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Losers amongst the losers: the welfare effects of the Great Recession across cohorts

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Abstract

This paper studies the effect of deep recessions on intergenerational inequality by quantifying the welfare effects on households at different phases of the life cycle. Deep recessionary episodes are characterized by large declines in the prices of real and financial assets and in employment. The former levies high welfare costs on older households who own financial wealth, the latter determines labour income losses and destroys the human capital of younger cohorts, lowering their productivity. The paper extends previous analyses in the literature by including permanent labour income losses in an OLG model calibrated to match the Great Recession. The analysis shows that younger households lose more than double of all other living cohorts, as younger household become unemployed and experience a decline in their future income. The dynamics of households' consumption and portfolio composition between 2007 and 2013 in the US are consistent with the predictions of the model.

JEL: E21, D31, D58, D63, D91

Keywords: Overlapping Generations, Inequality, Aggregate Risk, Youth Unemployment, Portfolio choice

Non-technical summary

Households of different ages are hit by economic downturns through different channels. At the beginning of the life-cycle households earn labour income and have a “human wealth”, i.e. the expected sum of future labour income, against which they can borrow and they do not owe financial assets. At the opposite, older households rely mainly on financial assets and don’t have neither labour income nor human wealth. Recessionary episodes are characterized by a fall of financial markets that hurts mainly older households and a huge increase of unemployment that hits active workers and usually levies an higher burden on younger cohorts. This paper uses an overlapping generations model calibrated over the Great Recession episode in the US to study how this two channels interacts for households at different ages and quantify the welfare costs of deep recessionary episodes for different cohorts.

In previous research it has been argued that younger cohorts are the least affected and may also benefit from economic downturns because they can start to cumulate risky assets (housing and stocks) that are relatively cheap by getting indebted with credit at low rates. We extend the existing literature by incorporating into our analysis the permanent losses coming from unemployment. Several empirical works have found that workers who suffered a lay-off have a lower labour income than a counterfactual worker who did not experienced a lay-off even several years after the unemployment spell. They have found that the loss is higher if the lay-off happened during a recession. This empirical result has been justified by the theoretical literature with human capital formation: during the time spent unemployed the worker misses the opportunity to increase her experience, becoming more productive and therefore earning an higher wage in the subsequent periods. What is more, a recession increases the length of unemployment spells and therefore the losses on future income are higher.

We find that younger households, who become active during an economic downturn, are the most severely hit by the recession. Their welfare losses are more than double in magnitude than any other cohort and that this result is mainly driven by the permanent losses of unemployment. There are several channels at work that determine this result.

Firstly, the analysis predicts a below-potential output for several years after the recession as a consequence of the loss in human capital experienced by living cohorts that reduce the amount of effective labour supply while they are active workers. Despite younger households are those that will spend more time in a stagnant economy, the quantitative analysis reveals that this channel plays a minor role in explaining their higher welfare loss.

On the contrary, the effects of unemployment on labour income and especially on human wealth are the most important factors in explaining the difference in welfare loss among cohorts. In particular, as a consequence of the Great Recession households' in their 20s experienced a loss in human wealth 25% higher than those of any other cohort in the model. These are the first-round losses, i.e. those arising directly from the calibration of the model.

The second effect is induced by the behaviour of individuals: the loss in human wealth suffered by young households increases their degree of risk-aversion and as a result they buy a lower share of risky assets and they are less leveraged than a counterfactual cohort born in normal times. This implication explain the difference between our result and the existing literature. Indeed, we show that their results depends crucially on the absence of long-term effects from unemployment.

Then we test the external validity of results by comparing predictions on consumption and portfolio choice of the model with actual data. Consumption data disaggregated by age groups show a reduction in consumption share of households in their twenties between 2007 and 2010 of a magnitude in line with model predictions. With respect to portfolio choices the model predicts that during a downturn young households, compared with their counterparts in normal times, have a lower share of risky asset and a lower leverage. Portfolio data for households exhibits the predicted pattern.

From a policy perspective our work has two main implications. Firstly, since labour market disruption is the main driver of welfare losses during a recessionary episode any policy intervention should be focused on reduction of unemployment and particularly on youth unemployment. Secondly, since our analysis reveals that future cohorts have a small welfare gain from the negative shock that hits living cohorts it is optimal to increase the debt burden to alleviate the welfare losses of those cohorts that were alive during the recessionary episode.

"The actual, unacceptable level of youth unemployment [...] conflicts with any notion of fairness, is the greatest waste of resources, inflicts damage on human capital, affects the potential of economies, pushing down growth for years to come."

— Mario Draghi, November 12^{th} 2014

"The legacies of the Great Recession are many and multifaceted; they not only affect current cyclical developments, but may also have permanent bearings on our economies. [...] Higher unemployment (or under-employment) may have had an impact extending well beyond the current cycle."

— Ignazio Visco, September 23^{rd} 2014

1 Introduction

Since the Great Financial Crisis there has been an increasing debate on which are the segments of the population that are most severely affected by deep recessions. The answer to this question is becoming more and more compelling. Indeed the elevated public debt burden has reduced the fiscal policy space in many western countries and the available resources should be concentrated to alleviate the welfare costs of those that suffer the most. This paper studies the welfare effects of an aggregate shock on households at different life-cycle phases.

Individuals at the beginning of their lives do not have financial wealth but only "human wealth" (Figure 1, left panel), i.e. the present discounted value (PDV) of the stream of future labour incomes. As a consequence, they can be negatively affected by a fall in employment (Figure 1, right panel) and the long-term effects associated to youth unemployment. On the other hand, older cohorts have accumulated assets throughout their lives but their human wealth is almost nil. They can be affected by the fall in asset prices (Figure 2). How these two channels affect different cohorts? Who are the "losers amongst the losers"? This paper studies the welfare effects of an aggregate shock on households at different life-cycle phases.

Kiyotaki et al. (2011) have highlighted that a recession creates a window of opportunity for young cohorts that can buy assets at a low price from older cohorts by leveraging with cheap credit and, in turn, they can enjoy a strong increase in their wealth in the future. Glover et al. (2020) use a calibrated OLG model to assess the intergenerational redistribution that took place during the Great Recession episode and found that higher labour income losses suffered by young generations would have been partially offset by "financial gains" while older households suffered the most from the recession. According to estimates in Glover et al. (2020) households, where the head is above 70 years old, experience significant losses from the

