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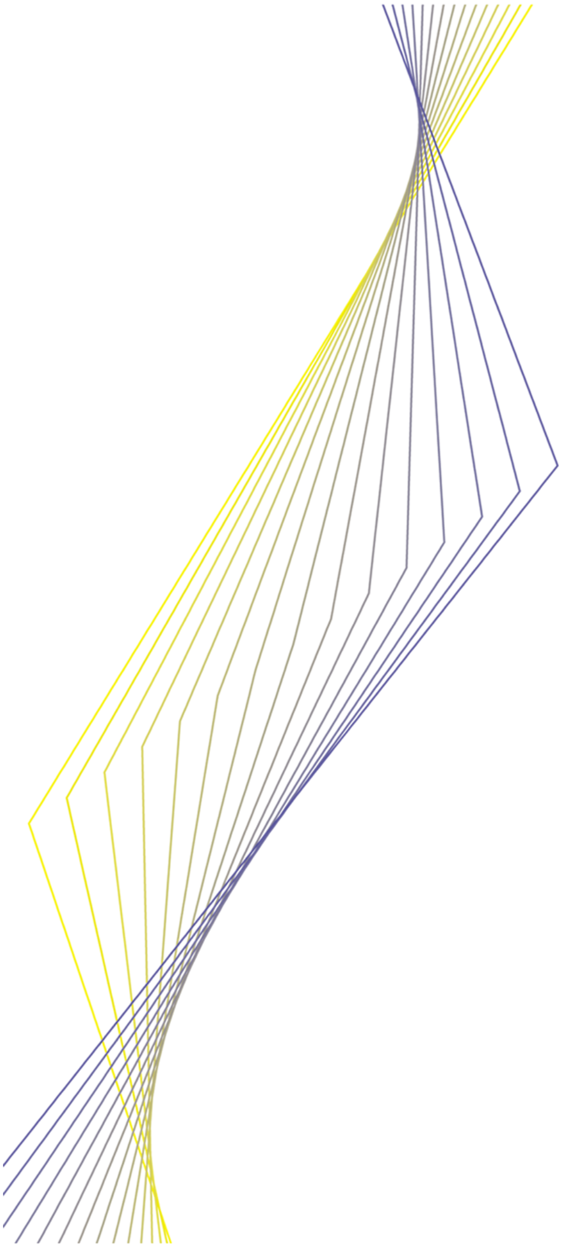

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CAN INDETERMINACY EXPLAIN THE SHORT-RUN NON-NEUTRALITY OF MONEY?

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Contents

Abstract

This paper analyzes the possibility to generate indeterminacy and equilibria with short-run non-neutrality of money in a model with flexible prices, constant returns to scale in production and constant money growth rules. The model recovers previous results in the literature as particular cases. It is shown that real effects of monetary shocks, as observed in the data, can arise in four regions of the parameter space. Two regions are characterized by unreasonable assumptions, which lead to inferiority of consumption or leisure. Two regions are characterized by reasonable assumptions and by normality of the goods. However, real effects of monetary shocks require implausible parameter values.

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1. Introduction

This paper analyzes the possibility to generate real indeterminacy and equilibria with short-run non-neutrality of money, as observed in the data, in a rational expectations model with flexible prices, constant returns to scale in production and constant money growth rules.

Monetary models often entail the possibility of indeterminacy, in the form of a continuum of equilibrium paths converging to the steady state. Under this kind of indeterminacy, a model with market-clearing and rational expectations can explain the short-run non-neutrality of money, because it generates equilibria that replicate the sequence of real effects observed in the data. Following an unexpected monetary injection, prices increase slowly, the interest rate declines and output temporarily rises above its stationary level. In the long run money is neutral, and the economy converges to its unchanged stationary state, where only prices are higher.

The possibility of indeterminacy in monetary models has been extensively analyzed in the literature, both in the form of self-fulfilling hyperinflationary or deflationary equilibria and of multiple stationary equilibria. Early contributions discuss indeterminacy in models where the authorities follow fixed monetary rules, as in Brock [8], Calvo [10], Kehoe and Levine [26], and Woodford [35]. More recent works have focused on models where the authorities adopt feedback monetary rules, which could themselves generate indeterminacy. This is discussed in Black [7], Leeper [27], Benhabib, Schmitt-Grohe and Uribe [6], and Carlstrom and Fuerst [12], [13]. These models can be used to evaluate the stability properties of alternative monetary policy rules and to select those rules which, by ensuring determinacy, eliminate unnecessary fluctuations in the economy.

There have also been several attempts to explain the short-run non-neutrality of money in general equilibrium models. Standard approaches are based on limited participation as in Lucas [28], Fuerst [25], and Christiano and Eichenbaum [16], [17], on price stickiness as in Christiano, Eichenbaum, and Evans [18], on nominal contracting as in Taylor [33], or on staggered price setting as in Calvo [11] and Chari, Kehoe, and McGrattan [15]. One limitation of these approaches is that, while they can easily generate real effects of monetary shocks, they are not able to replicate the persistence that is observed in the data.

Recently, an alternative approach to the short-run non-neutrality of money has been proposed, based on flexible price models that lead to indeterminate equilibria. Some authors add money to models where indeterminacy arises because of increasing returns to scale in production, as in Beaudry and Devereux [3] and Sussounov [32]. Indeterminacy, however, requires degrees of returns to scale that do not find support in the available empirical evidence.[1] Other authors obtain indeterminacy by adding money to models where externalities arise in preferences rather than in production, as in Benhabib and Farmer [5]. Finally, some authors claim that indeterminacy can also be obtained in models with flexible prices, no externalities, and constant money growth rules, as in Farmer [23] and Matheny [29]. Farmer [23] shows that indeterminacy can arise in a money-in-the-utility function version of a business cycle model, where preferences are such that the labor supply curve has a negative slope. Matheny [29] finds indeterminacy and persistent real effects of monetary shocks in a cash-in-advance model, where production is constant returns to scale. A crucial assumption in his analysis is that of Pareto substitutability between consumption and leisure, which implies that the marginal utility of leisure is a decreasing function of consumption. Both authors stress the importance of the non-separability of preferences, suggesting that the reason why most of the business cycle literature has found determinacy is the restrictive assumption of separable preferences.

In this paper, I argue that equilibria with short-run non-neutrality of money are not a plausible outcome in a model where there is a transactions role for money, in the spirit of Baumol [2] and Tobin [34]. The economy is such that prices are flexible, preferences are non-separable, no externalities arise in either production or preferences, and monetary policy follows a constant money growth rule. The cash-in-advance model used by Matheny [29] is a limiting case of the transactions technology model. Also, in the case with no capital, the money-in-the-utility function model used by Farmer [23] can be seen as a reduced form of the transactions technology

[1] See, for instance, Basu and Fernald [1].

model considered here. Therefore, it is possible to relate the results obtained in this paper to those previously obtained in the literature.

The paper proceeds as follows. In section 2, I describe the model economy. In section 2.1, I show that, under normality of consumption and leisure, if a steady state with valued money exists, it is unique. By showing uniqueness, I ensure that the analysis is global, so that there is no need to impose restrictions on the size of the shocks hitting the economy. In section 2.2, I derive the set of all possible perfect foresight equilibria, under normality of consumption and leisure.

Although inferiority is an unreasonable property of a representative agent model, previous works in the literature have not restricted the model specification to ensure normality. In section 3, I drop these restrictions in order to explain some of the previous results. The analysis remains global except when leisure is inferior, in which case uniqueness of the steady state with valued money cannot be ensured. I proceed by deriving the general conditions under which *persistent* real effects of monetary shocks, as observed in the the data, can arise. Then, I partition the parameter space into four regions, according to whether consumption and leisure are normal or inferior and Pareto complements or Pareto substitutes. I show that in each of these regions indeterminacy and persistent real effects of nominal shocks can possibly arise.

In section 4, I analyze the plausibility of equilibria with persistent real effects in the four regions of the parameter space. While doing so, I also obtain previous results in the literature as specific cases of the results derived in this paper. The main finding is that indeterminacy and persistent real effects require either unreasonable assumptions or implausible parameter values. In section 4.1, I consider the two regions of the parameter space where either consumption or leisure are inferior. In a reasonably specified model, these regions would be ruled out. In the absence of appropriate restrictions, however, indeterminacy can be found because of model assumptions that imply inferiority of one of the goods. Farmer [23] finds indeterminacy under a specification of preferences that leads to a backward sloping labor supply. I show that, if the money-in-the-utility-function model is seen as the reduced form of a transactions technology model, a negative slope of the labor supply implies inferiority of consumption. In section 4.2, I consider the region where consumption and leisure are normal and there is either separability of preferences or Pareto complementarity between consumption and leisure. These are common assumptions in the business cycle literature. I argue that the existence of equilibria with persistent real effects requires implausible parameter values. Finally, in section 4.3, I consider the region where consumption and leisure are normal and Pareto substitutes. These are the assumptions under which Matheny [29] finds equilibria with persistent real effects, in a cash-in-advance model. I show that these equilibria require implausible parameter values, in a more general transactions technology model that is calibrated to replicate the main facts about money demand.

Although indeterminacy is not found to be plausible in the set up of this paper, it is nonetheless possible. On the contrary, indeterminacy cannot arise in a correspondent real model where production is constant returns to scale. In section 5, I provide an intuitive explanation for this, based on the role of the nominal interest rate as a tax that distorts labor decisions. Section 6 contains the conclusions.

