the arbitrage relationship between the return rates on real government debt and real capital holdings

$$1 + r_t = \frac{r_{t+1}^k + p_{t+1}^k(1 - \delta)}{p_t^k}. \tag{30}$$

2.5.1 Policy rules

To close the system we assume that fiscal and monetary policy follow stylized feedback rules.[8] As regards *fiscal policy,* we consider a rule which stabilizes a certain target level b^* of the debt ratio (b_t/y_t) by variations in the remaining free fiscal instrument τ_t such that

$$\frac{\tau_t}{y_t} = \tau^* + \gamma_1 \left(\frac{b_t}{y_t} - b^*\right) + \gamma_2 \left(\frac{b_t}{y_t} - \frac{b_{t-1}}{y_{t-1}}\right), \tag{31}$$

where τ^* denotes the tax ratio (τ_t/y_t) which corresponds to b^* along a balanced growth path, γ_1 denotes the direct feedback parameter to counteract deviations of the debt ratio from its target and γ_2 controls the smoothness of this process. This specification is in line with the broad discussion given in Mitchell et al. (2000) and may qualify as a simple benchmark. However, alternative fiscal closures (in terms of residual fiscal instruments as well as target variables) can be imagined which could easily replace (31).

As regards *monetary policy*, we assume that the central bank has a target inflation rate which is equal to zero ($\pi^{p*} = 0$), while for the inflation rate π_t^p we use below for the log-linearized economy the approximation $\pi_t^p = \ln(P_t/P_{t-1})$. The reaction of the central bank is modelled through a Taylor-type feedback rule which sets the nominal interest rate as a function of the current inflation rate, the output gap ($\widetilde{y}_t = \ln(\frac{y_t}{\overline{y}})$), where $\overline{y}$ denotes the steady-state level of the detrended economy established below), and the previous value of the nominal interest rate (with weight ρ), i.e.

$$i_t = \rho i_{t-1} + (1-\rho)\left[r + \gamma_\pi \pi_t^p + \gamma_y \widetilde{y}_t\right]. \tag{32}$$

3 General Equilibrium

In equilibrium, government actions and optimizing decisions of workers, retirees, and firms must be mutually consistent at the aggregate level. In sum, an *equilibrium* consists for all periods t of sequences of endogenous variables $\{\epsilon_t,\ \pi_t,\ \Omega_t,\ l_t^w,\ l_t^r,\ l_t,\ h_t^w,\ h_t^r,\ d_t^w,\ d_t^r,\ c_t^w,\ c_t^r,\ c_t,\ y_t,\ k_t,\ f_t,\ mc_t,\ w_t,\ r_t^k, i_t^k, p_t^k, p_t^*,\ p_t,\ i_t,\ r_t,\ \lambda_t,\ a_t,\ b_t,\ m_t,\ \tau_t,\ e_t\}$ which satisfy the system of equations (2)-(32), taking as given exogenous sequences of policy-related variables $\{\tau^*,\ b^*,\ \mu_t\}$, demographic processes $\{n_t^w,\ \omega_t, \gamma_t\}$, productivity growth x, and appropriate initial conditions for N_t^w, N_t^r, X_t, and all endogenous state variables.[9]

As long as one assumes $x > 0$, $n > 0$, the economy is subject to ongoing exogenous growth. Hence, we consider from now on a detrended version of (2)-(32) which expresses all unbounded variables in terms of efficiency units per workers. For the detrended equation system, we use the following notational conventions. Consider generic variables $v_t \in \{c_t,\ y_t,\ k_t,\ f_t,\ r_t^k,\ i_t^k,\ a_t,\ b_t,\ m_t,\ \tau_t,\ e_t\}$, $v_t^w \in \{h_t^w,\ d_t^w,\ c_t^w, \}$, and $v_t^r \in \{\ h_t^r,\ d_t^r,\ c_t^r\}$. Then,

$$\frac{v_t}{N_t^w X_t} = \overline{v_t}, \qquad \frac{v_t^w}{N_t^w X_t} = \overline{v_t^w}, \qquad \frac{v_t^r}{N_t^w X_t} = \frac{v_t^r}{N_t^r X_t}\frac{N_t^r}{N_t^w} = \overline{v_t^r}\psi_t.$$

[8] For an early contribution in this spirit, see Leeper (1991). Recent and more detailed discussions can be found, for example, in Schmitt-Grohé and Uribe (2007) and Leith and von Thadden (2008).

[9] Endogenous state variables with predetermined initial conditions relate, in particular, to the level of aggregate non-human wealth, its breakdown across assets, and its distribution between workers and retirees.

Moreover, reflecting the properties of labour-augmenting technical progress, we specify the real wage and the variables related to the labour supply as

$$\frac{w_t}{X_t} = \overline{w_t}, \quad \frac{l_t}{N_t^w} = \overline{l_t}, \quad \frac{l_t^w}{N_t^w} = \overline{l_t^w}, \quad \frac{l_t^r}{N_t^w} = \frac{l_t^r}{N_t^r}\frac{N_t^r}{N_t^w} = \overline{l_t^r}\psi_t.$$

3.1 Detrended economy

Appendix II summarizes the detrended counterparts of all equations (2)-(32) which make up the dynamic system we study from now onwards. Based on this representation, it is straightforward to characterize steady states of the detrended economy. A compact summary can be established if one invokes a number of steady-state features. Let the variables without time subscript refer to steady-state values. In particular, $P^* = P = P(z) = 1, \overline{y} = \overline{y(z)}$, $\overline{l} = \overline{l(z)}$ and $\overline{k(z)} = \frac{\overline{k}}{(1+n)(1+x)}$, $mc = (\theta - 1)/\theta$ and $\overline{f} = (1/\theta)\overline{y}$. Moreover, $p^k = 1$, $r = i$, $r^k = r + \delta$.

$$\begin{aligned}
\epsilon\pi &= 1 - \left[(\frac{1}{1+x})^{v_3\rho}\right]^{\sigma} \beta^{\sigma}(1+r)^{\sigma-1}\gamma \\
\pi &= 1 - \left[(\frac{1}{1+x})^{v_3\rho}\right]^{\sigma} \beta^{\sigma}((1+r)\Omega)^{\sigma-1} \\
\Omega &= \omega + (1-\omega)\,\epsilon^{\frac{1}{1-\sigma}}(\frac{1}{\xi})^{v_3} \\
\overline{l^r} &= 1 - \frac{v_3}{v_1}\frac{\overline{c^r}}{\xi\overline{w}}, \quad \overline{l^w} = 1 - \frac{v_3}{v_1}\frac{\overline{c^w}}{\overline{w}}, \quad \overline{l} = \overline{l^w} + \xi\overline{l^r}\psi \\
\overline{h^r} &= \xi\overline{w}\overline{l^r} + \mu\overline{w} + \gamma\frac{1+x}{1+r}\overline{h^r} \\
\overline{h^w} &= \overline{w}\overline{l^w} + (1/\theta)\overline{y} - \overline{\tau} + \frac{\omega}{\Omega}\frac{1+x}{1+r}\overline{h^w} + (1-\frac{\omega}{\Omega})\frac{1+x}{1+r}\overline{h^r}
\end{aligned}$$

$$\begin{aligned}
\overline{c^r}(1+\frac{v_2}{v_1}) &= \epsilon\pi(\frac{1+r}{(1+n)(1+x)}\lambda\overline{a} + \overline{h^r}) \\
\overline{c^w}(1+\frac{v_2}{v_1}) &= \pi(\frac{1+r}{(1+n)(1+x)}(1-\lambda)\overline{a} + \overline{h^w}) \\
\overline{c} &= \overline{c^w} + \psi\overline{c^r}, \quad \overline{a} = \overline{k} + b^*\overline{y} + \frac{v_2}{v_1}\frac{1}{r}\overline{c} \\
\lambda\overline{a} &= \omega\left[(1-\epsilon\pi)\left(\frac{\lambda(1+r)}{(1+n)(1+x)}\overline{a} + \overline{h^r}\psi\right) - (\overline{h^r} - \xi\overline{w}\overline{l^r} - \mu\overline{w})\psi\right] + (1-\omega)\overline{a} \\
\overline{y} &= (\overline{l})^{\alpha}(\frac{\overline{k}}{(1+n)(1+x)})^{1-\alpha} = \overline{c} + \overline{g} + \left(1 - \frac{1-\delta}{(1+n)(1+x)}\right)\overline{k} \\
\frac{\theta-1}{\theta} &= \left(\frac{\overline{w}}{\alpha}\right)^{\alpha}\left(\frac{r+\delta}{1-\alpha}\right)^{1-\alpha}, \quad \frac{1}{(1+n)(1+x)}\frac{\overline{k}}{\overline{l}} = \frac{\overline{w}}{r+\delta}\frac{1-\alpha}{\alpha} \\
b^*\overline{y} &= \frac{(1+n)(1+x)}{1+r-(1+n)(1+x)}(\overline{\tau} - \overline{g} - \mu\overline{w}\psi) + \frac{(1+n)(1+x)-1}{1+r-(1+n)(1+x)}\frac{v_2}{v_1}\frac{1+r}{r}\overline{c}
\end{aligned}$$

These equations constitute a system in 18 equations and 18 unknown endogenous variables, i.e. $\{\epsilon,\ \pi,\ \Omega,\ \overline{l^r},\ \overline{l^w},\ \overline{l},\ \overline{h^r},\ \overline{h^w},\ \overline{c^r},\ \overline{c^w},\ \overline{c},\ \overline{y},\ \overline{a},\ \overline{k},\ \overline{w},\ r,\ \lambda, \overline{\tau}\}$. Finally, we log-linearize all detrended equilibrium conditions around the zero-inflation steady state. The linearized versions of the detrended equations will be used in Section 6 of the paper which discusses selected policy scenarios.

4 Calibration and demographic trends

4.1 Calibration

We calibrate the system of steady-state equations to match key features of annual euro area data, taking, in particular, recent demographic observations until 2008 as a benchmark, as provided by the comprehensive '2009 Ageing Report' prepared by the European Commission and published in European Economy (2009). Tables 1, 2, and 3 summarize our assumptions concerning the initial choices of demographic variables, the structural parameters, and the steady-state relevant policy-related variables, respectively. When used within the set of steady-state equations, these assumptions give rise to steady-state values of the endogenous variables (or ratios of them) as summarized in Table 4. While all our assumptions are quantitatively in line with the related literature, it is worth making a number of comments which focus, in particular, on the demographic aspects of the model.

