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Agostino Consolo, Matthias Hänsel **HANK faces unemployment**

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## Abstract

Since the advent of Heterogeneous Agent New Keynesian (HANK) models, countercyclical unemployment risk has been deemed an important amplification mechanism for business cycles shocks. Yet, the aggregate effects of such “unemployment fears” are hard to pin down. We thus revisit this issue in the context of a rich two-asset HANK model, proposing new ways to isolate their general equilibrium effects and tackle the long-standing challenge of modelling wage bargaining in this class of model. While unemployment fears can exert noticeable aggregate effects, we find their magnitude to depend importantly on the distribution of firm profits. Households’ ability to borrow stabilizes the economy. Our framework has also implications for policy: in the aftermath of an adverse energy price shock, fiscal policy can help reducing the hysteresis effects on unemployment and most households gain if the central bank accommodates an employment recovery at the cost of higher inflation.

*Keywords:* Heterogeneous models; search and matching models; alternating offer bargaining; monetary and fiscal policy.

*JEL Classification:* D52, E24, E52, J64.

## Non-Technical Summary

The post-pandemic wave of global supply-chain disruptions and energy price increases were unprecedented compared to the relatively smoother global trade conditions since early 2000s. On average between mid-2021 and mid-2022, oil prices increased by more than 70% (euro area gas prices grew by 400%) and global shipping costs by more than 30%. Supply shocks are a salient driver of business cycle fluctuations. Episodes of large spikes in energy prices have historically led to swings in price inflation as well as large and persistent increases in unemployment rates.

This brought back the memories of the 1970s and governments across advanced economies were concerned how energy price increase could have aggravated inequalities and threaten prosperity. In 2022, the G7 Leaders communique explicitly referred to *measures to reduce price surges and prevent further impacts on our economies and societies* as well as *providing short-term fiscal support to the most vulnerable groups to support affordability, as well as to businesses and industry*. For instance, euro area governments provided energy-related fiscal measures - in the order of about 5% of GDP in 2022 and 2023 - with a view to counteract the negative impact on the economy while avoiding the spikes in unemployment rates in the 1970s and the hysteresis effects of the 1980s.

While supply shocks are important drivers of the business cycle and this can be amplified by “unemployment fears”, the post-pandemic increase in global supply chain disruptions and energy prices have partly been smoothed by macroeconomic policies. Fiscal policy, especially, has reacted differently from what a typical terms-of-trade shock would call for and has provided relief to households and firms.

To study the relevance of these channels, we extend the two-asset HANK model from [Bayer et al. \(2024\)](#) with search and matching frictions and tackle two important modelling challenges. Firstly, we provide a micro-founded way of modelling wage setting in rich heterogeneous agents literature that is closely connected to the existing literature and provides for wage outcomes independent of worker wealth. Secondly, we propose a simple and consistent way to isolate and measure the general equilibrium impact of precautionary “unemployment fears” demand amplification in a complex HANK model.

Our paper contributes to the sprawling HANK literature, pioneered by [McKay and Reis \(2016\)](#) and [Kaplan et al. \(2018\)](#). Our work naturally relates to research analyzing the role of time-varying unemployment risk and related policies. [Ravn and Sterk \(2021\)](#), [Challe \(2020\)](#) and [Broer et al. \(2021\)](#) study “unemployment fears” amplification and their policy implications in so-called “zero liquidity” HANK models featuring a degenerate wealth distribution.

Firstly, we demonstrate that an Alternating Offer Bargaining (AOB) protocol in the veins of [Christiano et al. \(2016\)](#) and [Ljungqvist and Sargent \(2021\)](#) can yield wages not depending on individual wealth by eliminating dependence on workers' continuation value. Thus, our HANK model features a micro-founded bargaining set-up consistent with realistic unemployment volatility.

Secondly, in order to isolate the marginal effect from “unemployment fears”, we consider an alternative model version which only differs from our baseline model in that households have counterfactual beliefs regarding their idiosyncratic unemployment risk. The difference between this “naive” HANK model and our baseline economy provides a natural measure of the *precautionary* channel's general equilibrium effects in our rich heterogeneous agents environment. This differs from other HANK models in which unemployment benefits (UI) were turned off as that would also account for the lack of redistribution effects from UI.

We find that the precautionary-saving channel stemming from “unemployment fears” is not negligible, but its quantitative impact remains more moderate compared to the findings of previous studies employing “*zero liquidity*” HANK models. However, our results indicate the aggregate amplification to depend importantly on how firm profits are distributed, an insight we consider to be relevant for future work. Additionally, it implies that the way supply shock scenarios are modelled can be particularly important in HANK. Our results also indicate that households' ability to borrow to dampen an economy's response to aggregate shocks.

We then use our model to look at how macroeconomic policies affect main macroeconomic aggregates and, in particular, inflation. The supply shocks in 2021 and 2022 remained high in the agenda of global policymakers and fiscal policy was actively working to minimise the adverse effect on the economy and the most fragile households. Most of the fiscal measures, including those affecting the labour market such as unemployment benefits, were deficit and debt increasing - hence supporting aggregate demand - in an environment in which aggregate supply was constrained. In our HANK framework, we find that this has led to higher inflation persistence and more volatile investment. However, we also find that to the extent a central bank follows a strict inflation targeting rule, the longer-term effects on consumption, investment, unemployment and real wages are better off in the case of debt-financing fiscal policy.

# 1 Introduction

Supply shocks are a salient driver of unemployment fluctuations. For example, episodes of large spikes in energy prices have historically led to swings in price inflation as well as large and persistent increases in unemployment rates. The post-pandemic squeeze in commodity markets and in global supply chain were unprecedented compared to the relatively smoother global trade conditions since early 2000s. On average between mid-2021 and mid-2022, oil prices increased by more than 70% (euro area gas prices grew by 400%) and global shipping costs by more than 30%.

This brought back the memories of the 1970s and governments across advanced economies were concerned how energy price increase could have aggravated inequalities and threaten prosperity. In 2022, the G7 Leaders communique explicitly referred to *measures to reduce price surges and prevent further impacts on our economies and societies* as well as *providing short-term fiscal support to the most vulnerable groups to support affordability, as well as to businesses and industry*.<sup>1</sup> For instance, euro area governments provided energy-related fiscal measures - in the order of about 5% of GDP in 2022 and 2023 - with a view to counteract the negative impact on the economy while avoiding the spikes in unemployment rates in the 1970s and the hysteresis effects of the 1980s.

Of course, supply-side factors are also important from a modelling perspective, where they are typically found to constitute an important source of the exogenous variation driving business cycle fluctuations (see e.g. [Smets and Wouters, 2007](#)). Their actual aggregate effects depend, however, on how households react to variation in different types of incomes as well as unemployment risk. To account for the realistically unequal exposure to these, we set up a Heterogeneous Agent New Keynesian (HANK) model: A key feature of our work is the emphasis on the unemployment rate as a key business cycle outcome and on its endogenous time-varying risk that leads to stronger precautionary saving motives, amplifying the aggregate shock through demand effects.

This *unemployment fears*-channel has long been a concern for policy makers, with e.g. the Federal Open Market Committee (FOMC) worrying already in 2009 that “*fear of unemployment could well lead to further increases in the saving rate that would damp consumption growth in the near term.*”<sup>2</sup>. Previous research has also found such effects to be particularly relevant for the case of supply shocks (c.f. [Challe, 2020](#); [Broer et al., 2021](#)).

Our main contribution is to study the relevance of these effects in a rich HANK model à la [Bayer](#)

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<sup>1</sup>See [G7 Leader communique, June 2022](#)

<sup>2</sup><https://www.federalreserve.gov/monetarypolicy/fomcminutes20090318.htm>

et al. (2024) that we enrich with Search-and-Matching (SaM) frictions on the labor market. In this framework, households accumulate illiquid assets with relatively higher return and liquid assets to smooth consumption against idiosyncratic skill- and unemployment risk: The SaM frictions make the latter dependent on the business cycle, implying a time-varying demand for liquid assets such as government debt that can be used as a means of insurance.

We propose solutions to two important modelling challenges: First of all, modelling wage bargaining is seen as notoriously difficult in heterogeneous agents environments, given that standard bargaining protocols such as the Nash solution would imply wages to depend on a workers' asset holdings.<sup>3</sup> However, we demonstrate that an Alternating Offer Bargaining (AOB) protocol in the veins of Christiano et al. (2016) and Ljungqvist and Sargent (2021) can yield wages not depending on individual wealth by eliminating dependence on workers' continuation value. Thus, our HANK model features a micro-founded bargaining set-up consistent with realistic unemployment volatility instead of relying on ad hoc “wage norms” often used in this strand of literature.

Secondly, in a rich quantitative HANK model, pinning down the general equilibrium amplification through *unemployment fears* precisely appears essentially impossible: A model constructed to feature less unemployment risk, e.g. due to different unemployment insurance (UI), will also differ in terms of (re-)distribution. For example, if one finds that UI expansions dampen the aggregate effects of a contractionary shock, this might be due to either the *precautionary savings* channel we aim to isolate or just because the policy redistributes towards high MPC unemployed households, confounding insurance- and redistribution effects. We overcome this challenge by considering an alternative model version which only differs from our baseline model in that households have counterfactual beliefs regarding their idiosyncratic unemployment risk. The difference between this “naive” HANK model and our baseline economy provides a natural measure of the *precautionary* channel's general equilibrium effects in our rich heterogeneous agents environment.

Overall, we find that in our 2-asset HANK economy, the demand effects induced by the precautionary channel are non-negligible but moderated compared to simpler models. However, our results also point towards an important role for profits. While shocks to TFP and aggregate markups (so-called “cost-push” shocks) tend to have similar effects in representative agent models, they differ markedly regarding their implications for aggregate outcomes and particularly the strength of the *precautionary savings* amplification in the HANK model. The latter is due to profits *decreasing* after TFP shocks but *increasing* after markup shocks: In our baseline

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<sup>3</sup>While the methods recently proposed by Hänsel (2023) facilitate solving heterogeneous agent models with bilateral bargaining, including Nash bargaining into a rich 2-asset model remains costly.

model, the time-varying firm profits are received by rich “entrepreneurs” that are not subject to unemployment risk and have access to all asset markets. Intuitively, if their income decreases in response to a TFP shock, they reduce their substantial amount of liquid savings due to intertemporal substitution motives, *counteracting* the additional precautionary savings demand of regular “worker” households. The opposite happens if profits increase after the mark-up shock, in which case the rich are also induced to save, *amplifying* the demand effect.

We confirm this intuition by considering an alternative model version in which the cyclically varying profits are allocated to Hand-to-Mouth households not participating in the asset market. Thus, the aggregate amplification through “unemployment fears” depends importantly on assumptions of how firm profits are distributed, an insight we believe to be important for future modelling work. These results also mean that in HANK, it can have important consequences whether specific scenarios are modelled as TFP or cost push shocks.

Our model has also implications for other important questions: We investigate the role of borrowing, a feature many HANK models abstract from, and analyze how the impact of energy price shocks depends on macroeconomic policies.

Regarding the former, we find that even if borrowing is possible only at a substantial penalty rate, the ability of households to borrow stabilizes the aggregate response to supply shocks. On policy, allowing the monetary authority to respond to the unemployment rate improves welfare in the aftermath of an energy price shock, but there is a stark difference between households relying on labor income (welfare gain) compared to those relying on firm profit income (welfare loss). In terms of fiscal policy, we show that the way fiscal measures are financed – either via debt issuance or tax increases – affects the degree of inflation persistence with limited impact on the unemployment rate.

In what follows, we start surveying the literature. We then present the main model specification in Section 2 covering the household problem, the production side and the labour market. In Section 3 we provide the calibration of the model parameters and its validation by comparing untargeted model moments with some empirical counterparts. We then present our main results on the contribution of “unemployment fears” in Section 4. We also provide an analysis of inflation persistence following an energy price shock in Section 5 and how it is affected by monetary and fiscal policy rules. Section 6 concludes.

**Literature review:** Our paper contributes to the sprawling HANK literature, pioneered by [McKay and Reis \(2016\)](#) and [Kaplan et al. \(2018\)](#). Within that area, our work naturally relates most closely to other research analyzing the role of time-varying unemployment risk and related policies. In particular, the works of [Ravn and Sterk \(2017, 2021\)](#), [Challe \(2020\)](#) as well as

Broer et al. (2021) study “unemployment fears” amplification and their policy implications in so-called “zero liquidity” HANK models featuring a degenerate wealth distribution. Other papers employ quantitatively oriented one-asset HANK-SaM models to study the heterogeneous welfare effects of different monetary policy rules (Gornemann et al., 2021), the stabilization effects of unemployment insurance (Kekre, 2022) or the aggregate effects of deviations from rational expectations (Bardoczy and Guerreiro, 2023). Alves (2022) and Birinci et al. (2022) analyze the implications of job-to-job transitions in HANK models, an aspect many other HANK-SaM papers (including ours) abstract from. Lee (2021) also constructs a 2-asset HANK model with SaM frictions but only focuses on the transmission of monetary policy shocks.

The work closest to ours is thus the paper by Graves (2021), who also aims to study how cyclical unemployment risk amplifies aggregate shocks in a 2-asset HANK economy. Yet, he does so by comparing his baseline model to another framework featuring no UI benefits and thus answers the related but not identical question “*Does the presence of UI dampens aggregate shocks?*” instead of directly isolating the precautionary savings amplification *within* a given economy as we do. His work also does not analyze the role of profits but emphasizes the role of illiquid asset adjustment costs, a model aspect difficult to discipline that we abstract from.

Finally, our results on the importance of profits are reminiscent of Broer et al. (2020), who highlight a different way how the distribution of profits can crucially shape aggregate outcomes in HANK economies.

## 2 The model

This section describes our two-asset HANK model, many aspects of which are similar to the model studied by Bayer et al. (2024), except the way the labor market is modeled.

**Period timing** In every period, there is the following order of events, on which additional details will be provided below:

1. Aggregate shocks are revealed; job separations take place; Government policies are announced
2. The labor market opens: Labor agencies post vacancies; Unemployed search for jobs; matches are formed
3. The labor market closes; there is an even number of  $M$  subperiods during which production takes place and workers and labor agencies can negotiate over wages



4. Goods and asset markets open: Asset returns are paid out; consumption and investment decisions are made
5. Goods and asset markets close; shocks to idiosyncratic states  $s_{it}$  and  $\Xi_{it}$  are revealed

## 2.1 Households

### 2.1.1 Idiosyncratic states

There is an unit mass of ex-ante identical households, which we also refer to as “agents” interchangeably. These differ ex-post by several idiosyncratic states:

- First of all, households vary in terms of their holdings of liquid and illiquid assets  $a_{it}$  and  $k_{it}$ . The assets represent holdings of bonds and capital and we require that  $k_{it} \geq 0$  as well as  $a_{it} \geq \underline{a}$ , with  $\underline{a}$  representing an exogenous borrowing limit. Capital is illiquid in that an household can change her stock  $k_{it}$  only infrequently: In particular, following [Bayer et al. \(2024\)](#), we assume that the opportunity to do so arises randomly in an i.i.d. fashion, in that an households only gets to participate in the market for illiquid assets with probability  $\lambda \in (0, 1)$  every period.
- Secondly, the agents can be workers ( $\Xi_{it} = 0$ ) or “entrepreneurs” ( $\Xi_{it} = 1$ ). The former participate in the frictional labor market, while the latter don’t supply labor market but receive the profits generated by the firms (to be described below), which, for simplicity, are assumed to be shared equally among all  $\Xi_{it} = 1$  households. Transitions to and out of the “entrepreneur” state are exogenous with probabilities  $\zeta$  and  $\iota$ .
- Worker households ( $\Xi_{it} = 0$ ) additionally differ by their idiosyncratic labor productivity or “skill”  $s_{it} \in \mathcal{S} = \{s_1, s_2, \dots, s_{ns}\}$ , which evolves stochastically according to a discrete Markov chain. We allow for transition probabilities  $\Pi^s(s_{it+1}|s_{it}, e_{it})$  to depend on employment status  $e_{it}$  (see next bullet point) in order to parsimoniously allow for skill accumulation (depreciation) while employed (unemployed). Workers who are selected to become entrepreneurs lose their idiosyncratic  $s_{it}$  state as well as their job, while exiting entrepreneurs draw a new  $s_{it}$  according to exogenous probabilities  $p_{s_1}, p_{s_2}, \dots$  and enter unemployment.
- Finally, workers will either be employed ( $e_{it} = 1$ ) or unemployed ( $e_{it} = 0$ ). We assume there to be no disutility from either work or job search, so that all workers will be working or searching full time. Job finding rates  $p_t^{UE}$  and  $p_t^{EU}(s_{it})$  will be *endogenously* determined on the frictional labor market described in Section 2.3. Note that the latter may depend

on individual labor productivity.