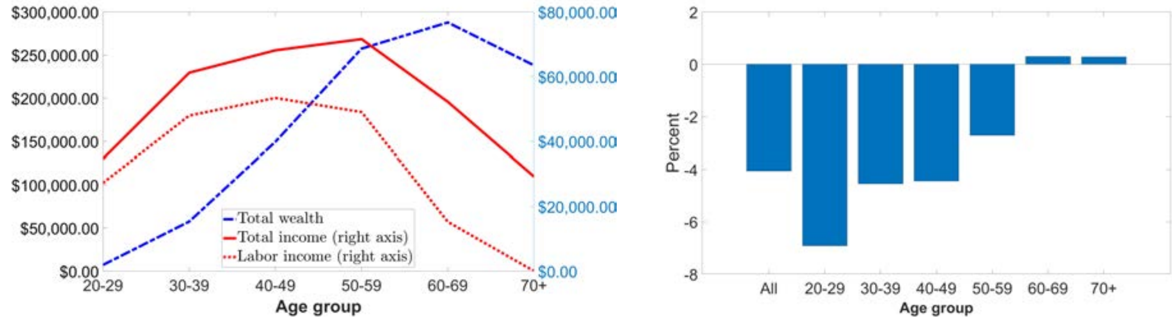

Figure 1: Life cycle profile of total wealth, labour income (right axis) and total income (right axis) computed with intra-cohort median values (left panel, source: Survey of Consumer Finances, 2007) and employment-population ratio change by age group 2007:2-2010:2 (right panel, source: author's elaboration of U.S. Bureau of Labor Statistics and Census Bureau data)

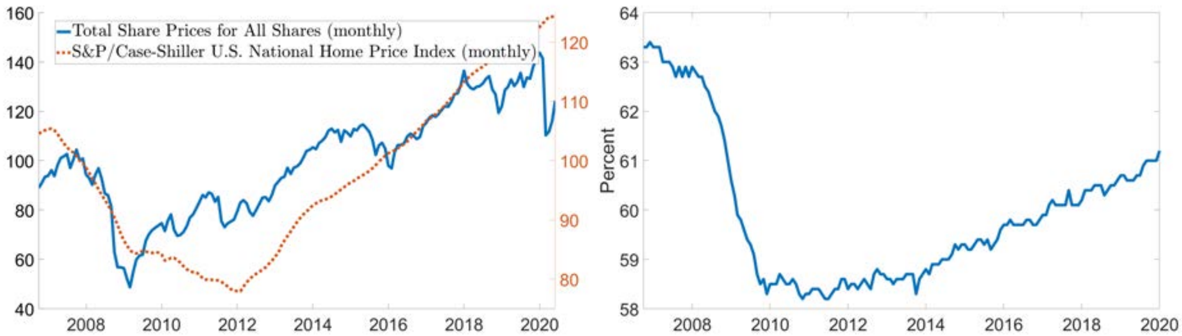

Figure 2: The effects of the Great Recession on stocks, housing prices (left panel, index scale, normalized to 100 on November 2007, source: FRED, Federal Reserve Bank of St. Louis) and civilian Employment-Population Ratio (right panel, source: U.S. Bureau of Labor Statistic).

Great Recession. By contrast, younger households who become active during a downturn suffer relatively less and, under some specific conditions, they may even enjoy net welfare gains if compared with those that become active in normal times.

Nonetheless, a crucial hypothesis for the above results is that the loss in labour income arising from the recession episode is temporary. In particular, they assume that unemployment impacts human wealth only through labour income missed during the recession and that it has no effect on the future stream of wages and employment. However, several empirical works (Ellwood, 1979; Jacobson et al., 1993; Gregg and Tominey, 2005; Bell and Blanchflower, 2011; Davis and von Wachter, 2011; Jarosch, 2014 among the others) have shown that earning losses from unemployment are persistent: a worker that suffers layoff and/or unemployment has, *ceteris paribus*, lower labour income even after decades. Theory suggests that the "permanent scar" on the stream of future labour income can be explained by different factors, including less experience and on-the-job training, and loss in firm specific human capital caused by the displacement.

The first channel is more relevant during severe economic downturns, like the Great Recession, due to higher duration of unemployment spell while the amplitude of the second effect is independent from the state of the economy.

This paper extends the previous analyses by taking into account the permanent losses in earnings arising from unemployment. Specifically, we extend the framework developed by Glover et al. (2020) to incorporate a labour market friction and human capital that cumulates through on-the-job experience. The labour provided by each cohort is determined exogenously: labour market is characterized by an entry friction that affects different cohorts in an heterogeneous manner and becomes less or more tight according to the state of the economy. Human capital affects risk-taking: a decrease in human capital by lowering the expected labour income in future periods makes the households poorer and therefore, with standard CRRA preferences, it increases the risk aversion of households.

The model is calibrated to match both macro and micro data. With respect to macro data the model matches the main empirical moments at the aggregate level observed during the Great Recession: the fall in GDP, the decrease in employment, the fall in the stock market. Long-term losses from unemployment for households of different age groups are calibrated using micro estimates from the empirical literature, and specifically those of Davis and von Wachter (2011). With these calibration the model exhibits a good fit of the fall in income of each cohort induced by the recession, a moment not explicitly targeted. The model is then used to compute the welfare loss of each cohort in terms of life cycle consumption equivalent compared with a counterfactual cohort that never experienced a recession.

The analysis reveals that households in their twenties are the most affected in a severe economic downturn as the Great Recession. Their welfare loss is more than double in magnitude than those of other living cohorts and amounts to around 23% of one-period consumption, that is the amount of consumption that they require over ten years (the length of our period in the model) to be indifferent with a counterfactual cohort that did not experienced the recession.

The quantitative model is then used to understand which channel is more relevant in explaining the difference in welfare losses among cohorts. The first channel involves a below-potential output for several years after the recession as a consequence of the loss in human capital experienced by living cohorts that reduces the amount of effective labour supply while they are active workers. Despite younger households are those that will spend most of their life-time in an economy with a lower output than the counterfactual, the quantitative analysis reveals that this channel plays a minor role in explaining their larger welfare loss. On the contrary, the effects of unemployment on labour income and especially on human wealth are the most important factors in explaining the difference in welfare loss among cohorts. In particular, as a consequence of the Great Recession, households' in their 20s experienced a loss in human wealth of 17%, a quarter higher than those of any other cohort in the model. These are the first-round losses, i.e. those arising from the partial-equilibrium effects. The second channel works through the behaviour of individuals: the loss in human wealth suffered by young households increases their degree of risk-aversion and , in turn, their purchases of risky assets and leverage with respect to a counterfactual cohort born in normal times. These results are opposed to those of Glover et al. (2020) on the Great Recession , which we show depend crucially on the absence of long-term effects from unemployment. Therefore gains identified by the theoretical work of Kiyotaki et al. (2011) fades away if permanent losses from unemployment are included into the model.

Having studied the working of the model and having assessed the channels of transmission of recessions, we test the validity of our results by comparing the predictions of the model for consumption and portfolio choices with actual data. Consumption data disaggregated by age groups from *Consumption Expenditure Survey* (CE) and *Panel Study of Income Dynamics* (PSID) show a reduction in consumption share of households in their twenties between 2007 and 2010 of a magnitude in line with model predictions. With respect to portfolio choices, the model predicts that during a downturn young households, compared with their counterparts in normal times, have a lower share of risky asset and a lower leverage. Data from *Survey of Consumer Finances* (SCF) exhibits the pattern predicted by the model.