Table 1: Demographic parameters

Growth rate of working age population	n	0.004
Retirement probability of workers	$1-\omega$	0.020
Implied average working period	$T^w = 1/(1-\omega)$	50
Probability of death of retirees	$1-\gamma$	0.069
Implied average retirement period	$T^r = 1/(1-\gamma)$	14.5
Implied old age dependency ratio	$\psi = \frac{1-\omega}{1+n-\gamma}$	0.27

First, the demographic assumptions in Table 1 closely match euro area characteristics reported for the year 2008.[10] Since our model features only working age and retirement age, the choice of n corresponds to the growth rate of the working age population which is reported as 0.4%. Reflecting well-known properties of the geometric distribution, the total average lifetime (T) in our model is given by $T = 1/(1-\omega)+1/(1-\gamma) = T^w + T^r$. In the data, working age covers the years 15-64, while retirement age is defined as 65 years and above. Life expectancy at birth is reported as 79.5 years, and our calibration of ω and γ corrects for the absence of young people below 15 in our model, i.e. $T^w + T^r = 64.5$. The (steady-state) old-age dependency ratio of $\psi = 0.27$ implied by the model matches exactly the old-age dependency ratio reported for the euro area in 2008.

[10] The benchmark calibration reproduces euro area data listed in the column for the year 2008 of the summary table 'Main demographic and macroeconomic assumptions' for the euro area (EA 16) in the Statistical Annex to European Economy (2009), p. 174.

Table 2: Structural parameters

Intertemporal elasticity of substitution	σ	1/3
Discount factor	β	0.99
Cobb-Douglas share of labour	α	2/3
Relative productivity of retirees	ξ	0.325
Depreciation rate of capital	δ	0.05
Growth rate of technological progress	x	0.01
Elasticity of demand (intermediate goods)	θ	10
Preference parameter: consumption	v_1	0.64
Preference parameter: real balances	v_2	0.002
Preference parameter: leisure	v_3	0.358

Second, the relative productivity parameter ξ has been set to ensure that the participation rate of workers is 0.70, in line with the empirical value reported for 2008, while the implied participation rate of retirees is approximately zero. The latter result may seem overly restrictive, but it does respect the cut-off feature of the empirical data set, namely to assume that all persons at age 65 or above are assumed to have retired. Third, in calibration exercises of this type there is some leeway to fix the long-run *level* of the real interest rate. Our numerical choices for the crucial parameters β, σ, x, δ are in line with the literature, as is the implied value of r which amounts to 3.9%.

Table 3 : Steady-state relevant policy parameters

Debt-to-output-ratio	b^*	0.7
Government spending share	g/y	0.18
Replacement rate	$\mu = e^{rj}/w$	0.47

Third, concerning the fiscal closure of the model, we specify the share of government spending and the debt-to-output ratio as 0.18 and 0.7 respectively. Combined with a value of 0.47 for the replacement rate, this leads to a share of total retirement benefits in output (e/y) of 0.11, in line with euro area evidence.[11] Reflecting the residual role of taxes in our fiscal specification, these assumptions imply a share of taxes in output (τ/y) of 0.31.

[11] The value of $e/y = 0.11$ corresponds to the most recent observation (reported for 2007) of 'social security pensions as % of GDP' in the Statistical Annex to European Economy (2009), page 174.

Table 4 : Endogenous variables

Real interest rate	r	0.039
Share of consumption in output	c/y	0.60
Share of investment in output	i^k/y	0.22
Share of taxes in output	τ/y	0.31
Share of total benefits in output	e/y	0.11
Capital-output ratio	k/y	3.50
Share of real balances in output	m/y	0.05
Distribution of wealth	λ	0.23
Participation rate of workers	$\overline{l^w} = l^w/N^w$	0.70
Participation rate of retirees	$\overline{l^r} = l^r/N^r$	0.01
Consumption share of workers	c^w/y	0.47
Consumption share of retirees	$\psi c^r/y = c/y - c^w/y$	0.13
Propensity to consume out of wealth (workers)	π	0.05
Propensity to consume out of wealth (retirees)	$\epsilon\pi$	0.09
Relative discount term	Ω	1.05

Finally, the transitional dynamics depend on the reaction functions of monetary and fiscal policy, the assumed degree of nominal rigidities and the specification of the adjustment costs of the investment function. Table 5 summarizes the benchmark specifications which we use in the remainder of the paper.

Table 5 : Parameters responsible for adjustment dynamics

Direct adjustment parameter in debt rule	γ_1	0.04
Smoothing parameter in debt rule	γ_2	0.3
Inertial parameter in interest rate rule	ρ_i	0.7
Inflation coefficient in interest rate rule	γ_π	1.5
Output gap coefficient in interest rate rule	γ_y	0
Calvo survival probability of contracts	ζ	0.2
Elasticity of investment function ($\eta = -\frac{\phi''(v)}{\phi'(v)}v$)	η	0.25

Concerning the choices made in Table 5, four comments are worth making. First, fiscal feedback rules, in general, are much more difficult to pin down than monetary rules, reflecting the wide range of conceivable fiscal instruments and closure specifications. Specifically, in our particular fiscal specification different pairs of γ_1 and γ_2 affect the speed of adjustment, although the shape of impulse responses is qualitatively robust to perturbations of the chosen parameter values. The particular numerical values of γ_1 and γ_2 are taken from the detailed analysis of Mitchell et al. (2000). Second, given the supply-side nature of demographic shocks and the slow materialization of their effects, the otherwise standard monetary policy rule is specified as a pure inflation targeting rule. Third, numerical specifications of the properties of the investment function $\phi(.)$ differ largely across the literature, as discussed, for example, in Gali et al. (2004). Our specification of $\eta = 0.25$ is in line with Dotsey (1999), but we comment on this choice in further detail below. Fourth, the assumption of $\zeta = 0.2$ implies an average duration of prices of 1.25 years, which is in accordance with euro area empirical evidence, as summarized in Altissimo et al. (2006).

4.2 Demographic trends

From the stylized perspective of the model developed in Section 2, two aspects of projected demographic developments in the euro area are particularly noteworthy, as summarized in Figure 1. First, the growth rate of the working age population is predicted to decline significantly over the next decades. Second, the average life expectancy at birth is predicted to further increase in the future. The magnitudes of these predicted developments are substantial. When comparing the years 2008 and 2030 (which defines the policy horizon considered in the final part of this paper), the growth rate of working age population is projected to decline by one percentage point from +0.4% to −0.6%, while life expectancy is projected to increase by 3.4 years, reaching 82.9 years in 2030. In combination, these two developments lead to a significant increase in the old-age dependency ratio which is projected to increase by 13 percentage points, reaching 40% in 2030. As verified below, our model matches this increase.[12]

5 Comparative statics effects of demographic changes: how does the model work?

Using our model, this section identifies general equilibrium repercussions of the above summarized demographic changes, focusing, in particular, on the implications for the long-run level of the equilibrium real interest rate. The section takes a comparative statics perspective and explains the key channels operating within the system of steady-state equations summarized in Section 3.1 by reporting long-run predictions for different scenarios. Our reasoning starts out from the well-known insight in economic growth theory that demographic variables like the population growth rate are a key driving force for the long-run real interest rate. However, while in models without a detailed life-cycle structure the relationship between demographics and the interest rate is rather mechanical, this is, by construction, different in our model. The two main demographic trends, namely the slowdown in population growth and the increase in longevity, tend to decrease the long-run interest rate. Yet, the magnitude of this predicted decline depends sensitively on various features, like the future design of pension payments, the retirement age of the working age population, or the overall degree of fiscal consolidation.[13]

[12] Projetions are taken from the Statistical Annex to European Economy (2009), p. 174. As shown in Figure 1, they are very similar to projections provided by the United Nations (see: 'World population prospects: The 2008 revision', available at http://esa.un.org/unpp). For closely related discussions of projected demographic changes in the euro area, see also European Economy (2006) and Maddaloni et al. (2006).

[13] Moreover, it is clear that the assumed trend path of labour productivity matters which itself may well depend on the age-structure of the economy.

In particular, policy assumptions concerning future pension systems turn out to be important. To illustrate this insight, we distinguish between two different types of scenarios in which the rising old-age dependency ratio does or does not lead to changes in the existing PAYGO pension system (modelled via adjustments in the replacement rate). For the first scenario type, we assume that the replacement rate decreases endogenously such that the aggregate benefits-output ratio remains unchanged. This assumption can be interpreted as a strengthening of privately funded elements, resulting from a pension system reform which introduces a ceiling on the overall tax-financed redistribution between workers and retirees. For the second scenario type, we make the opposite assumption that the replacement rate remains constant, leading to a rise in the aggregate benefits-output ratio. This assumption models in a simple way a 'no reform' scenario which extrapolates the existing pension system into the future, leading to a higher tax burden on workers. To distinguish between these two 'extreme' policy scenarios is instructive because the distinctly different incentives for savings at the individual level generate sizably different degrees of downward pressure on the long-run real interest rate.
All reported comparative statics exercises are driven by exogenous demographic changes which are taken from a comparison between the euro area values projected for the year 2030 and the benchmark values corresponding to the year 2008. This is done for simple illustration only, i.e. Figure 1 indicates that there is no reason to believe that in 2030 a new steady state will be reached. In other words, this section aims to illustrate how the model works, before we discuss empirical scenarios in Section 5.

5.1 Endogenous replacement rate

The likely decline in the long-run real interest rate is particularly pronounced in scenarios in which the replacement rate (μ) declines endogenously such that the aggregate benefits-output ratio (e/y) remains unchanged. To disentangle the contributions of the slowdown in working age population growth and the rise in longevity we consider first these two trends in isolation (Scenarios I, II) before we discuss them in combination (Scenario III). Finally, we also consider variations in the retirement age (Scenario IV).

Scenario I: Slowdown of working age population growth Scenario I isolates the effects resulting from the slowdown in working age population growth. As summarized in Table 5, Scenario I has the key feature that, relative to the benchmark calibration, the decline in population growth by 1 percentage point reduces the equilibrium interest rate by about 0.9 percentage points from 3.9% to 3.0%, with little movement in the other endogenous variables or ratios of variables reported in Table 5. Since in long-run comparison both groups of agents grow at the same (smaller) rate, there is little scope for life-cycle effects in individual consumption and savings behaviour. While the share of the consumption of workers in total output (c^w/y) stays unchanged, there is a small increase in the aggregate consumption share (c/y). The latter effect is largely driven via equation (1) by the increase in the old-age dependency ratio (ψ) by 5 percentage points (which implies a shift of consumption towards agents with a higher propensity to consume), and not by changes in individual savings patterns. Because of these features, the findings are similar to predictions from infinite horizon models with a single representative agent.

