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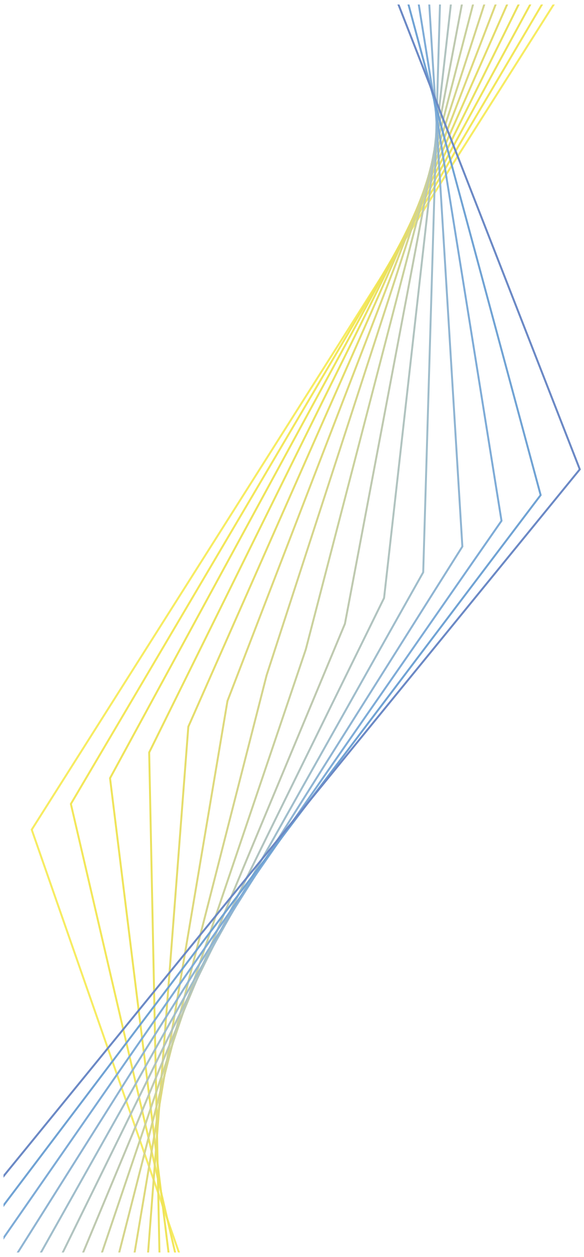

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MONETARY POLICY WITH UNCERTAIN PARAMETERS*

ULF SÖDERSTRÖM**

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** Address: Research Department, Sveriges Riksbank, SE-103 37 Stockholm, Sweden. E-mail: ulf.soderstrom@riksbank.se.

Address	**Kaiserstrasse 29**
	D-60311 Frankfurt am Main
	GERMANY
Postal address	**Postfach 16 03 19**
	D-60066 Frankfurt am Main
	Germany
Telephone	**+49 69 1344 0**
Internet	**http://www.ecb.int**
Fax	**+49 69 1344 6000**
Telex	**411 144 ecb d**

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Abstract

In a simple dynamic macroeconomic model, it is shown that uncertainty about structural parameters does not necessarily lead to more cautious monetary policy, refining the accepted wisdom concerning the effects of parameter uncertainty on optimal policy. In particular, when there is uncertainty about the persistence of inflation, it may be optimal for the central bank to respond more aggressively to shocks than under certainty equivalence, since the central bank this way reduces uncertainty about the future development of inflation. Uncertainty about other parameters, in contrast, acts to dampen the policy response.

Keywords: Optimal monetary policy, parameter uncertainty, Brainard conservatism, interest rate smoothing.

JEL Classification: E43, E52.

1 Introduction

It is widely accepted that policymakers facing uncertainty about the effects of policy on the economy should be more cautious when implementing policy than if acting under complete certainty (or certainty equivalence). The attractiveness of this result, named the 'Brainard conservatism principle' by Alan Blinder (1997, 1998) after the original analysis of William Brainard (1967), lies in both the simplicity of the original argument and in the underlying intuition. That the argument is well understood and used by central bankers in the practical policy process is made clear by, for example, Blinder (1998) and Goodhart (1999).

However, Brainard's analysis concerned only uncertainty about the transmission of policy to a target variable. It is less clear whether his result also applies to uncertainty concerning other parameters in the economy. The purpose of this paper, therefore, is to analyze the effects of multiplicative parameter uncertainty in a dynamic macroeconomic model typically used for monetary policy analysis, developed by Svensson (1997, 1999). Recently, Svensson (1999) has shown that the Brainard conservatism result holds in a special case of that model: when there is uncertainty about some of the structural parameters, the optimal policy response to current inflation and output (i.e., the coefficients in the policymaker's optimal reaction function) are shown to get smaller as the amount of uncertainty increases.[1] Due to the complexity of the model with parameter uncertainty, however, Svensson chooses to analyze a special case, where only inflation (and no measure of output) enters the central bank's objective function. In the present paper, Svensson's analysis will be extended to cover uncertainty about all structural parameters of the model, and the preferences of the central bank in the choice between stabilizing output and inflation are allowed to vary. In addition to the initial response of policy, the time path of policy after a shock is also examined.

Somewhat surprisingly, the results show that parameter uncertainty does not necessarily dampen the policy response, but may actually make policy more aggressive than under certainty equivalence. In particular, when the central bank puts some weight on stabilizing output in addition to inflation, uncertainty about the persistence of inflation increases the optimal reaction function coefficients. Uncertainty about other parameters, in contrast, always dampens the policy response. The reason is that when the dynamics of inflation are uncertain, the amount of

[1]Similar results have been reached by, e.g., Estrella and Mishkin (1998), Sack (1998a), and Wieland (1998).

uncertainty facing policymakers is greater the further away the inflation rate is from target. Consequently, to reduce the amount of uncertainty about the future path of inflation, optimal policy is more aggressive, pushing inflation closer to target.[2] In contrast, the persistence of output is a crucial part of the transmission of policy to inflation, so uncertainty concerning the dynamics of output makes policy less aggressive.

Perhaps less surprisingly, when parameter uncertainty does act to dampen the current policy response, it is optimal for the central bank to return to a neutral policy stance later than under certainty equivalence. This is due to the persistence of inflation and output: a smaller initial response leads to larger deviations of the goal variables from target in future periods, so policy needs to be away from neutral for a longer time to get the economy back on track. Thus, parameter uncertainty can lead to a smoother policy path in response to shocks, an issue analyzed in more detail by Sack (1998a) and Söderström (1999).

The paper is organized as follows. In Section 2 the theoretical framework is presented, and the optimal policy of the central bank is derived in a dynamic economy with stochastic parameters. Since analytical solutions of the model are difficult, if not impossible, to find, Section 3 presents numerical solutions for different configurations of uncertainty, to establish the effects of parameter uncertainty on the optimal policy response. Finally, the results are discussed and conclusions are drawn in Section 4.

2 The model

2.1 Setup

The basic model used in the analysis is the dynamic aggregate supply-aggregate demand framework developed by Lars Svensson (1997, 1999), which is similar to many other models used for monetary policy analysis, for example by Ball (1997), Cecchetti (1998), Taylor (1994), and Wieland (1998). The model consists of two equations relating the output gap (the percentage deviation of output from its 'natural' level) and the inflation rate to each other and to a monetary policy instrument,

[2] These results are closely related to those of Craine (1979), who shows that uncertainty about the impact effect of policy leads to less aggressive policy behavior, but uncertainty about the dynamics of the economy leads to more aggressive policy, albeit in a univariate model. Also, Sargent (1999) and Onatski and Stock (1998), using robust control theory, argue that a central bank trying to avoid bad outcomes in the future may respond more aggressively to shocks when uncertainty increases.

the short-term interest rate. Assuming a quadratic objective function for the central bank, one can solve for the optimal decision rule as a function of current output and inflation, similar to a Taylor (1993) rule.