Workers receive a wage  $w_t(s_{it})$ , while unemployed agents receive an unemployment insurance (UI) benefit  $b_t(s_{it})$ . As outlined above, wages  $w_t$  will be the outcome of an AOB bargaining protocol to be described in Section 2.3.3, while  $b_t(s_{it})$  is set by the government: its level is assumed to depend on  $s_{it}$  to introduce dependence on previous income without adding additional state variables to the household problem.

Below, we will denote by  $m_t(\cdot)$  the mass of households that, at the beginning of a period, are currently in the specified state, e.g.  $m_t(k, s, e)$  is the respective measure of households with capital holding  $k$ , skill  $s$  and employment status  $e$ . Additionally, we will use  $m_t^e$ ,  $m_t^u$  and  $m_t^\Xi$  to denote the masses of agents that feature states  $e_{it} = 1$ ,  $e_{it} = 0$  or  $\Xi_{it} = 1$  at the beginning of stage 4 of any period  $t$  (compare Section 2 above).

### 2.1.2 The Household problem

Households value an consumption stream  $\{c_t\}_{t=0}^\infty$  according to standard time-separable CRRA preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{it}^{1-\xi} - 1}{1-\xi} .$$

An agent who get to adjust her illiquid capital stock will face budget constraint (written in real terms)

$$c_{it} + q_t k_{it+1} + a_{it+1} = y_{it}(e_{it}, s_{it}, \Xi_{it}) + \frac{R_t^a(a_{it})}{\pi_t} a_{it} + (q_t + r_t^k) k_{it} \quad (1)$$

while non-adjusters, the constraint will be of the form

$$c_{it} + a_{it+1} = y_{it}(e_{it}, s_{it}, \Xi_{it}) + \frac{R_t^a(a_{it})}{\pi_t} a_{it} + r_t^k k_{it} . \quad (2)$$

Both budget constraint are already written in real terms, with  $\pi_t = \frac{P_t}{P_{t-1}}$  denoting gross inflation. Furthermore,  $q_t$  represents the time  $t$  price of capital goods,  $r_t^k$  the real net return of capital goods and  $R_t^a(a_{it})$  the gross nominal return on bonds  $a_{it}$ . The latter depends on  $a_{it}$  due to the presence of a borrowing penalty. In particular, we have

$$R_t^a(a_{it}) = \begin{cases} A_t R_t^B & \text{if } a_{it} \geq 0 \\ A_t R_t^B + \bar{R} \pi_t & \text{if } a_{it} < 0 \end{cases} \quad (3)$$

where  $R_t^B$  is the nominal return on liquid government bonds, determined by the central bank as described below.  $\bar{R}$  is a *real* borrowing penalty, while  $A_t$  is a shock to financial intermediation,

which [Bayer et al. \(2024\)](#) introduce as stand-in for the “risk premium”-shocks often included in quantitative DSGE models such as [Smets and Wouters \(2007\)](#).<sup>4</sup>

Finally,  $y_{it}$  represents an household’s labor-, transfer- or profit income so that

$$y_{it}(e_{it}, s_{it}, \Xi_{it}) = (1 - \tau^y) \begin{cases} w_t(s_{it}) & \text{if } e_{it} = 1, \Xi_{it} = 0 \\ b_t(s_{it}) & \text{if } e_{it} = 0, \Xi_{it} = 0 \\ \frac{\Pi_t}{m_t^{\Xi}} & \text{if } \Xi_{it} = 1 \end{cases} . \quad (4)$$

$\tau^y$  represents a proportional income tax.

Letting  $\Gamma_t$  denote a set containing the economy’s *aggregate* state at period  $t$ , we are now ready to state the Bellman equation corresponding to the households’ dynamic utility maximization problem, which are

$$V^a(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t) = \max_{c_{it}, k_{it+1}, a_{it+1}} \left\{ \frac{c_{it}^{1-\xi} - 1}{1 - \xi} + \beta \mathbb{E}_t V(a_{it+1}, k_{it+1}, e_{it+1}, s_{it+1}, \Xi_{it+1}; \Gamma_{t+1}) \right\} \\ \text{s.t. to (1), (4), } k_{it} \geq 0 \text{ and } a_{it} \geq \underline{a} \quad (5)$$

for an household able to adjust its capital stock and

$$V^{na}(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t) = \max_{c_{it}, a_{it+1}} \left\{ \frac{c_{it}^{1-\xi} - 1}{1 - \xi} + \beta \mathbb{E}_t V(a_{it+1}, k_{it}, e_{it+1}, s_{it+1}, \Xi_{it+1}; \Gamma_{t+1}) \right\} \\ \text{s.t. to (2), (4), } k_{it} \geq 0 \text{ and } a_{it} \geq \underline{a} \quad (6)$$

for an household that unable to do so. The ex-ante value function  $V(\cdot)$  is given by

$$V(a_{it+1}, k_{it}, e_{it+1}, s_{it+1}, \Xi_{it+1}; \Gamma_{t+1}) = \lambda V^a(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t) \\ + (1 - \lambda) V^{na}(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t) .$$

## 2.2 Production

The model’s supply side is similar to standard “medium scale” DSGE models, except the way the labor market is modelled: Production is vertically integrated. There is a final good that can either be consumed or used by capital goods producers to produce investment goods subject to adjustment costs. This final good is assembled by a representative final goods producer, that in turn requires differentiated inputs provided by a continuum of retailers. The latter set prices in a

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<sup>4</sup>Our specification for the borrowing wedge implies that every unit of debt held by an household incurs a real resource cost of  $\bar{R}$ , e.g. due to costly monitoring. Furthermore, given that bonds will be in positive supply in our economy, the [Bayer et al. \(2024\)](#) specification for the  $A_t$  shock implies that a positive (negative) “risk premium” creates (destroys) real resources proportionally to the aggregate bond supply.

monopolistic competitive fashion subject to nominal rigidities and require intermediate goods to produce their output. These are produced by a set of competitive intermediate goods producers that require capital, energy and labor services as inputs. However, the provision of the labor input requires hiring on a frictional labor market à la Diamond-Mortensen-Pissarides, which is handled by labor agencies.

As [Bayer et al. \(2024\)](#), we make the simplifying assumption that entrepreneurs don't make the dynamic decisions of the various firms directly but instead outsource them to a group of risk-neutral managers with aggregate measure 0, that do not have access to asset markets and discount the future at the same rate  $\beta$  as the households.<sup>5</sup>

### 2.2.1 Final Goods producer

The representative final good producers combines a continuum of differentiated inputs according to production function

$$Y_t = \left( \int_0^1 y_{jt}^{\frac{1}{\mu_t}} dj \right)^{\mu_t} . \quad (7)$$

Given prices  $p_{jt}$ , the first order conditions of the producers profit maximization problem give rise to the familiar demand schedule for any given variety as

$$y_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{\frac{-\mu_t}{\mu_t-1}} Y_t \quad (8)$$

where  $P_t$  is the aggregate price level given by

$$P_t = \left( \int_0^1 p_{jt}^{\frac{1}{1-\mu_t}} dj \right)^{1-\mu_t} .$$

We allow for exogenous time variation in  $\mu_t$ , so-called “markup shocks”.

### 2.2.2 Retailers

There is a unit mass of retailers, each of which produce a given variety as monopolist, taking into account demand schedule (8). Their only input are intermediate goods, which they purchase at real price  $mc_t$  (also referred to as “marginal cost”) from the competitive intermediate goods producers. However, they are subject to nominal rigidities à la Calvo with price indexation, i.e.

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<sup>5</sup>Since we will linearize our model with respect to aggregate shocks, only the steady-state value of the discount factor in the firms' dynamic problems will matter for the dynamic model responses. [Bayer et al. \(2019\)](#) and [Lee \(2021\)](#) report that using different specifications does not significantly affect results in their 2-asset HANK models with many similar features.

they can only re-set their price if chosen with an exogenous probability  $\lambda_Y$ .

If not receiving the re-set opportunity, a retailer's price is automatically adjusted by the steady inflation rate  $\pi_{SS}$ .<sup>6</sup> If receiving it, the retailer will choose a price to maximize the corresponding expected net present value of real profits

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1 - \lambda_Y)^t \left( \frac{p_{jt}^* \pi_{SS}^t}{P_t} - mc_t \right) \left( \frac{p_{jt}^* \pi_{SS}^t}{P_t} \right)^{\frac{-\mu_t}{\mu_t - 1}} Y_t .$$

Log-linearizing the first order conditions of the resulting price setting problem gives rise to the standard log-linear Phillips curve

$$\log \left( \frac{\pi_t}{\pi_{SS}} \right) = \kappa_Y \left( mc_t - \frac{1}{\mu_t} \right) + \beta \mathbb{E}_t \log \left( \frac{\pi_{t+1}}{\pi_{SS}} \right) \quad (9)$$

with  $\kappa_Y := \frac{(1-\lambda_Y)(1-\lambda_Y\beta)}{\lambda_Y}$ .

### 2.2.3 Intermediate goods producers

The homogeneous intermediate good is produced by a continuum of firms that operate with a constant-returns-to-scale technology represented by production function

$$F_t(u_t K_t, H_t, E_t) = Z_t F(u_t K_t, H_t, E_t) = Z_t \left( \varrho^{\frac{1}{\epsilon_e}} E_t^{\frac{\epsilon_e - 1}{\epsilon_e}} + (1 - \varrho)^{\frac{1}{\epsilon_e}} ((u_t K_t)^\alpha H_t^{1-\alpha})^{\frac{\epsilon_e - 1}{\epsilon_e}} \right)^{\frac{\epsilon_e}{\epsilon_e - 1}} . \quad (10)$$

$E_t$ ,  $K_t$  and  $H_t$  denote the input of energy goods, capital and labor services.  $u_t$  is the degree of capital utilization that determines capital depreciation according to

$$\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2$$

and  $Z_t$  is a shock to Total Factor Productivity (TFP). We chose to include an energy good input in (10) to be able to study shocks to energy prices, an arguably salient macroeconomic issue during 2022. The chosen functional form is inspired by [Hassler et al. \(2021\)](#).

Taking the prices  $p_t^E$  and  $h_t$  for energy and labor services as well as the capital rental rate  $r_t$  and its output price  $mc_t$  as given, an intermediate goods producer solves the static profit maximization problem

$$\max_{K_t, E_t, H_t, u_t} mc_t F_t(u_t K_t, H_t, E_t) - p_t^E E_t - h_t H_t - (r_t + q_t \delta(u_t)) K_t ,$$

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<sup>6</sup>This allows to normalize  $\pi_{SS} = 1$ .

the solution of which can be characterized using the following order conditions:

$$p_t^E = mc_t Z_t \left( \frac{E_t}{\varrho F(u_t K_t, H_t, E_t)} \right)^{-\frac{1}{\epsilon_e}} \quad (11)$$

$$h_t = (1 - \alpha) mc_t Z_t \left( \frac{(u_t K_t)^\alpha H_t^{1-\alpha}}{(1 - \varrho) F(u_t K_t, H_t, E_t)} \right)^{-\frac{1}{\epsilon_e}} \left( \frac{u_t K_t}{H_t} \right)^\alpha \quad (12)$$

$$r_t + q_t \delta(u_t) = \alpha mc_t Z_t \left( \frac{(u_t K_t)^\alpha H_t^{1-\alpha}}{(1 - \varrho) F(u_t K_t, H_t, E_t)} \right)^{-\frac{1}{\epsilon_e}} \left( \frac{u_t K_t}{H_t} \right)^{\alpha-1} u_t \quad (13)$$

$$q_t(\delta_1 + \delta_2(u_t - 1)) = \alpha mc_t Z_t \left( \frac{(u_t K_t)^\alpha H_t^{1-\alpha}}{(1 - \varrho) F(u_t K_t, H_t, E_t)} \right)^{-\frac{1}{\epsilon_e}} \left( \frac{u_t K_t}{H_t} \right)^{\alpha-1} . \quad (14)$$

## 2.2.4 Energy goods production

For simplicity, the production of energy goods is modeled in a particularly parsimonious fashion: We assume there to be a competitive energy producer that is endowed with a technology to transform  $Z_t^E$  units of the final good into one unit of the energy good, with  $Z_t^E$  itself being determined by an exogenous shock process. In turn, the real price of energy will be simply be  $p_t^E = Z_t^E$ .

Note that this setting is isomorphic to a case in which energy goods need to be purchased on a world market at an exogenously determined (and potentially fluctuating) real price  $p_t^E$ .

## 2.2.5 Capital goods producer

Capital goods producers use the final good as input and operate a technology subject to adjustment costs: Using  $I_t$  units of the final good, they can produce

$$Z_t^I \left[ 1 - \frac{\phi}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t$$

units of capital. Investment-specific productivity  $Z_t^I$  is exogenous and potentially following a time-varying shock process.

Taking the price of capital  $q_t$  as given, the producers choose  $I_t$  to maximize the net present value of real profits

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t \left( q_t Z_t^I \left[ 1 - \frac{\phi}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t - I_t \right)$$

and their optimal interior solution will fulfill first-order condition

$$1 + q_t Z_t^I \left( \frac{\phi}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - 1 + \phi \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right) = \beta q_{t+1} Z_{t+1}^I \phi \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 . \quad (15)$$

## 2.3 Labor market

### 2.3.1 Labor agencies

Labor services are produced by a continuum of homogeneous labor agencies, each of which is matched with at most one worker of productivity  $s$ . Such a match produces  $s_t$  units of the labor service output, the price of which are taken as given by an agency.<sup>7</sup> Job separations are exogenous and take place either if (1) the match is subject to a separation shock arriving with probability  $\delta(s)$  or if (2) the worker becomes an entrepreneur with probability  $\zeta$ . We allow for job separation rates  $\delta(s)$  to depend on skill, consistent with evidence that low-income workers face higher job separation risk (see e.g. [Birinci and See, 2021](#)). Given all the above, the recursive characterization of the value of a matched agency is

$$J(s_t; \Gamma_t) = h_t s_t - w_t(s_t) + (1 - \zeta)(1 - \delta(s_t))\beta \mathbb{E}_t J(s_{t+1}; \Gamma_{t+1}) \quad . \quad (16)$$

### 2.3.2 Job matching and vacancy creation

There is a single labor market, on which unmatched labor agencies can meet unemployed workers by posting vacancies. The number of meeting is governed by a Cobb-Douglas matching technology

$$M_t(V_t, U_t) = A_m U_t^\chi V_t^{1-\chi} \quad . \quad (17)$$

$V_t$  represents the total number of vacancies posted and

$$U_t = m_t(e = 0) + \sum_{s_i \in \mathcal{S}} \delta(s_i) m_t(e = 1, s = s_i)$$

the total mass of workers searching for a job. From (17), it follows that the period- $t$  job-finding probability  $p_t^{UE}$  and vacancy-filling probability  $p_t^{vf}$  are

$$p_t^{UE} = \frac{M_t(V_t, U_t)}{U_t} = A_m \theta_t^{1-\chi} \quad \text{and} \quad p_t^{vf} = \frac{M_t(V_t, U_t)}{V_t} = A_m \theta_t^{-\chi} \quad , \quad (18)$$

respectively.  $\theta := V_t/U_t$  is the labor market tightness.