This work contributes to welfare analysis of the severe recessionary episodes as the Great Recession. With respect to the analysis of Glover et al. (2020), our contribution is to include permanent income losses from unemployment. This channel proves to be very relevant for a proper assessment of the welfare costs. The quantitative analysis shows that neglecting the permanent effects of unemployment leads to an underestimation of the losses of the younger cohorts. Our findings complement those in Hur (2018), who extends the framework of Glover et al. (2020) by adding intra-cohort heterogeneity, a dimension not considered here. Hur (2018) shows that during the Great Recession, a large fraction of young households, who tend to be more indebted and liquidity-constrained than other cohorts, couldn't take advantage of cheap funding and that the amount of risky assets in their portfolios did not increase. His model predicts a much larger loss for the younger, up to 8% of lifetime consumption or 33% in one period consumption (over a ten years period). These losses are additional to those estimated in this paper.

More generally, the paper relates to the studies on the welfare costs of an aggregate shock on different age groups. Doepke and Schneider (2006) study the redistributive effect of a positive inflation shock in a quantitative OLG model and find that while the redistribution of wealth from borrowers to lenders is zero-sum, gainers and losers are characterized by different responses in terms of consumption which do not offset. They also conclude that, on aggregate, positive inflation surprises increase the welfare of the economy: redistributive gains quantitatively offset the losses coming from monetary frictions estimated in other work (e.g. Lucas 2000).

Menno and Oliviero (2014) study the welfare effects of financial tensions on borrowers and savers. A worsening in financial sector conditions forces borrowers to deleverage and generates a pure redistribution towards savers. Therefore, borrowers suffered higher welfare losses from the financial crisis with respect to the former. This paper is related to this work because being borrower or saver also relates to age heterogeneity but it extends the framework by taking into account the effects on human wealth.

Finally, the paper indirectly relates to the literature on the below potential growth after the Great Recession and the secular stagnation hypothesis (Hall 2014; Ball 2014; Fernald 2015; Summers 2014; Hansen 1939, among the others). In our model, the permanent loss in human capital of working cohorts caused by the recession reduces potential output for some decades after the end of the recession.

The rest of the paper is structured as follows: in the next subsection, we look at previous empirical findings on the permanent effects of a crisis at aggregate and individual level and; in section 2, we describe the model; in section 3, we explain the solution method and the calibration; in section 4, we present the welfare effects of the Great Recession and we provide a quantitative analysis of different channels at work; in section 5 we test the model's implications on micro data; finally, section 6 contains the conclusion.

1.1 Permanent effects of deep recessions: some stylized facts and a brief literature review

The welfare analysis in Glover et al. (2020) shows that the youngest cohort is the least affected in relative terms (but also in absolute terms depending on the calibration) in recession episodes like the Great Recession. This result relies on the assumption that a downturn, while being persistent and with sizeable redistribution effects between generations, does not have a permanent impact on the level of output and on the earning abilities of the agents.

This hypothesis conflicts with past experience and in particular with the analyses of the effects of the Great Recession. In the left panel of Figure 3 US real per capita GDP between 1919 and 1940 (solid line, interwar period) is compared to the fitted values of a linear regression over the 1919-1929 period, and in a similar way in the right panel the US real GDP between 1991:Q1 and 2019:Q2 is compared with the fitted values of a linear regression over the 1991:Q1 and 2007:Q2 subsample.

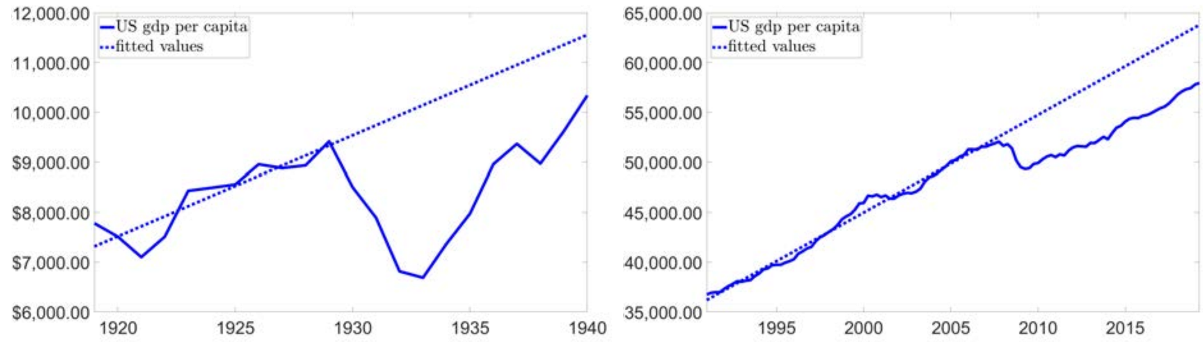

Figure 3: The long-term effects of deep recessions on per-capita output: the Great Depression (left panel, chained 2012 $, source: Jordà et al. (2016)) and the Great Recession (right panel, chained 2012 $, source: FRED, Federal Reserve Bank of St. Louis).

As we have discussed previously, our model with permanent human capital losses delivers this below potential output behaviour that progressively vanishes as cohorts hit by the recession exits the labour market.

In a more detailed and quantitatively accurate analysis Hall (2014) disaggregates U.S. output growth in its main components during the pre-crisis period (1990-2007) and measures the impact of the crisis on their long-run trends. His main results are summarized in Table 1. They show that GDP in 2013, i.e. 4 years after the end of the recession, was still significantly lower than the pre-crisis trend (13.3%) and confirm that, after a strong recessionary episodes, output remains below the previous trend for an extended period of time.

Year	Output	Productivity	Capital contribu-tion	Population	labour force par-ticipation	Employment rate	Hours per week	labour quality	Business fraction
2008	4.9	3.0	0.2	0.3	0.0	0.8	0.5	-0.3	0.3
2009	7.4	1.7	0.8	0.3	0.6	2.4	1.6	-0.4	0.4
2010	0.1	-1.6	1.0	0.3	0.6	0.3	-0.5	0.0	0.0
2011	0.5	0.3	0.8	0.4	0.5	-0.4	-0.2	0.1	-0.9
2012	-0.1	0.1	0.6	-0.1	0.4	-0.6	-0.4	0.0	-0.1
2013	0.5	0.1	0.5	0.3	0.3	-0.3	-0.2	0.3	-0.3
2007 through 2010	12.4	3.1	2.1	0.8	1.2	3.5	1.6	-0.6	0.7
2007 through 2013	13.3	3.5	3.9	1.3	2.4	2.2	0.8	-0.3	-0.5

Table 1: Components of the shortfall of output two and five years after the crisis. Source: Hall (2014)

Our analysis extends the available assessments on the impact of a crisis on households from different cohorts by incorporating another important channel, namely the long-term negative effects of unemployment. The relevance of this channel is supported by many empirical works which find a negative and persistent impact of unemployment on human wealth.

In this respect, the positive correlation between unemployment, and especially youth unemployment, and future poor labour market performances is highly documented. While this evidence could be associated with the presence of "bad type" workers who, due to their low productivity, suffer the most the effects of a crisis and permanently earn lower wages, there is an abundant literature which points to a direct exogenous negative impact from unemployment or layoff to low future earnings.