Important features of the model are the inclusion of control lags in the monetary transmission mechanism, and the fact that monetary policy only affects the rate of inflation indirectly, via the output gap. Monetary policy is assumed to affect the output gap with a lag of one period, which in turn affects inflation in the subsequent period.[3] Policymakers thus control the inflation rate with a lag of two periods. In the simplest version, including only one lag,[4] the output gap (the percentage deviation of output from its 'potential' level) in period $t + 1$, y_{t+1}, is related to the past output gap and the ex-post real interest rate in the previous period, $i_t - \pi_t$, by the relationship

$$y_{t+1} = \alpha_{t+1} y_t - \beta_{t+1}(i_t - \pi_t) + \varepsilon^y_{t+1}, \tag{1}$$

where ε^y_{t+1} is an i.i.d. demand shock with mean zero and constant variance σ^2_y. The rate of inflation between periods t and $t+1$, π_{t+1}, (or rather, its deviation from the long-run average inflation rate, given by the constant inflation target) depends on past inflation and the output gap in the previous period according to the Phillips curve relation

$$\pi_{t+1} = \delta_{t+1}\pi_t + \gamma_{t+1} y_t + \varepsilon^\pi_{t+1}, \tag{2}$$

where ε^π_{t+1} is an i.i.d. supply shock with zero mean and variance σ^2_π. Note that all variables are measured as deviations from their respective long-run averages. Thus, negative values of the interest rate are allowed.

In the model presented here, there are two important modifications to the original Svensson framework: the persistence parameter of the inflation process, δ_{t+1}, is allowed to take values different from unity; and the parameters of the model are stochastic, and therefore time-varying. When the central bank sets its interest rate instrument at time t, it is assumed to know all realizations of the parameters up to and including period t, but it does not know their future realizations, and thus

[3]In the simple one-lag model used here, one period can be thought of as equal to one year. The short interest rate could then be interpreted as the central bank's interest rate instrument, assumed to be held constant for a year at a time. See Svensson (1999).

[4]Rudebusch and Svensson (1999) and Söderström (1999) use a version of the model including four lags in each relationship, and estimate it on quarterly U.S. data. Söderström (1999) also formally tests the restrictions imposed by Svensson (1997, 1999).

cannot be certain about the effects of policy on the economy.[5] The parameters are assumed to be random variables with means $E(\alpha_{t+1}) = \alpha$, $E(\beta_{t+1}) = \beta$, $E(\gamma_{t+1}) = \gamma$, and $E(\delta_{t+1}) = \delta$, and variances σ_α^2, σ_β^2, σ_γ^2, and σ_δ^2. They are also assumed to be independent of each other and of the structural shocks ε_{t+1}^π and ε_{t+1}^y.[6] Furthermore, the realizations of the parameters are drawn from the same distribution in each period, so issues of learning and experimentation are disregarded in the analysis.[7]

For simplicity, the model (1)–(2) does not include any forward-looking elements, a feature which could be seen as unrealistic. Nevertheless, as shown by Estrella and Fuhrer (1998, 1999), purely forward-looking models of monetary policy are less successful in matching the data than backward-looking specifications, and they are not necessarily less sensitive to the Lucas critique. Also, hybrid models, including both forward- and backward-looking features, in many ways behave similarly to the purely backward-looking model used here.

2.2 Optimal policy

To determine the optimal path for the interest rate, the central bank is assumed to minimize the expected discounted sum of future values of a loss function, which is quadratic in output and inflation deviations from target (here normalized to zero).[8] Thus, the central bank solves the optimization problem

$$\min_{\{i_{t+\tau}\}_{\tau=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \phi^\tau L(\pi_{t+\tau}, y_{t+\tau}), \tag{3}$$

[5] That policymakers do not have complete information about the parameters in an economy is clearly not an unrealistic assumption. Holly and Hughes Hallett (1989) point to three reasons why a model's parameters may be seen as stochastic: (1) they are genuinely random; (2) they are really fixed, but are impossible to estimate precisely, due to the sampling variability in a finite data set; and (3) they vary according to some well-defined but imperfectly known scheme, e.g., because the model is a linearization around a trajectory of uncertain exogenous variables. Blinder (1997, 1998), Goodhart (1999), and Poole (1998) all stress the relevance of uncertainty for practical monetary policy.

[6] The assumption of independence is convenient for the derivation of optimal policy, and may be realistic if the model equations (1) and (2) are interpreted as structural relationships. If, on the other hand, one interprets the model as reduced-form relations derived from microeconomic foundations, the parameters might well be correlated if they are derived from the same micro relations.

[7] See Sack (1998b) or Wieland (1998) for similar models of monetary policy including learning and experimentation; or Balvers and Cosimano (1994), Başar and Salmon (1990), and Bertocchi and Spagat (1993) for models in slightly different contexts.

[8] The central bank is thus allowed to have explicit targets for both inflation and output. The output target is given by the potential level, so the central bank aims at a zero output gap (excluding the possibility of a systematic inflation bias). The inflation target pins down the long-run average inflation rate, so the target for π, the deviation of inflation from the average, is also zero.

subject to (1)–(2), where in each period the loss function $L(\pi_t, y_t)$ is given by

$$L(\pi_t, y_t) = \pi_t^2 + \lambda y_t^2, \tag{4}$$

and where ϕ is the central bank's (constant) discount factor.[9] The parameter $\lambda \geq 0$ specifies the relative weight of output to inflation stabilization, and is assumed to be known and constant.[10] In the simple case when parameters are non-stochastic, it is relatively straightforward to find an analytical solution for the optimization problem (3), as shown by Svensson (1997, 1999). When parameters are stochastic, however, finding an analytical solution is prohibitively difficult, so I shall here focus on numerical solutions.[11]

The inclusion of parameter uncertainty into this model will have an important effect on optimal policy. As is well known, optimal policy in a linear-quadratic framework with only additive uncertainty exhibits certainty equivalence. Consequently, the degree of uncertainty does not affect the optimal policy rule, which depends only on the expected value of the goal variables, so the central bank acts as in a non-stochastic economy. As will be clear below, when incorporating multiplicative parameter uncertainty into the model, certainty equivalence ceases to hold, and the variances of the state variables will affect the optimal policy rule. Thus, the amount of uncertainty facing policymakers has a decisive influence on their optimal behavior.

To solve the central bank's optimization problem it is convenient to rewrite the model (1)–(2) in state-space form as

$$x_{t+1} = A_{t+1} x_t + B_{t+1} i_t + \varepsilon_{t+1}, \tag{5}$$

[9] The quadratic specification of the objective function is very common in the literature. Some authors, e.g., Rudebusch and Svensson (1999) and Rudebusch (1999), include an interest rate smoothing objective in the loss function to capture the apparent preference of central banks for small persistent changes in the instrument. As shown by Sack (1998a) and Söderström (1999), however, such an *ad hoc* smoothing objective is not necessary to mimic policy behavior in the U.S., at least not in an unrestricted VAR framework.

[10] Typically, λ is positive also in regimes of inflation targeting, since central banks seemingly want to stabilize also short-term fluctuations in output. See Svensson (1998) for a discussion of 'strict' versus 'flexible' inflation targeting, and Fischer (1996) for a critique of central banks' tendency to only acknowledge price stability and not output stabilization as the goal of monetary policy.