Hiring is costly in that a) posting a vacancy incurs a real resource cost of  $\kappa_1$  per vacancy posted and b) upon meeting a worker a labor agency needs to pay a resource cost of  $\kappa_2$  before bargaining can begin. The latter may represent resources needed to “screen” the worker.  $\theta_t$  is in turn pinned

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<sup>7</sup>Due to CRS and the market for labor services being competitive, one could equivalently assume that intermediate goods firms produce labor services “in-house” and handle hiring themselves.

down by a free entry condition of the form

$$\kappa_1 = p_t^{vf} \left( \sum_{s \in \mathcal{S}} \frac{U_t(s)}{U_t} J(s; \Gamma_t) + \kappa_2 \right) \quad (19)$$

with

$$U_t(s_i) = m_t(e = 0, s = s_i) + \delta(s_i) m_t(e = 1, s = s_i)$$

denoting the mass of job-searchers of a given skill level  $s_i \in \mathcal{S}$ . These terms reflect labor agencies taking account which type of workers they are most likely to meet.

### 2.3.3 Wage determination

Wages are determined according to an *intra-period* Alternative Offer Bargaining (AOB) protocol in the veins of [Christiano et al. \(2016\)](#), imposing the restriction of no intra-period bargaining break-downs used by [Ljungqvist and Sargent \(2021\)](#):<sup>8</sup> During the  $M$  subperiods of a period's production stage (compare Section 2), the worker and the labor agency take turns extending wage offers: We will be denoting variables on a per-subperiod basis with a  $\Delta$ , e.g.  $h_{\Delta,t} := \frac{h_t}{M}$  is the revenue a labor agency for producing during one of the subperiods etc..

We assume that in any given period, the labor agency gets to make the first offer. If the worker rejects it, she can make a counter-offer in the next period that the firm can reject or accept, and so on. Once a wage agreement has been reached, the match starts producing labor services and the worker is paid the agreed wage rate for the remainder of the period. However, before that happens, an agency matched with a skill  $s$ -worker incurs a cost of delay  $\gamma_{\Delta}(s)$  per subperiod, while the worker will receive an outside income  $\tilde{b}_{\Delta,t}(s)$  per sub-period. Both these values will have to depend on the respective's workers productivity  $s$  to avoid high (low) productivity workers being able to bargain wages that are disproportionately low (high) comparatively to their skill. If no wage is accepted before the last period  $M$ , the worker gets to make a *take-it-or-leave-it* offer during the last subperiod ( $M$  is even). If rejected by the firm, the match irreparably dissolves and the worker enters the pool of the unemployed.

Now, to characterize the wage outcome, we first note that independently of worker wealth, a worker (agency) in our model would always like the wage to be as high (low) as possible.<sup>9</sup> In turn, it is optimal for each party to make offers barely acceptable to the other.<sup>10</sup> Hence, slightly

<sup>8</sup>[Ljungqvist and Sargent \(2021\)](#) find this restrictions to hardly affect model dynamics in the rich representative agent New Keynesian model studied by [Christiano et al. \(2016\)](#).

<sup>9</sup>This may not generalize to some settings with endogenous separations in which the firm has the opportunity to lay off the worker ex-post.

<sup>10</sup>If indifferent, a party is assumed to accept.



abusing notation, if a firm gets to make an wage offer  $w_{j,\Delta,t}^f$  to a worker in a subperiod  $j < M$ , this offer should fulfill

$$V\left((1-\tau^y)[(M-j+1)w_{j,\Delta,t}^f + (j-1)\tilde{b}_{\Delta,t}(s)], \dots\right) = V\left((1-\tau^y)[(M-j)w_{j+1,\Delta,t}^w(s) + j\tilde{b}_{\Delta,t}(s)], \dots\right) \quad (20)$$

with  $V$  being a value functions as in (5) or (6), having added  $y_t$  as additional input. The left-hand side is the value a worker would obtain from accepting the offer, while the right-hand side is the value of not accepting and making the equilibrium counter-offer  $w_{j+1,\Delta,t}^w(s)$  in the next subperiod (which will be accepted). Since  $V$  as in (5), (6) is strictly increasing in income (additional resources can always be consumed), (20) implies

$$(M-j+1)w_{j,\Delta,t}^f(s) + (j-1)\tilde{b}_{\Delta,t}(s) = (M-j)w_{j+1,\Delta,t}^w(s) + j\tilde{b}_{\Delta,t}(s) \quad . \quad (21)$$

Intuitively, worker wealth does not matter for in indifference condition (20), as any worker prefers higher period income in our setting. Similarly, if a worker gets to make an wage offer  $w_{j,\Delta,t}^f$  to a firm in a subperiod  $j < M$ , this offer should fulfill

$$(M-j+1)(h_{\Delta,t}s - w_{j,\Delta,t}^w(s)) = -\gamma_{\Delta}(s) + (M-j)(h_{\Delta,t}s - w_{j+1,\Delta,t}^f(s)) \quad . \quad (22)$$

Finally, if no wage is accepted until period  $j = M$ , the indifference condition for an agency contemplating a worker's offer  $w_{M,\Delta,t}^w$  would be

$$h_{\Delta,t}s - w_{M,\Delta,t}^w(s) + (1-\zeta)(1-\delta(s_t))\beta\mathbb{E}_t J(s_{t+1}, \Gamma_t) = 0 \quad (23)$$

as, if rejecting the offer, the firm would have to look for a new worker, the value of which is 0 due to free entry. We note that worker wealth does not enter indifference condition (23) either, as any worker would like to claim the maximum possible amount of income during the final bargaining period.

Since the equilibrium wage outcome can be characterized using equations (21), (22) and (23), it follows that our AOB bargaining scheme delivers wages that are independent of worker wealth:

**Proposition 1.** *The per-period wage of a matched worker with labor productivity  $s$  will be given by*

$$w_t(s) = \frac{1}{2}\left(h_t s + \tilde{b}_t(s)\right) + \frac{M-2}{2M}\gamma(s) + (1-\zeta)(1-\delta(s_t))\beta\mathbb{E}_t J(s_{t+1}, \Gamma_t) \quad . \quad (24)$$

*Proof.* See Appendix A. □

## 2.4 Government

### 2.4.1 Monetary Authority

The monetary authority sets the nominal interest rate on bonds, which follow a Taylor rule of the form

$$\frac{R_{t+1}^B}{R_{SS}^B} = \left( \frac{R_t^B}{R_{SS}^B} \right)^{\rho^B} \left[ \left( \frac{\pi_t}{\pi_{SS}} \right)^{\theta_\pi} \exp(m_t^u - m_{SS}^u)^{\theta_u} \right]^{1-\rho^B} . \quad (25)$$

The parameter  $\rho^B$  introduces rate smoothing and if  $\theta_u \neq 0$ , the rule reacts to unemployment in addition to inflation.

### 2.4.2 Fiscal Authority

The fiscal authority collects taxes, pays out unemployment insurance and engages in government consumption  $G_t$ . Its budget constraint (in real terms) is

$$B_{t+1} + \tau^y \left( \sum_{s \in \mathcal{S}} w_t(s) m_t^e(s) + \Pi_t \right) = G_t + \frac{R_t^B}{\pi_t} B_t + (1 - \tau^y) \sum_{s \in \mathcal{S}} b_t(s) m_t^u(s) . \quad (26)$$

In our benchmark application, we assume the real value of UI benefits to be constant over time and equal to a fixed replacement rate over steady state (post tax) wages, i.e.

$$b_t(s) = \Upsilon_b w_{ss}(s) . \quad (27)$$

Furthermore, we postulate government spending to follow the rule

$$\frac{G_t}{G_{SS}} = \left( \frac{G_{t-1}}{G_{SS}} \right)^{\rho_G} \left( \frac{B_t}{B_{SS}} \right)^{(1-\rho_G)\psi_B} , \quad (28)$$

also considered by [Bayer et al. \(2023\)](#), and assume the fiscal authority to issue any amount of bonds  $B_{t+1}$  necessary to fulfill its budget constraint (26). Intuitively, policy (28) means that the government will reduce spending in response to excess debt, but does so only slowly over time. Some of our quantitative exercises below will consider alternatives to specifications (27) and (28).

## 2.5 Market clearing conditions and equilibrium

The Definition of Equilibrium is standard, but tedious, given that our model features multiple markets and also requires keeping track of the evolution of measures  $m_t(\cdot)$ . In turn, we relegate these details to [Appendix B](#).

## 2.6 Numerical Approach

We approximate the dynamic equilibrium of the model using a version of the method used by [Bayer and Luetticke \(2020\)](#), which conducts first-order perturbation around the economy’s non-stochastic steady state, following a dimension reduction step.

For obtaining that steady state, we use a multi-dimensional Endogenous Grid Method similar to the algorithm described in [Bayer et al. \(2019\)](#) to solve the households’ dynamic programming problem. The joint income- and asset distribution is approximated as a histogram using the “lottery”-method proposed by [Young \(2010\)](#).

However, the representations of the (marginal) value functions as well as the joint distribution on a tensor grid are too large to be practically handled by standard perturbation algorithms. In turn, the dimensionality of the (marginal) value function is reduced by applying a Discrete Cosine Transform (DCT) and perturbing only the coefficients most important for the shape of the steady state marginal value function. Additionally, the joint distribution is split into a copula and marginals and we only perturb the marginals as well as the largest coefficients resulting from a similar DCT of the copula.

Further details on the numerical implementation are provided in [Appendix D](#).

## 3 Calibration

A model period is interpreted to be a quarter. We aim for our model to be consistent with the most relevant features of the US economy: For our calibration strategy, we first set a range of parameters exogenously, relying on the previous literature: In addition to standard preference- and technology parameters, this includes some parameters exclusively affecting the dynamic model response to aggregate shocks, for which we rely on previous papers estimating a HANK model. Afterwards, we choose the remaining parameters values to match various steady state distribution- and labor market moments.

### 3.1 Externally calibrated

The household’s risk aversion parameter is set to 1.0, a standard value also used by [Kaplan et al. \(2018\)](#). Regarding technology, we use the standard values of  $\alpha = 0.33$  for the Cobb-Douglas parameter for capital and set a quarterly depreciation rate for capital of  $\delta = 0.015$ . Similar, we set the steady state value for  $\mu_t$  to a conventional value of 1.1, resulting in a steady state markup of 10%. The elasticity of substitution between energy and the capital-labor-bundle is set to 0.1, the benchmark value used by [Pieroni \(2023\)](#). The number of subperiods during which

Parameter	Description	Value	Source
$\xi$	risk aversion	1.0	Standard
$\iota$	Exit prob. entrepreneurs	1/16	<a href="#">Bayer et al. (2022)</a>
$\alpha$	Cobb-Douglas parameter	0.33	Standard
$\delta_0$	Steady State depreciation	0.015	Standard
$\mu_{SS}$	SS goods markup	1.1	Standard
$\epsilon_E$	Elasticity of substitution energy	0.1	<a href="#">Pieroni (2023)</a>
$\phi$	investment adjustment cost	3.5	<a href="#">Bayer et al. (2022)</a>
$\delta_2/\delta_1$	utilization parameters	1.0	<a href="#">Bayer et al. (2022)</a>
$\kappa_Y$	Slope of NK Phillips curve	0.08	Standard
$\chi$	matching elasticity	0.5	<a href="#">Petrongolo and Pissarides (2001)</a>
$M$	no. bargaining periods	60	<a href="#">Christiano et al. (2016)</a>
$(\psi_G, \psi_B)$	Gov't spending rule	(0.94, -0.75, 0.0)	<a href="#">Bayer et al. (2023)</a>
$\tau^y$	Proportional income taxes	0.25	See text
$\Upsilon_b$	SS UI replacement rate	0.4	<a href="#">Shimer (2005)</a>
$(\rho_R, \theta_\pi, \theta_u)$	Taylor rule parameters	(0.5, 1.5, 0.0)	See text
$(R_{SS}^B, \pi_{SS})$	SS nominal rate & inflation	(1.0, 1.0)	<a href="#">Bayer et al. (2022)</a>

Table 1: Externally set parameters

bargaining can take place is set to  $M = 60$ , the same value as in [Christiano et al. \(2016\)](#): this reflects the typical number of business days within a quarter. We furthermore set  $\chi = 0.5$ , a standard value for the matching elasticity going back to [Petrongolo and Pissarides \(2001\)](#). The slope of the New Keynesian Phillips curve is 0.08, a standard value also used by [Graves \(2021\)](#). Several other parameters governing the economy are set according to [Bayer et al. \(2022\)](#): First of all, we also set the probability of exiting the  $\Xi = 1$  state within a given period to be 6.25%. The investment adjustment cost is chosen to be 3.5 and the ratio  $\delta_2/\delta_1$  set to be 1, reflecting the results of their model estimation. For a given  $\delta_2/\delta_1$ -ratio, we always set  $\delta_1$  and  $\delta_2$  to achieve  $u_t = 1.0$  in steady state. Finally, we also follow them in setting the Central Bank's target gross nominal rate for liquid assets to equal the long-run rate of inflation (i.e. liquid bonds don't yield a net real return) and set the target inflation rate to 1 (i.e. no inflation). The latter constitutes a normalization given the price indexation assumption and implies  $R_{SS}^B = \pi_{SS} = 1$ .

The remaining government policies are parameterized as follows: We follow [Shimer \(2005\)](#) by choosing an unemployment replacement rate of 0.4. For the Taylor rule, our benchmark calibration features a moderate nominal rate persistence of  $\rho_R = 0.5$ .<sup>11</sup> The coefficient on inflation has the “textbook” value  $\theta_\pi = 1.5$  and we chose  $\theta_u = 0.0$ , i.e. the central bank only reacts to inflation as in typical “textbook” models. Furthermore, we set the proportional income taxes

<sup>11</sup>While many authors consider rules with substantially more persistence, e.g.  $\rho_R = 0.8$ , [Consolo and Favero \(2009\)](#) argue this to be inconsistent with the low predictability of monetary policy rates.

$\tau^y$  to 25%: These values are close both to the average US tax rates on labor incomes as well as profits and are consistent with a realistic long-run government consumption-to-GDP ratio of  $G/Y = 0.158$ . Note that since our model does not feature an intensive labor supply margin, introducing tax progressivity would not have any related effects but be purely redistributive. Finally, the policy rule for government consumption follows Bayer et al. (2023) but with a somewhat lower persistence of  $\psi_G = 0.94$ , as our calibration is quarterly instead of monthly.

### 3.2 Internal calibration

Parameter	Description	Value	Target
$\beta$	Time discounting	0.9839	$K/Y = 11.22$
$\zeta$	prob. entrepreneur state	0.0003	Wealth share top 10
$\lambda$	prob. illiquid asset adjustment	0.087	$B/Y = 1.04$
$\bar{R}$	Borrowing penalty	0.044	16% borrower share
$\underline{a}$	Borrowing limit	-0.8809	100 % avg. quart. income
$\varrho$	Energy share parameter	0.0504	SS energy share 5%
$A_m$	Matching efficiency	0.6518	Unemployment rate 5.5%
$\frac{\kappa_1}{p_{ss}} + \kappa_2$	total hiring cost	0.0665	7% of avg. hire wage
$\kappa_2$	screening cost	0.0166	25% of hiring cost
$s$	Individual labor productivity	See Appendix E	See text
$\gamma(s)$	Costs of delay	See Appendix E	See text
$\delta(s)$	Separation rates	See Appendix E	See text

Table 2: Internally calibrated parameters

The remaining parameters are chosen so that the model matches various target moments in the non-stochastic steady state. To clarify how they come about, we present for each parameter the moment we use to identify it. While, if taking other *parameters* as given, any parameter will somewhat affect any of the stationary equilibrium’s target moments, it is often the case that if one assumes other *target moments* to have been realized, individual parameters can be identified just by the respective moments. For the parameters for which this is not true, it nevertheless turns out that achieving a good fit with the target relies mostly on the stated parameters.

Several parameter values are used to target moments related to the long-run wealth distribution: We choose the household discount factor  $\beta$  to match a ratio of average steady state capital holdings to output of 11.22 as in Bayer et al. (2024), resulting in  $\beta = 0.9839$ . As the probability  $\zeta$  determines the amount of “super rich” entrepreneur households, we use it to target a Top 10% wealth share of 67%, which requires a value of approx. 0.0003. The borrowing penalty  $\bar{R}$  determines the share of households with a negative liquid asset position: to get a share of 16%,

we set value of 0.044.<sup>12</sup> Notice that for the parameters above, our targets are the same as in [Bayer et al. \(2024\)](#).  $\lambda$  determines the (il-)liquidity of capital and thus how much liquid bonds wish to additionally hold for consumption smoothing purposes: We use it to target mean liquid asset holdings of 0.26 times annual output as in [Kaplan et al. \(2018\)](#). This requires  $\lambda = 0.087$ . We also follow these authors by setting the borrowing limit equal to the average quarterly labor- and transfer income (post-tax).