The first attempt to quantitatively measure this effect has been done by Ellwood (1979). Using the *National Longitudinal Survey of Youth* (1979) he finds that the impact of youth unemployment on the probability of future employment exists but fades away in few years while the effects on future wages are persistent and significant.

Jacobson et al. (1993) study the effect of displacement on "high tenure" workers (six or more years), those that are more likely to have long-term losses in labour earnings. In order to do so they construct a longitudinal data set by merging several administrative records. They found that displaced workers' earning losses are around 25% per year, which are independent from age and sex and inversely correlated to employment growth in the local labour market.

Gregg and Tominey (2005) study the effect of youth unemployment on future wages in UK using a dataset that contains data on family background and skill characteristics they control for unobservable heterogeneity. Furthermore, the authors use an IV approach based on the rate of youth unemployment in the area of residence when the individual is 16 years old and has a low mobility. They found that one year of youth unemployment (i.e. when aged between 16 and 23 as in the ILO definition) implies a decrease by 13-21% in earnings at 41. The fall is lower if unemployment is experienced only once.

Davis and von Wachter (2011) address the endogeneity issue focusing on mass layoff events. According to their definition a mass layoff is a reduction of employees in a firm of at least 30%. Moreover they focus on cases in which the reduction of activity is caused by "bad times" and, as a consequence, the probability of becoming unemployed is orthogonal to skills and other unobservable individual characteristics. They exploit a long panel dataset that allows to estimate the effects on earnings twenty years after. As it can be seen in Figure 4 older workers tend to have larger immediate losses than younger workers, nonetheless younger displaced workers have non-negligible negative effects that persist in all periods considered. What is more, if the layoff happened during a recession wage fall is considerably larger for those below 40 and especially for those in their twenties, suggesting that a correlation exists between labour market conditions and the magnitude of future earning loss. Finally, they try to use some variations of Diamond-Mortensen-Pissarides search and matching labour market model to explain numerical data but they couldn't replicate

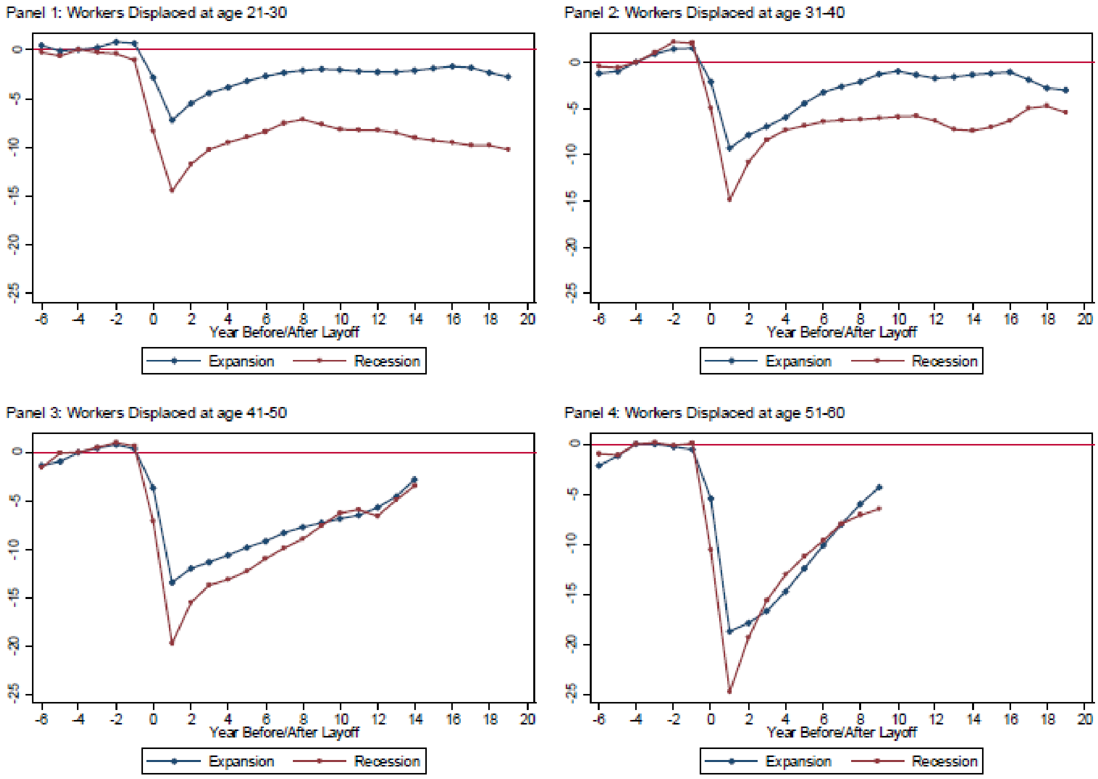

Figure 4: Effects on earnings of displacement during mass layoff. Expansions and recessions are determined using NBER definition. Source: Davis and von Wachter (2011).

the persistency of the losses suffered by the young. Their analysis does not allow to disentangle the effects of time spent unemployed from those associated with a layoff.

Jarosch (2014) uses the same identification strategy of Davis and von Wachter (2011) on German data and finds similar results: workers who suffered a layoff due to exogenous factors have a permanent decrease in earnings (effects are significant even 20 years after the unemployment spell). He estimates a fall in the present discounted value by 21.2%. He uses a search model with two dimensions (productivity and job security) to study the effects of unemployment benefits on labour market efficiency.

Overall the available evidence suggests that unemployment has a negative effect on future wages. There are no estimates on the effect of labour market tightness on human wealth loss, however the works of Jacobson et al. (1993), Davis and von Wachter (2011) and Jarosch (2014) suggest that losses are greater in magnitude for those being displaced during a recession. Therefore, it could be the case that bad labour market conditions determine a longer period of unemployment and therefore an higher deterioration (or less opportunity of accumulation) of human capital and in the end to higher losses in the future. Therefore, the deepness and the duration of the Great Recession suggests that a complete analysis on welfare effects can

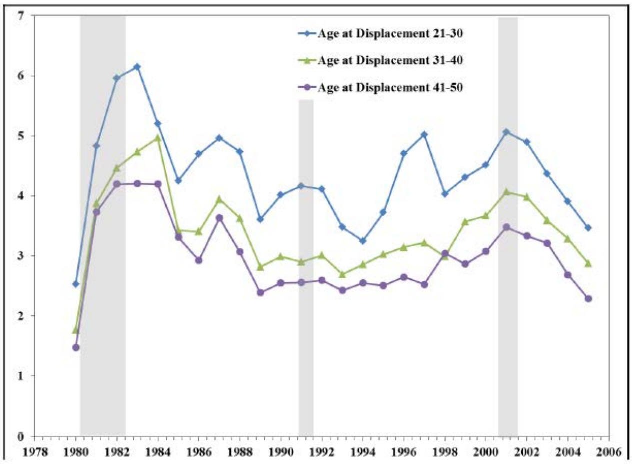

Figure 5: Displacement rate during mass layoff by age group. Source: Davis and von Wachter (2011)

not disregard the long-term costs associated to unemployment.