[11] Svensson (1999) analytically solves a very simple case of parameter uncertainty, where δ_{t+1} is non-stochastic and always equal to unity, and where $\lambda = 0$. Since the most interesting results are obtained when $\lambda > 0$ and δ_{t+1} is stochastic, I shall not follow his route. In independent research, Srour (1999) analyzes a similar model under parameter uncertainty, and solves the model analytically for the case of a finite time horizon. His results partly overlap with those presented here.

where $x_{t+1} = [\ y_{t+1} \quad \pi_{t+1}\ ]'$ is a state vector, and $\varepsilon_{t+1} = [\ \varepsilon^y_{t+1} \quad \varepsilon^\pi_{t+1}\ ]'$ is a vector of disturbances. The parameter matrices A_{t+1} and B_{t+1} are then stochastic with means

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}, \quad B = \begin{bmatrix} -\beta \\ 0 \end{bmatrix}, \tag{6}$$

and variance-covariance matrices

$$\Sigma_A = \begin{bmatrix} \sigma^2_\alpha & 0 & 0 & 0 \\ 0 & \sigma^2_\beta & 0 & 0 \\ 0 & 0 & \sigma^2_\gamma & 0 \\ 0 & 0 & 0 & \sigma^2_\delta \end{bmatrix}, \quad \Sigma_B = \begin{bmatrix} \sigma^2_\beta & 0 \\ 0 & 0 \end{bmatrix}, \quad \Sigma_{AB} = \begin{bmatrix} 0 & 0 \\ -\sigma^2_\beta & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{7}$$

Using the state-space formulation, the central bank's optimization problem can be written as the control problem

$$J(x_t) = \min_{i_t} \left[x_t' Q x_t + \phi E_t J(x_{t+1}) \right], \tag{8}$$

subject to (5), where Q is a (2×2) preference matrix of the central bank, with λ and 1 on the diagonal and zeros elsewhere. The loss function will in this framework be quadratic, so

$$J(x_{t+1}) = x_{t+1}' V x_{t+1} + w, \tag{9}$$

where the matrix V remains to be determined.

To illustrate the effects of including multiplicative uncertainty into the model, and show why certainty equivalence no longer holds, it is instructive to consider the expected value of the value function (9). In the general case, this expected value is

$$E_t J(x_{t+1}) = (E_t x_{t+1})' V (E_t x_{t+1}) + \mathrm{tr}(V \Sigma_{t+1|t}) + w, \tag{10}$$

where $\Sigma_{t+1|t}$ is the variance-covariance matrix of x_{t+1}, evaluated at time t, and the notation 'tr' denotes the trace operator. The variance-covariance matrix is given by

$$\Sigma_{t+1|t} = E_t \left[x_{t+1} - E_t x_{t+1} \right] \left[x_{t+1} - E_t x_{t+1} \right]', \tag{11}$$

where

$$x_{t+1} - E_t x_{t+1} = (A_{t+1} - A) x_t + (B_{t+1} - B) i_t + \varepsilon_{t+1}. \tag{12}$$

When the parameters are non-stochastic, so $A_{t+1} = A$ and $B_{t+1} = B$ for all t, $x_{t+1} - E_t x_{t+1} = \varepsilon_{t+1}$, so the variance-covariance matrix $\Sigma_{t+1|t}$ coincides with the variance matrix of the disturbance vector ε_{t+1}, and thus is independent of the instrument i_t. Therefore, although the expected value of the objective function depends on the variance of the disturbances, the optimal policy rule is independent of the degree of uncertainty, so optimal policy is certainty equivalent. In contrast, when the parameters are uncertain, the variance-covariance matrix depends on the state of the economy (x_t), the instrument (i_t), and the variances of the parameters as well as those of the additive disturbances. Optimal policy will then minimize not only the future deviation of the expected state variables from target (via the term $(E_t x_{t+1})' V (E_t x_{t+1})$), but also their variance. Thus, certainty equivalence ceases to hold, and optimal policy depends crucially on the degree of uncertainty in the economy.[12]

Appendix A shows that the optimal decision rule for the central bank is to set the short-term interest rate as a linear function of the state vector in each period, that is,

$$i_t = f x_t, \tag{13}$$

where

$$f = -\left[B'\left(V+V'\right)B + 2v_{11}\Sigma_B^{11}\right]^{-1}\left[B'\left(V+V'\right)A + 2v_{11}\Sigma_{AB}^{11\,\prime}\right]. \tag{14}$$

Here Σ_{AB}^{ij} denotes the covariance matrix of the ith row of A_{t+1} with the jth row of B_{t+1}, and v_{ij} denotes element (i,j) of the matrix V, which is given by iterating on the Ricatti equation

$$\begin{aligned} V &= Q + \phi(A+Bf)'V(A+Bf) \\ &+ \phi v_{11}\left(\Sigma_A^{11} + 2\Sigma_{AB}^{11} f + f'\Sigma_B^{11} f\right) + \phi v_{22}\Sigma_A^{22}. \end{aligned} \tag{15}$$

To obtain an analytical solution for this problem, one would need to solve equations (14)–(15) for the fixed-point value of V. For some simple configurations, for example, in the non-stochastic case, this is manageable (although tedious), since the system of equations obtained is relatively straightforward to solve. In this setup of multiplicative parameter uncertainty, however, the system of equations is highly non-linear and far too complicated to yield a usable solution. Therefore I proceed

[12] The same argument can be made by considering the expected future value of the loss function

$$\begin{aligned} E_t L(\pi_{t+\tau}, y_{t+\tau}) &= E_t\left[\pi_{t+\tau}^2 + \lambda y_{t+\tau}^2\right] \\ &= \left(E_t \pi_{t+\tau}\right)^2 + \mathrm{Var}_t(\pi_{t+\tau}) + \lambda\left[\left(E_t y_{t+\tau}\right)^2 + \mathrm{Var}_t(y_{t+\tau})\right]. \end{aligned}$$

by numerical methods to analyze the optimal behavior of the central bank in this setting.

3 The effects of parameter uncertainty on optimal policy

Having derived the optimal policy rule (13) for the central bank, this section will analyze how the rule, and the resulting path of the short-term interest rate, depends on the degree of uncertainty in the economy. I therefore choose some values for the parameter means α, β, γ, and δ, and for the discount factor ϕ, and then examine how optimal policy behaves for different configurations of the parameter variances $\sigma_\alpha^2, \sigma_\beta^2, \sigma_\gamma^2$, and σ_δ^2, varying the preference parameter λ.

Shocks to output and inflation in equations (1) and (2) will affect monetary policy on two different, but related, levels. First, there is an initial effect, as policy is adjusted to respond to current shocks. This effect is given by the vector f in the decision rule (13). Second, there is a dynamic effect of shocks, since these will not be completely offset in the initial period, but will partly be transmitted to subsequent periods through the dynamics of the economy. Thus policy will also need to respond to past shocks, as these remain in the economy. I will distinguish between these two effects, and begin by analyzing the initial response of policy in Section 3.1, followed by an analysis of the dynamic response over time in Section 3.2.

The exact parameter values used for this numerical exercise are chosen so as to best illustrate the results, but are also consistent with empirical studies of the monetary transmission mechanism both in the Euro area and the U.S.[13] The reported results do not depend on the exact configuration of parameter values, but hold for many different plausible and implausible configurations.

The mean of the persistence parameter of the output gap, α_{t+1}, is given a value of 0.85, taken from Cooley and Hansen (1995, Table 7.1). This value is the autocorrelation coefficient of the observed detrended output process, and as such would tend to overestimate the true persistence of the output gap, unaffected by active stabilization policy. To the parameter β_{t+1}, the elasticity of output with respect to the real interest rate, a mean value of 0.35 is assigned, taken from Fuhrer's estimate of output's sensitivity to the long real interest rate for the U.S. from 1966 to 1993 (Fuhrer, 1994, Table 3). The mean of the persistence parameter of the Phillips

[13] Orphanides and Wieland (1999) estimate a similar model on both Euro area and U.S. data, restricting the parameter δ to unity. Their parameter estimates (using OECD data) are $\alpha = 0.77$ (0.11), $\beta = 0.40$ (0.10), $\gamma = 0.34$ (0.13) for the Euro area, and $\alpha = 0.47$ (0.16), $\beta = 0.32$ (0.13), $\gamma = 0.39$ (0.09) for the U.S. (with standard errors in parentheses).