Table 5 : Comparative statics effects of demographic changes

	Benchmark	I	II	III	IV	V	VI
n	0.004	-0.006	0.004	-0.006	-0.006	-0.006	-0.006
T^w	50	50	50	50	53.4	50	50
T^r	14.5	14.5	17.9	17.9	14.5	17.9	17.9
e/y	0.11	0.11	0.11	0.11	0.11	0.16	0.16
μ	0.47	0.41	0.39	0.33	0.43	0.47	0.47
x	0.01	0.01	0.01	0.01	0.01	0.01	0.01
b^*	0.7	0.7	0.7	0.7	0.7	0.7	1
ψ	0.27	0.32	0.33	0.40	0.30	0.40	0.40
r	0.039	0.030	0.036	0.028	0.029	0.034	0.036
c/y	0.60	0.61	0.59	0.61	0.61	0.62	0.63
c^w/y	0.47	0.47	0.44	0.44	0.47	0.43	0.43
$\psi c^r/y$	0.13	0.15	0.15	0.17	0.14	0.20	0.20
i^k/y	0.22	0.21	0.23	0.21	0.21	0.20	0.19
τ/y	0.31	0.31	0.30	0.31	0.31	0.36	0.37
l^w/N^w	0.70	0.69	0.70	0.70	0.69	0.71	0.71
λ	0.23	0.24	0.26	0.28	0.22	0.29	0.30

Scenario II: Increase in longevity (at unchanged retirement age) Scenario II isolates the effects resulting from a rise in longevity. Importantly, it is assumed that the expected number of years spent in the workforce remains unchanged (as captured by maintaining $T^w = 50$), i.e. the expected retirement age is kept unchanged. Scenario II has the key feature that, relative to the benchmark, the increase in longevity by 3.4 years reduces the equilibrium interest rate by about 30 basis points from 3.9% to 3.6%. By construction, this decline in r results entirely from the life-cycle features of the model. In particular, the increase in longevity leads to a significant increase in the old-age dependency ratio by about 6 percentage points, while the replacement rate falls substantially by 8 percentage points to keep the benefits-output ratio constant. This drop in the replacement rate strongly increases savings incentives in working age, as can be inferred from the decline of c^w/y by 3 percentage points and an associated increase in the investment share (i^k/y). In sum, these changes in consumption and savings patterns over the life cycle reduce the long-run interest rate, and the mechanism behind this reduction differs from Scenario I.

Scenario III: Slowdown of working age population growth and increase in longevity (at unchanged retirement age) Scenario III combines the two partial demographic scenarios of scenarios I and II, i.e. it considers not only the slowdown in working age population growth but also the projected increase in life expectancy, assuming again an unchanged retirement age. Not surprisingly, the combined effect of these two demographic changes on the interest rate is stronger than in either of the first two scenarios in isolation, leading to a decrease of r by 1.1 percentage points from 3.9% to 2.8% in comparison with the benchmark. This finding reflects that the model is capable of adding life-cycle aspects to the predictions from conventional infinite horizon models.

Scenario IV: Slowdown of working age population growth and combined increase in longevity and retirement age To shed further light on the findings of Scenario III, Scenario IV maintains the assumptions concerning the slowdown in working age population growth and the rise in longevity, but it is now assumed that the additional 3.4 years are entirely spent in working age. In other words, the retirement age is adjusted upward in order to keep the expected number of years spent in retirement identical to the benchmark. This measure undoes to a large extent the life-cycle effects on r discussed in Scenario II, i.e. the variables r, ψ, μ, and λ revert largely back to the values established in Scenario I of a pure slowdown in population growth.

To conclude this subsection it is worth pointing out that in Scenarios I-IV the assumption of an endogenously determined decline in the replacement rate not only stabilizes the share of aggregate benefits in output (e/y) but also the tax share (τ/y). In all scenarios the government expenditure ratio (g/y) and the target level of the debt-to-output ratio (b^*) have been held constant. Since outstanding government debt needs to be backed by appropriate primary surpluses, the constancy of e/y leaves also the tax share τ/y unchanged.

5.2 Constant replacement rate

The implications of demographic changes are different under alternative scenarios in which the replacement rate remains unchanged. Given the rise in the old-age dependency ratio, this leads to a higher share of aggregate benefits in output and implies a higher tax burden on workers. In other words, the additional pension burden resulting from demographic changes is now financed out of additional transfers from workers to retirees within the unchanged PAYGO-system, while in Scenarios I-IV it is financed from higher savings of workers in anticipation of lower pensions. This replacement of additional savings by additional transfers in scenarios with a constant replacement rate mitigates the effects of demographic changes on the long-run interest rate, as shown in Scenario V.

Scenario V: Slowdown of working age population growth and increase in longevity (at unchanged retirement age) This scenario uses the same demographic assumptions as Scenario III. However, the full funding of pensions through the PAYGO-system mitigates the effect on r substantially. Compared with the benchmark, r decreases from 3.9% to 3.4%, i.e. the decrease in r by 50 basis points is about half of the decrease reported in Scenario III. Notice that the consumption share of workers in total output (c^w/y) declines by about the same amount as in Scenario III. Differently from Scenario III, this decrease in the consumption of workers does not lead to higher savings but to an increase in the tax share by 5 percentage points which funds the higher share of benefits in output.

This increase in the tax burden on workers counteracts the life-cycle features discussed in Scenario III, leading to overall smaller changes in the interest rate.

Finally, the model can be used to highlight another policy-related channel which is of key importance for the long-run effects of demographic changes on the real interest rate, namely the overall extent of fiscal consolidation in an ageing society. For simple comparability, we add this channel, ceteris paribus, to Scenario V.

Scenario VI: Slowdown of working age population growth and increase in longevity (at unchanged retirement age), combined with a lack of fiscal consolidation Scenario VI relaxes the assumption that the steady-state debt ratio b^* can be stabilized at the benchmark value of 0.7. Instead we assume a higher value of $b^* = 1$. The additional government debt crowds out physical capital, as to be inferred from the decline in the investment share. This crowding-out effect, ceteris paribus, further moderates the decline in r compared with Scenario III. To put it differently, this effect illustrates that Ricardian equivalence is not satisfied in our model because of life-cycle features, and this feature adds to the rich general equilibrium repercussions of demographic changes.

6 Scenarios for the euro area until 2030

Unlike typical shocks (such as productivity or government spending shocks), demographic developments are well predictable rather far into the future. Exploiting this feature, this section uses annual demographic projections for the euro area as a deterministic and time-varying input for the log-linearized version of the model which we solve under perfect foresight. Specifically, we use the two series for working age population growth and for life expectancy as summarized in Figure 1 to specify exogenously the paths for n_t^w and γ_t, leaving the old age dependency ratio ψ_t to be determined endogenously. In parallel with the analysis of the previous section, we report first findings from a policy scenario with an endogenously adjusting replacement rate, before then turning to an alternative scenario which keeps the replacement rate constant. As discussed above, the underlying assumptions are of deliberately opposite nature, generating plausible upper and lower bounds for the projected decline in the equilibrium real interest rate.

6.1 Endogenous replacement rate

Figure 2, which summarizes the results, shows not only the combined effects of the slowdown in population growth and the increase in longevity (at an unchanged retirement age), but also the two effects in isolation. This decomposition is done with the aim to facilitate a comparison with the discussion of comparative statics effects in Section 5.1 (i.e. Scenarios I, II, and III). According to the projections, the demographic effects change slowly over time and do not die out over the reported horizon until 2030, as to be inferred from the first subplot in Figure 2. This feature implies that the endogenous variables also respond rather slowly over time and reach their peak values at the end of the reported sample period.[14] For this reason, the results in Figure 2 are *qualitatively* close to the long-run predictions of Section 5.1, but the *quantitative* effects are smaller, since the demographic variables do not jump permanently to new levels but rather change slowly over time.

The main findings are as follows. First, for key variables the effects of a slowdown in working age population growth and an increase in longevity reinforce each other. In line with Table 5 this holds true for the real interest rate, the old-age dependency ratio, and the replacement rate. Further examples are the two factor prices, the inflation rate and the nominal interest rate. Second, exceptions to this pattern are, again in line with Table 5, the shares of aggregate consumption and investment, i.e. for these variables the isolated effects lead into opposite directions, thereby mitigating the combined effect. For this second group of variables the findings are consistent with the discussed differences between a pure reduction in n_t^w and the life-cycle effects associated with an increase in γ_t. Third, as regards the quantitative dimension of the reactions, the slow materialization of the demographic changes implies that the real interest rate declines towards the end of the sample by a total of about 50 basis points. In line with the active inflation reaction of the monetary policy rule ($\gamma_\pi = 1.5 > 1$), the nominal interest rate reacts more strongly, reflecting the deflationary impulse of the demographic changes which reaches 100 basis points at the end of the sample. However, these effects are spread out over a long horizon of about 20 years, implying that they are too small to be visible within the shorter time horizon relevant for monetary policy-making.

6.2 Constant replacement rate

Finally, drawing on our discussion of Scenario V in Section 5.1, we present a scenario until 2030 which uses the same demographic inputs as Figure 2, but keeps the replacement rate constant, implying endogenous adjustments in the aggregate share of benefits. Figure 3 reports the findings from this exercise. Again, we show not only the combined effects of the slowdown in population growth and the increase in longevity (at an unchanged retirement age), but also the two effects in isolation.

The main differences compared with Figure 2 are as follows. First, in Figure 3 the share of aggregate benefits in output increases (and replaces, so to speak, the decline in the replacement rate in Figure 2). Related to this, the debt ratio also increases, reflecting that additional benefits payments are on impact financed via deficit spending before taxes are gradually increased to stabilize the debt ratio. Second, the consumption share of workers decreases more strongly. However, this does not lead to an increase, but rather to a fall in the investment share, in line with the increase in the tax-financed redistribution within the unchanged PAYGO-structure. Third, in accordance with the discussion of Scenario V, the real interest falls towards the end of the sample by about 35 basis points which is less than in Figure 2. The deflationary impact of the shock is also smaller, reaching about 50 points towards the end of the sample.