$\varrho$ , which governs the importance of energy for production, is set to 0.0504, targeting a GDP share of aggregate energy spending of 5%. This value is roughly consistent with recently observed values for the US (e.g., 5.7% in 2019 and 4.8% in 2020). We choose matching productivity  $A_m = 0.6522$  to achieve an average unemployment rate of 5.5%.

Following [Christiano et al. \(2016\)](#), we target steady state hiring costs  $\kappa_1/p_{SS}^{vf} + \kappa_2$  to be 7% of the average wage of newly hired workers and furthermore assume the screening cost  $\kappa_2$  to account for 25% of this cost. The relative size of this fixed cost is an important determinant of the sensitivity of unemployment to aggregate shocks and we choose it so that the unemployment peak after a TFP shock is of a similar magnitude as in [Broer et al. \(2021\)](#). Finally, it is necessary to set the parameters connected to the individual labor productivity levels  $s$ . To calibrate the values for  $s$  and  $\gamma(s)$ , we build on the literature estimating income processes: In particular, the recent paper by [Braxton et al. \(2021\)](#) estimates a process in which the permanent component  $z_{i,t}$  of log individual income has an AR(1) form with labor market status-specific parameters

$$z_{i,t+1} = \mu_z(e_{it}) + \rho_z z_{i,t} + \sigma_z(z_{i,t}) \varepsilon_{i,t} \quad ,$$

i.e. the drift and the innovation variance of the process depend on whether an individual works or not. [Braxton et al. \(2021\)](#) argue that such a set-up captures on-the-job skill accumulation as well as human capital depreciation during unemployment. Since we wish to account for such phenomena, we use their annual estimates  $\rho_z = 0.94$ ,  $(\mu_z(1), \mu_z(0)) = (0.0038, -0.1472)$  as well as  $(\sigma_z(1), \sigma_z(0)) = (0.2261, 0.4171)$ , transform them into quarterly values and discretize the process onto a grid of 11 points following the methodology outlined in their paper.<sup>13</sup>

This, however, provides us only with a discretized process for household's *labor earnings*, i.e. the wages  $w_t(s_{it})$ , while the calibration requires the primitives determining them. Conveniently, the linearity of bargaining outcome (24) provides an easy way of backing them out: To reduce

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<sup>12</sup>While this value may appear very high (approx. 18% annually), note that the convention in the 2-asset HANK literature is to treat secured borrowing such as mortgage debt as part of household's net illiquid asset stock. Various forms of unsecured borrowing such as credit card debt or overdraft facilities commonly feature borrowing rates of similar magnitude.

<sup>13</sup>For the transformation, we follow an approach similar to [Krueger et al. \(2016\)](#): The persistence of the quarterly process is set to  $\hat{\rho}_z = \rho_z^{1/4}$  and we replicate the cross-sectional variance of the AR(1) processes by setting  $\hat{\sigma}_z(e_{it}) = \frac{1-\hat{\rho}_z^2}{1-\rho_z^2} \sigma_z(e_{it})$ . We adapt the drift so that the quarterly processes have the same mean as the annual one, i.e.  $\hat{\mu}_z(e_{it}) = \frac{1-\hat{\rho}_z}{1-\rho_z} \mu_z(e_{it})$

the number of parameters, we first restrict  $\gamma(s) = \bar{\gamma}h_{ss}s$ , i.e. that a labor agency's costs of delay are proportionally to the revenue generated by the match in steady state. Then, together with a linear rule relating the level of worker outside income to the steady state wage level (27), the steady state match revenue  $h_{ss}s$  necessary to induce the wage levels  $w_t(s_{it})$  can be backed out by solving a linear system. We choose  $\bar{\gamma}$  by targeting a steady state vacancy filling rate of 0.71 (den Haan et al., 2000) and the  $s$  levels themselves are subsequently obtained by using that other target moments provide the steady state level  $h_{ss}$ . The actually realized values for  $s$  and  $\gamma(s)$  are provided in Appendix E.

For the worker outside income, we follow the paper by Christiano et al. (2016) inspiring our bargaining solution by using  $\tilde{b}_t = b_t$ , i.e. impose that the bargaining outside income is equal to the income received during regular unemployment.

Finally, for the separation rates  $\delta(s)$ , we follow Birinci and See (2021) by using the functional form  $\delta(s) = \bar{\delta}\exp(\eta_s(s - 1))$  and choosing  $\bar{\delta}$  to target an average monthly EU flow rate of  $\bar{p}_{SS}^{EU} = 3.5\%$  and an EU flow rate ratio of  $p^{EU}(s_{ns})/p^{EU}(s_1) = 0.2$ . i.e. the richest workers are 5 times less likely to lose their jobs than the poorest workers.<sup>14</sup>

### 3.3 Distributional Moments

In this section, we validate our internal calibration by analyzing various model-generated moments not directly targeted by the calibration.

Table 3 compares various untargeted moments of the model's Steady State income- and wealth distributions with their empirical counterparts as reported by Krueger et al. (2016). The latter are based on the 2006 Panel Survey of Income Dynamics (PSID), with Disposable Income defined as the sum of after-tax earnings plus unemployment benefits plus income generated by assets held. In both model and data, Net Worth relates to both liquid and illiquid assets. While the model can naturally not achieve a perfect fit, it matches both the distributions' quintiles as well as their Gini coefficients well.

Since we are employing a two-asset model, it is not only relevant how closely our framework matches data moments related to the distribution of net worth, but also the asset portfolios held by the households. We do so in Table 4: First, we are considering moments of the illiquid- and liquid wealth distribution separately. In particular, we compare them with statistics reported by Kaplan et al. (2018), who use the 2004 Survey of Consumer Finance (SCF). As in the data, we generate a more unequal distribution of liquid assets and ownership of both asset classes is concentrated in their respective Top 10%, with the bottom 50% holding hardly any. However, compared to the reported data moments, we generate a somewhat more equal asset

<sup>14</sup>Birinci and See (2021) target  $\bar{p}_{SS}^{EU} = 1.2\%$  and  $p^{EU}(s_{ns})/p^{EU}(s_1) = 1/5.54 \approx 0.18$  for a monthly calibration.

distributions, with the shares held by the Top 10% not as high and the share of the Next 40% substantially larger than in the SCF data. It should be noted that this is at least partly due to the targets we adopt: As [Bayer et al. \(2024\)](#), we target a Top 10% net wealth share of 67%, which is the average 1954-2019 in the World Inequality Database but, due to secular trends in inequality, lower than in more recent years.

	Disposable Income		Net Worth	
	Model	Data	Model	Data
Quint. 1	4.5	4.5	0.0	-0.9
Quint. 1	8.4	9.9	1.7	0.8
Quint. 3	13.4	15.3	5.1	4.4
Quint. 4	21.0	22.8	13.0	13.0
Quint. 5	52.6	47.5	79.9	82.5
Gini	0.48	0.42	0.77	0.77

Note: “Data” refers to moments computed by [Krueger et al. \(2016\)](#) using PSID.

Table 3: Distributional moments comparison

Moments	Model	Data (incl. source)
<i>Illiquid asset shares</i>		(from <a href="#">Kaplan et al., 2018</a> )
Top 10%	61.8	70
Next 40%	35.6	27
Bottom 50%	2.6	3
<i>Liquid asset shares</i>		(from <a href="#">Kaplan et al., 2018</a> )
Top 10%	73.2	86
Next 40%	25.4	18
Bottom 50%	1.4	-4
<i>Hand-to-Mouth (HtM) Status</i>		(from <a href="#">Kaplan et al., 2014</a> )
Share HtM	28.8	31.2
Share Wealthy HtM	20.2	19.2
Share Poor HtM	8.6	12.1

Table 4: Portfolio moments comparison

Finally, we also analyze whether households are *Hand-to-Mouth* (HtM) in the sense of [Kaplan et al. \(2014\)](#), i.e. whether their liquid asset holdings are a) positive but amount to less than 2 weeks (1/6 of a model period) of income or b) negative and amount to less than the borrowing



constraint plus the 2 weeks of income. We also classify them as “Wealthy HtM” if they additionally hold illiquid assets and “Poor HtM” if they do not. Our model matches the empirical evidence on the size of either group of agents reasonably well, although we generate slightly too much Wealthy HtM and a little bit too few poor HtM. As we can see in Figure 1, these households close to kinks in their budget set would increase their consumption particularly much if endowed with additional income or liquid wealth. In turn, our model features an average quarterly MPC of 15.4%, implying an average annualized MPC of 40.6%.<sup>15</sup>

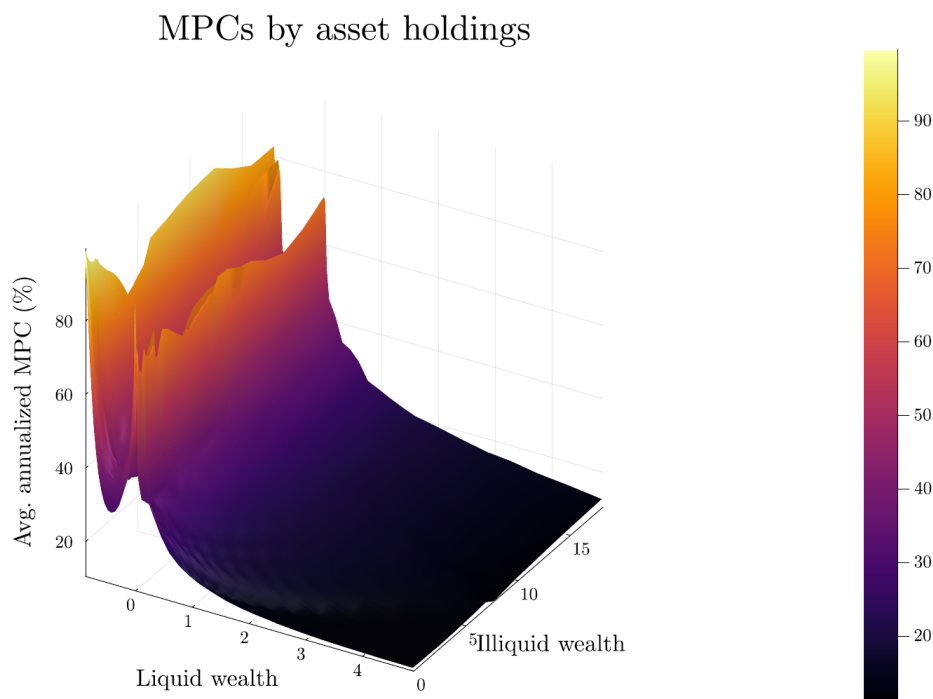


Figure 1: MPCs by wealth holding

## 4 Cyclical amplification from *unemployment fears*

This section analyzes the amplification effects of the unemployment risk-induced precautionary savings motives - the “*unemployment fears*”.

<sup>15</sup>We annualize quarterly MPCs  $qMPC$  as  $aMPC = 1 - (1 - qMPC)^4$  following Carroll et al. (2017). Note that these *annualized* MPCs do not exactly equal the *annual* MPCs. For Figure 1, we average the annualized MPCs of the mass of agents that are at the particular grid points in steady state, resulting in a somewhat “rugged” look due to composition effects.

## 4.1 Isolating the effects of *unemployment fears*

As alluded to above, a key challenge in quantitative HANK settings is to pin down the aggregate effect of the risk-induced demand amplification, given that any type of insurance policy also features *redistributive* in addition to pure *insurance* effects. To overcome this challenge, we consider a version of the model in which policy is unchanged and aggregate dynamics unrestricted, but households' expectation deviate from rational expectations in the following way: Households always believe that their idiosyncratic employment transition probabilities are always fixed at their steady state levels, i.e. don't realize their unemployment risk has changed. Technically, this only involves *not* perturbing the  $p_{t+1}^{UE}$  and  $p_{t+1}^{EU}(s)$  terms entering the Euler equations of the household problem when computing the derivatives for the linearized model solution: The result then provides the first-order solution of the general equilibrium of a model in which workers are "naive" about their individual unemployment risk. Note that said "naive" model will be based on *exactly* the same steady state and *distributional* impacts of any shock will be preserved, e.g. it will still be the case that low income households transit at relatively higher rates into unemployment, but they will not take into account that their risk has gone up for making their consumption-savings decision so that the *precautionary* channel is shut down. The difference the responses of the baseline and the "naive" model can then be seen as the general equilibrium impact of the precautionary *unemployment fears* demand effect.

While other authors have conducted *partial equilibrium* decompositions to separate the effects of precautionary savings motives and realized employment outcomes, e.g. [Harmenberg and Öberg \(2021\)](#) or [Fernandes and Rigato \(2022\)](#), we are, to the best of our knowledge, the first to generalize this simple trick to a *general equilibrium* setting.

## 4.2 *Unemployment fears* after a TFP shock

We are now ready to isolate the equilibrium effects of the *unemployment fears* for a supply side shocks and begin with a standard contractionary TFP shock, normalized to a size of 1% and reverting back according to an AR(1) in logs with persistence 0.9.

Figure 2 displays the the aggregate response to the shock of both the Baseline (blue solid line) and the "naive" model (red dashed line). Overall, the the model results are as expected: Consumption and investment drop in the response to the shock, with the investment response following a hump-shape due to the presence of the aggregate adjustment costs. Unemployment begins to rise substantially, as the AOB bargaining protocol induces only limited real wage adjustments. Inflation increases on impact and remains elevated persistently: This result is due to the upwards pressure increased levels of public debt exert on the "natural" interest rate

in HANK models, an issue studied in detail by Hänsel (2024). We briefly return to this issue below.

Comparing the results for the results of the Baseline and “naive” models, we see that that the *unemployment fears* have the same effects as predicted by simpler analytical models: In the equilibrium with fixed expectations, consumption and real wages drop relatively less, the increase in unemployment is reduced and inflation is higher on impact. Investment is hardly affected, as the “fears” have offsetting effects: While higher idiosyncratic risk calls for a portfolio rebalancing towards liquid assets, they also decrease the real return on liquid assets which counteracts the former effect. Additionally, a lot of the illiquid capital stock is held by richer households who, both in the model and the data, face only limited unemployment risk.

To quantify the actual impact of the “fears”, Figure 3 displays the the difference between the baseline and “naive” model relative to the steady state value of different aggregate aggregate outcomes: At the impact of the shock, the general equilibrium effect of the precautionary motive depresses Consumption by 0.175% of steady state the consumption. The relative impact on investment is substantially smaller, resulting in an overall effect on output in between.