Table 1: Numerical values of parameter means and variances

	Stochastic parameters								Non-stochastic parameters
	Mean	Variance							Value
α_{t+1}	0.85	{0.10,	0.00,	0.00,	0.00,	0.01,	0.01}	ϕ	0.95
β_{t+1}	0.35	{0.00,	0.10,	0.00,	0.00,	0.01,	0.01}	λ	[0,2]
γ_{t+1}	0.4	{0.00,	0.00,	0.10,	0.00,	0.01,	0.01}		
δ_{t+1}	1.0	{0.00,	0.00,	0.00,	0.10,	0.10,	0.20}		

curve, δ_{t+1}, is assigned a value of unity, leading to a standard accelerationist Phillips curve, on average. Finally, for γ_{t+1}, the inflation rate's sensitivity to the output gap, I assign a mean value of 0.4, which is approximately what Romer (1996, Table 2) finds for the U.S. economy for the period 1952–73, and which is also consistent with the correlation coefficient reported by Cooley and Hansen (1995, Table 7.1).

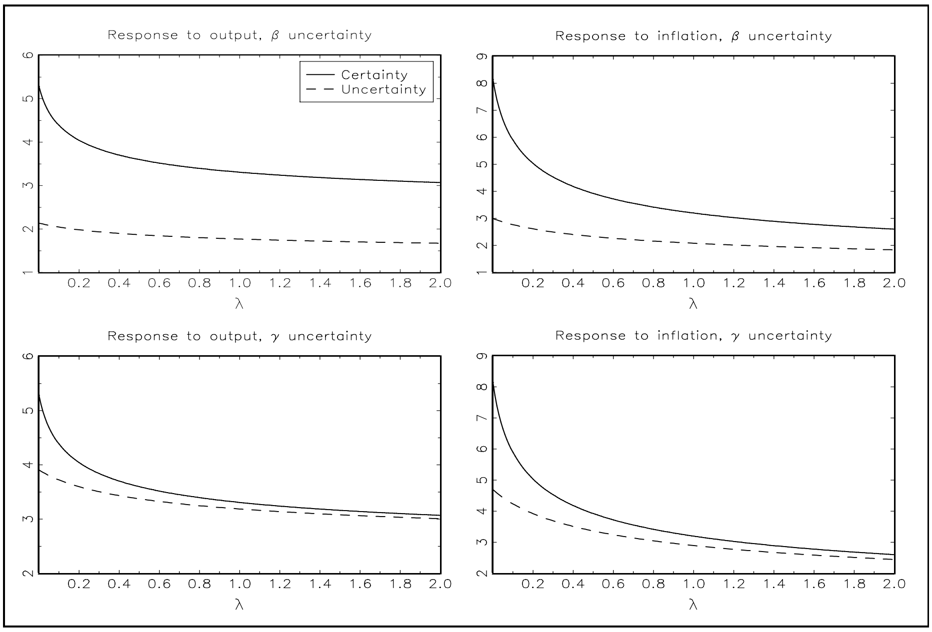
To analyze the effects of parameter uncertainty on policy, I begin by considering uncertainty about each of the four parameters separately. Thus, the variance of the uncertain parameter is set to 0.1, and the other variances to zero. At a second stage, I consider combinations of uncertainty about the parameters, always with the variances of $\alpha_{t+1}, \beta_{t+1}$, and γ_{t+1} set to 0.01, but first when the degree of uncertainty about the persistence parameter of inflation δ_{t+1} is relatively small (so its variance is 0.1), and secondly when it is relatively large (and its variance is 0.2). In each case, optimal policy will be compared to the certainty equivalence case, when all parameters are constant and equal to their means. The actual degree of uncertainty assigned through the parameter variances is chosen to make clear the effects of parameter uncertainty on policy. The qualitative results remain irrespective of the actual size of the parameter variances.

The resulting values for the means and variances of the stochastic parameters are given in the left-hand panel of Table 1. As shown in the right-hand panel, the discount factor ϕ is assigned a value of 0.95, implying a discount rate of 5% per period. Finally, since the effects of uncertainty on policy depend crucially on the value of the preference parameter λ, this will be allowed to take values varying from 0, that is, 'strict inflation targeting', to 2, with a larger weight on stabilizing output than on fighting inflation.

3.1 The initial policy response

As a first step, let us analyze the initial policy response to current inflation and output (i.e., the coefficients of the f-vector in equation (13)) when there is uncertainty about each of the four parameters separately. These responses are shown in Figures 1

Figure 1: Initial response to output and inflation, impact uncertainty



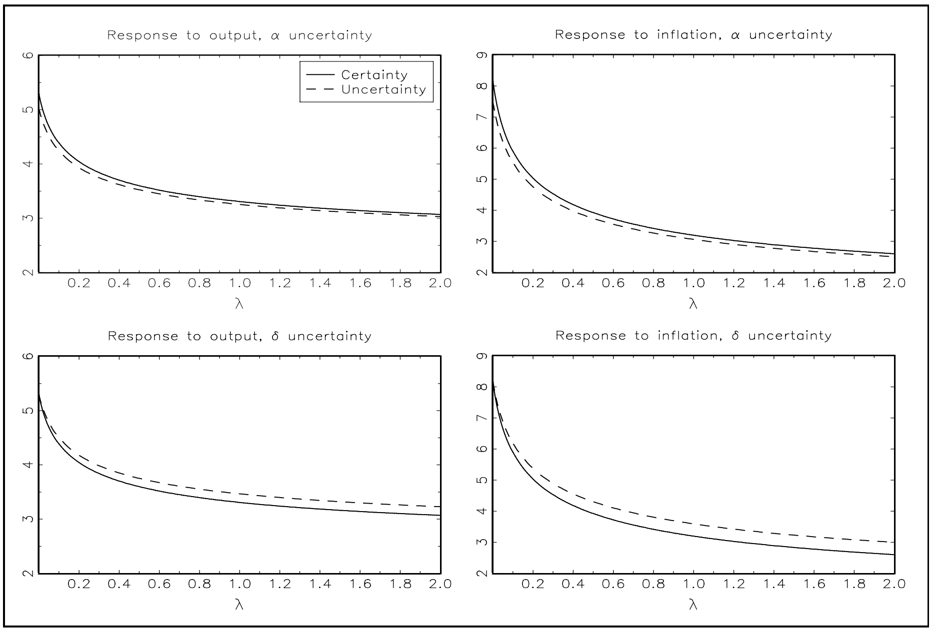
and 2, letting λ vary between 0 and 2. In each figure, the left-hand graph shows the response to output (or demand shocks) and the right-hand graph the response to inflation (supply shocks), with the solid line representing the certainty equivalent case, and the dashed line representing the response under parameter uncertainty.[14]

First, Figure 1 shows the two cases of 'impact uncertainty', i.e., when there is uncertainty about the parameters in the transmission mechanism, β_{t+1} and γ_{t+1}. As is clear, the Brainard conservatism result is confirmed: when there is uncertainty concerning the impact parameters, the optimal response coefficients are smaller than under certainty equivalence, so optimal policy is less aggressive. Increasing the variance of either parameter will weaken the response of the central bank, and in the limit, as the variances tend to infinity, the optimal response is to do nothing. This is true for all values of λ, although the effect of uncertainty is larger for small λ. Furthermore, the effect of uncertainty about β_{t+1}, the elasticity of the output gap with respect to the real interest rate, has a larger effect on policy than uncertainty concerning γ_{t+1}, the parameter of transmission from output to inflation.

Second, consider Figure 2, which shows the response coefficients under uncertainty about the two persistence parameters, α_{t+1} and δ_{t+1}. As seen in the two top graphs, uncertainty about the persistence of output affects policy in the same

[14]Note that the response coefficients to both output and inflation are decreasing in λ. This is because policy offsets shocks to both output and inflation by creating a recession. As the weight on output stabilization increases, optimal policy creates a smaller recession in response to a given shock.