[14]The simulations use the demographic projections until 2050 as an input. We have checked that the reported results for the projection horizon until 2030 are not sensitive with respect to the assumptions made for the years close to 2050 and beyond.

7 Conclusion

This paper develops a small-scale DSGE model which embeds a demographic structure within a monetary policy framework. We extend the tractable, though non-monetary overlapping-generations model of Gertler (1999) and present a small synthesis model which combines the set-up of Gertler with a New-Keynesian structure, implying that the short-run dynamics related to monetary policy are in line with the paradigm summarized in Woodford (2003). Reflecting the underlying overlapping generations structure, the dynamics of the model are critically affected by fiscal policy (including aspects of the social security system), as we show under a number of different policy experiments. In sum, the model offers a New-Keynesian platform which can be used to investigate in a closed economy set-up the response of macroeconomic variables to demographic shocks, similar to technology, government spending or monetary policy shocks. Empirically, we use a calibrated version of the model to discuss a number of macroeconomic scenarios for the euro area with a horizon of around 20 years. The main finding is that demographic changes, while contributing slowly over time to a decline in the equilibrium interest rate, are not visible enough within the shorter time horizon relevant for monetary policy-making to require monetary policy reactions.

In future work, the analysis can be extended in a number of directions. We find three aspects particularly important. First, the fiscal specification should be enriched by allowing for a broader set of taxes. In particular, it remains to be checked to what extent distortionary taxes affect the quantitative predictions of the analysis. Second, in line with the non-standard requirements of the Gertler-economy, the current labour supply specification is restrictive and it would be interesting to redo the analysis with a more flexible specification of preferences. Third, from a normative perspective, optimal policies should be linked to the welfare of the representative agents in the economy. Because of the heterogeneity of agents this raises interesting distributional questions which go beyond the analysis of optimal policies typically carried out in monetary policy frameworks.

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Appendix I: Decision problems of retirees and workers

Problem of the representative retiree

$$V_t^{rj} = \left[\left[\left(c_t^{rj}\right)^{v_1} (m_t^{rj})^{\nu_2}(1-l_t^{rj})^{v_3}\right]^\rho + \beta\gamma_t \left[V_{t+1}^{rj}\right]^\rho\right]^{\frac{1}{\rho}} \tag{33}$$

subject to

$$c_t^{rj} + \frac{i_t}{1+i_t} m_t^{rj} + a_t^{rj} = \frac{1+r_{t-1}}{\gamma_{t-1}} a_{t-1}^{rj} + \xi w_t l_t^{rj} + e_t^j \tag{34}$$

First-order conditions

i) w.r.t. consumption c_t^{rj} :

$$v_1 \left(c_t^{rj}\right)^{v_1\rho-1} (m_t^{rj})^{\nu_2\rho}(1-l_t^{rj})^{v_3\rho} = \beta\gamma_t \left(V_{t+1}^{rj}\right)^{\rho-1} \frac{\partial V_{t+1}^{rj}}{\partial a_t^{rj}} \tag{35}$$

To obtain $\frac{\partial V_{t+1}^{rj}}{\partial a_t^{rj}}$ invoke the envelope theorem, i.e.:

$$\frac{\partial V_t^{rj}}{\partial a_{t-1}^{rj}} = \left(V_t^{rj}\right)^{1-\rho} v_1 \frac{1+r_{t-1}}{\gamma_{t-1}} \left(c_t^{rj}\right)^{v_1\rho-1} (m_t^{rj})^{\nu_2\rho}(1-l_t^{rj})^{v_3\rho}$$

Shifting this expression one period forward leads to

$$\frac{\partial V_{t+1}^{rj}}{\partial a_t^{rj}} = \left(V_{t+1}^{rj}\right)^{1-\rho} v_1 \frac{1+r_t}{\gamma_t} \left(c_{t+1}^{rj}\right)^{v_1\rho-1} (m_{t+1}^{rj})^{\nu_2\rho}(1-l_{t+1}^{rj})^{v_3\rho} \tag{36}$$

Combining (36) and (35) yields

$$\left(c_t^{rj}\right)^{v_1\rho-1} (m_t^{rj})^{\nu_2\rho}(1-l_t^{rj})^{v_3\rho} = \beta(1+r_t) \left(c_{t+1}^{rj}\right)^{v_1\rho-1} (m_{t+1}^{rj})^{\nu_2\rho}(1-l_{t+1}^{rj})^{v_3\rho} \tag{37}$$

ii) w.r.t. labour supply l_t^{rj} :

$$v_3 \left(c_t^{rj}\right)^{v_1\rho} (m_t^{rj})^{\nu_2\rho}(1-l_t^{rj})^{v_3\rho-1} = \beta\gamma_t \left(V_{t+1}^{rj}\right)^{\rho-1} \frac{\partial V_{t+1}^{rj}}{\partial a_t^{rj}} \xi w_t$$

Combine with (35) to obtain

$$1 - l_t^{rj} = \frac{v_3}{v_1} \frac{c_t^{rj}}{\xi w_t} \tag{38}$$

iii) w.r.t. real balances m_t^{rj} :

$$v_2 \left(c_t^{rj}\right)^{v_1\rho} (m_t^{rj})^{\nu_2\rho-1}(1-l_t^{rj})^{v_3\rho} = \beta\gamma_t \left(V_{t+1}^{rj}\right)^{\rho-1} \frac{\partial V_{t+1}^{rj}}{\partial a_t^{rj}} \frac{i_t}{1+i_t}$$

Combine with (35) to obtain

$$m_t^{rj} = \frac{v_2}{v_1}\frac{1+i_t}{i_t}c_t^{rj} \tag{39}$$

iv) Derivation of Euler equation in c_t^{rj}-terms:
Combining (37), (38), and (39) yields

$$c_{t+1}^{rj} = [\beta(1+r_t)(\frac{\frac{1+i_{t+1}}{i_{t+1}}}{\frac{1+i_t}{i_t}})^{\nu_2\rho}(\frac{w_t}{w_{t+1}})^{v_3\rho}]^{\sigma}c_t^{rj}, \tag{40}$$

where $\sigma = \frac{1}{1-\rho}$.

Solution of V_t^{rj}

i) Conjecture for the combined consumption term $c_t^{rj} + \frac{i_t}{1+i_t}m_t^{rj}$:

$$c_t^{rj} + \frac{i_t}{1+i_t}m_t^{rj} = c_t^{rj}(1+\frac{v_2}{v_1}) = \epsilon_t\pi_t\left(\frac{1+r_{t-1}}{\gamma_{t-1}}a_{t-1}^{rj} + h_t^{rj}\right), \tag{41}$$

using

$$\begin{aligned} h_t^{rj} &= d_t^{rj} + \frac{\gamma_t}{1+r_t}h_{t+1}^{rj} \\ d_t^{rj} &= \xi w_t l_t^{rj} + e_t^j \end{aligned}$$

Using (41) within the Euler equation (40) yields

$$a_t^{rj} + \frac{\gamma_t}{1+r_t}h_{t+1}^{rj} = [\beta(\frac{\frac{1+i_{t+1}}{i_{t+1}}}{\frac{1+i_t}{i_t}})^{\nu_2\rho}(\frac{w_t}{w_{t+1}})^{v_3\rho}]^{\sigma}(1+r_t)^{\sigma-1}\gamma_t\frac{\epsilon_t\pi_t}{\epsilon_{t+1}\pi_{t+1}}\left(\frac{1+r_{t-1}}{\gamma_{t-1}}a_{t-1}^{rj} + h_t^{rj}\right)$$

Use (41) to rewrite the budget constraint (34) as

$$a_t^{rj} + \frac{\gamma_t}{1+r_t}h_{t+1}^{rj} = [1-\epsilon_t\pi_t]\left(\frac{1+r_{t-1}}{\gamma_{t-1}}a_{t-1}^{rj} + h_t^{rj}\right),$$

implying

$$\epsilon_t\pi_t = 1 - [\beta(\frac{\frac{1+i_{t+1}}{i_{t+1}}}{\frac{1+i_t}{i_t}})^{\nu_2\rho}(\frac{w_t}{w_{t+1}})^{v_3\rho}]^{\sigma}(1+r_t)^{\sigma-1}\gamma_t\frac{\epsilon_t\pi_t}{\epsilon_{t+1}\pi_{t+1}} \tag{42}$$

ii) Conjecture for value function V_t^{rj}:

$$V_t^{rj} = \Delta_t^r c_t^{rj}(\frac{v_2}{v_1}\frac{1+i_t}{i_t})^{\nu_2}(\frac{v_3}{v_1}\frac{1}{\xi w_t})^{v_3}$$

This implies within (33)

$$\begin{aligned} &\Delta_t^r c_t^{rj}(\frac{v_2}{v_1}\frac{1+i_t}{i_t})^{\nu_2}(\frac{v_3}{v_1}\frac{1}{\xi w_t})^{v_3} \\ = &\left[[c_t^{rj}(\frac{v_2}{v_1}\frac{1+i_t}{i_t})^{\nu_2}(\frac{v_3}{v_1}\frac{1}{\xi w_t})^{v_3}]^{\rho} + \beta\gamma_t\left[\Delta_{t+1}^r c_{t+1}^{rj}(\frac{v_2}{v_1}\frac{1+i_{t+1}}{i_{t+1}})^{\nu_2}(\frac{v_3}{v_1}\frac{1}{\xi w_{t+1}})^{v_3}\right]^{\rho}\right]^{\frac{1}{\rho}} \end{aligned}$$

Use (40) to write this as

$$(\Delta_t^r)^\rho = 1 + \gamma_t(\Delta_{t+1}^r)^\rho(1+r_t)^{\sigma-1}[\beta(\frac{\frac{1+i_{t+1}}{i_{t+1}}}{\frac{1+i_t}{i_t}})^{\nu_2\rho}(\frac{w_t}{w_{t+1}})^{v_3\rho}]^\sigma,$$

where we have used: $\sigma = \frac{1}{1-\rho} \Leftrightarrow \rho = 1 - \frac{1}{\sigma} = \frac{\sigma-1}{\sigma} \Rightarrow \rho\sigma = \sigma - 1 \Rightarrow (1+\rho\sigma)\rho = \rho\sigma$.
Let

$$(\Delta_t^r)^\rho = (\epsilon_t\pi_t)^{-1}$$

Then,

$$\epsilon_t\pi_t = 1 - \gamma_t(1+r_t)^{\sigma-1}[\beta(\frac{\frac{1+i_{t+1}}{i_{t+1}}}{\frac{1+i_t}{i_t}})^{\nu_2\rho}(\frac{w_t}{w_{t+1}})^{v_3\rho}]^\sigma\frac{\epsilon_t\pi_t}{\epsilon_{t+1}\pi_{t+1}},$$

which completes the conjectures for the consumption function and the value function V_t^{rj}.