Figure 2: TFP SHOCK AND UNEMPLOYMENT FEARS

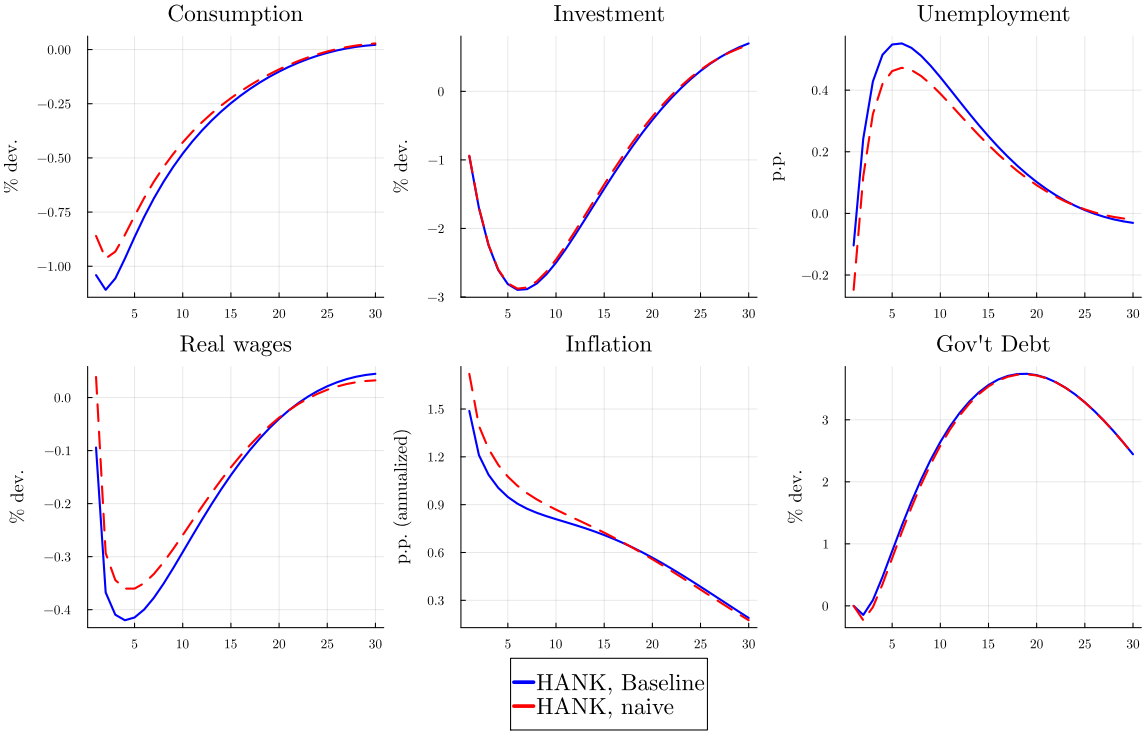
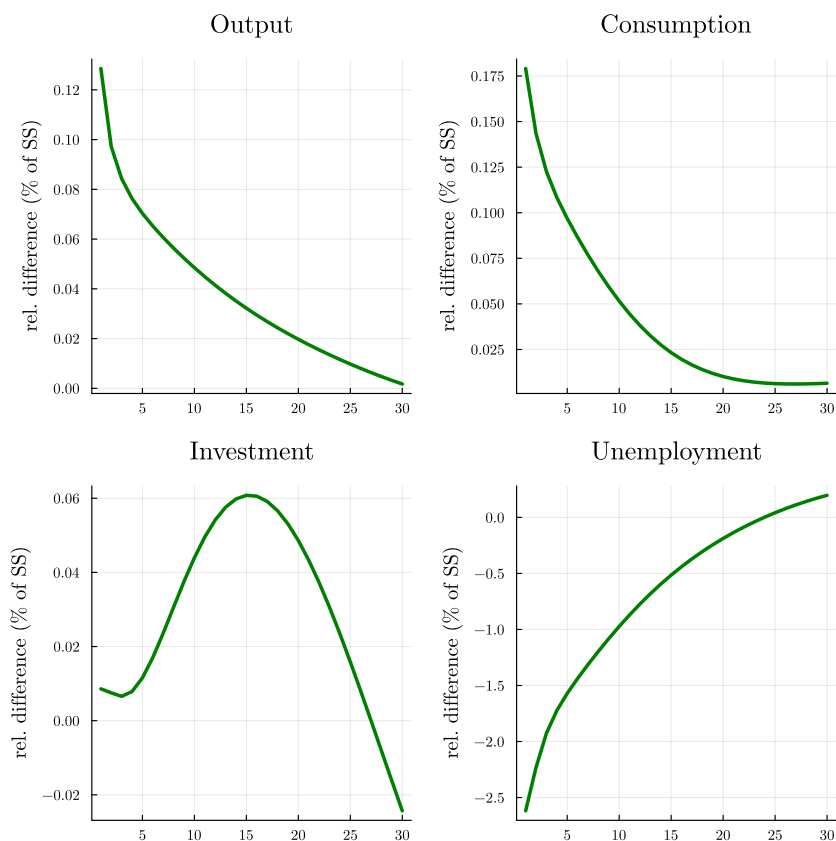


Figure 3: TFP SHOCK: CONTRIBUTION OF UNEMPLOYMENT FEARS



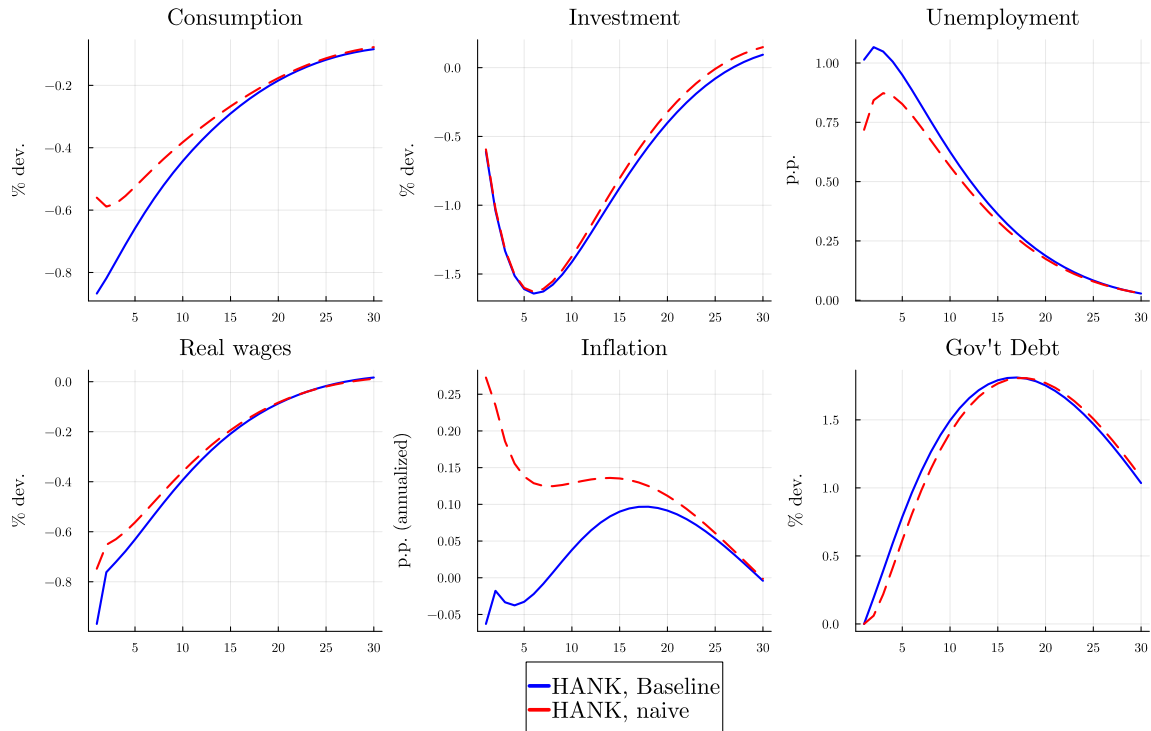
### 4.3 Unemployment fears after a markup shock

We now turn towards analyzing the aggregate response to so-called “cost-push” shock that increases the aggregate markup  $\mu_t$ , again normalized to a size of 1% and assumed to revert back according to an AR(1) in logs with persistence 0.9. As mentioned before, TFP and markup shocks tend to have similar aggregate effects in simple Representative Agents models, even though they have different implications for optimal policy.

However, inspecting Figure 4, we observe that this is not the case for the HANK model. Indeed, in the baseline model the shock actually results in a *decrease* in inflation, defying conventional wisdom on their effects but consistent with the results of “zero-liquidity” Hank-SaM models as e.g. in Challe (2020): So, we observe the “deflationary spirals” emphasized in the previous literature for the “cost-push” but not productivity shocks. Comparing the baseline results with “naive” equilibrium, we also observe a more pronounced impact of the precautionary motive on various other aggregates, in particular consumption. In particular, the quantitative magnitudes displayed in Figure 5 indicate that the relative effect of the “fears” on consumption is larger even though aggregate consumption drops less compared to the TFP shock. While somewhat

more pronounced effects might be expected given unemployment (risk) rises relatively more after the “cost-push” shock, it is puzzling to see this affecting the inflation and real liquid returns so starkly.<sup>16</sup>

Figure 4: MARKUP SHOCKS AND UNEMPLOYMENT FEARS

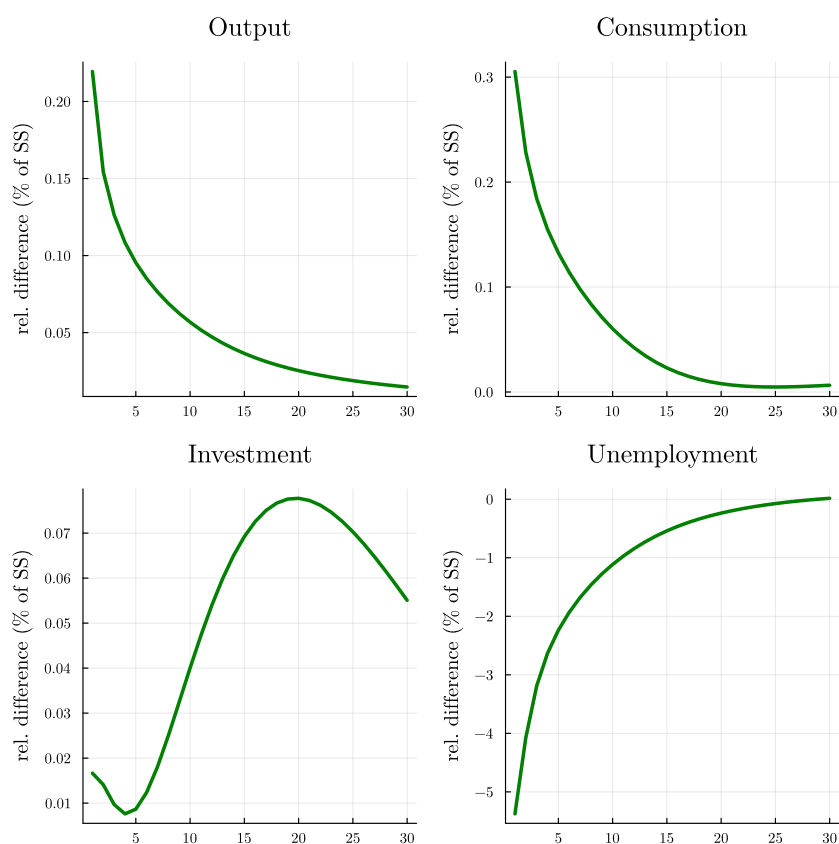


#### 4.4 Deflationary spirals: The role of profits

As demonstrated above, in our model the general equilibrium amplification due to the precautionary savings motive is substantially more pronounced for markup- than for TFP-shocks. While one may be tempted to assign these differences solely to the different unemployment risk, these two scenarios differ in another relevant way: In response to TFP shocks, aggregate profits in the economy go down, while they increase substantially in response to the “cost push” shock. Now, recall that in our baseline model, profits are received by a group of rich “entrepreneurs” that can participate in the asset market and face only limited unemployment risk (if they exit the  $\Xi = 1$  state). Moreover, these entrepreneurs account for a substantial share of overall liquid asset holdings and their savings decision are shaped by intertemporal substitution motives, i.e. they will choose to save less if their income is temporarily low and choose to save more if it is

<sup>16</sup>Recall that the nominal liquid return only depends on inflation due to the CB’s policy rules, i.e. different time paths for inflation translate directly into different paths for nominal and real liquid returns.

Figure 5: MARK-UP SHOCK: CONTRIBUTION OF UNEMPLOYMENT FEARS



temporarily high.

Thus, in response to a TFP shocks, the declining liquid savings demand of the “entrepreneurs” will *counteract* the precautionary motives of the  $\Xi = 0$  households, who want to hold more liquid assets to insure themselves against the resulting higher unemployment risk. The overall liquid saving demand thus increases less, reducing the downward pressure on real interest rates and hence also inflation. The opposite is the case for the markup shock, when not only workers but also  $\Xi = 1$  households would like to save more, *increasing* the downward pressure on real rates and inflation. So, one would expect the overall extent of “unemployment fears” demand amplification to depend on whether the nature of the shock induces the savings motives of workers and “entrepreneurs” to be aligned or not.

To test this hypothesis, we consider yet another model variant that differs from the Baseline as follows: The  $\Xi = 1$  households are assumed to be “super homeproducers” instead of entrepreneurs, i.e. their income consists entirely of home production independent of the state of the business cycle. Additionally, the actual firm profits are transferred to a group of Hand-to-Mouth “entrepreneurs” that cannot participate in any asset market but have to consume any income on the spot. The latter assumption is in line with those made in [Graves \(2021\)](#)

or “tractable” Hank-SaM models such as [Ravn and Sterk \(2021\)](#). Moreover, if the  $\Xi = 1$  household’s home production is assumed to equal their profit income in the Baseline model, the seemingly odd first alternative assumption ensures that this “*alternative profits*” framework will have exactly the same steady state state wealth distribution as the baseline model, making the mechanism at work more transparent and sidestepping the need to for another model calibration.

In [Figure 6](#), we present the model responses of this alternative model variant and its “naive” counterpart to the same shock as in [Section 4.3](#): There is less general amplification of unemployment.<sup>17</sup> But most noticeable, the deflationary pressure induced by the “fears” is reduced substantially. While the shock was able to *overturn* the sign of the initial inflation response in the Baseline model, it merely reduces it by a substantially smaller amount in the “alternative profits” version. This is despite the latter featuring an unemployment response that is still substantially larger than the one in [Section 4.2](#) for the TFP shock, further supporting our conjecture that the unemployment response alone does not drive the differences in “fears”-induced deflationary pressure between [Sections 4.2](#) and [4.3](#). We also confirmed that for TFP shocks, the “alternative profits” model results in *stronger* demand amplification, consistent with the above arguments.

## 4.5 Discussion

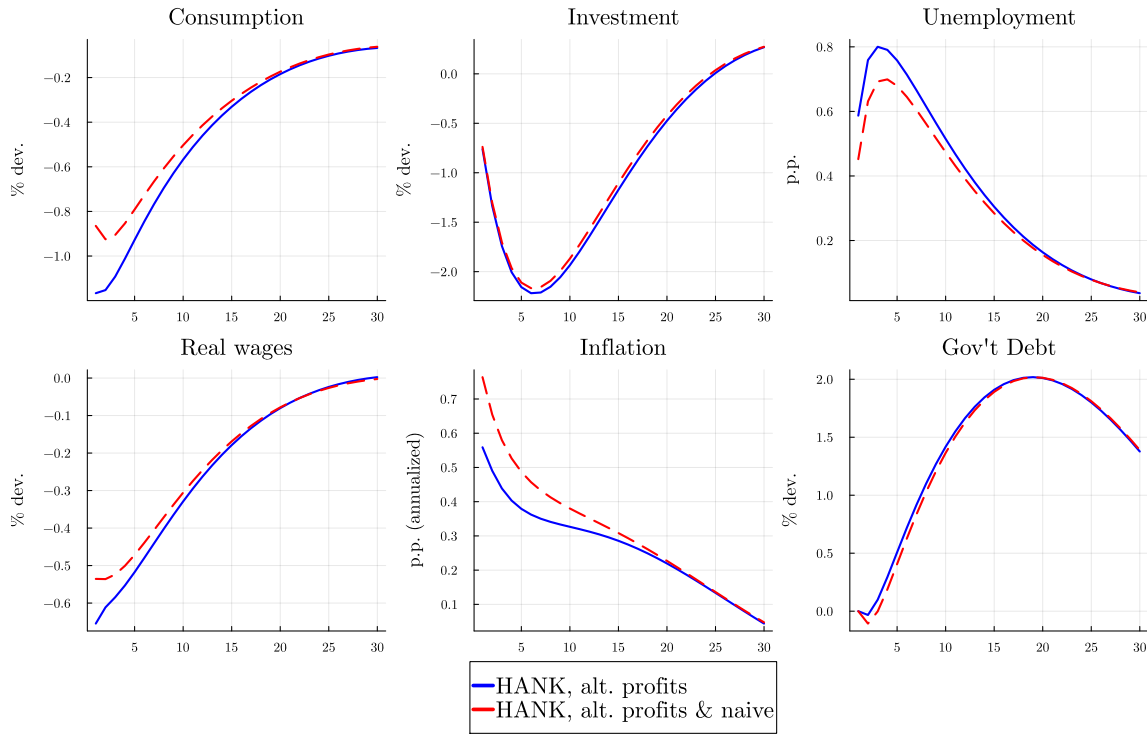
Above, we noticed substantial differences in precautionary demand amplification for different type of supply shocks. In [Section 4.4](#), we demonstrated that these are due to how the model’s profit response shape the savings demand of the upper end of the income distribution. Thus, we establish another important channel through which the distribution of profits can exert influence on the aggregate outcomes of HANK models: This complements the previous work of [Broer et al. \(2020\)](#) who argued profits to be important for cyclical variation in intensive margin labor supply, a channel that our HANK-SaM model abstracts from.