Figure 2: Initial response to output and inflation, dynamic uncertainty



manner as uncertainty about the transmission parameters (albeit to a smaller degree): the initial response gets less aggressive than under certainty equivalence. In contrast, uncertainty about the persistence of inflation in the two bottom graphs affects the optimal policy coefficients in the opposite direction. For $\lambda > 0$, optimal policy is *more* aggressive under uncertainty than under certainty equivalence, in contradiction to the Brainard intuition. When $\lambda = 0$, however, uncertainty about δ_{t+1} has no effect on optimal policy.[15]

Since these results may be counterintuitive at first glance, they may need some further consideration. The model used here differs from that of Brainard (1967) in two respects: it is dynamic rather than static, and it incorporates uncertainty concerning not only the impact effect of policy, but also concerning the dynamic development of the economy. As mentioned above, the central bank wants to minimize the future deviation of expected inflation and output from target as well as their variance. When parameters are non-stochastic, so there is only additive uncertainty in the model, the variance of inflation and output is constant, and thus independent of their distance from target. Under multiplicative parameter uncertainty, however, when the dynamics of the variables are uncertain, their variances increase with the distance from target, so when inflation and output are further away from target, the uncertainty about their future development is greater. Since the persistence

[15]Since writing the first version of this paper, I have discovered independent work by Srour (1999) and Shuetrim and Thompson (1999) who both demonstrate versions of this results.

of inflation only affects the dynamics of the economy, optimal policy reduces the amount of uncertainty about future inflation by acting more aggressively to push inflation closer to target.[16] On the other hand, although a similar effect is present for uncertainty about the dynamics of output, the persistence of the output process is also a crucial part of the transmission mechanism from policy to inflation. Therefore, uncertainty about the persistence of output has the traditional effect of making policy less aggressive.

It is also noteworthy that the effect on policy of uncertainty concerning the dynamics of inflation only operates when the central bank gives some weight to output in its loss function, so $\lambda > 0$. When the central bank cares only about stabilizing inflation (when $\lambda = 0$), it is always optimal to push inflation to target as quickly as possible (i.e., after two periods). Then uncertainty about the dynamics of inflation has no effect on optimal policy. When $\lambda > 0$, on the other hand, optimal policy closes only a fraction of the gap between expected inflation and target in each period, and with a larger λ, the size of this fraction is smaller, so inflation is returned to target more slowly (see Svensson, 1997). As a consequence, uncertainty about the dynamics of inflation affects policy more strongly when λ increases, a pattern that is clear from the bottom graphs of Figure 2.[17]

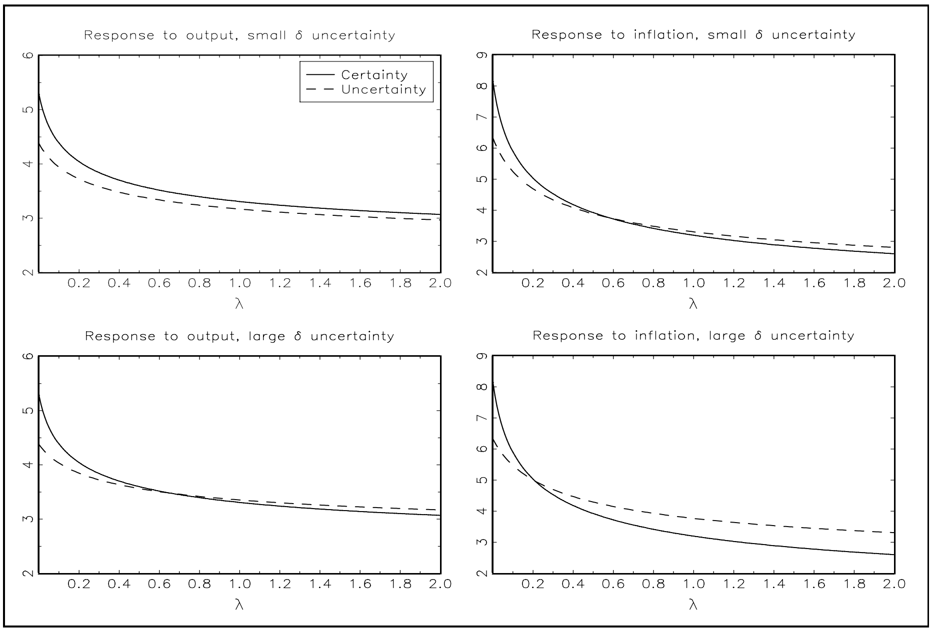
Finally, consider the case when there is uncertainty about all four parameters, shown in Figure 3. Now we have two different possibilities: when λ is low, optimal policy under uncertainty is more cautious than under certainty, since the uncertainty about $\alpha_{t+1}, \beta_{t+1}$, and γ_{t+1} dampens the response, but the uncertainty about δ_{t+1} has no or little effect. As λ increases, the uncertainty about δ_{t+1} starts to affect the response positively, and eventually the response under uncertainty might get stronger than under certainty. For a given λ, whether the initial response is more or less aggressive under uncertainty depends on the relative variances of $\alpha_{t+1}, \beta_{t+1}$, and γ_{t+1} on the one hand and δ_{t+1} on the other. When the degree of uncertainty about δ_{t+1} is relatively small ($\sigma_\delta^2 = 0.1$) in the top graphs of Figure 3, the response to supply shocks is larger under uncertainty for $\lambda \geq 0.58$, whereas the response to demand shocks is always smaller under uncertainty.[18] When uncertainty about δ_{t+1} gets relatively more important, however, in the lower part of Figure 3 (where $\sigma_\delta^2 = 0.2$),

[16]It should be noted that the qualitative effects of uncertainty concerning δ_{t+1} do not hinge on its mean value being equal to unity. For smaller values of the mean, uncertainty still makes policy more aggressive, although quantitatively the effects get smaller.

[17]In practice, the case where $\lambda = 0$ is probably less realistic than that with a positive λ, since central banks typically want to avoid excessive real fluctutations. See, e.g., Svensson (1998).

[18]For these parameter values, this is true for all λ at least up to 50,000.

Figure 3: Initial response to output and inflation, all parameters uncertain



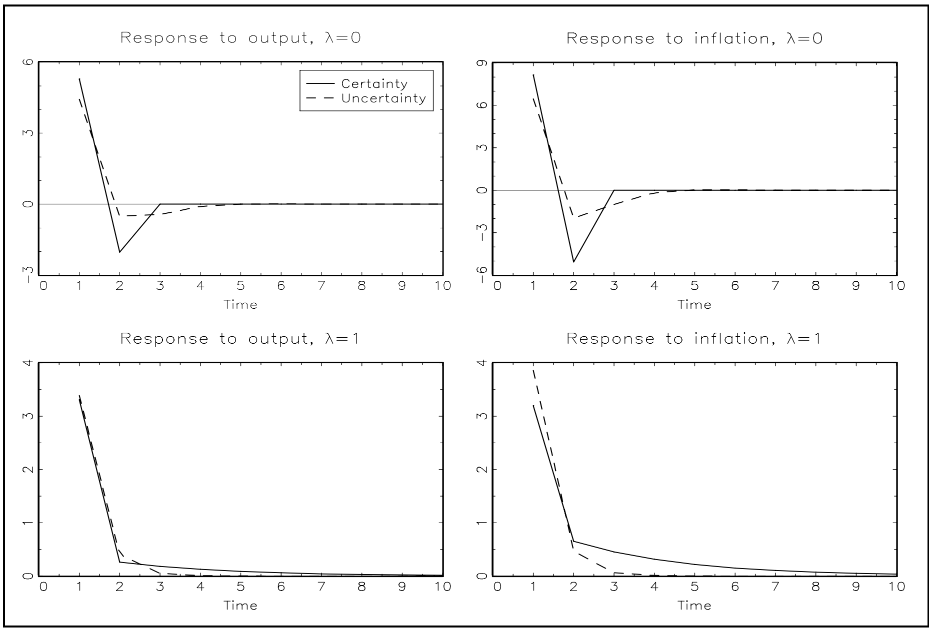
policy is more likely to be more aggressive under uncertainty; the corresponding cutoff values are now $\lambda \geq 0.66$ for demand shocks and $\lambda \geq 0.22$ for supply shocks . As a consequence, the net effect on policy of parameter uncertainty depends not only on the relative variances of the shocks, but also on the weight of output stabilization in the central bank's loss function.