2. The Model

The economy is described by a transactions technology monetary model as in Chari, Christiano and Kehoe [14], Correia and Teles [19], De Fiore and Teles [22], and Mulligan and Sala-i-Martin[30].

There is a large number of identical households, whose preferences are defined over a consumption good, c_t , and leisure, h_t. Each household maximizes

$$\sum_{t=0}^{\infty} \beta^t U(c_t, h_t), \tag{2.1}$$

where U is an increasing concave function. Thus, $U_c > 0$, $U_h > 0$, $U_{cc} < 0$, $U_{hh} < 0$. There is no prior restriction on the sign of U_{ch}. When $U_{ch} > 0$ consumption and leisure are Pareto complements, when $U_{ch} < 0$ they are Pareto substitutes.

The household is endowed with one unit of time that can be allocated to labor n_t, leisure h_t, or transactions s_t,

$$n_t + h_t + s_t = 1. \tag{2.2}$$

There are two assets, money and nominal bonds. Agents need to transact in order to acquire the consumption good. This activity is costly, because it requires time that could otherwise be spent for production. The amount of time spent transacting is assumed to increase with consumption and to decrease with real balances, according to the following transactions technology:

$$s_t \geq l\left(c_t, m_t\right), \tag{2.3}$$

where $m_t = \frac{M_t}{P_t}$, and P_t is the price of the good. It is assumed that:

i) The function l is homogeneous of degree k, and it can be written as

$$l(c_t, m_t) = \Gamma(\frac{m_t}{c_t})c_t^k, \tag{2.4}$$

where $\Gamma : A \to R^+$, $A \subseteq R^+$; $\Gamma' \leq 0$, $\Gamma'' \geq 0$. This implies that $l_c \geq 0$, $l_m \leq 0$, $l_{mm} \geq 0$.

ii) The function l is such that the problem is concave. This implies that k cannot be too low. The minimum value of the degree of homogeneity, k, depends on the curvature of the utility function. Only non negative values for k are considered, because when $k < 0$, l_c can be negative.

The point of *full liquidity,* $\frac{\overline{m}}{c} = \sup A$, is defined as the level at which an additional unit of real balances does not reduce transactions time. At that point, $l_m\left(\overline{m}, c\right) = 0$.

At the beginning of each period t, the household chooses nominal money holdings M_t, to be used for transactions in that same period, and nominal bond holdings B_t.

These bonds entitle him to $(1+i_t)B_t$ units of money in period $t+1$. At the end of period t, the household receives nominal wage income $P_t w_t n_t$, where w_t is the real wage, which he can then spend in $t+1$. The government redistributes revenue from money creation through lump-sum nominal transfers Z_t.

The budget constraints are given by the conditions

$$P_t c_t + M_{t+1} + B_{t+1} \leq M_t + (1+i_t)B_t + P_t w_t n_t + Z_t, \; t \geq 0, \tag{2.5}$$

$$M_0 + B_0 \leq \overline{W}_0, \tag{2.6}$$

where $\overline{W}_0$ is initial nominal wealth at time zero. The household's decisions also need to satisfy a no-Ponzi games condition

$$\lim_{t \to \infty} \frac{P_{t+1}}{P_j} \prod_{k=j}^{t} \frac{1}{(1+i_{k+1})} \left(\frac{M_{t+1} + B_{t+1}}{P_{t+1}} \right) \geq 0, \tag{2.7}$$

where $\frac{P_{t+1}}{P_j} \prod_{k=j}^{t} \frac{1}{(1+i_{k+1})}$ is the value at date j of a unit of consumption in period $t+1$.

There is a linear production technology,

$$y_t = A_t n_t. \tag{2.8}$$

For simplicity, I assume zero growth, so that $A_t = 1$ for all t, although a similar analysis would extend to an economy where A_t is growing over time, as long as preferences and the transactions technology are restricted to ensure that a balanced growth path exists.

The government's monetary policy is given by a constant money growth rule and an initial value of the money stock M_0. Money supply evolves according to

$$M_{t+1} = \mu M_t. \tag{2.9}$$

For simplicity, I assume that fiscal policy is given by

$$B_t = 0, \; \text{for all } t. \tag{2.10}$$

In the analysis below, I will focus on a perfect foresight model. The problem of the household is defined by the maximization of (2.1), subject to (2.2), (2.3), (2.5), (2.6), and (2.7). The optimality conditions include

$$\frac{U_c\left(c_t, h_t\right)}{U_h\left(c_t, h_t\right)} = \frac{1 + w_t l_c\left(c_t, m_t\right)}{w_t}, \tag{2.11}$$

$$-w_t l_m\left(c_t, m_t\right) = i_t, \tag{2.12}$$

$$\frac{U_h(c_t, h_t)}{P_t w_t} = \beta \frac{U_h(c_{t+1}, h_{t+1})}{P_{t+1} w_{t+1}} \left[1 - w_{t+1} l_m\left(c_{t+1}, m_{t+1}\right)\right], \tag{2.13}$$

$$\lim_{t \to \infty} \frac{P_{t+1}}{P_j} \prod_{k=j}^{t} \frac{1}{\left(1 + i_{k+1}\right)} \left(\frac{M_{t+1} + B_{t+1}}{P_{t+1}}\right) = 0. \tag{2.14}$$

2.1. Steady State Analysis

In this section, I analyze the conditions under which one or more steady states exist with valued money. Notice that in this economy there also exists a steady state where money has no value, so that $m = 0$.

Define the marginal rate of substitution as

$$D(c, h) \equiv \frac{U_c\left(c, h\right)}{U_h\left(c, h\right)}. \tag{2.15}$$

Variables with no time subscripts denote steady state values. Given our assumptions that $A_t = 1$, for all t, and that firms are perfectly competitive, it must be that in equilibrium

$$w_t = 1, \text{ for all } t. \tag{2.16}$$

A steady state with valued money has to satisfy the following system of equations:

$$\frac{\mu}{\beta} - 1 = -l_m\left(c, m\right), \tag{2.17}$$

$$D\left(c, 1 - c - l\left(c, m\right)\right) = 1 + l_c\left(c, m\right). \tag{2.18}$$

Because the focus of the analysis is on business cycle fluctuations, I restrict the attention to monetary policies that achieve a strictly positive nominal interest rate, as implied by the following assumption:

$(A1)$ $\mu > \beta$.

Under $(A1)$, there is a single positive value of m that satisfies equation (2.17). Since in the steady state $\frac{\mu}{\beta} - 1 = i$, (2.17) can be rewritten as $i = -l_m(c, m)$. This can be used to express the stationary level of real balances as a function of consumption and the nominal interest rate,

$$m = L(c, i), \tag{2.19}$$

where $L_c = -\frac{l_{cm}}{l_{mm}}$.

I will also make the following assumption:

$(A2)$ $l_{cm} \leq 0$ and $l_c - \frac{l_m l_{cm}}{l_{mm}} \geq 0$.

Assumption $(A2)$ relates to the way transactions are produced in the economy. To understand what it implies, consider a transactions technology $s = l(c, m)$. The inverse of this function gives the technology which describes the production of transactions, $c = f(s, m)$. The technical rate of substitution between s and m can be obtained by implicit differentiation, and is given by $-f_m/f_s$. From $s = l(c, m)$, we know that $-f_m/f_s = l_m = -i$. Assumption $(A2)$ requires that, for given i, which is the relative price of s and m, the amount of transactions produced c cannot increase when one input (time or real balances) is decreased. In fact, $l_{cm} \leq 0$ ensures that in the steady state, where the relative price of s and m is constant, $\frac{dm}{dc} \geq 0$. This implies that the elasticity of money demand with respect to consumption is positive, as required in the literature. Also, $l_c - \frac{l_m l_{mc}}{l_{mm}} \geq 0$ ensures that in the steady state $\frac{ds}{dc} \geq 0$. To see why, rewrite the transactions technology as $s = l(c, L(c, i))$. Under $(A2)$, $\frac{ds}{dc} = \left(l_c - \frac{l_m l_{mc}}{l_{mm}}\right) \geq 0$.[2]

[2] When $f(s, m)$ is homothetic, assumption $(A2)$ is always satisfied. In fact, homotheticity implies that, for the same relative price of s and m, an increase in the amount of transactions c requires an increase in the same proportion of both inputs. It can easily be checked that $f(s, m)$ is homothetic when, for instance, $l(c, m)$ is homogeneous of degree one. When the transactions technology is homogeneous of degree different from 1, $f(s, m)$ can still be homothetic, as for the Baumol-Tobin specification, where $s = \theta \frac{c}{m}$ and θ is a constant. However, this is not necessarily the case.

Since i is exogenously fixed in the steady state, I can define $l_c^*(c) \equiv l_c(c, L(c,i))$ and $D^*(c) \equiv D(c, 1-c-l(c, L(c,i)))$. Also, I define

$$\gamma^*(c) \equiv D^*(c) - l_c^*(c) - 1.$$

The number of steady states with valued money in this economy is given by the number of intersections of the function $\gamma^*(c)$ with the x-axis. A unique intersection is ensured by monotonicity of $\gamma^*(c)$.