Our results also clarify that previous results as in [Graves \(2021\)](#) or “zero liquidity” models such as [Challe \(2020\)](#) depend importantly on these authors’ specific assumptions regarding which types of agents receive time-varying profits. While we believe our setting in which profits are received by a group of rich households with asset market access to be more plausible than one in which they are paid to Hand-to-Mouth households, it is of course also imperfect: In reality, also households outside the very top of the income distribution receive profits and the income risk of actual entrepreneurs is likely more time-varying: For example, firm level volatility may increase recessions, a channel not captured by our model. Overall, our results thus suggest a

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<sup>17</sup>The former excludes the resources going to HtM “entrepreneurs” and thus drop a bit more on impact

Figure 6: PROFIT ASSUMPTION AND UNEMPLOYMENT FEARS



Note: Under the alternative assumption on profits, they are rebated to Hand-to-Mouth agents not participating in any asset market.

need for richer empirically grounded theories of cyclical profits and their distribution in HANK models.

## 5 Policy responses to energy shocks

In this section we present the quantitative implications of the model focusing on the role of monetary and fiscal policy. We will focus on exogenous shocks to the price of energy in our model, the size of which is normalized to 10%.<sup>18</sup>

First, we show the transmission mechanism of an energy price shock in our baseline HANK specification and we compare it with an equivalent Representative Agent New Keynesian (RANK).<sup>19</sup>

<sup>18</sup>As shown in sub-section, 2.2.3, we work with the energy price being exogenous and unrelated to global demand - i.e. it is not implicitly driven by the domestic output effects on global demand - which would imply different responses to the domestic economy (see [Delle Chiaie et al. \(2022\)](#)).

<sup>19</sup>The RANK model has the same features of our HANK model (capital accumulation, SAM, AOB, etc.) apart from household heterogeneity.



In this way, we would like to highlight the contribution from working with a heterogeneous agent model and its implications for macroeconomic policy. Secondly, we analyze how household’s ability to borrow affects the aggregate response to such shocks: While typically imposed exogenously in HANK models such as ours, in practice borrowing limits depend on policies such as financial regulation or the government’s willingness to enforce private debt contracts (c.f. [Bhandari et al., 2017](#)). Next, we zoom into the role of fiscal policy and to the automatic-stabilising effects of higher public deficit to cover increased unemployment benefit spending and lower tax revenues during an economic downturn. We analyse two cases of budget financing: (i) an increase in public debt which are, over time, undone by expenditure reductions or (ii) via higher taxes proportionally shared across households and entrepreneurs. Finally, we move to monetary policy. In models with equilibrium unemployment and search and matching frictions, simple monetary policy rules that mimic optimal policy include a reaction to the unemployment rate (see [Blanchard and Galí \(2010\)](#); [Abbritti and Consolo \(2022\)](#)). We consider this feature in a HANK model assuming the driver of the business cycle is an energy shock and compare its macroeconomic stabilisation properties as well as the overall welfare impact.

## 5.1 Energy shocks in HANK and RANK

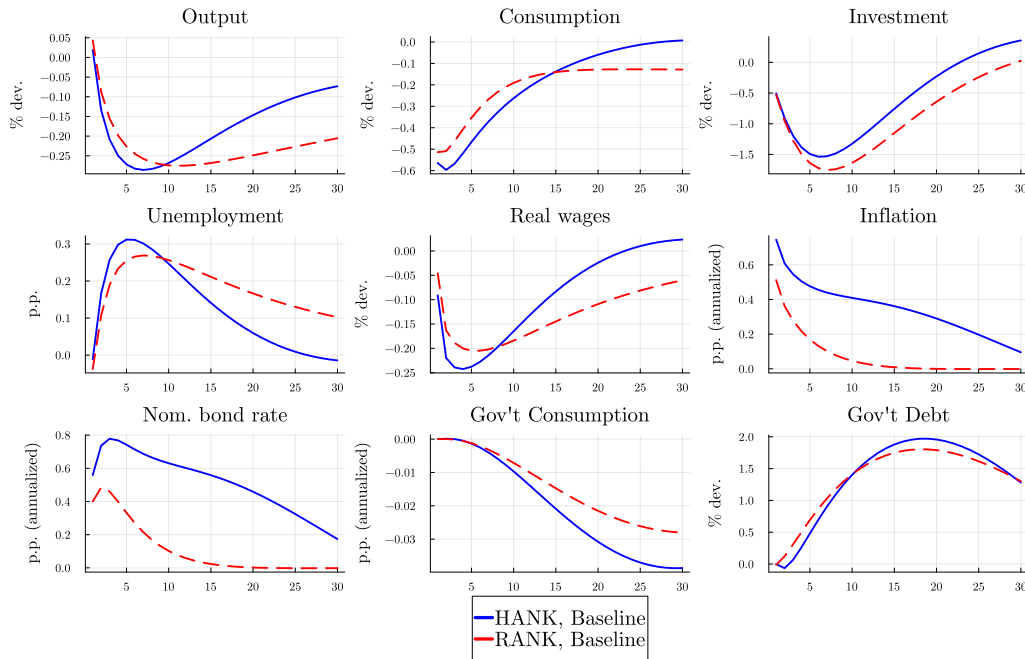
We start with a comparison of the impulse response functions of key variables in a HANK and a RANK model to gauge the differences in the transmission mechanism and highlight some characteristics of the HANK setting. In [Figure 7](#), the impact of a negative energy shock on output, consumption and unemployment is larger under HANK than RANK. The opposite is true for investment and we see a more pronounced and long-lived response of inflation and the nominal rate in HANK. These difference reflect various economic forces present in the incomplete markets setting but not the RANK model: For example, the HANK model features a high average MPC and the “unemployment fears” contributing to the more pronounced consumption drop. Additionally, in HANK models, the public debt increase following the shock induces a persistent rise in the “neutral” rate of the neutral rate of interest. Intuitively, additional government debt crowds out private demand less than in RANK as it is beneficial for risk-sharing and some households behave less forward-looking due to the possibility of binding borrowing constraints. As analyzed in detail by [Hänsel \(2024\)](#), this effect can yield persistently elevated inflation in the presence of a standard Taylor rule.<sup>20</sup> Also, the resulting higher real interest rates on liquid government debt induces a more pronounced increase of public debt, necessitating a stronger spending reduction by the government. Overall, the positive fiscal effects on demand

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<sup>20</sup>As this model follows the standard assumption of *segmented* asset markets for liquid and illiquid assets, this effect is particular pronounced in our setting. Again, see [Hänsel \(2024\)](#) for details.

present in HANK but absent in RANK (which features Ricardian equivalence) accommodate a relatively faster recovery in the former setting.

Figure 7: ENERGY SHOCK: HANK vs RANK

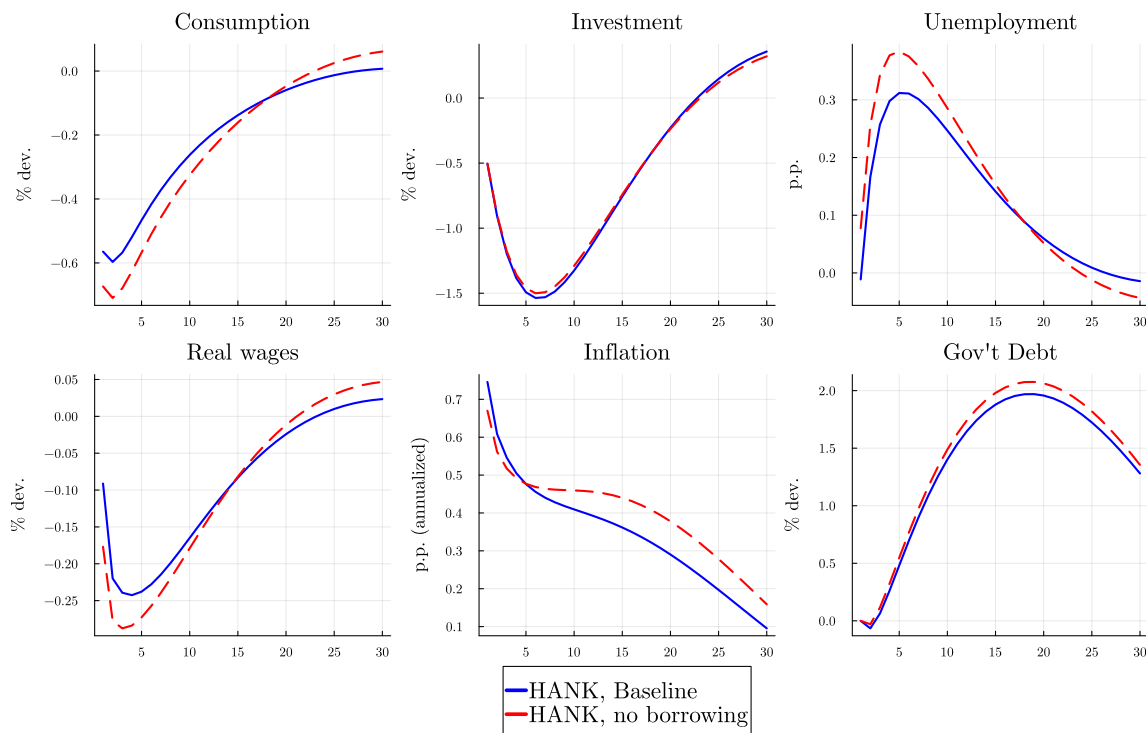


## 5.2 Borrowing over the business cycle

Until now, we have always assumed in the baseline model that individuals have access to borrowing, i.e. they can hold a limited amount of negative liquid assets at a penalty rate. This mechanism helps consumption smoothing and may alleviate precautionary savings-driven demand effects. To gauge the importance of these channels, we solve a version of the model requiring households to hold a non-negative amount of liquid savings. This requires solving for a new steady state, which we restrict to feature the same government debt-to-GDP ratio as the baseline. Figure 8 displays the dynamics of no-borrowing economy, comparing it with the baseline: The absence of borrowing leads to larger fall in aggregate consumption, real wages and unemployment fluctuations, while leaving the investment profile quite unaffected. This is partly due to a composition effect, as the no-borrowing economy features more households close to the now tight borrowing constraint at 0 and higher aggregate MPCs. Investment is affected by different factors which tend to balance each other (see Figure 8). On the one hand, a higher contraction in aggregate demand should lead to lower investment. But as already discussed above, higher unemployment risk exerts downward pressure on liquid returns and thus induces

adjuster households to choose a larger illiquid portfolio share which has a positive impact on private investment. Also, most illiquid assets are held by relatively rich households who face a low risk of hitting the borrowing constraint or becoming unemployed. Thus, we conclude

Figure 8: THE ROLE OF BORROWING CONSTRAINTS



that private borrowing, even if only possible at a penalty rate, provides noticeable aggregate stabilization effects.

### 5.3 Debt- vs tax-financing of budget deficits

In our baseline specification of the HANK model, we assume that the additional fiscal expenditure also related to higher outlays for unemployment benefits is financed by issuing new government bonds. In the subsequent years, debt stabilisation is ensured by a reduction in government consumption (which has no /utility value in our model). Now, Figure 9 compares our baseline specification with a model in which the government keeps debt and its consumption  $G$  fixed, with a balanced budget achieved via higher taxes - in particular,  $\tau^y$  immediately adjusts to clear the budget for constant  $B$  and  $G$ . Such a fiscal policy based on tax financing (red dashed line) results in larger and more persistent negative effects on consumption. While results for investment, unemployment and real wages are approximately similar in the short-term, over

the adjustment period they remain substantially subdued - making the initial transitory energy price shock more persistent. A key difference is the path for price inflation in our HANK model between debt- and tax-financed fiscal policy. With tax financing, inflation is less persistent and converges more rapidly to the inflation target: Again, this is due to the ability of public debt to affect the neutral interest rate in HANK models.

Summing up, the intratemporal redistribution between higher payments for unemployment benefits and debt service and higher taxes on workers and entrepreneurs to balance the budget dampens aggregate demand, output and inflation. As already alluded to above, the impact of debt financing on the inflation process is indeed a feature of incomplete markets HANK models. To highlight the importance of heterogeneity, we compare the inflation responses of the HANK model (with debt- and tax-financing policy) and a RANK model with debt-financing (RANK baseline, green dashed line). The role of debt-financing fiscal policy on inflation is very much visible in the HANK model, but is non-existent in a RANK model whose inflation response resemble the HANK model with tax-financed deficits. The results of this exercise are important in the context of deciding the fiscal-monetary policy mix after the economy is hit by an adverse shock. According to our results, given the substantial impact of fiscal policy can exert on stabilising the economy and increasing inflation persistence, monetary policy should remain attentive and consider the impact of unconventional fiscal policies following global supply chain disruptions and energy shocks when setting interest rates. This could help reducing inflation persistence.

Figure 9: FISCAL POLICY VIA TAX FINANCING OF UI

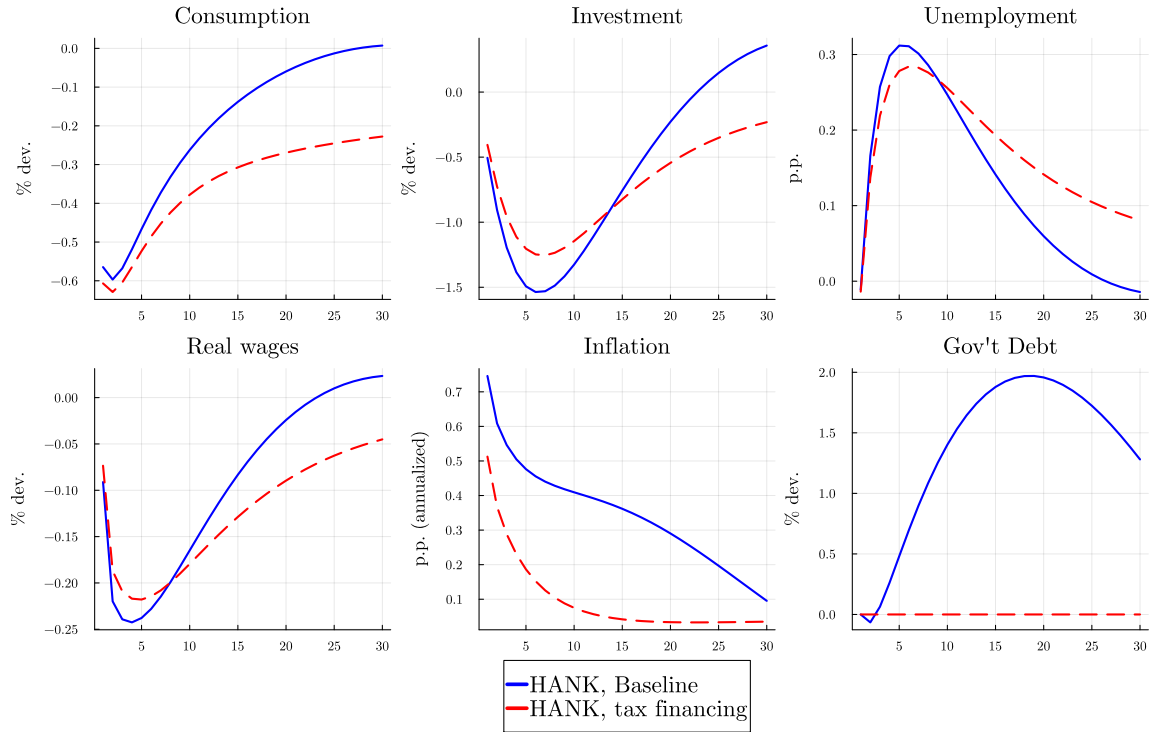
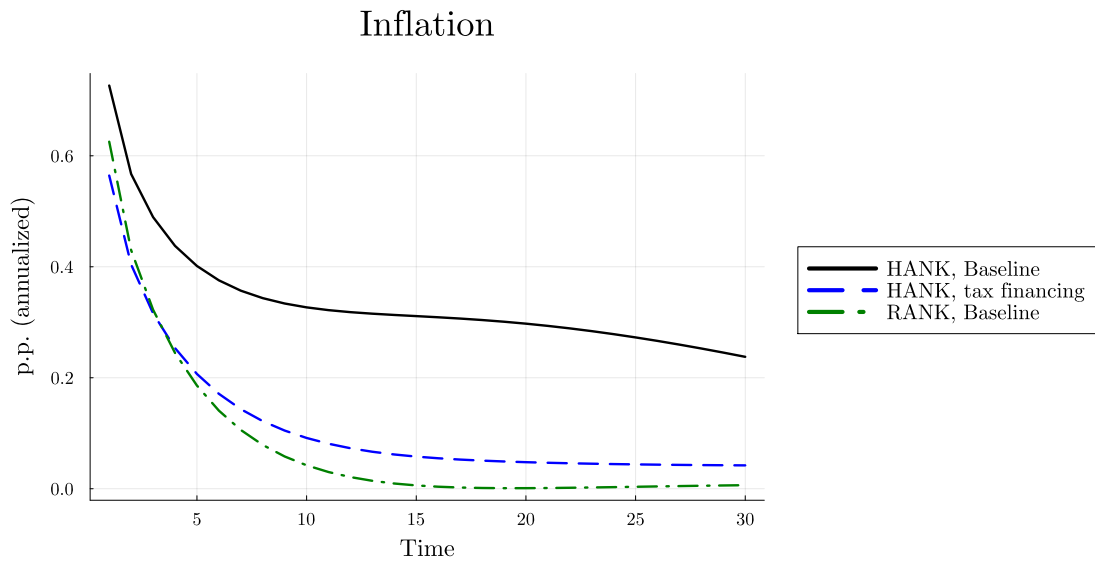