In related work, Craine (1979) comes to a similar conclusion, using a dynamic model with one target variable: uncertainty about the impact of policy on the economy leads to less aggressive policy in response to shocks, but uncertainty about the dynamics of the economy leads to more aggressive policy.[19] In that simple setup, it is straightforward to separate uncertainty about the transmission of policy from uncertainty about the dynamics of the economy. In the Svensson model, this separation is less clear-cut. Thus, the analysis above shows that the Craine (1979) result is valid also in the Svensson setup, but with one qualification: since policy affects inflation via output, the dynamics of the output process is an important part of the transmission of policy to inflation. Uncertainty about the dynamics of output therefore makes policy less aggressive.

Also, Onatski and Stock (1998) and Sargent (1999) make a similar point, using robust control theory: when the policymaker chooses policy to minimize the risk of bad outcomes under model uncertainty, particular configurations of uncertainty lead to more aggressive policy than under certainty equivalence. Intuitively, 'cau-

[19]See also Holly and Hughes Hallett (1989).

Figure 4: Policy response over time, all parameters uncertain



tious' policy can also mean that bad future outcomes are avoided by acting more aggressively today.

3.2 The time path of policy

The introduction of multiplicative parameter uncertainty also has interesting implications for the dynamic response of monetary policy, that is, the response of policy to past shocks to output and inflation. Figures 4–6 show the response of monetary policy to supply and demand shocks over the first ten periods following a shock, for $\lambda = 0$ and $\lambda = 1$. Figure 4 illustrates the case where there is uncertainty about all parameters, with $\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_\gamma^2 = 0.01$, and $\sigma_\delta^2 = 0.2$, so uncertainty about δ_{t+1} strongly dominates. Figure 5 illustrates the case where there is uncertainty only about the transmission parameters $\alpha_{t+1}, \beta_{t+1}$, and γ_{t+1}, with $\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_\gamma^2 = 0.1$, and $\sigma_\delta^2 = 0$. Figure 6 shows the case where there is uncertainty only about the persistence parameter in the inflation equation, δ_{t+1}, with $\sigma_\delta^2 = 0.2$ and $\sigma_\alpha^2 = \sigma_\beta^2 = \sigma_\gamma^2 = 0$.

As noted by Ellingsen and Söderström (1999), in the simple Svensson model under certainty equivalence, the response of monetary policy over time varies substantially with the preference parameter λ. In particular, for small values of λ, the optimal policy response to an inflationary shock under certain parameter configurations is to raise the interest rate instrument in the first period, but then lower it below the initial level and move back to a neutral policy (with $i = 0$) from below. This is shown by the solid lines in the top two graphs of Figures 4–6.

Figure 5: Policy response over time, impact uncertainty only

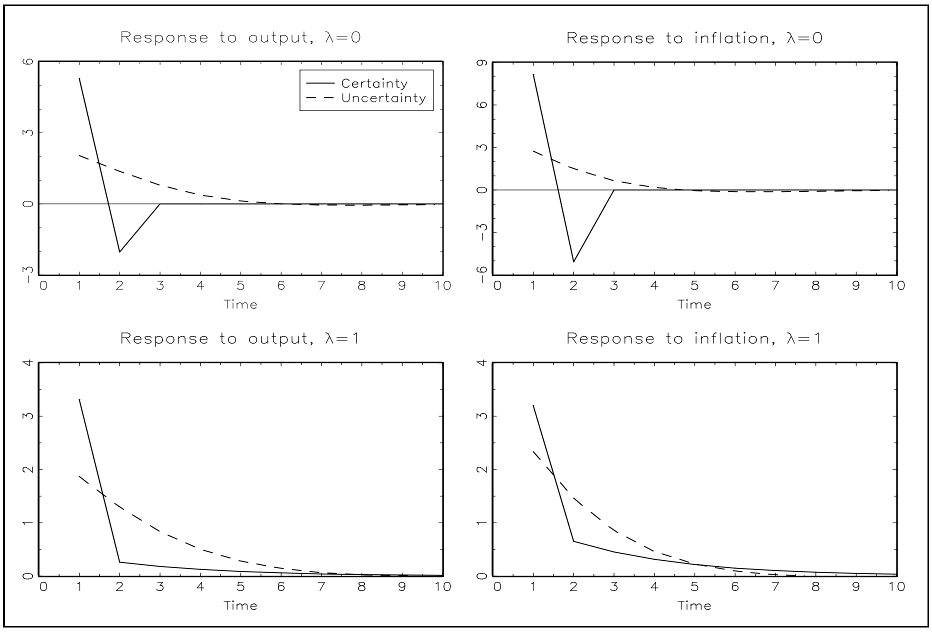
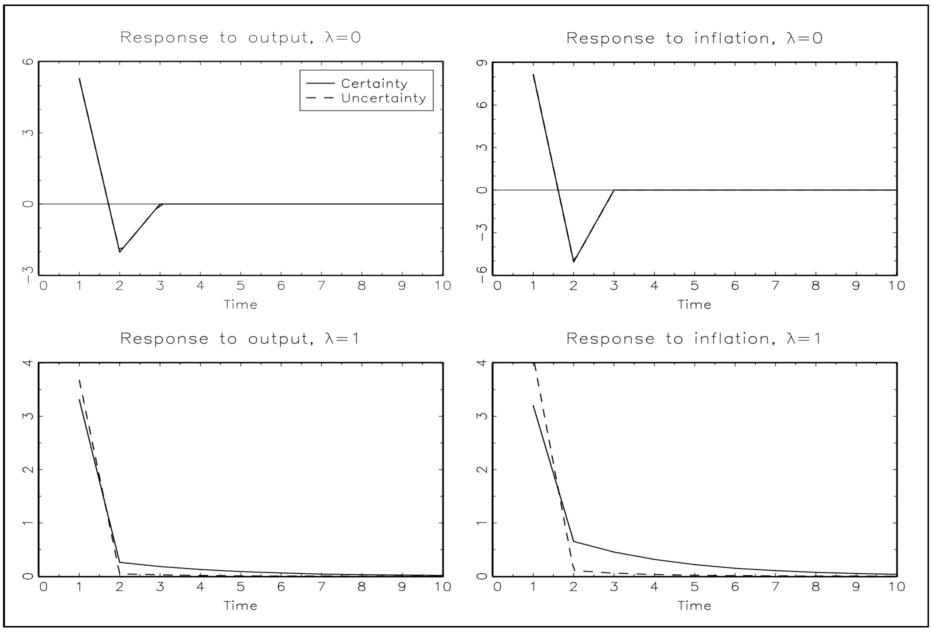


Figure 6: Policy response over time, adjustment uncertainty only



When parameters are uncertain, this behavior can be mitigated or magnified, depending on whether the initial response is dampened or strengthened. When, as in the bottom graphs of Figure 4, uncertainty about δ_{t+1} dominates (since $\lambda = 1$), so that the initial policy response is more aggressive under uncertainty, policy in later periods is closer to neutral, since the strong initial move has neutralized a larger part of the shock. If, on the other hand, uncertainty about $\alpha_{t+1}, \beta_{t+1}$, and γ_{t+1} dominates, as in Figure 5, so that the policy response is initially dampened, policy stays away from neutral longer, to compensate for the weaker initial response.