Problem of the representative worker

$$V_t^{wj} = \left[\left[\left(c_t^{wj}\right)^{v_1}(m_t^{wj})^{\nu_2}(1-l_t^{wj})^{v_3}\right]^\rho + \beta\left[\omega_t V_{t+1}^{wj} + (1-\omega_t)V_{t+1}^{rj}\right]^\rho\right]^{\frac{1}{\rho}} \tag{43}$$

subject to

$$c_t^{wj} + \frac{i_t}{1+i_t}m_t^{wj} + a_t^{wj} = (1+r_{t-1})a_{t-1}^{wj} + w_t l_t^{wj} + f_t^j - \tau_t^j \tag{44}$$

First-order conditions

i) w.r.t. consumption c_t^{wj} :

$$v_1\left(c_t^{wj}\right)^{v_1\rho-1}(m_t^{wj})^{\nu_2\rho}(1-l_t^{wj})^{v_3\rho} = \beta[\omega_t V_{t+1}^{wj} + (1-\omega_t)V_{t+1}^{rj}]^{\rho-1}[\omega_t\frac{\partial V_{t+1}^{wj}}{\partial a_t^{wj}} + (1-\omega_t)\frac{\partial V_{t+1}^{rj}}{\partial a_t^{rj}}] \tag{45}$$

To obtain $\frac{\partial V_{t+1}^{wj}}{\partial a_t^{wj}}$ and $\frac{\partial V_{t+1}^{rj}}{\partial a_t^{rj}}$ invoke the envelope theorem:

$$\frac{\partial V_t^{wj}}{\partial a_{t-1}^{wj}} = \left(V_t^{wj}\right)^{1-\rho}v_1(1+r_{t-1})\left(c_t^{wj}\right)^{v_1\rho-1}(m_t^{wj})^{\nu_2\rho}(1-l_t^{wj})^{v_3\rho}$$

$$\frac{\partial V_t^{rj}}{\partial a_{t-1}^{rj}} = \left(V_t^{rj}\right)^{1-\rho}v_1(1+r_{t-1})\left(c_t^{rj}\right)^{v_1\rho-1}(m_t^{rj})^{\nu_2\rho}(1-l_t^{rj})^{v_3\rho}$$

Shifting these expressions one period forward leads to

$$\frac{\partial V_{t+1}^{wj}}{\partial a_t^{wj}} = \left(V_{t+1}^{wj}\right)^{1-\rho}v_1(1+r_t)\left(c_{t+1}^{wj}\right)^{v_1\rho-1}(m_{t+1}^{wj})^{\nu_2\rho}(1-l_{t+1}^{wj})^{v_3\rho}$$

$$\frac{\partial V_{t+1}^{rj}}{\partial a_t^{rj}} = \left(V_{t+1}^{rj}\right)^{1-\rho}v_1(1+r_t)\left(c_{t+1}^{rj}\right)^{v_1\rho-1}(m_{t+1}^{rj})^{\nu_2\rho}(1-l_{t+1}^{rj})^{v_3\rho}$$

Using these expressions within (45) yields

$$v_1 \left(c_t^{wj}\right)^{v_1\rho-1} (m_t^{wj})^{\nu_2\rho}(1-l_t^{wj})^{v_3\rho} = \beta[\omega_t V_{t+1}^{wj}+(1-\omega_t)\, V_{t+1}^{rj}]^{\rho-1}[\omega_t \frac{\partial V_{t+1}^{wj}}{\partial a_t^{wj}}+(1-\omega_t)\, \frac{\partial V_{t+1}^{rj}}{\partial a_t^{rj}}], \tag{46}$$

with:

$$\begin{aligned} \omega_t \frac{\partial V_{t+1}^{wj}}{\partial a_t^{wj}} + (1-\omega_t)\, \frac{\partial V_{t+1}^{rj}}{\partial a_t^{rj}} \quad &= \quad \omega_t \left(V_{t+1}^{wj}\right)^{1-\rho} v_1(1+r_t) \left(c_{t+1}^{wj}\right)^{v_1\rho-1} (m_{t+1}^{wj})^{\nu_2\rho}(1-l_{t+1}^{wj})^{v_3\rho} \\ &+(1-\omega_t) \left(V_{t+1}^{rj}\right)^{1-\rho} v_1(1+r_t) \left(c_{t+1}^{rj}\right)^{v_1\rho-1} (m_{t+1}^{rj})^{\nu_2\rho}(1-l_{t+1}^{rj})^{v_3\rho} \end{aligned}$$

ii) w.r.t. labour supply l_t^{wj} :

$$v_3 \left(c_t^{wj}\right)^{v_1\rho} (m_t^{wj})^{\nu_2\rho}(1-l_t^{wj})^{v_3\rho-1} = \beta[\omega_t V_{t+1}^{wj}+(1-\omega_t)\, V_{t+1}^{rj}]^{\rho-1}[\omega_t \frac{\partial V_{t+1}^{wj}}{\partial a_t^{wj}}+(1-\omega_t)\, \frac{\partial V_{t+1}^{rj}}{\partial a_t^{rj}}]w_t$$

Combine with (45) to obtain

$$1-l_t^{wj} = \frac{v_3}{v_1}\frac{c_t^{wj}}{w_t} \tag{47}$$

iii) w.r.t. real balances m_t^{wj} :

$$v_2 \left(c_t^{wj}\right)^{v_1\rho} (m_t^{wj})^{\nu_2\rho-1}(1-l_t^{wj})^{v_3\rho} = \beta[\omega_t V_{t+1}^{wj}+(1-\omega_t)\, V_{t+1}^{rj}]^{\rho-1}[\omega_t \frac{\partial V_{t+1}^{wj}}{\partial a_t^{wj}}+(1-\omega_t)\, \frac{\partial V_{t+1}^{rj}}{\partial a_t^{rj}}]\frac{i_t}{1+i_t}$$

Combine with (45) to obtain

$$m_t^{wj} = \frac{v_2}{v_1}\frac{1+i_t}{i_t}c_t^{wj} \tag{48}$$

iv) Substituting out for l_t^{wj} and m_t^{wj}-terms:
Use (47) and (48) in (46) to simplify the consumption Euler equation:

$$\begin{aligned} \left(c_t^{wj}\right)^{\rho-1} \quad &= \quad [\omega_t V_{t+1}^{wj} + (1-\omega_t)\, V_{t+1}^{rj}]^{\rho-1}\beta(1+r_t)(\frac{\frac{1+i_{t+1}}{i_{t+1}}}{\frac{1+i_t}{i_t}})^{\nu_2\rho}(\frac{w_t}{w_{t+1}})^{v_3\rho} \\ &\cdot[\omega_t \left(V_{t+1}^{wj}\right)^{1-\rho} \left(c_{t+1}^{wj}\right)^{\rho-1} + (1-\omega_t) \left(V_{t+1}^{rj}\right)^{1-\rho} (\frac{1}{\xi})^{v_3\rho} \left(c_{t+1}^{rj}\right)^{\rho-1}] \end{aligned} \tag{49}$$

Solution of V_t^{wj}

i) Conjecture for value function V_t^{wj}:

$$V_t^{wj} = \Delta_t^w c_t^{wj}(\frac{v_2}{v_1}\frac{1+i_t}{i_t})^{\nu_2}(\frac{v_3}{v_1}\frac{1}{w_t})^{v_3}, \quad \Delta_t^w = \pi_t^{-\frac{1}{\rho}}$$

Moreover, recall from above:

$$V_t^{rj} = \Delta_t^r c_t^r(\frac{v_2}{v_1}\frac{1+i_t}{i_t})^{\nu_2}(\frac{v_3}{v_1}\frac{1}{w_t\xi})^{v_3}, \quad \Delta_t^r = (\epsilon_t\pi_t)^{-\frac{1}{\rho}}$$

Use these conjectures within (49) to further simplify the consumption Euler equation

$$\begin{aligned}\left(c_t^{wj}\right)^{\rho-1} &= \left[\omega_t c_{t+1}^{wj} + (1-\omega_t)\left(\epsilon_{t+1}\right)^{-\frac{1}{\rho}} (\frac{1}{\xi})^{v_3} c_{t+1}^{rj}\right]^{\rho-1} \beta(1+r_t)(\frac{\frac{1+i_{t+1}}{i_{t+1}}}{\frac{1+i_t}{i_t}})^{\nu_2\rho}(\frac{w_t}{w_{t+1}})^{v_3\rho} \\ &\cdot \left[\omega_t + (1-\omega_t)\left(\epsilon_{t+1}\right)^{-\frac{1-\rho}{\rho}} (\frac{1}{\xi})^{v_3}\right]\end{aligned}$$

Recall from above $\sigma = \frac{1}{1-\rho} \Leftrightarrow \rho = 1 - \frac{1}{\sigma} = \frac{\sigma-1}{\sigma}$ and let

$$\begin{aligned}\Lambda_{t+1} &= \frac{\Delta_{t+1}^r}{\Delta_{t+1}^w} = \left(\epsilon_{t+1}\right)^{-\frac{1}{\rho}} = \left(\epsilon_{t+1}\right)^{\frac{\sigma}{1-\sigma}}, \\ \chi &= (\frac{1}{\xi})^{v_3} \\ \Omega_{t+1} &= \omega_t + (1-\omega_t)\,\epsilon_{t+1}^{\frac{1}{1-\sigma}} \chi\end{aligned}$$

to further simplify the consumption Euler equation, yielding

$$\omega_t c_{t+1}^{wj} + (1-\omega_t)\,\Lambda_{t+1}\chi c_{t+1}^{rj} = \left[\beta(1+r_t)\Omega_{t+1}(\frac{\frac{1+i_{t+1}}{i_{t+1}}}{\frac{1+i_t}{i_t}})^{\nu_2\rho}(\frac{w_t}{w_{t+1}})^{v_3\rho}\right]^{\sigma} c_t^{wj} \tag{50}$$

ii) Conjecture for the combined consumption term $c_t^{wj} + \frac{i_t}{1+i_t} m_t^{wj}$:
Conjecture for the worker that $c_t^{wj} + \frac{i_t}{1+i_t} m_t^{wj}$ is given by:

$$c_t^{wj} + \frac{i_t}{1+i_t} m_t^{wj} = c_t^{wj}(1+\frac{v_2}{v_1}) = \pi_t\left((1+r_{t-1})a_{t-1}^{wj} + h_t^{wj}\right), \tag{51}$$

while the 'just retired person' consumes according to

$$c_t^{rj} + \frac{i_t}{1+i_t} m_t^{rj} = c_t^{rj}(1+\frac{v_2}{v_1}) = \epsilon_t\pi_t\left((1+r_{t-1})a_{t-1}^{wj} + h_t^{rj}\right) \tag{52}$$

since his financial assets are pre-determined from his previous savings as a worker. Moreover, define the human capital of workers as follows:

$$\begin{aligned}h_t^{wj} &= d_t^{wj} + \frac{\omega_t}{\Omega_{t+1}}\frac{h_{t+1}^{wj}}{1+r_t} + \left(1 - \frac{\omega_t}{\Omega_{t+1}}\right)\frac{h_{t+1}^{rj}}{1+r_t} \\ d_t^{wj} &= w_t l_t^{wj} + f_t^j - \tau_t^j\end{aligned}$$