Now, differentiate $\gamma^*(c)$ to get

$$\gamma_c^*(c) = D_c - DD_h - l_{cc} + \frac{l_{cm}^2}{l_{mm}} + \left(\frac{l_m l_{cm}}{l_{mm}}\right) D_h,$$

where each function is evaluated at the steady state level of c. As shown in Appendix A.1, the second order conditions for the household's problem require that

$$\Delta = -(D_c - DD_h) + l_{cc} - \frac{l_{cm}^2}{l_{mm}} > 0. \tag{2.20}$$

It follows that

$$\gamma_c^*(c) = -\Delta + \left(\frac{l_m l_{cm}}{l_{mm}}\right) D_h. \tag{2.21}$$

Notice that the expression in brackets in (2.21) is positive, while D_h can be either positive or negative depending on the sign of U_{ch}, since $D_h = \frac{1}{U_h}(U_{ch} - U_{hh}D)$.

To analyze the sign of $\gamma_c^*(c)$, it is useful to derive the conditions under which c and h are normal goods. Consider the static formulation of the household's problem described above:[3]

$$\max \; U(c,h)$$

subject to

$$c = wn - im + z,$$

and

$$h = 1 - n - l(c,m).$$

Here $z \equiv \frac{Z}{P}$ denotes the amount of transfers in units of consumption, or real wealth net of wage income. The first order conditions are given by

$$D(c, 1-n-l(c,m)) = \frac{1}{w} + l_c(c,m), \tag{2.22}$$

[3] For simplicity, I assume zero initial wealth, $\overline{W}_0 = 0$.

$$i = -wl_m\left(c, m\right), \tag{2.23}$$

$$c = wn - im + z. \tag{2.24}$$

From the first order condition for real balances, it follows that

$$m = L\left(c, \frac{i}{w}\right), \tag{2.25}$$

where $L_c = -l_{cm}/l_{mm}$. Notice that equation (2.19) takes into account the equilibrium condition $w = 1$. On the contrary, equation (2.25) is simply a first order condition of the household's problem, where w is taken as given.

To derive the conditions under which consumption is normal, rewrite the system of equations (2.22)-(2.24) as

$$D\left[c, 1 - n - l\left(c, L\left(c, \frac{i}{w}\right)\right)\right] = \frac{1}{w} + l_c\left(c, L\left(c, \frac{i}{w}\right)\right), \tag{2.26}$$

$$n = \frac{1}{w}\left[c + iL\left(c, \frac{i}{w}\right) - z\right]. \tag{2.27}$$

Implicit differentiation gives that

$$\left[D_c - D_h\left(l_c + l_m L_c\right) - l_{cc} - l_{cm}L_c\right]\frac{dc}{dz} - \frac{D_h}{w}\left(\left(1 + iL_c\right)\frac{dc}{dz} - 1\right) = 0.$$

Rearranging, we obtain that

$$\frac{dc}{dz} = \frac{D_h}{w\Delta}. \tag{2.28}$$

To derive the conditions under which leisure is normal, recall that what appears in the household's utility is leisure defined as time endowment net of all market activities, $h = 1 - n - s$. By implicit differentiation of (2.26), and using (2.28), we get that

$$\frac{dn}{dz} = \left(\frac{1}{w} + \frac{l_m l_{cm}}{l_{mm}}\right)\frac{D_h}{w\Delta} - \frac{1}{w}. \tag{2.29}$$

From $s = l\left[c, L\left(c, \frac{i}{w}\right)\right]$, and using (2.28), we obtain that

$$\frac{ds}{dz} = \left(l_c - \frac{l_m l_{cm}}{l_{mm}}\right)\frac{D_h}{w\Delta}. \tag{2.30}$$

From the definition of leisure, it follows that

$$\frac{dh}{dz} = \frac{\Delta - DD_h}{w\Delta}. \tag{2.31}$$

Proposition 1. *Under* $(A1)$, *consumption is normal if and only if* $D_h > 0$, *while leisure is normal if and only if* $\Delta - DD_h > 0$. *Under* $(A1), (A2)$ *and normality of consumption, a necessary condition for leisure to be normal is that*

$$-\Delta + \left(\frac{l_m l_{cm}}{l_{mm}}\right) D_h < -\frac{D_h}{w}. \qquad (2.32)$$

Proof. Under $(A1)$, normality of consumption and leisure follows directly from (2.28) and (2.31). Under $(A1)$, $(A2)$ and normality of consumption, $\frac{ds}{dz} \geq 0$. Since $\frac{dh}{dz} = -\frac{dn}{dz} - \frac{ds}{dz}$, a necessary condition for leisure to be normal is that $\frac{dn}{dz} < 0$. Rearranging (2.29), one obtains condition (2.32). ■

Notice that, when consumption is inferior, $D_h < 0$, so that $\Delta - DD_h > 0$ and leisure must be normal. When consumption is normal, $D_h > 0$ and leisure can be either normal or inferior, depending on the steady state values of Δ and D_h.

It is now possible to characterize the sign of $\gamma_c^* (c)$ as in (2.21), and to state the conditions under which there is a unique steady state or multiple steady states with valued money.

Proposition 2. *Under* $(A1) - (A2)$ *and normality of consumption and leisure, if a steady state with valued money exists, it is unique.*

Proof. When consumption and leisure are normal, $D_h > 0$ and $\Delta - DD_h > 0$. Under $(A1)$ and $(A2)$, a necessary condition for normality of leisure is given by (2.32). Together with normality of consumption, this condition implies that $-\Delta + \left(\frac{l_m l_{cm}}{l_{mm}}\right) D_h < -\frac{D_h}{w} < 0$. In this case $\gamma_c^* (c) < 0$ and, if a steady state with valued money exists, it is unique. ■

2.2. Perfect Foresight Equilibria

I analyze a perfect foresight version of the model, under the assumption that consumption and leisure are normal and that a steady state with strictly positive real balances exists, and I derive the set of all possible equilibria along which money is valued. I consider both non-stationary equilibria, where m_t becomes either arbitrarily large or arbitrarily close to zero, and stationary equilibria, where m_t either stays at the steady state with valued money or converges to it.

To simplify the system of equilibrium conditions, use the fact that $\frac{P_t}{P_{t+1}} = \frac{m_{t+1}}{m_t \ \mu_{t+1}}$, the production function (2.8), the wage condition (2.16), the monetary and fiscal policy rules (2.9)-(2.10), and the resources constraint for $t \geq 0$

$$h_t = 1 - c_t - l\left(c_t, m_t\right),$$

to rewrite the first order conditions of the private problem (2.11) and (2.13). Also, denote $x \equiv \frac{\mu}{\beta}$.

A perfect foresight equilibrium is any sequence for c_t and m_t that satisfies the following system:

$$D\left[c_t, 1 - c_t - l\left(c_t, m_t\right)\right] = 1 + l_c\left(c_t, m_t\right) \tag{2.33}$$

$$\begin{aligned} & x \ m_t U_h\left[c_t, 1 - c_t - l\left(c_t, m_t\right)\right] \\ = \ & m_{t+1} U_h\left[c_{t+1}, 1 - c_{t+1} - l\left(c_{t+1,} m_{t+1}\right)\right]\left[1 - l_m\left(c_{t+1}, m_{t+1}\right)\right], \end{aligned} \tag{2.34}$$

$$\lim_{t \to \infty} \beta^{t+1-j} \frac{U_h\left[c_{t+1}, 1 - c_{t+1} - l\left(c_{t+1}, m_{t+1}\right)\right]}{U_h\left[c_j, 1 - c_j - l\left(c_j, m_j\right)\right]} m_{t+1} = 0. \tag{2.35}$$

The last equation is derived by plugging (2.12), (2.13), (2.16) and the condition that $B_t = 0$, for all t, into condition (2.14). Equation (2.33) implicitly defines consumption as a function of real balances

$$c_t = \varphi\left(m_t\right).$$

By implicit differentiation, we obtain that

$$\frac{dc_t}{dm_t} = \varphi_m\left(t\right) = \frac{l_{cm}\left(t\right) + l_m\left(t\right) D_h\left(t\right)}{D_c\left(t\right) - D\left(t\right) D_h\left(t\right) - l_{cc}\left(t\right)}, \tag{2.36}$$

where, for simplicity, t replaces the arguments of each function. Under normality of consumption, $D_h > 0$ and the numerator is negative. From the second order conditions (2.20), we also know that the denominator is negative. Therefore, when $D_h > 0$, also $\varphi_m\left(t\right) > 0$. Now, define

$$F\left(m_t\right) \equiv U_h\left[\varphi\left(m_t\right), 1 - \varphi\left(m_t\right) - l\left(\varphi\left(m_t\right), m_t\right)\right], \tag{2.37}$$

$$G\left(m_t\right) \equiv 1 - l_m\left(\varphi\left(m_t\right), m_t\right). \tag{2.38}$$

Then equation (2.34) can be rewritten as

$$xm_t \ F(m_t) - m_{t+1}F(m_{t+1})\,G(m_{t+1}) = 0. \tag{2.39}$$

Any equilibrium path for m_t must also satisfy

$$\lim_{t\to\infty} \beta^{t+1-j} \frac{F(m_{t+1})}{F(m_j)} m_{t+1} = 0. \tag{2.40}$$

From (2.39), it follows that

$$\frac{dm_{t+1}}{dm_t} = \frac{x\left[F(m_t) + m_t F_m(m_t)\right]}{G(m_{t+1})\left[F(m_{t+1}) + m_{t+1}F_m(m_{t+1})\right] + m_{t+1}F(m_{t+1})\,G_m(m_{t+1})}. \tag{2.41}$$

Notice that, at the steady state with strictly positive real balances, $x \equiv \frac{\mu}{\beta} = 1 + i = 1 - l_m \equiv G$. Differentiate (2.37) and (2.38), and evaluate (2.41) at the steady state value m to get:

$$\left.\frac{dm_{t+1}}{dm_t}\right|_m \equiv \alpha = \frac{a}{a-b}, \tag{2.42}$$

where

$$a \equiv 1 - ml_m \frac{U_{hh}}{U_h} + m\varphi_m D_h, \tag{2.43}$$

$$b \equiv \frac{m}{x}\left(l_{cm}\varphi_m + l_{mm}\right), \tag{2.44}$$

and each function is evaluated at the steady state value m.