Thus, as is clear from Figure 5, parameter uncertainty can lead to smoother paths of the interest rate than under certainty equivalence, without introducing an explicit smoothing objective into the central bank's loss function. Casual observation suggests that central banks tend to respond to shocks by first slowly moving the interest rate in one direction, and then gradually moving back to a more neutral stance. When parameters are certain, the model suggests a large initial move, and then a quick return to the original level, unless λ is very large. Under certain configurations of parameter uncertainty, however, the central bank behaves in a more gradual way: although the initial response is always the strongest, it is more modest under these cases of uncertainty, and the policy move is drawn out longer over time. In particular, the tendency of the bank to 'whipsaw' the market by creating large swings in the interest rate is mitigated.[20]

Finally, for completeness, Figure 6 shows the optimal time path of policy when only the persistence of inflation is uncertain. When $\lambda = 0$ in the top panels, the paths under parameter uncertainty and certainty equivalence coincide, since uncertainty about the persistence of inflation has no effect on optimal policy. When $\lambda = 1$, policy is initially more aggressive under parameter uncertainty, which allows the central bank to return to a neutral stance earlier.

4 Concluding remarks

This paper demonstrates how uncertainty about parameters in a dynamic macroeconomic model can lead the central bank to pursue *more* aggressive monetary policy, providing a counterexample to the common wisdom following the results of Brainard (1967). When a policymaker is uncertain about the dynamics of the economy, he

[20]This issue of parameter uncertainty leading to more plausible paths of policy is examined more carefully by Sack (1998a) and Söderström (1999). The latter shows, however, that the Svensson model always implies excessive volatility of the policy instrument, whereas optimal policy from an unrestricted VAR model comes very close to mimicking the actual behavior of the Federal Reserve.

might find it optimal to move more aggressively in response to shocks, so as to reduce uncertainty about the future path of the economy. Uncertainty about the impact effect of policy still leads to less aggressive policy, in accordance with Brainard's original analysis.

It should be stressed that the model and the examples used are highly stylized and may not be entirely satisfactory from an empirical point of view, so any serious implications for policy are difficult to estimate. However, the qualitative points obtained from this simple model are also present in a more general empirical framework, similar to that of Rudebusch and Svensson (1999), and are likely to remain also in models incorporating partially forward-looking behavior.

It is possible that configurations of uncertainty in the real world are such that the Brainard result is always valid, or to quote Blinder (1998, p. 12), "My intuition tells me that this finding is more general—or at least more wise—in the real world than the mathematics will support." Using the standard errors of econometric parameter estimates as proxies for the degree of uncertainty concerning each parameter in a more complete econometric formulation of the Svensson model, Söderström (1999) shows that in the resulting configuration of variances, transmission uncertainty dominates uncertainty about the dynamics, so parameter uncertainty does act to dampen policy. Also, Rudebusch (1999) argues that multiplicative parameter uncertainty has made Federal Reserve policy less aggressive, although it is not sufficient as an explanation for the Fed's cautious behavior. Nevertheless, the main point in this paper is that the effects on policy of parameter uncertainty may be less clear-cut than previously recognized. Determining the relevance of this result for actual policy should be an interesting topic for future research.

A Solving the control problem

First, the state vector x_{t+1} has expected value

$$E_t x_{t+1} = Ax_t + Bi_t, \tag{16}$$

and covariance matrix

$$\Sigma_{t+1|t} = \begin{bmatrix} \Sigma^{y}_{t+1|t} & \Sigma^{y,\pi}_{t+1|t} \\ \Sigma^{\pi,y}_{t+1|t} & \Sigma^{\pi}_{t+1|t} \end{bmatrix}, \tag{17}$$

evaluated at t. Since all parameters are assumed independent, the off-diagonal elements of $\Sigma_{t+1|t}$ are zero. The diagonal elements are

$$\begin{aligned} \Sigma^{y}_{t+1|t} &= \mathrm{Var}_t[\alpha_{t+1} y_t - \beta_{t+1}(i_t - \pi_t) + \varepsilon^{y}_{t+1}] \\ &= x_t' \Sigma^{11}_{A} x_t + 2x_t' \Sigma^{11}_{AB} i_t + i_t' \Sigma^{11}_{B} i_t + \Sigma^{11}_{\varepsilon}, \end{aligned} \tag{18}$$

and

$$\begin{aligned} \Sigma^{\pi}_{t+1|t} &= \mathrm{Var}_t[\delta_{t+1}\pi_t + \gamma_{t+1} y_t + \varepsilon^{\pi}_{t+1}] \\ &= x_t' \Sigma^{22}_{A} x_t + \Sigma^{22}_{\varepsilon}, \end{aligned} \tag{19}$$

where Σ^{ij}_{AB} is the covariance matrix of the ith row of A_{t+1} with the jth row of B_{t+1}, that is,

$$\Sigma^{11}_{A} = \begin{bmatrix} \sigma^2_{\alpha} & 0 \\ 0 & \sigma^2_{\beta} \end{bmatrix}, \quad \Sigma^{22}_{A} = \begin{bmatrix} \sigma^2_{\gamma} & 0 \\ 0 & \sigma^2_{\delta} \end{bmatrix}, \tag{20}$$

$$\Sigma^{11}_{B} = \sigma^2_{\beta}, \quad \Sigma^{11}_{AB} = \begin{bmatrix} 0 \\ -\sigma^2_{\beta} \end{bmatrix}, \tag{21}$$

and

$$\Sigma^{11}_{\varepsilon} = \sigma^2_{y}, \quad \Sigma^{22}_{\varepsilon} = \sigma^2_{\pi}. \tag{22}$$

The trace term in equation (10) is then

$$\begin{aligned} \mathrm{tr}(V\Sigma_{t+1|t}) &= v_{11}\left(x_t' \Sigma^{11}_{A} x_t + 2x_t' \Sigma^{11}_{AB} i_t + i_t' \Sigma^{11}_{B} i_t + \Sigma^{11}_{\varepsilon}\right) \\ &+ v_{22}\left(x_t' \Sigma^{22}_{A} x_t + \Sigma^{22}_{\varepsilon}\right), \end{aligned} \tag{23}$$

where v_{11} and v_{22} are the diagonal elements of the matrix V.

Using equations (9), (10), and (16) in the control problem (8), we can express the Bellman equation as

$$
\begin{aligned}
& x_t' V x_t + w \\
& = \min_{i_t} \left\{ x_t' Q x_t + \phi \left(A x_t + B i_t \right)' V \left(A x_t + B i_t \right) + \phi \mathrm{tr}(V \Sigma_{t+1|t}) + \phi w \right\},
\end{aligned} \tag{24}
$$

which gives the necessary first-order condition as[21]

$$
\phi \left[B'(V+V')A x_t + B'(V+V')B i_t + \frac{d\mathrm{tr}(V\Sigma_{t+1|t})}{d i_t} \right] = 0, \tag{25}
$$

where, from (23),

$$
\frac{d\mathrm{tr}(V\Sigma_{t+1|t})}{d i_t} = 2 v_{11} \left(\Sigma_{AB}^{11\,\prime} x_t + \Sigma_B^{11} i_t \right). \tag{26}
$$

Thus we get the optimal policy rule

$$
\begin{aligned}
i_t &= -\left[B' \left(V + V' \right) B + 2 v_{11} \Sigma_B^{11} \right]^{-1} \left[B' \left(V + V' \right) A + 2 v_{11} \Sigma_{AB}^{11\,\prime} \right] x_t \\
&= f x_t.
\end{aligned} \tag{27}
$$