Figure 10: FISCAL POLICY EFFECTS IN HANK AND RANK



## 5.4 Energy shocks, monetary policy and inequality

The impact of large energy shocks such as the recent surge in commodity prices in 2022 has led to unequal and regressive impacts across individuals (Del Canto et al. (2023); Pallotti et al. (2023)). In this section of the paper we look at the role of monetary policy in addressing inequality and supporting inclusion in the labour market. This is somehow complementary to the role of fiscal policy analysed in the previous section. The main idea here is to have monetary policy reacting to the unemployment rate while tolerating a higher or more persistent inflation rate. This is in line with the latest findings on the review of the monetary policy strategy at the Federal Reserve (as well as other major central banks around the world) which have emphasised the importance of running the labour market hot with a view to reducing slack and include marginal workers into the labour market (Alves and Violante, 2023). The Fed has substantially reviewed its framework by introducing an asymmetric average inflation targeting (Clarida (2022)), especially with a view to avoiding another Great Recession. While the main shocks driving the great financial crisis were of an aggregate demand nature, here we will look closely at the trade-off from energy shocks that move inflation and unemployment in the same direction. We focus on what happens to inflation dynamics when monetary policy reacts to unemployment (instead of following a strict inflation targeting), whether such a different reaction function is welfare improving and how monetary-fiscal policy interaction is affected.

Figure 11 shows that, indeed, by responding to the unemployment rate monetary policy can partly smooth the response of consumption and investment while making the labour market tighter. However, this comes at the cost of higher and more persistent inflation. To assess which types of households gain or lose from the alternative policy option, we conduct a simple welfare calculation whether households would prefer the unemployment targeting rule to be implemented after the energy price shock. The exercise is based on a perfect foresight scenario and thus does *not* account for the possibility of other shocks occurring.

Figure 12 visualizes the results, presenting the consumption-equivalent utility gains the unemployment-conscious central bank policy provides households relative to the baseline policy. The focus is on income heterogeneity, plotting the average gains of all agents with the same initial labor market status: Compared to the case of monetary policy implementing a strict inflation targeting rule, welfare gains are positive both for employed and unemployed households along the labor productivity distribution. Perhaps unsurprisingly, initially unemployed households benefit more from the policy stimulating aggregate employment. More interestingly though, we also see that it is not necessarily the poorest households gaining the most: Indeed, agents in the middle of the skill distribution reap the highest welfare gains. This is due to the possibility of skill losses during unemployment: While poor workers face more pronounced job separation risks, the middle class

has more to lose from more likely and longer unemployment spells.<sup>21</sup> These results indicate the modelling of such labor market aspects to be important for assessing the distributional effects of monetary policy.

Overall, only the  $\Xi = 1$  “entrepreneur” households face substantially decreasing welfare. This has two reasons: Firstly, in our model the “entrepreneurs” hold particularly large amounts of nominal assets and are thus more exposed to higher inflation at the impact of the shock. Secondly, the relatively higher inflation under unemployment targeting is accompanied by higher marginal costs and thus relatively lower profit incomes.

We also provide a comparison exercise in our HANK framework regarding the role of accommodative monetary and fiscal policy. We start from a case in which monetary policy follows strict inflation targeting and fiscal policy operates via tax increases in case of an adverse shock that affects government budget (see red dashed line in Figure 13). We then ask ourselves whether it would be better to use fiscal or monetary policy to stabilise the macroeconomy. First we assume that monetary policy responds to the unemployment rate while fiscal policy continues to rely on tax-based financing of the deficit. Figure (13) shows the response when the central bank reaction function responds to the unemployment rate (with a coefficient of  $\theta_u = 0.2$ ). We observe that monetary policy improves short-term responses of consumption, investment, unemployment and real wages, but it is not able to generate a quicker recovery in the medium to longer term. At the same time, the response of inflation is higher in the short-term. Alternatively, we can have fiscal policy using debt to finance the budget while monetary policy follows a strict inflation targeting rule (this is our baseline specification of the HANK model, blue line). In our framework, fiscal policy makes sure the economy returns quicker to the previous steady-state equilibrium, but short-run fluctuations tend to be worse than the ones achieved via monetary policy. Both investment and unemployment become more volatile, while the persistence of price inflation continues to remain broadly in line with the model with monetary policy responding to the unemployment rate.

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<sup>21</sup>In model versions not assuming skill depreciation, we indeed found the agents with the lowest skill to gain most from the policy.

Figure 11: MONETARY POLICY AND UNEMPLOYMENT TARGETING

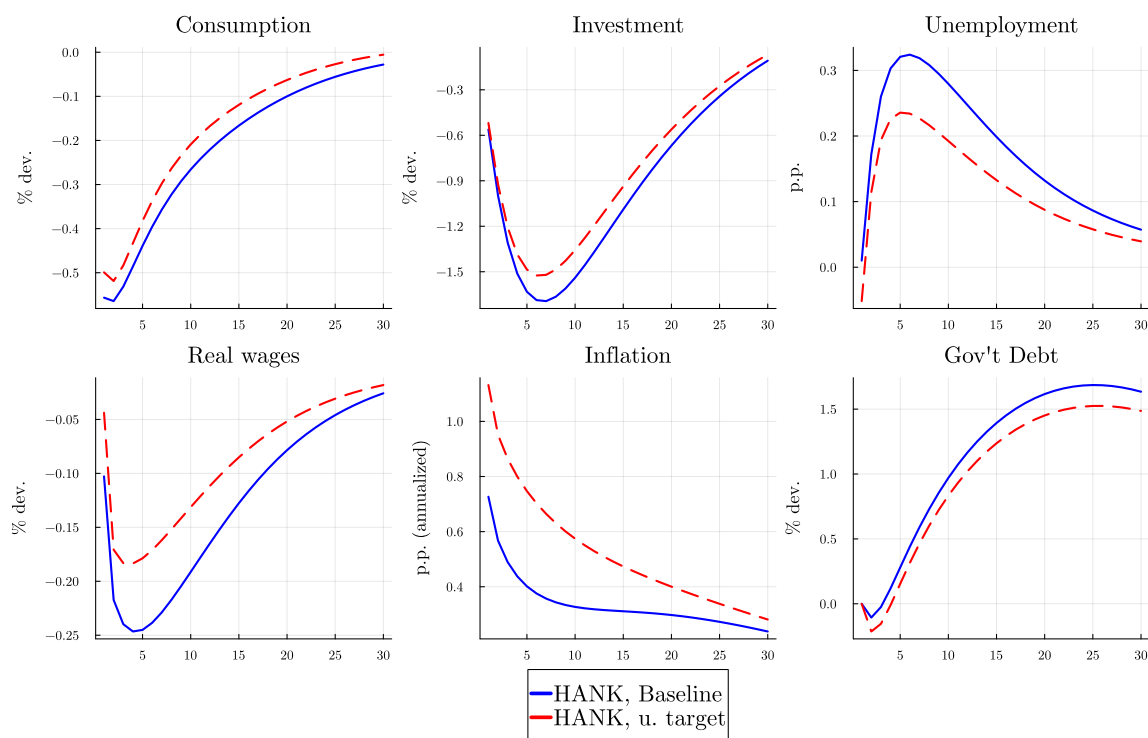


Figure 12: WELFARE GAINS

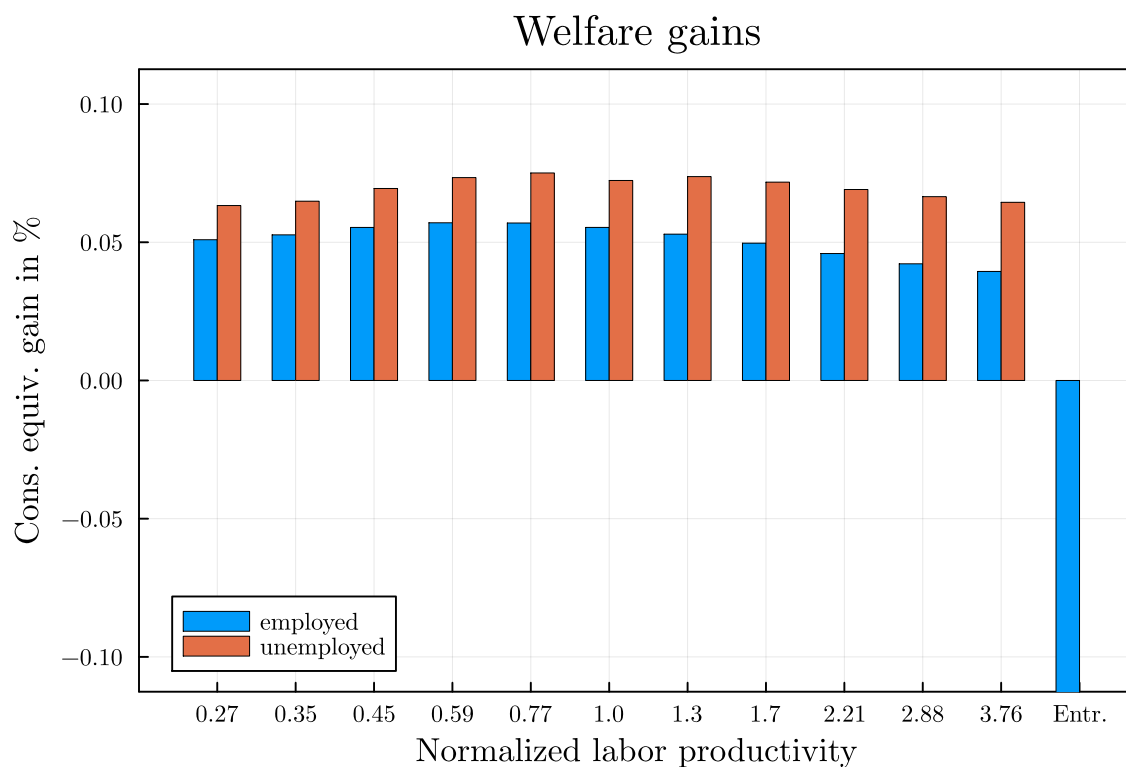
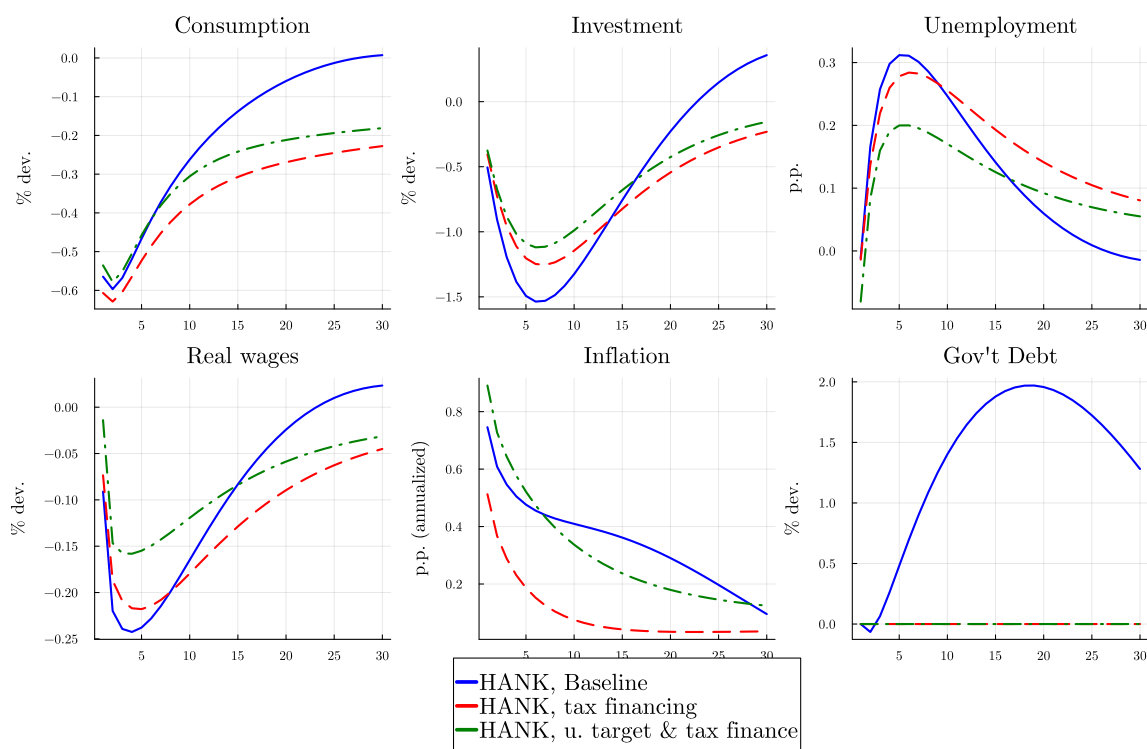




Figure 13: MONETARY VS FISCAL POLICY ACCOMMODATION



## 6 Conclusions

Supply shocks are important drivers of the business cycle and are argued to be amplified by precautionary savings motives. Additionally, the post-pandemic increase in global supply chain disruptions and energy prices has been addressed by a range of macroeconomic policies: Fiscal policy, especially, has reacted strongly and has provided support to households and firms.

To study the relevance of these channels, we develop a two-asset HANK model with search and matching frictions and tackle two important modelling challenges. Firstly, we provide a micro-founded way of modelling wage setting in rich heterogeneous agents models that is closely connected to the existing literature and provides for wage outcomes independent of worker wealth. Secondly, we propose a simple and consistent way to isolate and measure the general equilibrium impact of precautionary “unemployment fears” demand amplification in a complex HANK model.

We find that the precautionary-saving channel stemming from “unemployment fears” is not negligible, but its quantitative impact remains more moderate compared to the findings of previous studies employing “*zero liquidity*” HANK models. However, our results indicate the aggregate amplification to depend importantly on how firm profits are distributed, an insight we consider to be relevant for future work. Additionally, it implies that the way supply shock scenarios are modelled can be particularly important in HANK. Our results also indicate that households’ ability to borrow to dampen an economy’s response to aggregate shocks.

We furthermore use our model to look at how macroeconomic policies affect main macroeconomic aggregates and, in particular, inflation. The supply shocks in 2021 and 2022 were high on the agenda of global policymakers and fiscal policy was actively employed to minimise the adverse effect on the economy and the most vulnerable households. Most of these fiscal measures, including those affecting the labour market such as unemployment benefits, were debt-financed - hence supporting aggregate demand - in an environment in which aggregate supply was constrained. In our HANK framework, we find that this can lead to higher inflation persistence and more volatile investment. However, we also find that to the extent a central bank follows a strict inflation targeting rule, the longer-term effects on consumption, investment, unemployment and real wages are better off in the case of debt-financed fiscal policy.