Substituting the conjectures for the consumption of workers (51) and (52) into the Euler equation (50) yields

$$\frac{a_t^{wj} + h_t^{wj} - d_t^{wj}}{(1+r_{t-1})a_{t-1}^{wj} + h_t^{wj}} = \frac{\pi_t}{\pi_{t+1}}\beta^{\sigma}((1+r_t)\Omega_{t+1})^{\sigma-1}\left[(\frac{\frac{1+i_{t+1}}{i_{t+1}}}{\frac{1+i_t}{i_t}})^{\nu_2\rho}(\frac{w_t}{w_{t+1}})^{v_3\rho}\right]^{\sigma}$$

Next, rewrite the budget constraint (44) to obtain an alternative expression for the LHS of the previous equation:

$$a_t^{wj} + h_t^{wj} - d_t^{wj} = (1-\pi_t)\left((1+r_{t-1})a_{t-1}^{wj} + h_t^{wj}\right),$$

implying

$$\pi_t = 1 - \left[\left(\frac{\frac{1+i_{t+1}}{i_{t+1}}}{\frac{1+i_t}{i_t}} \right)^{\nu_2 \rho} \left(\frac{w_t}{w_{t+1}} \right)^{v_3 \rho} \right]^{\sigma} \beta^{\sigma} ((1+r_t)\Omega_{t+1})^{\sigma-1} \frac{\pi_t}{\pi_{t+1}},$$

which ensures that all conjectures add up to consistent solutions across all equations characterising optimal decisions of retirees and workers.

Appendix II: Summary of detrended equilibrium conditions

As stressed in the main text, consider generic variables $v_t \in \{c_t, y_t, k_t, f_t, r_t^k, i_t^k, a_t, b_t, m_t, \tau_t, e_t\}$, $v_t^w \in \{h_t^w, d_t^w, c_t^w, \}$, and $v_t^r \in \{ h_t^r, d_t^r, c_t^r\}$. Then,

$$\frac{v_t}{N_t^w X_t} = \overline{v_t}, \quad \frac{v_t^w}{N_t^w X_t} = \overline{v_t^w}, \quad \frac{v_t^r}{N_t^w X_t} = \frac{v_t^r}{N_t^r X_t} \frac{N_t^r}{N_t^w} = \overline{v_t^r} \psi_t$$

Moreover, let variables related to the labour market be detrended as follows:

$$\frac{w_t}{X_t} = \overline{w_t}, \quad \frac{l_t}{N_t^w} = \overline{l_t}, \quad \frac{l_t^w}{N_t^w} = \overline{l_t^w}, \quad \frac{l_t^r}{N_t^w} = \frac{l_t^r}{N_t^r} \frac{N_t^r}{N_t^w} = \overline{l_t^r} \psi_t$$

Using these definitions, the detrended counterpart of the system (2)-(32) can be summarized as follows, where we report for all equations first the general and then the steady-state version.

Propensity to consume of retirees (equation 2):

$$\begin{aligned}
\epsilon_t \pi_t &= 1 - \left[\left(\frac{1+i_{t+1}}{i_{t+1}} \frac{i_t}{1+i_t} \right)^{\nu_2 \rho} \left(\frac{\overline{w_t}}{\overline{w_{t+1}}} \frac{1}{1+x} \right)^{v_3 \rho} \right]^{\sigma} \beta^{\sigma} (1+r_t)^{\sigma-1} \gamma_t \frac{\epsilon_t \pi_t}{\epsilon_{t+1} \pi_{t+1}} \\
\epsilon \pi &= 1 - \left[\left(\frac{1}{1+x} \right)^{v_3 \rho} \right]^{\sigma} \beta^{\sigma} (1+r)^{\sigma-1} \gamma
\end{aligned}$$

Definition of omega (equation 3):

$$\begin{aligned}
\Omega_{t+1} &= \omega_t + (1-\omega_t)\, \epsilon_{t+1}^{\frac{1}{1-\sigma}} \left(\frac{1}{\xi} \right)^{v_3} \\
\Omega &= \omega + (1-\omega)\, \epsilon^{\frac{1}{1-\sigma}} \left(\frac{1}{\xi} \right)^{v_3}
\end{aligned}$$

Propensity to consume of workers (equation 4):

$$\begin{aligned}
\pi_t &= 1 - \left[\left(\frac{1+i_{t+1}}{i_{t+1}} \frac{i_t}{1+i_t} \right)^{\nu_2 \rho} \left(\frac{\overline{w_t}}{\overline{w_{t+1}}} \frac{1}{1+x} \right)^{v_3 \rho} \right]^{\sigma} \beta^{\sigma} ((1+r_t)\Omega_{t+1})^{\sigma-1} \frac{\pi_t}{\pi_{t+1}} \\
\pi &= 1 - \left[\left(\frac{1}{1+x} \right)^{v_3 \rho} \right]^{\sigma} \beta^{\sigma} ((1+r)\Omega)^{\sigma-1}
\end{aligned}$$

Aggregate labour supply of retirees, workers, and total labour force (equations 5-7):

$$
\begin{aligned}
1-\overline{l_t^r} &= \frac{v_3}{v_1}\frac{\overline{c_t^r}}{\xi\overline{w_t}}, \qquad 1-\overline{l_t^w} = \frac{v_3}{v_1}\frac{\overline{c_t^w}}{\overline{w_t}}, \qquad \overline{l_t} = \overline{l_t^w} + \xi\overline{l_t^r}\psi_t \\
1-\overline{l^r} &= \frac{v_3}{v_1}\frac{\overline{c^r}}{\xi\overline{w}}, \qquad 1-\overline{l^w} = \frac{v_3}{v_1}\frac{\overline{c^w}}{\overline{w}}, \qquad \overline{l} = \overline{l^w} + \xi\overline{l^r}\psi
\end{aligned}
$$

Aggregate human capital of retirees (equation 8):

$$
\begin{aligned}
\overline{h_t^r} &= \overline{d_t^r} + \gamma_t \frac{1+x}{1+r_t}\overline{h_{t+1}^r} \\
\overline{h^r} &= \overline{d^r} + \gamma \frac{1+x}{1+r}\overline{h^r}
\end{aligned}
$$

Aggregate human capital of workers (equation 9):

$$
\begin{aligned}
\overline{h_t^w} &= \overline{d_t^w} + \frac{\omega_t}{\Omega_{t+1}}\frac{1+x}{1+r_t}\overline{h_{t+1}^w} + (1-\frac{\omega_t}{\Omega_{t+1}})\frac{1+x}{1+r_t}\overline{h_{t+1}^r} \\
\overline{h^w} &= \overline{d^w} + \frac{\omega}{\Omega}\frac{1+x}{1+r}\overline{h^w} + (1-\frac{\omega}{\Omega})\frac{1+x}{1+r}\overline{h^r}
\end{aligned}
$$

Aggregate disposable income of retirees and workers (equations 10 and 11):

$$
\begin{aligned}
\overline{d_t^r} &= \xi\overline{w_t}\overline{l_t^r} + \frac{\overline{e_t}}{\psi_t}, \qquad \overline{d_t^w} = \overline{w_t}\overline{l_t^w} + \overline{f_t} - \overline{\tau_t} \\
\overline{d^r} &= \xi\overline{w}\overline{l^r} + \frac{\overline{e}}{\psi}, \qquad \overline{d^w} = \overline{w}\overline{l^w} + \overline{f} - \overline{\tau}
\end{aligned}
$$

Aggregate consumption of retirees (equation 12):

$$
\begin{aligned}
\overline{c_t^r}(1+\frac{v_2}{v_1}) &= \epsilon_t\pi_t\left(\frac{1+r_{t-1}}{\left(1+n_{t-1}^r\right)(1+x)}\lambda_{t-1}\overline{a_{t-1}} + \overline{h_t^r}\right) \\
\overline{c^r}(1+\frac{v_2}{v_1}) &= \epsilon\pi\left(\frac{1+r}{(1+n)(1+x)}\lambda\overline{a} + \overline{h^r}\right)
\end{aligned}
$$

Aggregate consumption of workers (equation 13):

$$
\begin{aligned}
\overline{c_t^w}(1+\frac{v_2}{v_1}) &= \pi_t\left(\frac{1+r_{t-1}}{\left(1+n_{t-1}^w\right)(1+x)}(1-\lambda_{t-1})\overline{a_{t-1}} + \overline{h_t^w}\right) \\
\overline{c^w}(1+\frac{v_2}{v_1}) &= \pi\left(\frac{1+r}{(1+n)(1+x)}(1-\lambda)\overline{a} + \overline{h^w}\right)
\end{aligned}
$$

Aggregate consumption and aggregate real balances (equations 14 and 15):

$$
\begin{aligned}
\overline{c_t} &= \overline{c_t^w} + \overline{c_t^r}\psi_t, \qquad \overline{m_t} = \overline{m_t^w} + \overline{m_t^r}\psi_t = \frac{1+i_t}{i_t}\frac{v_2}{v_1}\overline{c_t} \\
\overline{c} &= \overline{c^w} + \overline{c^r}\psi, \qquad \overline{m} = \overline{m^w} + \overline{m^r}\psi = \frac{1+i}{i}\frac{v_2}{v_1}\overline{c}
\end{aligned}
$$