A perfect foresight approximate solution is any deterministic sequence of the form

$$\widetilde{m}_{t+1} = \alpha^{t+1}\widetilde{m}_0, \tag{2.45}$$

where $\widetilde{m}$ denotes deviations of m_t from the steady state value m. It is now possible to characterize the class of equilibria with valued money that arise in this economy.

Proposition 3. *Under $(A1) - (A2)$ and normality of consumption and leisure, when $|\alpha| < 1$ there is a continuum of equilibrium paths converging to the unique steady state with valued money.*

Proof. Define $H(m_t, m_{t+1}) \equiv xm_t \ F(m_t) - m_{t+1}F(m_{t+1})\,G(m_{t+1})$. Under $(A1) - (A2)$ and normality of consumption and leisure, there is a unique steady state with valued money. When $|\alpha| < 1$, any equilibrium path described by (2.45) satisfies both conditions (2.39) and (2.40). Therefore, there is a continuum of equilibrium paths converging to the unique steady state with valued money, one for each initial $\widetilde{m}_0$. ■

Proposition 4. *Under* $(A1)-(A2)$ *and normality of consumption and leisure, when* $|\alpha|>1$ *the unique stationary equilibrium path with valued money is the steady state path* $m_t = m > 0$*, for all* t*. There is also a continuum of perfect foresight equilibria, where real balances converge asymptotically to zero.*

Proof. When $|\alpha|>1$, the locus $H(m_t, m_{t+1}) = 0$ intersects the 45 degree line with a slope greater than one, at the unique steady state with valued money. A path where $m_t = m > 0$, for all t, is an equilibrium path, since it satisfies both conditions (2.39) and (2.40). It is also the unique stationary path with valued money. There also exist non-stationary equilibrium paths. First, consider a path $\{m_0, m_1, m_2, ...\}$ originating from $m_0 > m$. As long as $\lim_{m_t\to\infty} F(m_t) \in (0,\infty)$, such paths can be ruled out because they are explosive and they violate the transversality condition (2.40). Now consider a path originating at $m'_0 < m$. All paths originating at points below m are hyperinflationary paths, where real balances approach zero over time. They are equilibrium paths, since they satisfy both conditions (2.39) and (2.40). ■

The locus $H(m_t, m_{t+1}) = 0$ intersects the 45 degree line from below or from above, according to whether $\left.\frac{dm_{t+1}}{dm_t}\right|_{m\to 0}$, as given by equation (2.41), is greater or smaller than unity. Figure 1, panel a, shows the locus when it intersects from below, for the case when $|\alpha|<1$. Similarly, panel b shows when it intersects from below, for the case when $|\alpha|>1$.

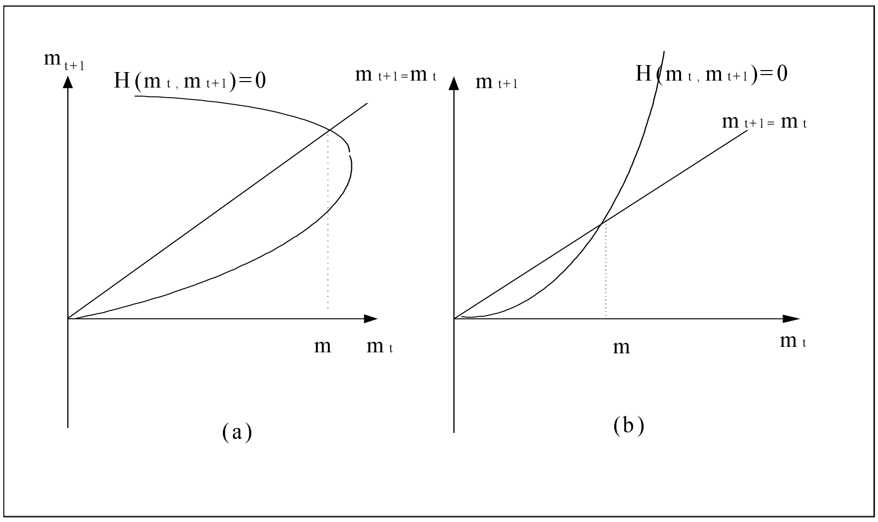

Figure 1

Notice that, when $|\alpha| < 1$, there is no possibility of pinning down a single stationary equilibrium among the continuum of stationary perfect foresight equilibrium paths. In this case, the possibility of there being paths along which monetary shocks have persistent real effects cannot be ruled out.

On the contrary, when $|\alpha| > 1$, there exists a unique stationary equilibrium with valued money under perfect foresight. In this case, money is neutral both in the short and in the long run. In a stochastic version of the model, it would be possible to build stationary stochastic paths, where real balances would converge to the steady state with valued money with some positive probability. However, the existence of these paths could hardly be reconciled with the evidence on the short-run non-neutrality of money, since the adjustment of real balances along these paths would generally be non-monotonic. Moreover, some authors have shown that hyperinflationary paths can be ruled out in a number of ways. For instance, Brock [8], [9] shows that these paths can be ruled out in models with pure fiat money by restricting agents' preferences. Obstfeld and Rogoff [31] argue that the restrictions on preferences necessary in a model with pure fiat money are unintuitive, since they require agents to have infinitely negative utility at zero real balances. They show that, in a model augmented with physical capital, speculative paths can be eliminated if the government is ready to redeem each unit of currency for a small amount of capital. When hyperinflationary paths can be ruled out, the steady state with strictly positive real balances remains the unique stationary equilibrium with valued money also in a stochastic environment.

3. Conditions for Indeterminacy and the Short-Run Non-Neutrality of Money

In the previous section, I have characterized the steady state and the dynamic properties of the economy under restrictions that ensure normality of consumption and leisure. These restrictions have not been imposed in previous papers, where indeterminacy and real effects of monetary shocks have been found to be possible. To show that indeterminacy can emerge as a consequence of inferiority, I drop these restrictions. Then, I derive all possible conditions under which indeterminacy and equilibria with short-run non-neutrality of money, as observed in the data, can arise.

Proposition 2 ensures uniqueness of the steady state with valued money (if it exists), under normality of consumption and leisure. Uniqueness could also be ensured under inferiority of consumption, so that the analysis would remain global. In this case, $D_h < 0$ and $\Delta - DD_h > 0$. Then, $\gamma_c^* (c) = -\Delta + \left(\frac{l_m l_{cm}}{l_{mm}}\right) D_h < 0$, and there would still be a unique steady state with strictly positive real balances. Uniqueness could not be ensured under inferiority of leisure, so that the analysis would only be local. In fact, in this case $D_h > 0$ and $\Delta - DD_h < 0$. Although inferiority of leisure always arises when $-\Delta + \left(\frac{l_m l_{cm}}{l_{mm}}\right) D_h > -\frac{D_h}{w}$, it could also arise when $-\Delta + \left(\frac{l_m l_{cm}}{l_{mm}}\right) D_h < -\frac{D_h}{w} < 0$, as long as $\frac{ds}{dz}$ is large enough. Therefore, $\gamma_c^* (c)$ could take either sign and the possibility of multiple steady states with valued money could not be ruled out.

The question addressed in this paper is whether money supply shocks can have real effects in the short run, as observed in the data, due to the possibility of multiple stationary equilibrium paths. The thought experiment is the following: suppose there is a once and for all change in the *level* of the money supply, and suppose that prices slowly adjust to the unchanged stationary level m, implying that real balances, production and consumption temporarily increase. Is this outcome consistent with a rational expectations equilibrium? An affirmative answer requires the model to deliver a multiplicity of stationary equilibria, for realistic parameter values. Moreover, the coefficient α needs to be positive, so that the equilibrium paths of the real variables do not oscillate around their steady state values. Therefore, the relevant cases are those where $\alpha \in (0, 1)$, so that each equilibrium path displays *persistent* real effects of monetary shocks. This can happen when a and b in expression (2.42) have opposite signs. Using the definition of D and (2.36), a and b can be rewritten as

$$a \equiv 1 - ml_m \frac{U_{hh}}{U_h} + \frac{m\varphi_m}{U_h} (U_{ch} - U_{hh} D), \tag{3.1}$$

$$b \equiv \frac{ml_{mm}}{x} \left[\frac{\Delta - \frac{l_m l_{cm}}{l_{mm}} D_h}{\Delta + \frac{l_{cm}^2}{l_{mm}}} \right]. \tag{3.2}$$

Under $(A1)$ and $(A2)$, it is possible to partition the parameter space into the following four regions, which are summarized in table 1.

1. $D_h < 0$, which implies that $U_{ch} < 0$ and $\Delta - DD_h > 0$. In this region, c is inferior, h is normal, and there is Pareto substitutability between c and h. In fact, from $D_h = \frac{1}{U_h}(U_{ch} - U_{hh}D)$, it must be that when $D_h < 0$, $U_{ch} < 0$. From (2.28), c must be inferior so that h is normal. From (2.43), the sign of a cannot be determined. From (3.2), $b > 0$ since $D_h < 0$. Notice that in this region φ_m can be either positive or negative.