2 The Model

The model is an OLG with T cohorts, in each period the agents have to choose between consumption and savings. Savings can be stored using a risky asset or a risk-free bond[1]. Agents would like to inelastically supply labour in each period but they face a state-dependent probability of being employed or not due to labour market frictions. Cumulated human capital augments effective labour supply through experience: the more agents work the more they are productive and earn in future periods. Firms are perfectly competitive: they pay wages according to productivity. There is one unit of physical capital, fixed in all periods. The capital share goes to the owners of financial assets. Bonds are in fixed supply. The bond-holders receive a fixed return established in the previous period, the equity holders are residual claimants.

The model is an OLG *á la* Samuelson, as in the seminal work of Auerbach and Kotlikoff (1987). This choice is made to capture the life cycle behaviour in the savings-consumption choice that is not incorporated in the simpler Blanchard-Yaari setup (where all the individuals face the same probability of dying in each period)[2].

There are T cohorts. Each cohort has a representative agent, i.e. the model entails inter-cohort but not intra-cohort heterogeneity. This modelling strategy is justified by the research question: assess the effect of the same aggregate shock at different moments of the life cycle, disregarding the other dimensions of

[1]The model allows borrowing, indeed households can choose a negative amount of each assets.

[2]This modelling feature comes at a cost: for this family of models (usually) closed form solution cannot be computed.

heterogeneity among households. Agents live for T periods and then die with certainty.

Throughout the paper we use the following notation: x_t^i refers to the variable x of cohort with age i in period t[3].

2.1 The stochastic structure

The model entails only aggregate uncertainty. There is a random variable $\omega \in \Omega := \{\omega_L, \omega_H\}$ which follows a Markov process $\Gamma_{\omega'|\omega}$ and that represents the aggregate state of the economy (which can only be good or bad). All the other cohort-specific shocks of the model are functions of the aggregate state: this is a parsimonious representation that reduces the dimensionality of the problem justified by the focus of this work on the effects of the same aggregate shock on different cohorts.

2.2 Households

Households have standard time-separable preferences over stochastic consumption streams $(c_i)_{i=1}^T$ represented by:

$$U\left((c_i)_{i=1}^T\right) = \mathbb{E}\left[\sum_{i=1}^{T}\left(\prod_{j=1}^{i}\beta_j\right)\frac{c_i^{1-\sigma}-1}{1-\sigma}\right]$$

There is no uncertainty on lifetime, thus future periods consumption is simply discounted by the time-preference factors β. The utility function allows for different discount factors at different ages. This specification allows to calibrate $(\beta_i)_{i=1}^T$ to match life cycle profile of consumption. To keep the model tractable utility of future generations does not enter into the utility function and there is no uncertainty about death, thus there are neither voluntary nor involuntary bequests.[4]

Labour market

Households have one unit of labour in each period that they supply inelastically since it does not have a cost in the utility function. Nonetheless there is a labour market friction and agents face an idiosyncratic probability of being employed. This probability is cohort and state specific, capturing the fact that younger cohorts suffer a disproportionately higher probability of becoming unemployed during recessions and have a lower probability of finding a new job. Households of the same age are inside the same family and as in

[3] To make an example: x_t^2 refers to the cohort born in period $t-1$ which is now in her second period of life.

[4] Adding a bequest motive through a "warm-glow" as in De Nardi (2004) or De Nardi and Yang (2014), i.e. adding a "taste" for bequests in the utility function of parents, would affect the calibration of $(\beta_i)_{i=1}^T$ and would complicate the quantitative analysis but won't change the results of the analysis.

Heathcote et al. (2017) the head of family pools resources across family members insuring from idiosyncratic risk but being unable to provide insurance from cohort-specific risk (in this case the employment rate of the cohort).[5] The probability for an agent of i-cohort of getting employed in period t is:

$$\varphi^i(\omega_t)$$

Given that agents of the same cohort are entirely homogeneous, $\varphi^i(\omega_t)$ also represents the labour supply of the cohort representative agent, i.e. $l_t^i = \varphi^i(\omega_t)$.

The labour market friction is a one-to-one function of the aggregate state of the economy, i.e.

$$\Phi(\omega_t) := \begin{bmatrix} \varphi^1(\omega_t) \\ \varphi^2(\omega_t) \\ \vdots \\ \varphi^T(\omega_t) \end{bmatrix} : \Omega \longrightarrow [0,1]^T$$

Then, even if the shock is cohort specific, the model entails only aggregate uncertainty as discussed in the previous paragraph.

Human capital

Agents accumulate human capital through experience, human capital of i-cohort in period t is:

$$h_t^i = h_{t-1}^{i-1}\left(1 + \chi^i l_{t-1}^i\right) \tag{1}$$

Where χ^i represents the cohort-specific return from experience that captures the fact that experience has different returns at different stages of the life cycle ($h^1 = \overline{h}$ is a scale parameter that can be calibrated to normalize the total amount of labour earnings). We choose the multiplicative cumulation because the χ^i has a more direct mapping with the long-term loss due to unemployment estimated by Davis and von Wachter (2011) that we use as a target for the calibration. Using a different functional form, as for example the additive representation adopted by Michelacci and Pijoan-Mas (2008), would not alter the results but would make the calibration less straightforward. With this formulation χ^i, captures the loss in future earnings coming from unemployment.

[5] The labour market representation is similar to Krusell et al. (1998), but here there is no idiosyncratic risk and the aggregate exogenous state is a sufficient statistic of the state of the economy.

The modelization of human capital growth is simplified. It does not allow for endogenous investment by households through education or training nor for an increase in labour supply to raise its accumulation. Nonetheless it captures the effects of unemployment on future earnings without increasing the number of endogenous variable of the model. With this specification h^i is a function of the previous $i-1$ realizations of ω and, in turn, the exogenous state of the economy is not captured by ω but from the last T realizations of this variable.

The empirical literature[6] shows that the damage of unemployment on future earnings comes from two sources: lower re-employment wages and higher probability of becoming unemployed and\or remaining unemployed for a longer period. The modelization adopted here does not take a stand on the origin of future earnings losses: the parameter χ capture the losses coming from both channels making any mix of the two observationally equivalent (and it is calibrated accordingly).

2.3 Representative firm

In the economy there is an infinite amount of identical firms with a constant returns to scale production technology:

$$Y_t = z(\omega_t) K_t^{\alpha} \mathfrak{L}_t^{1-\alpha}$$

where $z(\omega_t)$ is the aggregate TFP, which is a function of the aggregate state of the economy (ω), K_t is physical capital and $\mathfrak{L}_t$ is the aggregate labour supply, that is the sum of effective hours of different cohorts:

$$\mathfrak{L}_t := \sum_{i=1}^{T} \varepsilon^i h_t^i l_t^i = \sum_{i=1}^{T} \varepsilon^i h_t^i \varphi^i(\omega_t)$$

Where ε^i is the cohort specific productivity shock. K_t is fixed and is equal to 1 in all periods.