Evolution of aggregate wealth of retirees (equation 16):

$$\begin{aligned}\lambda_t\overline{a_t} &= \omega_t\left[(1-\epsilon_t\pi_t)\left(\lambda_{t-1}\frac{(1+r_{t-1})}{(1+n^w_{t-1})(1+x)}\overline{a_{t-1}}+\overline{h^r}\psi_t\right)-(\overline{h^r_t}-\overline{d^r_t})\psi_t\right]+(1-\omega_t)\overline{a_t} \\ \lambda\overline{a} &= \omega\left[(1-\epsilon\pi)\left(\frac{\lambda(1+r)}{(1+n)(1+x)}\overline{a}+\overline{h^r}\psi\right)-(\overline{h^r}-\overline{d^r})\psi\right]+(1-\omega)\overline{a}\end{aligned}$$

Output (equation 17):

$$\begin{aligned}\overline{y_t} &= \left[\int_0^1 \overline{y_t(z)}^{\frac{\theta-1}{\theta}}dz\right]^{\frac{\theta}{\theta-1}}, \quad \text{with } \overline{y_t(z)}=\left(\overline{l_t(z)}\right)^{\alpha}\left(\overline{k_t(z)}\right)^{1-\alpha} \\ \overline{y(z)} &= \overline{y}=\overline{l}^{\alpha}\left(\frac{\overline{k}}{(1+n)(1+x)}\right)^{1-\alpha}, \quad \text{using } \overline{l(z)}=\overline{l} \text{ and } \overline{k(z)}=\frac{\overline{k}}{(1+n)(1+x)}\end{aligned}$$

Profits (equation 18):

$$\begin{aligned}\overline{f_t} &= \int_0^1\left(\frac{P_t(z)}{P_t}-mc_t\right)\overline{y_t(z)}dz \\ \overline{f} &= (1-mc)\,\overline{y}\end{aligned}$$

Aggregate capital-labour ratio and marginal costs (equations 19 and 20):

$$\begin{aligned}\frac{1}{(1+n^w_{t-1})(1+x)}\frac{\overline{k_{t-1}}}{\overline{l_t}} &= \frac{\overline{w_t}}{r^k_t}\frac{1-\alpha}{\alpha}, \quad mc_t=\left(\frac{\overline{w_t}}{\alpha}\right)^{\alpha}\left(\frac{r^k_t}{1-\alpha}\right)^{1-\alpha} \\ \frac{1}{(1+n)(1+x)}\frac{\overline{k}}{\overline{l}} &= \frac{\overline{w}}{r^k}\frac{1-\alpha}{\alpha}, \quad mc=\left(\frac{\overline{w}}{\alpha}\right)^{\alpha}\left(\frac{r^k}{1-\alpha}\right)^{1-\alpha}\end{aligned}$$

Optimal price setting and evolution of aggregate price level (equations 21 and 22):

$$\begin{aligned}\frac{P^*_t}{P_t} &= \frac{\theta}{\theta-1}\frac{E_t\sum_{i=0}^{\infty}(\zeta\beta)^i\left(\frac{1}{P_{t+i}}\right)^{1-\theta}\overline{y_{t+i}}N^w_{t+i}X_{t+i}mc_{t+i}\frac{P_{t+i}}{P_t}}{E_t\sum_{i=0}^{\infty}(\zeta\beta)^i\left(\frac{1}{P_{t+i}}\right)^{1-\theta}\overline{y_{t+i}}N^w_{t+i}X_{t+i}} \\ P_t &= \left(\zeta P^{1-\theta}_{t-1}+(1-\zeta)P^{*1-\theta}_t\right)^{\frac{1}{1-\theta}}\end{aligned}$$

$$mc=\frac{\theta-1}{\theta} \quad \text{using } P^*/P=1$$

Aggregate capital stock dynamics (equation 23):

$$\begin{aligned}\overline{k_t} &= \phi\left(\frac{\overline{i^k_t}}{\overline{k_{t-1}}}(1+n^w_{t-1})(1+x)\right)\cdot\frac{\overline{k_{t-1}}}{(1+n^w_{t-1})(1+x)}+(1-\delta)\frac{\overline{k_{t-1}}}{(1+n^w_{t-1})(1+x)} \\ \overline{k} &= \overline{i^k}+(1-\delta)\frac{\overline{k}}{(1+n)(1+x)}, \quad \text{using } \overline{i^k}=\overline{i^k(u)} \text{ and } \frac{\overline{k}}{(1+n)(1+x)}=\overline{k(u)}\end{aligned}$$

Price of capital, resource constraint and interest rate relations (equations 24, 25, 26):

$$
\begin{aligned}
1 &= p_t^k \phi' \Big(\frac{\overline{i_t^k}}{\overline{k_{t-1}}} \left(1+n_{t-1}^w\right)(1+x) \Big), \qquad \overline{y_t} = \overline{c_t} + \overline{g_t} + \overline{i_t^k}, \qquad 1+i_t = (1+r_t)\left(\frac{P_{t+1}}{P_t}\right) \\
1 &= p^k, \qquad \overline{y} = \overline{c} + \overline{g} + \overline{i^k} \qquad i = r
\end{aligned}
$$

Consolidated public sector budget constraint (equation 27):

$$
\begin{aligned}
\overline{b_t} &= (1+r_{t-1})\left(\frac{\overline{b_{t-1}}}{\left(1+n_{t-1}^w\right)(1+x)} + \frac{\frac{1}{1+i_{t-1}}\overline{m_{t-1}}}{\left(1+n_{t-1}^w\right)(1+x)}\right) + \overline{g_t} + \overline{e_t} - \overline{\tau_t} - \overline{m_t} \\
\overline{b} &= (1+r)\left(\frac{\overline{b}}{(1+n)(1+x)} + \frac{\frac{1}{1+i}\overline{m}}{(1+n)(1+x)}\right) + \overline{g} + \overline{e} - \overline{\tau} - \overline{m}
\end{aligned}
$$

Aggregate pension benefits and aggregate non-human wealth (equations 28 and 29):

$$
\begin{aligned}
\overline{e_t} &= \mu_t \overline{w_t} \psi_t, \qquad \overline{a_t} = p_t^k \overline{k_t} + \overline{b_t} + \frac{1}{1+i_t}\overline{m_t} \\
\overline{e} &= \mu \overline{w} \psi, \qquad \overline{a} = p^k \overline{k} + \overline{b} + \frac{1}{1+i}\overline{m}
\end{aligned}
$$

No-arbitrage relationship (equation 30):

$$
\begin{aligned}
1+r_t &= \frac{r_{t+1}^k + p_{t+1}^k(1-\delta)}{p_t^k} \\
r &= r^k - \delta
\end{aligned}
$$

Fiscal and monetary policy rules (equations 31 and 32):

$$
\begin{aligned}
\frac{\overline{\tau_t}}{\overline{y_t}} &= \tau^* + \gamma_1\left(\frac{\overline{b_t}}{\overline{y_t}} - b^*\right) + \gamma_2\left(\frac{\overline{b_t}}{\overline{y_t}} - \frac{\overline{b_{t-1}}}{\overline{y_{t-1}}}\right), \\
\frac{\overline{\tau}}{\overline{y}} &= \tau^*, \qquad \frac{\overline{b}}{\overline{y}} = b^*, \qquad i = r
\end{aligned}
$$