2. $D_h > 0, U_{ch} < 0$, $\Delta - DD_h > 0$. In this region, c and h are normal and Pareto substitutes. From (2.36) we know that, when $D_h > 0$, also $\varphi_m > 0$. From (3.1), the sign of a cannot be determined. Under $(A1)$ and $(A2)$, a necessary condition for leisure to be normal is that $-\Delta + \frac{l_m l_{cm}}{l_{mm}} D_h < -\frac{D_h}{w}$. Normality of consumption implies that $D_h > 0$, so that $\Delta - \frac{l_m l_{cm}}{l_{mm}} D_h > 0$. It follows that $b > 0$.

3. $D_h > 0, U_{ch} \geq 0$, $\Delta - DD_h > 0$. In this region, c and h are normal. When $U_{ch} > 0$, c and h are Pareto complements. When $U_{ch} = 0$, preferences are separable in c and h. From (3.1), the sign of a cannot be determined. Combining (2.32) and $D_h > 0$, we know that $b > 0$.

4. $D_h > 0, U_{ch} \geq 0$, $\Delta - DD_h < 0$. In this region, c is normal, h is inferior, and there is Pareto complementarity between the two. From (3.1), the sign of a cannot be determined. When $\Delta - DD_h < 0$, $\Delta - \frac{l_m l_{cm}}{l_{mm}} D_h$ can take either sign. Thus, also the sign of b cannot be determined.

Table 1 lists the four regions. The first column gives the sets of restrictions on the specification of the model. The second column gives the sign that a and b can possibly take. The third column states the implications of each set of restrictions for consumption and leisure. The table shows that indeterminacy and persistent real effects of monetary shocks may possibly arise in each of the four regions, since α can fall between zero and one.

Table 1: Regions of the parameter space

Regions	a and b	c and h
1. $D_h < 0$ $U_{ch} < 0$ $\Delta - DD_h > 0$	$a \gtreqless 0$ $b > 0$	c inferior h normal c and h PS[a]
2. $D_h > 0$ $U_{ch} < 0$ $\Delta - DD_h > 0$	$a \gtreqless 0$ $b > 0$	c normal h normal c and h PS[a]
3. $D_h > 0$ $U_{ch} \geq 0$ $\Delta - DD_h > 0$	$a \gtreqless 0$ $b > 0$	c normal h normal c and h PC[b]
4. $D_h > 0$ $U_{ch} \geq 0$ $\Delta - DD_h < 0$	$a \gtreqless 0$ $b \gtreqless 0$	c normal h inferior c and h PC[b]

[a]PS: Pareto substitutes. [b]PC: Pareto complements.

4. Is the Short-Run Non-Neutrality of Money a Plausible Outcome?

In this section, I argue that indeterminacy and persistent real effects of monetary shocks require unreasonable assumptions in region 1 and 4 of the table, while they require implausible parameter values in regions 2 and 3, under common specifications of preferences and of the transactions technology. I also derive previous results in the literature as specific cases of the results derived in this paper.

4.1. Inferiority of Consumption or Leisure

Table 1 shows that indeterminacy and persistent real effects of monetary shocks may arise in regions 1 and 4. It also shows that these regions are characterized by inferiority of consumption or leisure, which is an unreasonable property of a representative agent macro model. In a carefully specified model, where either preferences or the transactions technology are restricted to ensure normality, regions 1 and 4 would be ruled out. In the absence of these restrictions, however, indeterminacy can be found because of assumptions that lead to inferiority of one of the goods.

To see this, consider the result obtained by Farmer [23]. The author finds indeterminacy in a money-in-the-utility function model with capital, under constant returns to scale in production and either a pure interest rate peg or a constant money growth rule. The key to this result is a simple specification of preferences such that the labor supply curve (defined as a function of the real wage, holding consumption constant) has a negative slope. Since the model is not able to replicate the main facts about money demand under that simple specification of preferences, the author adopts a more general specification, which leads to indeterminacy under a realistic calibration. Sossounov [32] finds an error in Farmer's solution of the general model and shows that, under the author's preferred parameterization, the model displays saddle-path stability. However, this finding does not rule out the possibility of a reasonable parameterization existing, under which the labor supply curve would have a negative slope and real indeterminacy could indeed arise.

In what follows, I consider a version of Farmer's model with no capital. The presence of capital would add one more dimension to the system without changing the main conclusions. If the money-in-the-utility function model is justified as the reduced form of a model in which money reduces transactions costs, as suggested by Farmer, the analysis of section 2 and 3 can be applied. Then, we know that indeterminacy may arise under a constant money growth rule. Nonetheless, I show that a specification of preferences such that the labor supply curve has a negative slope corresponds to the case of region 1 in table 1, where consumption is inferior.

Money-in-the-utility function (MIUF) models are widely used in monetary economics for their analytical convenience. However, they are often seen as reduced forms of more accurate models, where real balances only affect utility indirectly by facilitating transactions. Feenstra [24] shows that it is possible to establish an equivalence between a transactions costs technology (TCT) model, where utility only depends on consumption, and a MIUF model. Correia and Teles [20] extend this equivalence to a model where preferences in the TCT model are defined over both consumption and leisure, and where transactions costs are measured in units of time. A MIUF model equivalent to the TCT model presented in section 2 can be built by defining $V(c_t, m_t, h_t^b) \equiv U(c_t, h_t^b - l\,(c_t, m_t))$, where h_t^b is time endowment minus labor activity. Thus, $h_t^b = h_t + l\left(c_t, m_t\right)$. Correia and Teles show that, under the assumptions made in section 2 on the transactions technology, the MIUF model is the reduced form of the TCT model if $V\left(c, m, h^b\right)$ satisfies the following conditions: V is concave, $V_{cc} \leq 0$, $V_{mm} \leq 0$, $V_m \geq 0$ and $V_{mh^b} \geq 0$. Therefore, the economy analyzed in section 2 can be described by an identical model, the only difference being that the representative household maximizes $\sum\limits_{t=0}^{\infty} \beta^t V(c_t, m_t, h_t^b)$, subject to (2.5), (2.6), and (2.7). This MIUF model also describes the economy considered in Farmer [23], in the case with no capital. Farmer chooses a different timing for households' decisions. In Appendix A.2, it is argued that this difference is not relevant for the analysis.[4]

When the model underlying the MIUF is a TCT model, a negative slope of the labor supply curve requires setting D_h negative. To see this, notice that the labor supply curve is the function that relates the amount of labor supplied to the real wage, for given prices, holding the level of consumption constant. In the model of section 2, this function is implicitly defined by equation (2.26). To characterize its slope, implicitly differentiate (2.26) to get:

$$\frac{dn}{dw} = \frac{1}{w^2}\left[\frac{1}{D_h} - \frac{i}{l_{mm}}\left(l_m + \frac{l_{cm}}{D_h}\right)\right].$$

A necessary condition for the labor supply curve to slope down is that $D_h < 0$. From our previous analysis, we know that $D_h < 0$ can induce a real indeterminacy. It also implies inferiority of consumption.

4.2. Normality and Pareto Complementarity

In this section, I evaluate whether equilibria with persistent real effects are a plausible outcome in region 3 of table 1, which is characterized by normality of the goods and either separability of preferences ($U_{ch} = 0$) or non-separability of preferences and Pareto complementarity between consumption and leisure ($U_{ch} > 0$). These are common assumptions in business cycle models, whose general result is determinacy and short-run neutrality.

Is it possible to obtain indeterminacy and persistent real effects of monetary shocks,

[4] It can also be shown that, under the choice of timing adopted here, a real indeterminacy can never arise under a pure interest rate peg.

or $\alpha \in (0,1)$, under the model assumptions of region 3? When consumption and leisure are normal, we know that $b > 0$. Therefore, α can only lie between zero and one if $a < 0$. From (2.43), a can be negative only if $ml_m \frac{U_{hh}}{U_h}$, evaluated at the steady state, is sufficiently large. The term l_m reflects the marginal effect of a change in real balances on the time spent transacting and - indirectly - on production. This term takes a low value under parameterizations that replicate historical data on the costs of holding money as a share of national income. Data on the US over the period 1915-1995[5] suggest an approximate relation of $im = .01\ y$. This imposes a low value on ml_m in the model of section 2, since in equilibrium $-ml_m = im$.

To see that the low historical value of im prevents $ml_m \frac{U_{hh}}{U_h}$ from being large enough, I compute the value of a under the most frequently used specifications of preferences and of the transactions technology, for a reasonable calibration of the parameters. Table 2 shows the results.

The first column lists the functional forms adopted for preferences. The first is a specification with constant relative risk aversion (CRRA); the second is a separable formulation; the third is a specification with constant elasticity of substitution (CES). Except for the separable specification, Pareto complementarity is ensured by imposing $\rho < 1$. In the computations reported in the table I set $\rho = .9$, although similar values for a arise for $\rho = .5$. For each specification of preferences, the parameter λ is set such that the amount of labor supplied at the steady state is around one fifth of the time endowment.