All factors of production are paid accordingly to their marginal productivity, thus the representative agent of i-cohort gets the following wage:

$$w_t^i = \frac{\partial Y_t}{\partial l_t^i} = (1-\alpha)\varepsilon^i z(\omega_t) K_t^{\alpha} \frac{h^i}{\mathfrak{L}_t^{\alpha}}$$

And thus, using also the fact that $K_t = 1$, the labour income of the i-cohort is:

$$l_t^i w_t^i = \varphi^i(\omega_t)(1-\alpha)\varepsilon^i z(\omega_t) \frac{h^i}{\mathfrak{L}_t^{\alpha}}$$

[6] A review of the empirical literature on permanent losses from unemployment is provided in section 1.1.

2.4 Financial markets

The firm issues two financial assets: risk-free bonds and equity. Marginal productivity of capital is:

$$\begin{aligned}\frac{\partial Y_t}{\partial K_t} &= \alpha z_t K_t^{\alpha-1}\mathfrak{L}_t^{1-\alpha} \\ &= \alpha Y_t\end{aligned}$$

The return of the risk-free bond is endogenously established in the previous period, the return of the equity is state specific.

The amount of debt B issued by the firm is exogenous and equal in all periods. One unit of debt in period t costs q and pays back 1 in period $t+1$ whatever the state of the economy.

There is one unit of equity and shareholders are residual claimants of firm profits. Equity gives right to a dividend in each period:

$$d_t = \alpha Y_t - B\left(1 - q_t\right)$$

The price of equity is p_t and it is endogenously determined in the model. The gross return of equity in period $t+1$ is given by:

$$\frac{p_{t+1} + d_{t+1}}{p_t}$$

Given the state of the economy in period t, only two states are possible in period $t+1$. Since the returns on bond and equity are independent, financial markets are complete.

2.5 History and "steady state"

The dynamics of human capital implies that the exogenous state of the model is given from the last T realizations of the random variable ω. Indeed, aggregate labour supply is a function of human capital of all alive cohorts (since they are all active in the labour market) and the human capital of cohort T is function of the previous $T-1$ realizations of ω while z and φ are functions of its realization at t. Then, to simplify notation (and the stochastic structure in the resolution algorithm), we introduce the variable $\eta_t = [\omega_{t-T+1}, \omega_{t-T+2}, \ldots, \omega_t]$. Notice that $\eta \in \underset{j=1}{\overset{T}{\times}} \Omega_j$ and clearly $\left| \underset{j=1}{\overset{T}{\times}} \Omega_j \right| = |\Omega|^T$.

A Markov process on η , $\Gamma_{\eta'|\eta}$ can be defined starting from $\Gamma_{\omega'|\omega}$. Notice that $|\eta'|\eta| = |\Omega|$.

Since the aim of this paper is to study the effect of a large but infrequent aggregate shocks on different cohorts, we do not analyze the Markovian equilibrium of the economy but we focus on a negative aggregate realization after a sequence, potentially infinite, of good realizations.

The model allows to compute the welfare losses of different cohorts and to study when it is most harmful to live through a big negative shock: if during youth, when the probability of becoming unemployed is really high, or later in life when financial and housing wealth are severely hit from market collapses.

Notationally we will use $\eta^{SS} := [\omega_H, \omega_H, \ldots, \omega_H, \ldots]$.

2.6 Recursive problem and equilibrium

The value function $v_i(\eta, A)$ of i-cohort representative agent is function of the two states of the economy: the exogenous shock η and the endogenous share of wealth $\mathcal{W}$ owned by each cohort denoted by A (indeed the pricing of the two financial assets depends on how the wealth is shared among cohorts), therefore the recursive formulation of the problem of the i-cohort representative agent is:

$$v_i(\eta, A) = \max_{c_i, s_i, \varphi_i, a'_i} \left\{ u(c_i) + \beta_i \sum_{\eta' \in |\Omega|^T} \Gamma_{\eta'|\eta} v_{i+1}(\eta', A') \right\}$$

$$s.t.$$

$$c_i + s_i = w_i(\eta) + \mathcal{W}(\eta, A)\, a_i \tag{2}$$

$$a'_i = \frac{\left(\varphi_i \frac{[p(\eta', A') + d(\eta', A')]}{p(\eta, A)} + (1 - \varphi_i) \frac{1}{q(\eta, A)}\right) s_i}{\mathcal{W}(\eta', A')} \tag{3}$$

$$A' = \Phi(\eta, A, \eta') \tag{4}$$

Where c_i is consumption of the i-cohort, s_i are savings, $w_i(\eta)$ represents labour income, φ_i denotes the fraction of wealth invested in risky assets by the cohort. $A \in S^T$ is the vector that contains the share of wealth owned by cohort each one denoted by a^i, where $\sum_i a^i = 1$. $\mathcal{W}$ measures the total amount of wealth in the economy and is defined by:

$$\mathcal{W}(\eta, A) = p(\eta, A) + d(\eta) + B$$

The problem has three constraints: Equation 2 is the budget constraint, Equation 3 is the law of motion of cohort wealth (and is function of the portfolio allocation in the previous period), Equation 4 is the law of motion of the share of the endogenous state variable.

Now we can fully characterize the recursive competitive equilibrium of the economy.

Recursive competitive equilibrium *A recursive competitive equilibrium is a set of value functions* $\{v_i(\eta, A)\}_{i\in T}$ *and a set of policy functions* $\{a'_i(\eta, A)\}_{i\in T}$, $\{c_i(\eta, A)\}_{i\in T}$, $\{s_i(\eta, A)\}_{i\in T}$, $\{\varphi_i(\eta, A)\}_{i\in T}$, *pricing functions* $w_i(\eta)$, $d(\eta, A)$, $p(\eta, A)$, $q(\eta, A)$ *and the aggregate law of motion* $\Phi(\eta, A, \eta')$ *such that* $\forall i \in T$*:*

1. *Given the pricing functions and the aggregate laws of motion, a set of value functions* $\{v_i\}_{i\in T}$ *solve the recursive problem of the households, and* $\left\{c_i, s_i, a_i^{'}, \varphi_i\right\}_{i\in T}$ *are the associated policy functions.*

2. *Wages and dividends satisfy:*

$$\begin{aligned} \forall i \in T: \; w_i(\eta) &= \varepsilon_i l_i(\eta)\frac{\partial Y}{\partial l^i} = \varepsilon_i l_i(\eta)\left[Y(1-\alpha)\frac{h^i}{\mathcal{L}^\alpha}\right] && (5) \\ d(\eta, A) &= \alpha Y(\eta) - [1 - q(\eta, A)]B && (6) \end{aligned}$$