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## A Derivation of the AOB bargaining outcome

Re-arranging indifference conditions (21) and (22) yields

$$w_{j,\Delta,t}^f(s) = \frac{\tilde{b}_{\Delta,t}(s)}{(M-j+1)} + \frac{M-j}{M-j+1} w_{j+1,\Delta,t}^w(s) \quad (29)$$

and

$$w_{j,\Delta,t}^w(s) = \frac{\gamma_{\Delta}(s) + h_{\Delta,t}s}{M-j+1} + \frac{M-j}{M-j+1} w_{j+1,\Delta,t}^f(s) \quad (30)$$

which we can combine to obtain

$$w_{j,\Delta,t}^w(s) = \frac{\gamma_{\Delta}(s) + h_{\Delta,t}s + \tilde{b}_{\Delta,t}(s)}{M-j+1} + \frac{M-j-1}{M-j+1} w_{j+2,\Delta,t}^w(s) . \quad (31)$$

Iterating (31) forward  $M/2 - 1$  times, we obtain

$$w_{2,\Delta,t}^w(s) = \frac{M-2}{2} \frac{\gamma_{\Delta}(s) + h_{\Delta,t}s + \tilde{b}_{\Delta,t}(s)}{M-1} + \frac{1}{M-1} w_{M,\Delta,t}^w(s)$$

which we can use in (29) for  $j = 1$  to get

$$w_{1,\Delta,t}^f(s) = \tilde{b}_{\Delta,t}(s) + \frac{M-2}{2} \frac{\gamma_{\Delta}(s) + h_{\Delta,t}s + \frac{\tilde{b}_{\Delta,t}(s)}{1-\tau^w}}{M} + \frac{1}{M} w_{M,\Delta,t}^w(s) . \quad (32)$$

Substituting (23) and re-arranging, we obtain the equilibrium subperiod 1 offer extended by the firm

$$w_{1,\Delta,t}^f(s) = \frac{1}{2} \left( h_{\Delta,t}s + \tilde{b}_{\Delta,t}(s) \right) + \frac{M-2}{2M} \gamma_{\Delta}(s) + \frac{1}{M} (1-\zeta)(1-\delta(s_t)) \beta \mathbb{E}_t J(s_{t+1}, \Gamma_t) \quad (33)$$

which will be accepted. In turn, the period wage is this wage times  $M$ , i.e.

$$w_t(s) = \frac{1}{2} \left( h_t s + \tilde{b}_t(s) \right) + \frac{M-2}{2M} \gamma(s) + (1-\zeta)(1-\delta(s_t)) \beta \mathbb{E}_t J(s_{t+1}, \Gamma_t)$$

## B Definition of equilibrium

Below, we define the equilibrium of our baseline model. The definitions for other model version are analogous. For the “naive equilibrium” versions, household value functions and -policies will be consistent with labor market transitions as determined by steady state labor market tightness  $\theta_{SS}$  instead of the labor market tightness consistent with (19).

**Definition 1.** A *Recursive Equilibrium* of our model consists of

- value functions  $V^a(a_{it}, k_{it}, e_{it}, s_{it}, \Psi_{it}; \Gamma_t)$ ,  $V^{na}(a_{it}, k_{it}, e_{it}, s_{it}, \Psi_{it}; \Gamma_t)$  and  $J(s_{it}, \Gamma_t)$ ,

- household policies  $a^a(a_{it}, k_{it}, e_{it}, s_{it}, \Psi_{it}; \Gamma_t)$ ,  $a^{na}(a_{it}, k_{it}, e_{it}, s_{it}, \Psi_{it}; \Gamma_t)$ ,  $k(a_{it}, k_{it}, e_{it}, s_{it}, \Psi_{it}; \Gamma_t)$  and  $c^a(a_{it}, k_{it}, e_{it}, s_{it}, \Psi_{it}; \Gamma_t)$ ,  $c^{na}(a_{it}, k_{it}, e_{it}, s_{it}, \Psi_{it}; \Gamma_t)$ ,
- firm sector policies  $I_t, K_t, H_t, Y_t, E_t, u_t, \theta_t, \Pi_t, y_{jt} \forall j \in [0, 1]$
- prices  $p_t^E, h_t, r_t, q_t, R_t^a, mc_t$
- a wage schedule  $w_t(s) \forall s \in \mathcal{S}$ ,
- government policies  $b_t(s), G_t, B_{t+1}, R_{t+1}^B$ ,
- measures  $m_t(\cdot)$ ,

so that

1. Given prices  $R_t^a, r_t, q_t$ , wage schedule  $w_t(s)$  and profits  $\Pi_t$  as well as labor market tightness  $\theta_t$ , the value functions  $V^a(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t)$ ,  $V^{na}(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t)$  solve the households' Bellman equations in (5) and (6) and  $a(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t)$ ,  $k(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t)$ ,  $c(a_{it}, k_{it}, e_{it}, s_{it}, \Xi_{it}; \Gamma_t)$  are the resulting optimal policy functions.
2.  $y_{jt} \in [0, 1]$  are consistent with demand schedule (8) and final output  $Y_t$  given by (7).
3. Inflation  $\pi_t$  is consistent with Phillips curve (9).
4. Given prices  $p_t^E, h_t, r_t, q_t, mc_t$  and technology shock  $Z_t$  the intermediate goods producers choices  $K_t, E_t, H_t, u_t$  are consistent with optimality conditions (11)-(14).
5. Given price  $q_t$  and technology shock  $Z_t^I$ , the intermediate goods producers choices  $I_t$  are consistent with optimality condition (15).
6. Given prices  $h_t$  and wage schedule  $w_t(s)$ , labor agency value functions  $J(s_{it}, \Gamma_t)$  are consistent with (16).
7. The wage schedule  $w_t(s)$  is consistent with bargaining outcome (24).
8. Labor market tightness  $\theta_t$  is consistent with free-entry condition (19).
9. Given inflation  $\pi_t$  and unemployment  $u_t$ , the monetary authority set  $R_{t+1}^B$  according to (25).
10. Taking the remaining values as given, the government issues debt  $B_{t+1}$  so that (26) holds.
11. The bond market clears, i.e.

$$B_t = \int_{\underline{a}}^{\infty} a_{it} m_t(a_{it}) da_{it} \ .$$

12. Capital market clearing requires, i.e.

$$K_t = \int_0^\infty k_{it} m_t(k_{it}) dk_{it} .$$

13. The market for investment good clears, i.e.

$$K_{t+1} = (1 - \delta(u_t))K_t + Z_t^I \left[ 1 - \frac{\phi}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t$$

14. The market for labor services clears, i.e.

$$H_t = \sum_{s \in \mathcal{S}} s m_t^e(s) .$$

15. The market for intermediate goods clears, i.e.

$$\int_0^1 y_t(j) dj = F_t(u_t K_t, H_t, E_t)$$

16. The final good market clears, i.e.

$$Y_t = C_t + G_t + I_t + \frac{\phi}{2} \left[ \frac{I_t}{I_{t-1}} - 1 \right]^2 + \kappa V_t + \bar{R} \int_{\underline{a}}^0 a_{it} m_t(a_{it}) da_{it} + \frac{E_t}{Z_t^E} .$$

17. The dynamics of measures  $m_t(\cdot)$  is consistent as described in Appendix C

## C Details on measures $m$

Formally,  $m_t$  describes a probability measure on the measurable space  $(\mathcal{X}, \mathcal{A})$ , with  $\mathcal{X} := [\underline{a}, \infty) \times \mathbb{R}_+ \times [0, 1] \times \mathcal{S} \times [0, 1]$  and  $\mathcal{A} := \mathbf{B}([\underline{a}, \infty)) \times \mathbf{B}(\mathbb{R}_+) \times \mathbf{P}([0, 1]) \times \mathbf{P}(\mathcal{S}) \times \mathbf{P}([0, 1])$ , where  $\mathbf{P}(\cdot)$  denotes the power set and  $\mathbf{B}(\cdot)$  the Borel  $\sigma$ -algebra of a given set.

Practically, with some abuse of notation, we have  $m_t(\cdot)$  describe the masses of households in a particular state at the beginning at period  $t$ , i.e.  $m_t(a = a_i, k = k_i, e = e_i, s = s_i, \Xi = \Xi_i)$  is the mass of households with assets  $a_i, k_i$ , employment status  $e_i$ , skill  $s_i$  and “entrepreneur status”  $\Xi_i$ . For ease of notation above, we suppress states that are fully integrated over, e.g.

$$m_t(a = a_i, e = e_i) = \sum_{\Xi_i \in \{0,1\}} \sum_{s_i \in \mathcal{S}} \int_0^\infty m_t(a = a_i, k = k_i, e = e_i, s = s_i, \Xi = \Xi_i) dk_i \quad (34)$$

denotes the mass of households with employment status  $e_i$  and bond holdings  $a_i$ . Additionally, we suppress the annotation of non-suppressed inputs whenever it does not cause any confusion,



i.e. we may write  $m_t(e_i) = m_t(e = e_i)$ .

Naturally, to be consistent with a unit mass of households, we require

$$\sum_{e_i \in \{0,1\}} \sum_{\Xi_i \in \{0,1\}} \sum_{s_i \in \mathcal{S}} \int_0^\infty \int_{\underline{a}}^\infty m_t(a = a_i, k = k_i, e = e_i, s = s_i, \Xi = \Xi_i) da_i dk_i = 1 \quad .$$

Additionally, the evolution of measures also need to be consistent with household choices. Defining

$$\begin{aligned} \tilde{X}^{na}(a', k, e, s, \Psi; \Gamma_t) &:= \{a \in [\underline{a}, \infty) : a^{na}(a, k_{it}, e_{it}, s_{it}, \Psi_{it}; \Gamma_t) = a'\} \\ \tilde{X}^a(\{a', k'\}, e, s, \Psi; \Gamma_t) &:= \{\{a, k\} \in [\underline{a}, \infty) \times \mathbb{R}_+ : a^a(a, k, s, \Psi; \Gamma_t) = a' \text{ and} \\ &\quad k(a, k, e, s, \Psi; \Gamma_t) = k'\} \end{aligned}$$

as well as the “middle of period” measure  $\tilde{m}_t(a, k, e, s, \Xi)$  fulfilling

$$\begin{aligned} \tilde{m}_t(a, k, e = 1, s, \Xi = 0) &= p_t^{UE} m_t(a, k, e = 0, s, \Xi = 0) + (1 - \delta(s) + \delta(s)p_t^{UE}) m_t(a, k, e = 1, s, \Xi = 0) \\ \tilde{m}_t(a, k, e = 0, s, \Xi = 0) &= (1 - p_t^{UE}) m_t(a, k, e = 0, s, \Xi = 0) + \delta(s)(1 - p_t^{UE}) m_t(a, k, e = 1, s, \Xi = 0) \\ \tilde{m}_t(a, k, e, s, \Xi = 1) &= m_t(a, k, e, s, \Xi = 1) \end{aligned}$$

means they must follow

$$\begin{aligned} m_{t+1}(a, k, e, s, \psi = 0) &= \\ (1 - \zeta) \sum_{s_t \in \mathcal{S}} \Pi^s(s, |s_t, e) &\left( \lambda \int_{\tilde{X}^a(\{a,k\}, e, s_t, \Xi=0; \Gamma_t)} d\tilde{m}_t(a_{it}, k_{it}, s_t, \Xi_t = 0) \right. \\ &+ (1 - \lambda) \int_{\tilde{X}^{na}(a,k,e,s_t,\Xi=0; \Gamma_t)} d\tilde{m}_t(a_{it}, k, s_t, \Xi_t = 0) \left. \right) \\ &+ \iota p_s \left( \lambda \int_{\tilde{X}^a(\{a,k\}, e, s, \Xi=1; \Gamma_t)} d\tilde{m}_t(a_{it}, k_{it}, \Xi_t = 1) + (1 - \lambda) \int_{\tilde{X}^{na}(a,k,e,s,\Xi=1; \Gamma_t)} d\tilde{m}_t(a_{it}, k, \Xi_t = 1) \right) \end{aligned}$$

and

$$\begin{aligned} m_{t+1}(a, k, e = 0, \psi = 1) &= \\ \zeta &\left( \lambda \int_{\tilde{X}^a(\{a,k\}, e, s, \Xi=0; \Gamma_t)} d\tilde{m}_t(a_{it}, k_{it}, \Xi_t = 0) + (1 - \lambda) \int_{\tilde{X}^{na}(a,k,e,s,\Xi=0; \Gamma_t)} d\tilde{m}_t(a_{it}, k, \Xi_t = 0) \right) \\ &+ (1 - \iota) \left( \lambda \int_{\tilde{X}^a(\{a,k\}, e, s, \Xi=1; \Gamma_t)} d\tilde{m}_t(a_{it}, k_{it}, \Xi_t = 1) + (1 - \lambda) \int_{\tilde{X}^{na}(a,k,e,s,\Xi=1; \Gamma_t)} d\tilde{m}_t(a_{it}, k, \Xi_t = 1) \right) \end{aligned}$$

Finally measures  $m_t^e, m_t^u$  and  $m_t^\Psi$  will fulfill

$$m_t^e = \sum_{s \in \mathcal{S}} [(1 - \delta(s) + \delta(s)p_t^{UE}) m_t(e = 1, s) + p_t^{UE} m_t(e = 0, s)]$$

as well as

$$m_t^u = \sum_{s \in \mathcal{S}} [\delta(s)(1 - p_t^{UE}) m_t(e = 1, s) + (1 - p_t^{UE}) m_t(e = 0, s)] \quad .$$

## D Details on numerical implementation

The household problem needs to be solved on a discretization of the state space: We choose 70 grid points for both  $a$  and  $k$ , either of which are non-linearly spaced as household decision functions tend to be more non-linear for lower levels of assets. In particular, the grid points for both  $a$  for  $k$  are spaced according to the “double exponential” rule, i.e.

$$\mathcal{X} = x_{min} + \exp(\exp(\mathbf{u}(0, x_{max}))) - 1$$

where  $x_{min}$  is the minimum value on the grid for variable  $x$ ,  $x_{max}$  the maximum value and  $\mathbf{u}(0, x_{max})$  a vector of equidistant points on the interval  $[0, x_{max}]$ . Since household value- and policy functions will feature an additional kink around  $a = 0$  when the borrowing penalty kicks in, we add 5 additional grid points in the immediate vicinity of that point. Given that individual labor productivity is discretized to 11 points, this means that the household problem is solved on a tensor grid of  $70 \times 70 \times (2 \times 11 + 1) = 112700$  points (the “entrepreneur” status adds an additional “income” state to the  $2 \times 11$  for employed and unemployed workers). The discretization of the individual labor productivity process is described in the main text, Section 3. Whenever interpolation is needed off the grid, we use linear interpolation.

For the implementation of the multidimensional EGM algorithm, we follow the replication codes for Bayer et al. (2020) closely.<sup>22</sup> Given the random illiquid asset adjustment, the EGM scheme only iterates over marginal value functions (i.e. the derivatives of  $V$  with respect to  $m$  and  $k$ ) and does not compute  $V$  directly.

Solving for the stationary steady state is relatively easy due to the fixed central bank nominal rate- and inflation targets: As these central bank policies targets effectively fix the steady state return on liquid assets, finding the stationary equilibrium can be reduced to a root-finding problem for a single variable, the steady state return on capital.

## E Additional model parameters

The model parameters not explicitly stated in Section 3 are provided in Table 5.

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<sup>22</sup>As of March 2024, these replication codes are available under <https://github.com/BASEforHANK/BASEtoolbox.jl>.

	$s$	$\gamma(s)$	$\delta(s)$
$s_1$	0.1296	0.1283	0.1269
$s_2$	0.1692	0.1674	0.1223
$s_3$	0.2205	0.2182	0.1165
$s_4$	0.2874	0.2844	0.1094
$s_5$	0.3744	0.3705	0.1007
$s_6$	0.4879	0.4828	0.0905
$s_7$	0.6357	0.6290	0.0787
$s_8$	0.8281	0.8193	0.0656
$s_9$	1.0786	1.0673	0.0517
$s_{10}$	1.4050	1.3902	0.0380
$s_{11}$	1.8341	1.8148	0.0254

Table 5: Skill-specific parameters

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