The first row of the table lists various specifications of the transactions technology, which are obtained by choosing specific parameter values for the function

$$l\left(c_t, m_t\right) = \left[\eta \frac{c_t}{m_t} + \zeta \frac{m_t}{c_t} - \varepsilon\right] c_t^k. \tag{4.1}$$

The parameters are restricted in such a way that the assumptions on the transactions technology made in section 2 are satisfied. This implies that $\varepsilon = 2\sqrt{\eta\,\zeta}$. The point of full liquidity is then given by $\frac{\overline{m}}{c} = \sqrt{\frac{\eta}{\zeta}}$, so that it changes with the values of the parameters. The parameter k gives the degree of homogeneity of the transactions

[5] See De Fiore [21].

technology. The first case considered is the function (4.1) with $k = 1$. As it is common in the literature, I assume that the point of full liquidity is unitary, which requires that $\eta = \zeta$. The second case is the function (4.1) with $k = 0$. Again the point of full liquidity is set to one. The last case is the function (4.1) with $k = 0$ and $\zeta = 0$, which collapses to the specification suggested by Baumol [2] and Tobin [34], where satiation only occurs as real balances converge to infinity. For each of the three cases, the parameter η is chosen such that the steady state value of $\frac{im}{y}$ is approximately .01. This choice also implies that, at a 6 percent nominal interest rate, the steady state interest elasticity of money demand and velocity are around -0.5 and 6 respectively.

To compute the coefficient a, I need to set the value of the remaining parameters in the model. The rate of time preference is set to $\beta = .98$. The growth rate of the money supply is chosen at $\mu = 1.01$, which corresponds to a steady state nominal interest rate of 3 percent. Table 2 shows that the coefficient a remains always positive and above unity.

Table 2: Values of the coefficient a

	general technology $(k = 1)$	general technology $(k = 0)$	Baumol-Tobin $(k = 0)$
$CRRA$, $\rho \in (0, 1)$ $U = \frac{c^{1-\rho}h^{\lambda(1-\rho)}-1}{1-\rho}$	1.066	1.067	1.063
CES, $\rho \in (0, 1)$ $U = (c^{\rho} + \lambda h^{\rho})^{\frac{1}{\rho}}$	1.059	1.064	1.061
$SEPARABLE$ $U = \ln c + \lambda\ h$	1.066	1.067	1.063

4.3. Normality and Pareto Substitutability

It remains to evaluate the plausibility of equilibria with persistent real effects in region 2 of table 1, where consumption and leisure are normal and Pareto substitutes.

Matheny [29] finds that indeterminacy and persistent real effects of monetary shocks are possible in a cash-in-advance (CIA) model with constant returns to scale in production, if consumption and leisure are Pareto substitutes.[6] In a calibrated example, where preferences are assumed to be CRRA, the author finds that real effects of monetary shocks require unrealistic values of the coefficient of relative risk aversion, in the absence of taxes. However, the author shows that taxes enhance the effect of Pareto substitutability, when the economy reacts to a monetary shock. After calibrating taxes at their actual values, he finds persistent real effects of monetary shocks for values of the coefficient of relative risk aversion that are considered acceptable in this literature, i.e. between 2 and 5.

In the general formulation of the CIA model, Matheny does not explicitly impose the restrictions necessary to ensure normality of consumption and leisure. In Appendix A.3, I show that the general conditions derived by the author under which money is non-neutral also include cases where consumption is inferior. Nonetheless, the CRRA specification of preferences used in the calibrated model is such that consumption and leisure are normal for all possible parameter values. The CIA is a limiting case of the TCT model, when the technology is Leontief. Therefore, a CIA model under a CRRA specification of preferences belongs to region 2 of table 1.

Matheny's result suggests that it should be possible to find equilibria with short-run non-neutrality of money under plausible parameter values, in region 2 of table 1. The problem with the CIA model is that it is unable to replicate the main facts about money demand, such as the interest elasticity of money demand, velocity and, in particular, the cost of holding money as a share of GDP. These features might have strong implications for the dynamic properties of the system, as the analysis of section 4.2 suggest. In what follows, I ask whether equilibrium paths with persistent

[6] In Matheny (1998), the constant returns to scale assumption is a specific case of a general technology that can also display increasing returns.

real effects of monetary shocks can arise in the same model used by Matheny, with the only difference that the CIA constraint is replaced by a more general transactions technology, which is calibrated to replicate the main facts about money demand.

The model is identical to that of section 2, except that the government now finances a constant flow of government expenditure, g, through a tax on consumption, τ_c, and a tax on income, τ_n. The household's budget constraints (2.5) become

$$P_t c_t \left(1+\tau_c\right) + M_{t+1} + B_{t+1} \leq M_t + (1+i_t)B_t + P_t w_t \left(1-\tau_n\right) n_t + Z_t,\ t \geq 0.$$

The presence of the tax on consumption also affects the transactions costs technology (2.3), which is now given by

$$s_t \geq l\left(c_t, \widetilde{m}_t\right),$$

where $\widetilde{m}_t = \frac{m_t}{1+\tau_c}$. This is discussed in De Fiore and Teles [22]. In Appendix A.4, I derive the dynamic properties of the TCT model in the presence of distortionary taxes, so that I can compute numerical results when taxes are at their actual value.

In the calibrated example, I adopt the same specification of preferences used by Matheny,

$$U\left(c,h\right) = \frac{c^{1-\rho}\, h^{\lambda(1-\rho)} - 1}{1-\rho}, \tag{4.2}$$

where the assumption of Pareto substitutability is reflected in the restriction that $\rho > 1$. I also adopt the same parameter values: $g_y = g/y = .25$, $\beta = .98$, $\mu = 1.01$, $\lambda = 3.45$. The parameter λ is selected so that, in Matheny's preferred case, where $\tau_c = .05$ and $\tau_n = .3$, the steady state labor supply equals one fifth of the time endowment.

Finally, I consider different specifications of the transactions technology. The first is a Leontief technology, which corresponds to the limiting case of a CIA constraint. Then, I consider the same three particular cases of the technology (4.1) that were analyzed in section 4.2. For these cases, I choose the parameter η such that, under Matheny's preferred case with $\tau_c = .05$, $\tau_n = .3$ and under a monetary policy that sets the nominal interest rate at 6 percent ($\mu = 1.04$), the steady state values of the cost of holding money as a share of GDP, the interest elasticity of money demand,

and velocity approach $.01, -0.5$ and 6 respectively. This implies that $\eta = .0009$, when $k = 0$, and $\eta = .006$, when $k = 1$.

Table 3 gives the results of the numerical computations. The first four columns describe the case of the model without taxes. The last four columns describe the results under the highest level of taxes used by Matheny, $\tau_c = .05$, $\tau_n = .46$, which is also the most favorable case to indeterminacy. I define ρ_A as the minimum integer for which $\alpha \in (0,1)$, such that persistent real effects of money supply shocks arise, and ρ_B as the minimum value for which indeterminacy arises at all, i.e. such that $\alpha \in (-1,1)$. The table also shows the corresponding values of α, as from (2.42).

The first row replicates Matheny's results in the limiting case of a CIA model, where velocity is fixed at one. With no taxes, values of ρ as high as 36 are necessary to explain the short-run non-neutrality of money. With taxes, however, the critical coefficient is 3, which is within the range considered acceptable in this literature. The second row lists the results in a TCT model with the same degree of homogeneity as the CIA model, when satiation occurs at unity. The coefficient required in order to explain real persistence rises to 84, in the case without taxes, and to 56 in the case with taxes. The third row gives the results when satiation is unitary and $k = 0$. The critical value for the coefficient of risk aversion remains unreasonably high, both in the case with and without taxes. The last row shows that similar results arise with a transactions technology such that satiation occurs at an arbitrarily large level of real balances, as in the Baumol-Tobin specification.

Table 3: Critical values for ρ_A and ρ_B .[a]

	$\tau_c = 0 \quad \tau_n = 0$				$\tau_c = .05 \quad \tau_n = .46$			
	ρ_A	α	ρ_B	α	ρ_A	α	ρ_B	α
CIA $(k = 1)$	36	.03	15	-.93	3	.09	2	-.51
TCT $(k = 1)$	84	.09	81	-.73	56	.09	54	-.79
TCT $(k = 0)$	95	.03	92	-.81	62	.04	60	-.75
BT $(k = 0)$	89	.05	87	-.50	63	.02	61	-.97

[a] The values are obtained holding constant β, λ, μ, g_y.

These results do not ensure that persistent real effects would not arise for reasonable parameter values in region 2 of table 1, under alternative specifications of preferences or of the transactions technology. However, table 3 shows that the level of ρ_A is similar across different assumptions on the transactions technology. Moreover, table 2 shows that both the sign and the value of the coefficient a are robust to alternative specifications of preferences, as long as the parameter λ is calibrated to replicate the same labor share in the time endowment. This suggests that the results obtained in this section should also extend to the adoption of alternative functional forms.

5. Indeterminacy in Real and Monetary Models

The analysis above suggests that indeterminacy and equilibria with short-run non-neutrality of money are not likely to be found in the framework considered, under reasonable model specifications and calibrations. However, the analysis shows that indeterminacy is possible in a monetary model with constant returns to scale in production. On the contrary, in a neoclassical model with a real sector only, indeterminacy cannot arise when production is constant returns to scale. This is shown in Benhabib and Farmer [4]. Here, I provide an intuition for why multiple stationary equilibria are possible in monetary models, even when they are not possible in corresponding real models.

Benhabib and Farmer [4] show that the necessary and sufficient conditions for indeterminacy to arise in their real neoclassical model have an equivalent in terms of slopes of the static labor demand and supply curves. In particular, the condition for indeterminacy requires the slope of the labour demand curve to be greater than the slope of the labour supply curve. In their model, this happens when there are large increasing returns to scale in production, because in this case the labour demand curve has a positive slope that can be larger than the slope of the labor supply curve, as represented in figure 2, panel a.