Finally, using equation (23) and the policy rule (27) in the Bellman equation (24) gives

$$
\begin{aligned}
x_t' V x_t + w &= x_t' Q x_t + \phi \left[(A x_t + B f x_t)' V (A x_t + B f x_t) + w \right] \\
&+ \phi v_{11} \left(x_t' \Sigma_A^{11} x_t + 2 x_t' \Sigma_{AB}^{11} f x_t + x_t' f_t' \Sigma_B^{11} f x_t + \Sigma_\varepsilon^{11} \right) \\
&+ \phi v_{22} \left(x_t' \Sigma_A^{22} x_t + \Sigma_\varepsilon^{22} \right) \\
&= x_t' \left[\begin{array}{l} Q + \phi (A + Bf)' V (A + Bf) \\ + \phi v_{11} \left(\Sigma_A^{11} + 2 \Sigma_{AB}^{11} f + f_t' \Sigma_B^{11} f \right) + \phi v_{22} \Sigma_A^{22} \end{array} \right] x_t \\
&+ \phi \left[w + v_{11} \Sigma_\varepsilon^{11} + v_{22} \Sigma_\varepsilon^{22} \right],
\end{aligned} \tag{28}
$$

so the matrix V is determined by

$$
\begin{aligned}
V &= Q + \phi (A + Bf)' V (A + Bf) \\
&+ \phi v_{11} \left(\Sigma_A^{11} + 2 \Sigma_{AB}^{11} f + f' \Sigma_B^{11} f \right) + \phi v_{22} \Sigma_A^{22}.
\end{aligned} \tag{29}
$$

See also Chow (1975).

[21] Use the rules $\partial x' A x / \partial x = (A + A') x$, $\partial y' B z / \partial y = B z$, and $\partial y' B z / \partial z = B' y$, see, e.g., Ljungqvist and Sargent (1997). Note also that V is not necessarily symmetric in this setup with multiplicative uncertainty.

References

Başar, Tamer and Mark Salmon, "Credibility and the value of information transmission in a model of monetary policy and inflation," *Journal of Economic Dynamics and Control* 14 (1), 97–116, February 1990.

Ball, Laurence, "Efficient rules for monetary policy," Working Paper No. 5952, National Bureau of Economic Research, March 1997.

Balvers, Ronald J. and Thomas F. Cosimano, "Inflation variability and gradualist monetary policy," *Review of Economic Studies* 61 (4), 721–738, October 1994.

Bertocchi, Graziella and Michael Spagat, "Learning, experimentation, and monetary policy," *Journal of Monetary Economics* 32 (1), 169–183, August 1993.

Blinder, Alan S., "What central bankers could learn from academics—and vice versa," *Journal of Economic Perspectives* 11 (2), 3–19, Spring 1997.

———, *Central banking in theory and practice*, The MIT Press, Cambridge, Mass., 1998.

Brainard, William, "Uncertainty and the effectiveness of policy," *American Economic Review* 57 (2), 411–425, May 1967.

Cecchetti, Stephen G., "Policy rules and targets: Framing the central banker's problem," *Federal Reserve Bank of New York Economic Policy Review* 4 (2), 1–14, June 1998.

Chow, Gregory C., *Analysis and control of dynamic economic systems*, John Wiley & Sons, New York, 1975.

Cooley, Thomas F. and Gary D. Hansen, "Money and the business cycle," in Cooley, Thomas F. (ed.), *Frontiers of business cycle research*, Princeton University Press, 1995.

Craine, Roger, "Optimal monetary policy with uncertainty," *Journal of Economic Dynamics and Control* 1 (1), 59–83, February 1979.

Ellingsen, Tore and Ulf Söderström, "Monetary policy and market interest rates," Working Paper in Economics and Finance No. 242 (revised version), Stockholm School of Economics, March 1999.

Estrella, Arturo and Jeffrey C. Fuhrer, "Dynamic inconsistencies: Counterfactual implications of a class of rational expectations models," Working Paper No. 98-5, Federal Reserve Bank of Boston, July 1998.

———, "Are 'deep' parameters stable? The Lucas critique as an empirical hypothesis," manuscript, Federal Reserve Bank of New York, May 1999.

Estrella, Arturo and Frederic Mishkin, "Rethinking the role of NAIRU in monetary policy: Implications of model formulation and uncertainty," Working Paper No. 6518, National Bureau of Economic Research, April 1998.

Fischer, Stanley, "Why are central banks pursuing long-run price stability?" in *Achieving price stability*, Federal Reserve Bank of Kansas City, 1996.

Fuhrer, Jeffrey C., "Optimal monetary policy and the sacrifice ratio," in Fuhrer, Jeffrey C. (ed.), *Goals, guidelines, and constraints facing monetary policymakers*, Federal Reserve Bank of Boston, 1994.

Goodhart, Charles A. E., "Central bankers and uncertainty," *Bank of England Quarterly Bulletin* 39 (1), 102–114, February 1999.

Holly, Sean and Andrew Hughes Hallett, *Optimal control, expectations and uncertainty*, Cambridge University Press, 1989.

Ljungqvist, Lars and Thomas J. Sargent, *Recursive macroeconomics*, manuscript, Stockholm School of Economics, 1997.

Onatski, Alexei and James H. Stock, "Robust monetary policy under model uncertainty in a small model of the U.S. economy," manuscript, Harvard University, December 1998.

Orphanides, Athanasios and Volker Wieland, "Inflation zone targeting," manuscript, Board of Governors of the Federal Reserve System, June 1999.

Poole, William, "A policymaker confronts uncertainty," *Federal Reserve Bank of St. Louis Review* 80 (5), 3–8, September/October 1998.

Romer, Christina D., "Inflation and the growth of output," Working Paper No. 5575, National Bureau of Economic Research, May 1996.

Rudebusch, Glenn D. and Lars E. O. Svensson, "Policy rules for inflation targeting," in Taylor, John B. (ed.), *Monetary policy rules*, Chicago University Press, 1999.

Rudebusch, Glenn D., "Is the Fed too timid? Monetary policy in an uncertain world," manuscript, Federal Reserve Bank of San Francisco, March 1999.

Sack, Brian, "Does the Fed act gradually? A VAR analysis," Finance and Economics Discussion Paper No. 1998-17, Board of Governors of the Federal Reserve System, March 1998a.

———, "Uncertainty, learning, and gradual monetary policy," Finance and Economics Discussion Paper No. 1998-34, Board of Governors of the Federal Reserve System, August 1998b.

Sargent, Thomas J., "Comment," in Taylor, John B. (ed.), *Monetary policy rules*, Chicago University Press, 1999.

Shuetrim, Geoffrey and Christopher Thompson, “The implications of uncertainty for monetary policy,” Research Discussion Paper No. 1999-10, Reserve Bank of Australia, November 1999.

Söderström, Ulf, “Should central banks be more aggressive?” Working Paper in Economics and Finance No. 309, Stockholm School of Economics, March 1999.

Srour, Gabriel, “Inflation targeting under uncertainty,” Technical Report 85, Bank of Canada, April 1999.

Svensson, Lars E. O., “Inflation forecast targeting: Implementing and monitoring inflation targets,” *European Economic Review* 41 (6), 1111–1146, June 1997.

———, “Inflation targeting in an open economy: Strict or flexible inflation targeting?” *Victoria Economic Commentaries* 15 (1), March 1998.

———, “Inflation targeting: Some extensions,” *Scandinavian Journal of Economics* 101 (3), 337–361, September 1999.

Taylor, John B., “Discretion versus policy rules in practice,” *Carnegie-Rochester Conference Series on Public Policy* 39, 195–214, December 1993.

———, “The inflation/output variability trade-off revisited,” in Fuhrer, Jeffrey C. (ed.), *Goals, guidelines, and constraints facing monetary policymakers*, Federal Reserve Bank of Boston, 1994.

Wieland, Volker, “Monetary policy and uncertainty about the natural unemployment rate,” Finance and Economics Discussion Paper No. 1998-22, Board of Governors of the Federal Reserve System, April 1998.

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