3. *Markets clear:*

$$\begin{aligned} \sum_{i\in T} c_i(\eta, A) &= Y(\eta) && (7) \\ \sum_{i\in T} \varphi_i(\eta, A)\, s_i(\eta, A) &= p(\eta, A) && (8) \\ \sum_{i\in T} [1 - \varphi_i(\eta, A)]\, s_i(\eta, A) &= Bq(\eta, A) && (9) \end{aligned}$$

4. *The law of motion for the distribution of financial wealth is consistent with equilibrium decision rules:*

$$\phi_1\left(\eta, A, \eta'\right) = 0 \qquad \forall \eta' \qquad (10)$$

$$\phi_i\left(\eta, A, \eta'\right) = a_{i-1}^{'}\left(\eta, A, \eta'\right) \qquad \forall \eta', i = 2, \ldots, T \qquad (11)$$

The first condition requires that, taking prices as given $\left\{c_i, s_i, a_i^{'}, \varphi_i\right\}_{i\in T}$ solve the household's problem. Equation 5 and Equation 6 have been discussed previously in 2.3. Equation 7 is the aggregate resource constraint: capital is fixed and therefore all the output is shared among cohorts for consumption. Equation 8 and Equation 9 are the clearing market conditions of financial markets. The former is the clearing condition for equity market and states that the sum of shares of savings invested in risky asset by living cohorts must be equal to the price of equity (multiplied by one, that is the normalized quantity of equity in fixed supply). Analogously, Equation 9 is the clearing condition of the bond markets. Finally, Equation 10 states that any newborn cohort has no wealth while Equation 11 characterize the law of motion of the share of wealth of all other cohorts.

3 Numerical computation

The state space of the problem includes $T-2$ continuous state variables and $|\Omega|^T$ discrete states. As we previously pointed out in Section 2.5, $|\eta_{t+1}|\eta_t| = |\Omega|$, i.e. given the exogenous state of period t there are only $|\Omega| = 2$ possible states in period $t+1$. Indeed, the number of discrete states is high due to the fact that human capital accumulation depends on the history of exogenous shock realizations (and clearly the higher the number of cohorts the longer the history and the higher the number of discrete states) but the exogenous state realizations are always two. In the economy there are two financial assets, stocks and bonds, with linearly independent returns: the bond provides a fixed return while stocks pays dividends depending on the state of the economy. Then the two independent financial assets span all the states of the world, markets are complete and there is perfect aggregate risk sharing among living households (they are subject only to the natural borrowing constraint). Nonetheless, this is true only for those cohorts that are active in financial markets before the shock realizes. At the opposite, the newborn generation cannot insure against being born in a specific state of the economy and therefore suffer a very specific market incompleteness.

Following Brumm and Kubler (2013), under this financial market setup we can use the Negishi approach: we solve the dual problem and we look for a policy function of the weights that a utilitarian social planner would give to each cohort that would guarantee an optimal allocation identical to the competitive equilibrium.[7] Given the perfect aggregate risk-sharing the relative weights of the living cohorts are always predetermined and we need to numerically define only the policy function that, given the distribution of wealth among the other cohorts, assigns the right amount of resources to the new generation.

In order to do that we use the algorithm of Brumm and Kubler (2013) that shows that the problem reduces to computing the policy function that satisfy the intertemporal budget constraint of newborn agents, thus reducing the burden of numerical computation.

The algorithm is explained in detail in appendix A.

3.1 Calibration

The aim of the welfare analysis is to compute the effects of big but infrequent aggregate shocks on different cohorts. One period of the model lasts 10 years, agents enter the economy when they are 20 and die when they are 80, therefore $T = 6$ (they live for 6 periods). The transition probabilities among the two states are calibrated to have that in expected terms a newborn will live only one period of his life in a downturn.

[7] The algorithm for the infinitely lived representative household has been developed by Negishi (1960). See for example Ljungqvist and Sargent (2012) for a textbook explanation.

Parameters of the households sector are risk aversion σ, time-preferences $\left(\beta^i\right)_{i=1}^T$, human capital accumulation factor $\left(\chi^i\right)_{i=1}^T$ and cohort specific productivity $\left(\varepsilon^i\right)_{i=1}^T$. labour market is driven by $\left(\varphi^i\right)_{i=1}^T$ that determines cohort-specific employment-to-population. The share of capital income is determined by α and how is split between equity and bond depends on B. The TFP process has two possible levels $(z(\omega))_{\omega\in\Omega}$ and is driven by the transition probability matrix $\Gamma_{\omega'|\omega}$ that determines the state of the economy ω.

For the pre-crisis state the second quarter of 2007 is taken as a reference. This quarter is the last in which SCF data on wealth are available before the acceleration in house prices' fall and the beginning of financial crisis[8]. The trough of the crisis is calibrated using the second quarter of 2010. This is not consistent with NBER recession definition but is consistent with Hall (2014) who argues that this is a more natural end date when considering the dynamics of labour market, an important channel as discussed before and therefore a more appropriate target for this analysis.

The calibration of financial markets is the same as in Glover et al. (2020). This choice keeps unchanged the magnitude of the channel that provides welfare gains to younger generations. There are four parameters to be calibrated α (the capital share of the production function), p (the steady state price of equity), q (the price of the bond), B (the amount of bonds in the economy). They are calibrated simultaneously to match three empirical moments in the data: the aggregate share of risky assets in households' net worth (which we denote with $\overline{\lambda}$), the aggregate wealth to labour income ratio ($\overline{W}$) and the gross interest rate ($\overline{R}$). The system is closed adding a non arbitrage condition in the steady-state equilibrium between equity and the riskless bonds. Calibration is detailed in Appendix B.1.

The set of parameters $\left(\varphi^i(\omega)\right)_{i=1}^T$ represents the probabilities that an agent of the i-cohort is employed in different phases of her life and states of the economy. Given the unity mass of each cohort population these probabilities can be calibrated using the cohort-specific employment-to-population ratios in the two states of the world (as explained at the beginning of the section 2007 Q2 is taken as a reference for the good state while 2010 Q2 is used for the bad state).[9]

The vector $\left(\chi^i\right)_{i=1}^T$ determines the human capital accumulation for workers of different age. Its natural target is the loss in PDV of labour income over the life cycle. Indeed it can be formally shown that for an agent of age i the sum of discounted losses in labour income throughout the life cycle, as compared with

[8]On August 9, 2007 French bank BNP Paribas suspended three of its funds as problems in the U.S. subprime mortgage sector were preventing it from calculating their value. This event is considered to be the beginning of the financial crisis.