Farmer [23] argues that a similar route to indeterminacy follows in his simple MIUF model, where both labor and capital are inputs in the production of the good. He argues that, when the equilibrium condition for money is used to substitute out real balances, the resulting reduced form of the model is similar in structure to the real model considered by Benhabib and Farmer [4]. Therefore, he concludes that the condition for indeterminacy remains the same, namely that the labor demand and supply schedules cross with the wrong slopes. In a model with constant returns to scale, the labor demand curve has a negative slope, as depicted in figure 2, panel *b*. Thus, the only case in which the two curves can cross with the wrong slope is when preferences are such that the labor supply curve has a negative slope, which is steeper (in absolute value) than that of the labor demand curve.

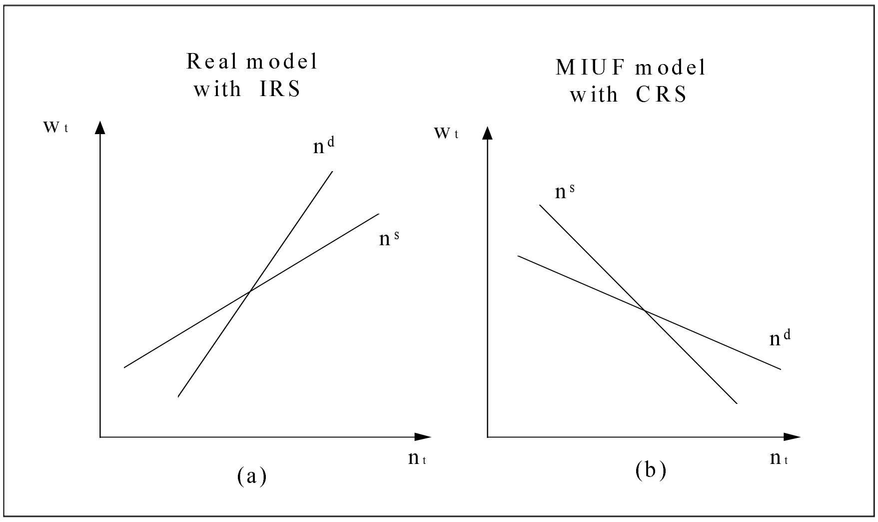

Figure 2

From table 1, we know that indeterminacy can arise in a wide range of cases in a monetary model with constant returns to scale. It can also arise when the slope of the labor demand curve is lower than the slope of the reduced form labor supply curve.

To realize this, consider the CIA model, when consumption and leisure are normal and Pareto substitutes. The assumption of a linear technology in labor implies that the labor demand curve is a flat line, $w_t = 1$, which is not shifted by movements in real balances. In this economy, lifetime utility is described by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, 1 - n_t),$$

while the budget constraints and the CIA constraints are given by

$$P_t c_t (1 + \tau_c) + M_{t+1} + B_{t+1} \leq M_t + (1 + i_t) B_t + P_t (1 - \tau_n) w_t n_t + Z_t,$$

$$P_t c_t (1+\tau_c) \leq M_t + Z_t,$$

and (2.6). Here τ_c is the tax on consumption and τ_n is the tax on income. Tax revenues are used to finance a constant flow of government expenditures, g. The first order conditions of the household include

$$\frac{U_c(c_t, 1-n_t)}{U_h(c_t, 1-n_t)} = \left(\frac{1+\tau_c}{1-\tau_n}\right)\frac{(1+i_t)}{w_t}. \tag{5.1}$$

Condition (5.1) shows that the nominal interest rate acts as a tax on consumption in distorting the consumption-leisure choice. Also, the condition implicitly defines the labor supply curve, i.e. the function that relates labor to the real wage, keeping consumption constant. Now, define the after-tax real wage as $\omega_t = \frac{(1-\tau_n)w_t}{(1+\tau_c)(1+i_t)}$. By implicitly differentiating the steady state version of (5.1), $D(c, 1-n) = \omega$, we obtain

$$\frac{dn}{d\omega} = \frac{1}{\omega^2 D_h},$$

$$\frac{dn}{dc} = \frac{D_c}{D_h}.$$

Under normality of consumption and leisure, $D_h > 0$ and $D_c < 0$, so that the labor supply curve has a positive slope, which is greater than the slope of the labor demand curve. Also, it shifts to the left following an increase in consumption, as represented in figure 3. From the analysis of sections 3 and 4.3, we know that indeterminacy and persistent real effects of monetary shocks can arise in this CIA model.

An intuition for why indeterminacy can arise in such a model should outline how a path with real effects of monetary shocks can be consistent with rational expectations and market clearing. When consumption and leisure are normal, the initial increase in m would induce an increase in c. The increase in c would in turn shift the labor supply curve to the left. This would be inconsistent with an increase in the equilibrium level of labor and with market clearing if the real wage were constant, as in a corresponding model with a real sector only. However, in a monetary model, the after-tax real wage is not constant on an equilibrium path with persistent real effects. From the equilibrium condition, $D(c, 1-c-g) = \frac{(1+\tau_c)(1+i)}{(1-\tau_n)}$, it follows that $\frac{di}{dc} = \left(\frac{1-\tau_n}{1+\tau_c}\right)(D_c - D_h) < 0$, so that the equilibrium nominal interest rate has to decrease. If we start from a point where the nominal interest rate is strictly positive, a shift to the left of the labor supply curve can be consistent with a higher equilibrium level of labor and with market clearing.

Consider the situation depicted in figure 3. The presence of taxes on money, consumption and income generates a wedge between the real wage paid by the firm, $w_t = 1$, and the real wage received by the agent. The initial situation is at a point like A. For a given level of the nominal interest rate, an increase in m and c would decrease labor supply (from A to B). However, the reduction in the nominal interest rate would lower the distortion in the consumption-labor choice, so that the equilibrium would be found by shifting upwards along the new labor supply curve. If the initial level of the distortion is high, and if a change in the after-tax real wage has a big impact on the decision to supply labor, the equilibrium quantity of labor can still increase (point C in figure 3).

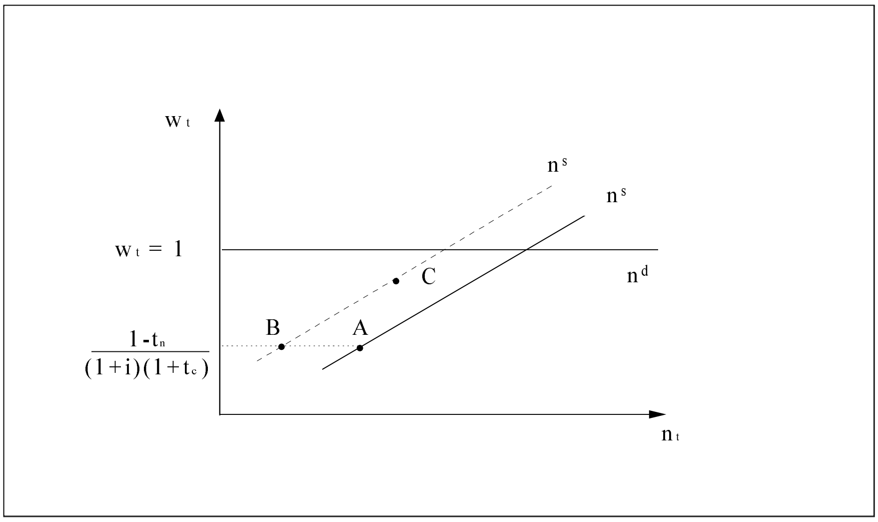

Figure 3

6. Conclusions

This paper has analyzed the possibility of real indeterminacy and equilibria with short-run non-neutrality of money, as observed in the data, in a transactions technology model with flexible prices, constant returns to scale in production and constant money growth rules.

It has been shown that persistent real effects of monetary shocks can arise in four regions of the parameter space. Two regions are characterized by inferiority of consumption or leisure and should therefore be ruled out by restricting the specification of the model. The remaining two regions are characterized by normality of the goods. However, persistent real effects require implausible parameter values, under common specifications of preferences and of the transactions technology.

It has also been argued that indeterminacy and equilibria with short-run non-neutrality have previously been found in the literature under specific model assumptions, which imply either inferiority of consumption or an implausibly high cost of holding money as a share of GDP. Under more reasonable assumptions, the model leads to determinacy and short-run neutrality.

Finally, an intuitive explanation has been given for why real indeterminacy can arise in monetary models, even when it cannot arise in corresponding real models. The intuition is based on the role of the nominal interest rate as a tax that distorts labor decisions.

A large number of authors have analyzed the possibility of indeterminacy in alternative versions of the standard business cycle model without money. This literature has shown that the degree of increasing returns to scale that is necessary to obtain indeterminacy is substantially lower when, for instance, we allow for capacity utilization or for the presence of different sectors in production. One possible extension of this paper would be to introduce capital and capacity utilization, or multiple sectors, into a monetary model with constant returns to scale.