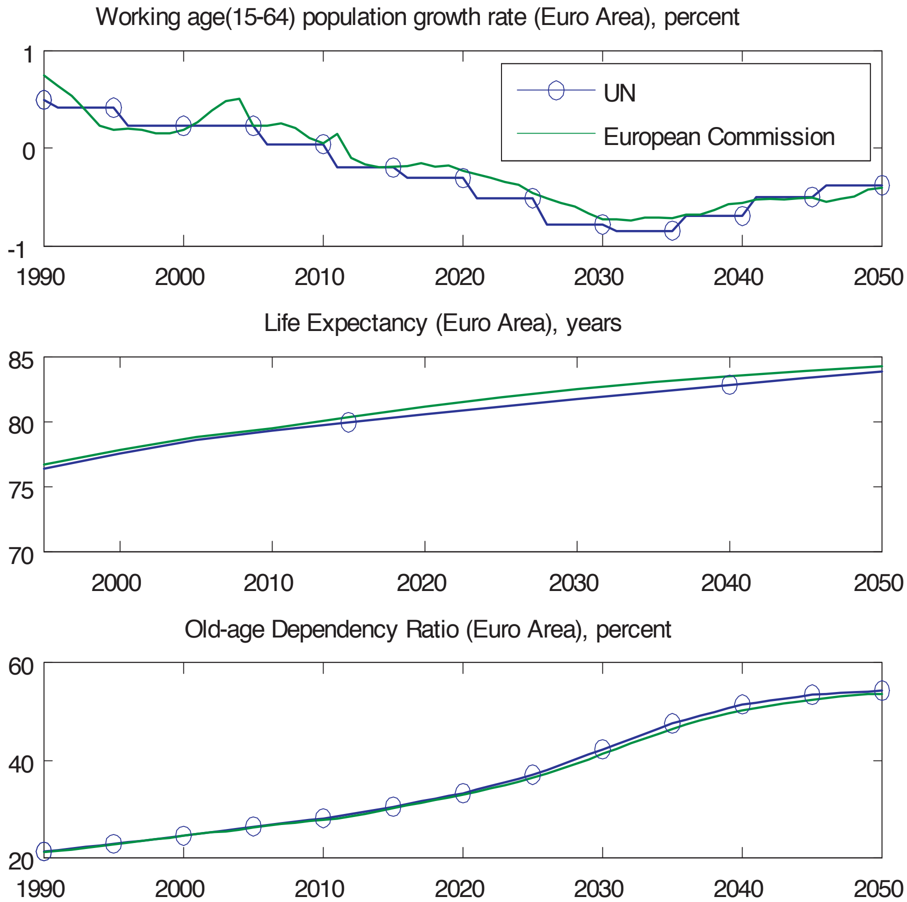


Figure 1: Past and Projected Demographic Developments in the Euro Area

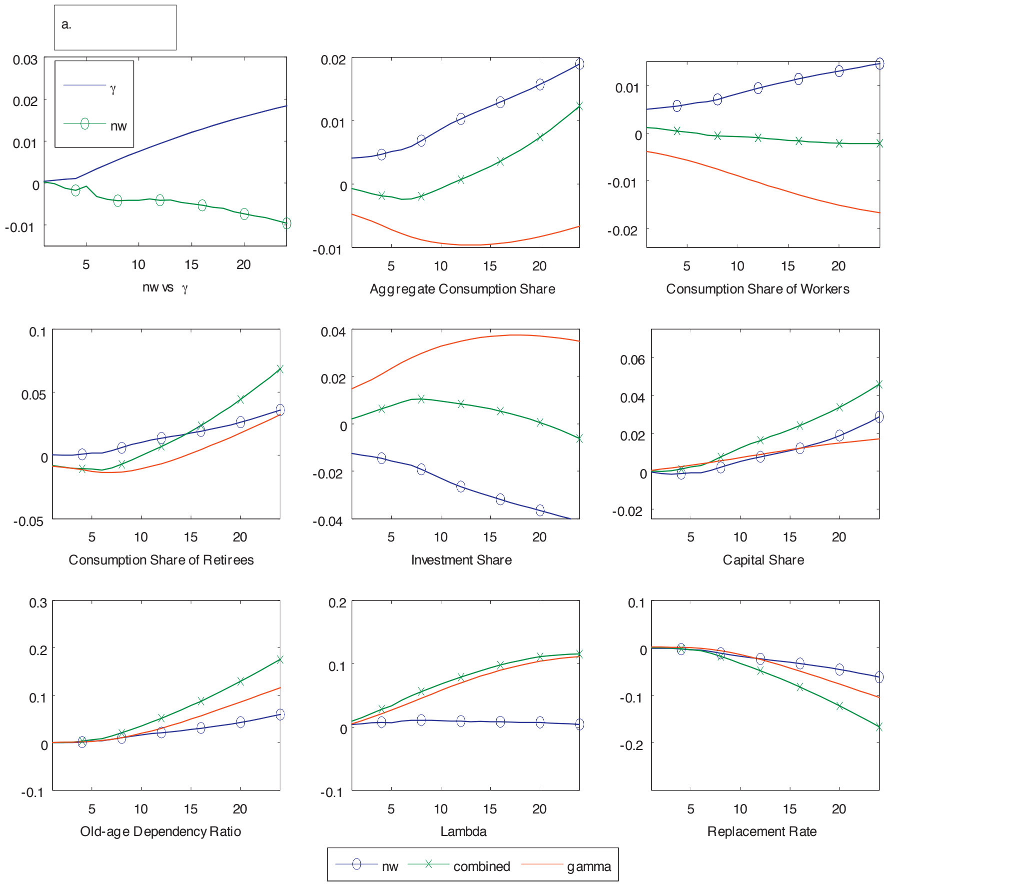
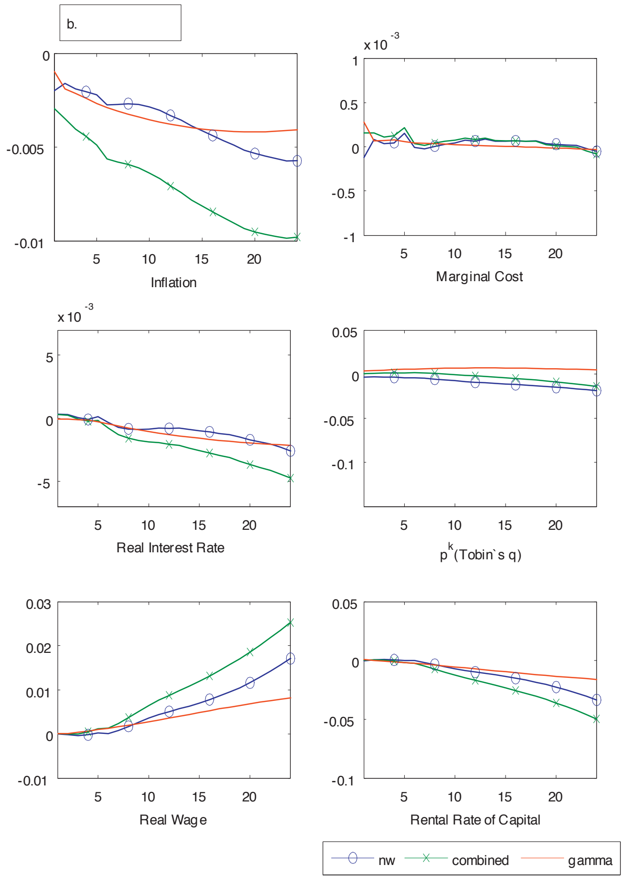


Figure 2: Effects of Euro Area Demographic Changes until 2030 with an Endogenous Replacement Rate in percent from intial steady state. The series on inflation and the nominal and real interest rates report level changes in percentage points.



b.
Inflation
Marginal Cost
Nominal Interest Rate
Real Interest Rate
p^k (Tobin`s q)
Labour Supply of Workers (l^w/N^w)
Real Wage
Rental Rate of Capital
Debt Share
nw
combined
gamma

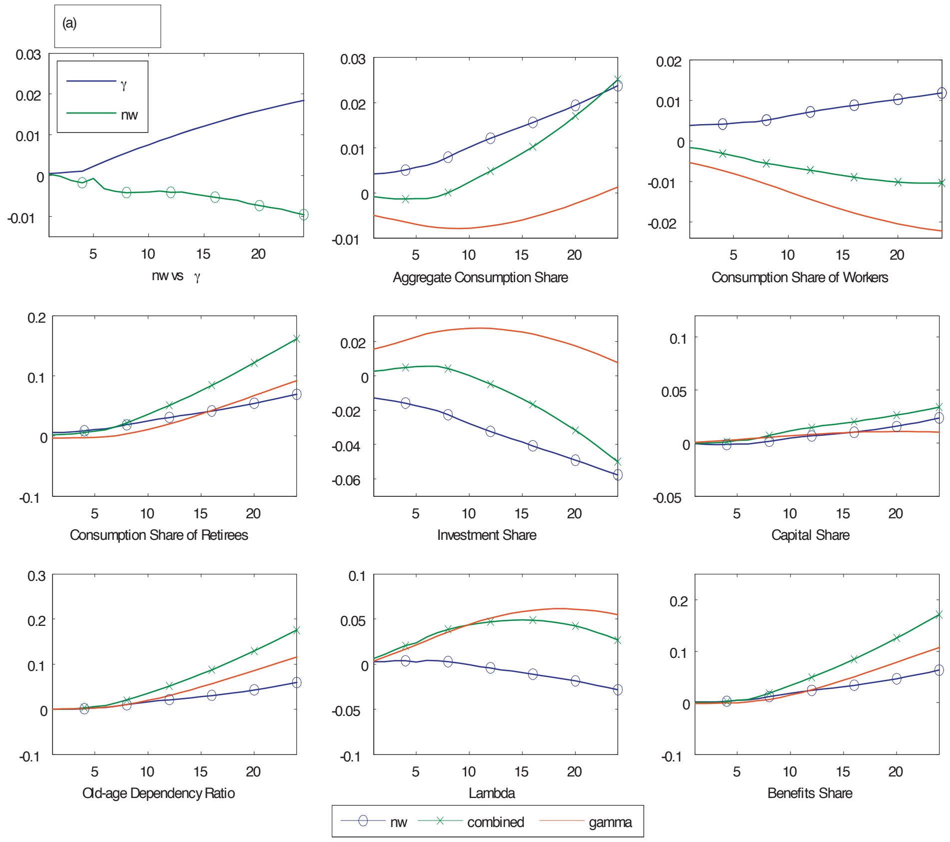
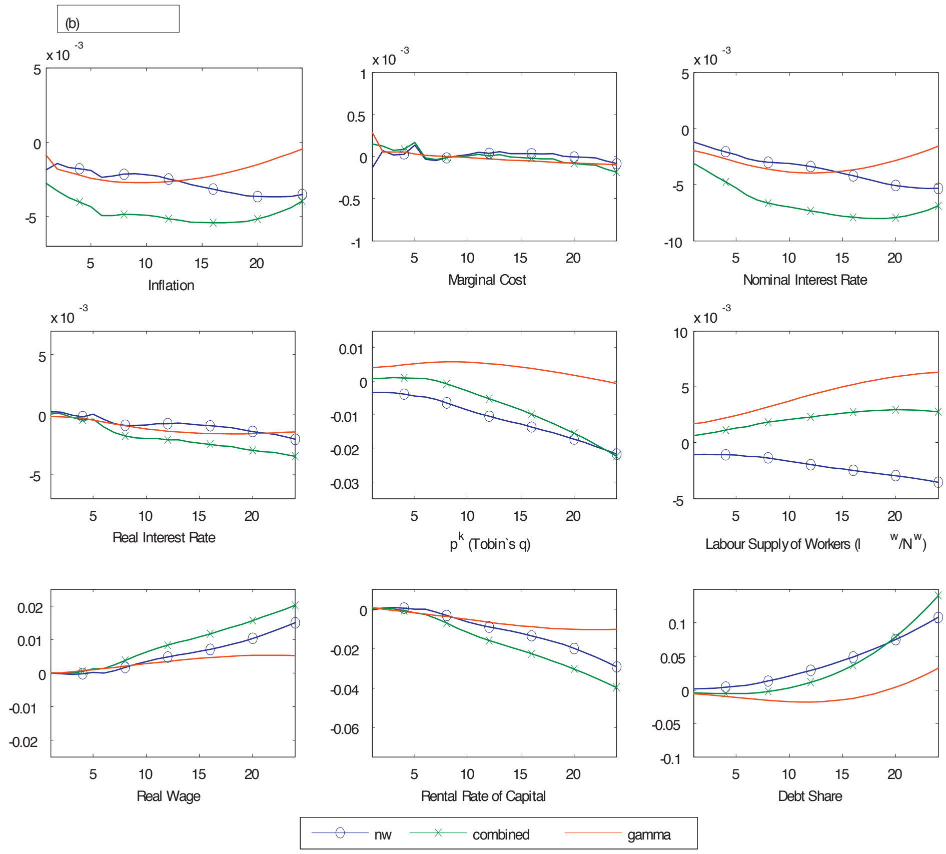


Figure 3: Effects of Euro Area Demographic Changes until 2030 with a Constant Replacement Rate. Changes in percent from intial steady state. The series on inflation and real interest rates report changes in percentage points.


(b)
Inflation
Marginal Cost
Nominal Interest Rate
Real Interest Rate
p^k (Tobin`s q)
Labour Supply of Workers (l^w/N^w)
Real Wage
Rental Rate of Capital
Debt Share
nw
combined
gamma