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A. Appendix

A.1. Second Order Conditions

In the steady state, the household budget constraint can be rewritten as

$$c + im = 1 - h - l\,(c, m)\,.$$

Using this constraint to substitute out leisure in the utility function, the household's problem can be simplified to

$$\max_{c,m}\; U\,(c, 1 - c - im - l\,(c, m))$$

The second derivatives, evaluated at the steady state level, are

$$\frac{\partial^2 U}{\partial c^2} = U_h\,(D_c - DD_h - l_{cc})\,,$$

$$\frac{\partial^2 U}{\partial m^2} = -U_h l_{mm},$$

$$\frac{\partial^2 U}{\partial c \partial m} = -U_h l_{mc}.$$

For the steady state to be a maximum we need the upper-left element of the Hessian to be negative and the determinant to be positive, that is:

$$D_c - DD_h - l_{cc} < 0,$$

$$-\,(D_c - DD_h) + l_{cc} - \frac{l_{cm}^2}{l_{mm}^2} > 0.$$

A.2. Timing

I show that the timing of households' decisions is not crucial to the dynamic analysis and to the conclusions of section 4.1. With the timing adopted in Farmer [23], the household's budget constraint (2.5) takes the form

$$P_t c_t + M_t + B_t \leq M_{t-1} + (1 + i_{t-1})B_{t-1} + P_t w_t n_t + Z_t,\; t \geq 1.$$

By deriving the first-order conditions of the household and by imposing the equilibrium conditions, it can be shown that the steady state analysis is unchanged. The only difference is that the opportunity cost of holding one unit of real balances in this model is given by $I_t = \frac{i_t}{1+i_t}$, rather than by i_t. Thus, the number of steady states is unaffected, as well as the conditions for normality of consumption and leisure.

It can also be shown that the stability properties of the steady state with valued money change in a non essential way. In particular, the expression for α in equation (2.42) is now given by $\alpha = \frac{a+b}{a}$, where a and b are still defined by (2.43) and (2.44). Therefore, $\alpha \in (0,1)$ requires that either $a > 0$, $b < 0$, and $-b > a$, or $a < 0$, $b > 0$, and $-b < a$. These conditions are more restrictive than those obtained in section 2. Therefore, the choice of the timing in this paper does not restrict the region of parameter values under which indeterminacy can arise, neither it changes the relation between indeterminacy and normality of consumption and leisure.

A.3. The CIA Model

I show that the general conditions derived by Matheny under which money is non-neutral in a CIA model also include cases where consumption is inferior. In his economy, lifetime utility is described by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, 1 - n_t), \tag{A.1}$$

while the budget constraints and the CIA constraints are given by

$$P_t c_t (1 + \tau_c) + M_{t+1} \leq M_t + P_t (1 - \tau_n) w_t n_t + Z_t, \tag{A.2}$$

$$P_t c_t (1 + \tau_c) \leq M_t + Z_t, \tag{A.3}$$

and (2.6). τ_c is the tax on consumption, τ_n is the tax on income, while the remaining variables have the same meaning as in section 2. In Matheny [29], constant returns to scale are a specific case of a technology that can also display increasing returns. Here, I focus on the case with constant returns to scale, $y_t = n_t$. It is assumed that i) the constant flow of government spending g, τ_c, and τ_n are set independently from the money growth rate μ; ii) the nominal interest rate i is strictly positive; iii) $B_t = 0$, for all t.[7]

The problem of the household is to maximize (A.1), subject to (A.2) and (A.3), taking M_0 as given. Using the equilibrium conditions $c_t + g = y_t = n_t$ and $\mu m_t =$

[7] Bonds and the nominal interest rate do not enter the budget constraints in Matheny's model. Under assumptions i)-iii), the dynamic properties of the system are unchanged in a formulation with or without bonds.

$(1-\tau_n)\, w_t n_t,$ and the household's optimality condition, a unique equilibrium condition can be derived as

$$n_t U_h(n_t - g, 1 - n_t) = \frac{\beta}{\mu}\left(\frac{1-\tau_n}{1+\tau_c}\right) n_{t+1} U_h(n_{t+1}, 1 - n_{t+1}) D(n_{t+1}, 1 - n_{t+1}). \quad (A.4)$$

Differentiating equation (A.4), and using the condition that $D = \frac{(1+\tau_c)(1+i)}{(1-\tau_n)} = \left(\frac{1+\tau_c}{1-\tau_n}\right)\frac{\mu}{\beta}$ at the unique steady state where money is valued, it follows that

$$\left.\frac{dn_{t+1}}{dn_t}\right|_n \equiv \alpha_c = \frac{1}{1 + \frac{(D_c - D_h) n U_h}{D[n(U_{ch} - U_{hh}) + U_h]}}. \quad (A.5)$$

In the CIA model, where the transactions technology is Leontief, the second order conditions are simply given by the requirement of convex indifference curves, $\Delta = -(D_c - DD_h) > 0$. It follows that in this model $\frac{dc}{dz} = \frac{D_h}{w\Delta}$ and $\frac{dh}{dz} = \frac{-D_c}{w\Delta}$. Thus, normality of consumption and leisure requires respectively that $D_h > 0$ and $D_c < 0$.

Indeterminacy and persistent real effects of monetary shocks can only arise when $\alpha_c \in (0,1)$. Consider first the case where $U_{ch} > 0$. Then, $D_c - D_h < 0$, while $D\left[n\left(U_{ch} - U_{hh}\right) + U_h\right] > 0$, so that $\alpha_c \notin (0,1)$. As pointed out by Matheny, $U_{ch} < 0$ is a necessary condition for indeterminacy and real effects to arise in the CIA model. However, when $U_{ch} < 0$, α_c can lie between zero and one in three different cases:

1. $U_{ch} < 0$, $D_c < 0$, $D_h < 0$, $D_c - D_h \gtreqless 0$. Then, c is inferior and h is normal.
2. $U_{ch} < 0$, $D_c < 0$, $D_h > 0$, $D_c - D_h < 0$. Then, c and h are normal.
3. $U_{ch} < 0$, $D_c > 0$, $D_h > 0$, $D_c - D_h < 0$. Then, c is normal and h is inferior.

Matheny imposes a zero elasticity of the long run labor supply with respect to the real wage (Assumption 1 in his paper), because this ensures that preferences are consistent with balanced growth path. This restriction requires that $(n-g)\frac{U_{ch}}{U_c} = \frac{\beta}{\mu}\left(\frac{1-\tau_n}{1+\tau_c}\right)\left(1 + \frac{cU_{cc}}{U_c}\right)$. Using the definition of D, the steady state condition $\frac{\beta}{\mu}\left(\frac{1-\tau_n}{1+\tau_c}\right) = D^{-1}$, and rearranging, it follows that $D_c = -\frac{1}{c}D < 0$. Therefore, the zero elasticity of the long run labor supply ensures normality of leisure, but it does not ensure normality of consumption.

A.4. The TCT Model with Taxes

The economy is identical to the one described in section 2, with the exception that the government finances a given flow of expenditures g by raising revenues from a tax on income, τ_n, and a tax on consumption, τ_c. Again, I impose that $B_t = 0$, for all

t. To minimize the differences relative to Matheny [29], I make the same assumption that g, τ_c, and τ_n are set independently from the money growth rate. Since the consumption tax is paid with cash, the transactions technology becomes

$$s_t \geq l\left(c_t, \widetilde{m}_t\right), \tag{A.6}$$

where $\widetilde{m}_t = \frac{m_t}{1+\tau_c}$. Similar assumptions to those made in section 2.1 on $l\left(t\right)$ hold. The household's budget constraints for $t \geq 0$ are given by

$$P_t c_t \left(1+\tau_c\right) + M_{t+1} + B_{t+1} \leq M_t + (1+i_t)B_t + P_t w_t \left(1-\tau_n\right) n_t + Z_t,\ t \geq 0. \tag{A.7}$$

The problem of the household is defined by the maximization of (2.1), subject to (2.2), (A.6), (A.7), (2.6), and (2.7). The first order conditions include

$$\frac{U_c\left(c_t, h_t\right)}{U_h\left(c_t, h_t\right)} = \frac{1+\tau w_t l_c\left(c_t, \widetilde{m}_t\right)}{\tau w_t},$$

$$-\tau w_t\, l_{\widetilde{m}}\left(c_t, \widetilde{m}_t\right) = i_t,$$

$$\frac{U_h(c_t, h_t)}{P_t w_t} = \beta \frac{U_h(c_{t+1}, h_{t+1})}{P_{t+1} w_{t+1}} \left[1 - \tau w_{t+1} l_{\widetilde{m}}\left(c_{t+1}, \widetilde{m}_{t+1}\right)\right],$$

where $\tau \equiv \frac{1-\tau_n}{1+\tau_c}$. Following a similar analysis as in section 2.2, expressions equivalent to (2.36), (2.42), (2.43), and (2.44) can be derived. They are respectively:

$$\varphi_m^\tau\left(t\right) = \frac{1}{\left(1+\tau_c\right)} \left[\frac{l_{c\widetilde{m}}\left(t\right) + l_{\widetilde{m}}\left(t\right) D_h\left(t\right)}{D_c\left(t\right) - D\left(t\right) D_h\left(t\right) - \left(1-\frac{1}{\tau}\right) D_h - l_{cc}\left(t\right)}\right],$$

$$\alpha_\tau = \frac{a_\tau}{a_\tau - b_\tau},$$

$$a_\tau = 1 - m\left[\frac{l_{\widetilde{m}}}{\left(1+\tau_c\right)} + \left(1-\frac{1}{\tau}\right)\varphi_m^\tau\right]\frac{U_{hh}}{U_h} + m\varphi_m^\tau D_h,$$

$$b_\tau = \frac{\left(1-\tau_n\right) m}{x}\left(l_{c\widetilde{m}}\varphi_m^\tau + \frac{l_{\widetilde{m}\widetilde{m}}}{\left(1+\tau_c\right)}\right),$$

where the subscript (for variables) or superscript (for functions) τ denotes the variable or function in the case with taxes, which corresponds to the variable or function obtained in section 2.2.

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