[9]The Bureau of Labor Statistics does not provide employment to population data for all the age brackets used in the model. For those that are missing we compute the data using the absolute number of employed (available for 5 years brackets) and population estimate by cohorts taken from Census Bureau.

an agent of the same age that was not unemployed, is equal to $\frac{\chi_i}{1+\chi_i}$.[10] For the calibration of cumulated discounted losses we take as a reference the estimates of Davis and von Wachter (2011).[11]

Despite not being an explicit target of their calibration, $\left(\varphi^i\right)_{i=1}^T$ and $\left(\chi^i\right)_{i=1}^T$ replicate with a good degree of approximation the gross labour income across households of different age observed in SCF data.[12] The $\left(\varepsilon^i\right)_{i=1}^T$ are calibrated to match labour income net of taxes and Social Security and Retirement transfers in the good state. This is a shortcut used in Glover et al. (2020) to match net income without introducing a government sector with a fiscal policy that would increase the burden of numerical computation. Two calibration tests are performed in Appendix D.2. Firstly, even if not targeted by the calibration, the model is able to fairly replicate the cohort specific net income after the recession observed in 2010 SCF micro data. Then, a government sector with social security is properly calibrated to match income in the good state is introduced instead of $\left(\varepsilon^i\right)_{i=1}^T$ and it is shown that the fit of the model with respect to post-crisis cohort net income is not improved by the more complex modelization.

Parameter(s)	Calibration	Moment(s) to be matched
$\Gamma_{\omega'\vert\omega}$	$\begin{bmatrix} \pi_{\omega_h'\omega_h} = 0.85 & \pi_{\omega_l'\omega_h} = 0.15 \\ \pi_{\omega_h'\omega_l} = 0.85 & \pi_{\omega_l'\omega_l} = 0.15 \end{bmatrix}$	One expected deep recessionary episode during the life cycle
$\{\left(\varphi^i\right)_{i=1}^T(\omega)\}_{\omega\in\Omega}$	$\left\{ \begin{array}{c} (0.74, 0.80, 0.81, 0.74, 0.42, 0.10) \\ (0.67, 0.76, 0.77, 0.71, 0.42, 0.10) \end{array} \right\}$	Employment to population ratios Q2 2007 and Q2 2010
$\left(\chi^i\right)_{i=1}^T$	$(0.2821, 0.1905, 0.2804, 0.4514, 0)$	Loss in future incomes from unemployment
$\left(\varepsilon^i\right)_{i=1}^T$	$(0.68, 0.91, 0.97, 1.02, 1.06, 2.18)$	Age specific wage-productivity gap and Social Security
$\left(z\left(\omega\right)\right)_{\omega\in\Omega}$	$(1, 0.947)$	Fall in output
σ	2	Fall in stock prices relative to fall output
$\{\left(\beta^i\right)_{i=1}^T\}$	$(1.24, 0.76, 0.77, 0.73, 0.51)$	Consumption profile
$\{\alpha, B, p, q\}$	$(0.30, 0.08, 0.51, 0.65)$	Gross interest rate, wealth-to-labour income ratio, share of risky assets

Table 2: Summary of the calibration

[10] Proof is in Appendix C.1.

[11] A detailed discussion with some examples is provided in Appendix B.2 while a calibration check is performed in Appendix D.1.

[12] After a series of six good states they are able to match the cohort-average gross labour income as measured in SCF 2007 data.

4 Welfare analysis

4.1 Welfare loss

The welfare analysis is conducted by comparing the steady state of the model as defined by η_{SS} with a counterfactual economy where the steady state is perturbed by one negative shock in the period $t = 0$ ($\omega_0 = \omega_L$) followed by a series of positive shocks ($\omega_t = \omega_H \ \forall t > 0$) leading the economy eventually back to the steady state.

Due to the OLG structure of the model the standard definition of welfare costs in terms of life-time consumption equivalent as in Lucas (2000) cannot be used directly to measure the effect of Great Recession on each cohort. The OLG structure of the model requires some modification to the methods used for infinitely lived representative households since households at different age face a different "life-time". We compute welfare losses in terms of *one period consumption equivalent*.

Definition 1 The *one period consumption equivalent* welfare loss of the cohort born at time k and experiencing the recession at age i (i.e. the i-cohort when the recession comes) is the $\delta^i_{t_0}$ such that:

$$u\left(c_t^{t+1-k}\left(\eta_t\right)\left(1+\delta^i_{t_0}\right)\right) + \sum_{t=k+i+1}^{T+k}\left(\prod_{j=t-k}^{T}\beta_j\right) u\left(c_t^{t+1-k}\left(\eta_t\right)\right) = \sum_{t=k+i}^{T+k}\left(\prod_{j=t-k}^{T}\beta_j\right) u\left(c_t^{t+1-k}\left(\eta_{ss}\right)\right)$$

where $\eta_t = \left[(\omega_j)_{j=t-T+1}^{t}\right]$ and the sequence is such that:

$$\omega_t = \begin{cases} \omega_L & t = k-1+i \\ \omega_H & else \end{cases}$$

This measure has been used in OLG setup by Hur (2018) and in infinitely lived households setup by Asturias et al. (2016), Sims and Wolff (2013) and Ganelli and Tervala (2015). This method computes the percentage increase in consumption today that is necessary to make the household indifferent with the no-recession scenario. An assessment on the robustness of welfare analysis results with respect to the measure used is performed in section E.

The *one period consumption equivalent* welfare losses for different cohorts are shown in Table 3. The younger generation is the most severely affected, her losses are between two and three times in magnitude those of older cohorts. The quantitative model allows to dissect the results by computing the losses arising

i-cohort	$\delta^i_{t_0}$
20-29	22.78%
30-39	9.52%
40-49	9.62%
50-59	8.89%
60-69	8.39%
70+	8.14%

Table 3: One period consumption equivalent for cohorts experiencing recession at different ages.

from the different channels. In the next subsection we explore the three main channels that determine a welfare loss and we look at their effects on the different cohorts to understand which one is more relevant in determining the results.

4.2 Inspecting the mechanism

Long-term harm to the economy

The red line in Figure 6 shows the output behaviour when the big recessionary episodes occurs while the dotted blue line is the counterfactual scenario where the negative shock does not realize: effects of the shock do not disappear immediately after the recession. The "scar" on the economy is generated by those households that were active when the recession hit and have suffered a permanent loss in human capital that will keep the economy below its potential until all cohorts will be replaced. This persistent fall in output determines a decrease in consumption for all the living cohorts and the longer a cohort lives through this lower output environment the higher the welfare loss that it faces.

In order to quantify the impact of this channel on welfare losses of different cohorts we use the model to simulate an alternative scenario where output does not suffer long-term harm from the recession. To do so we increase the TFP level z_t as to neutralize the aggregate human capital loss experienced in the recession. The counterfactual production function is

$$Y_t = \tilde{z}\left(\eta_t\right) K_t^{\alpha} \mathfrak{L}_t^{1-\alpha}$$

Where $\tilde{z}\left(\eta_t\right) := \left(1+\gamma_t\right) z\left(\omega_t\right)$ is the new TFP that contains the wedge γ_t. We set $\gamma_t \geq 0 \; \forall t \in [1,5]$, i.e. a positive wedge in all those periods that would see a below-potential output. More formally the sequence must satisfy:

$$\forall t > 0 : \; \left(\gamma_t\right)_{t>0} \; s.t. \; Y_t\left(\eta_t\right) = Y\left(\eta_{SS}\right)$$

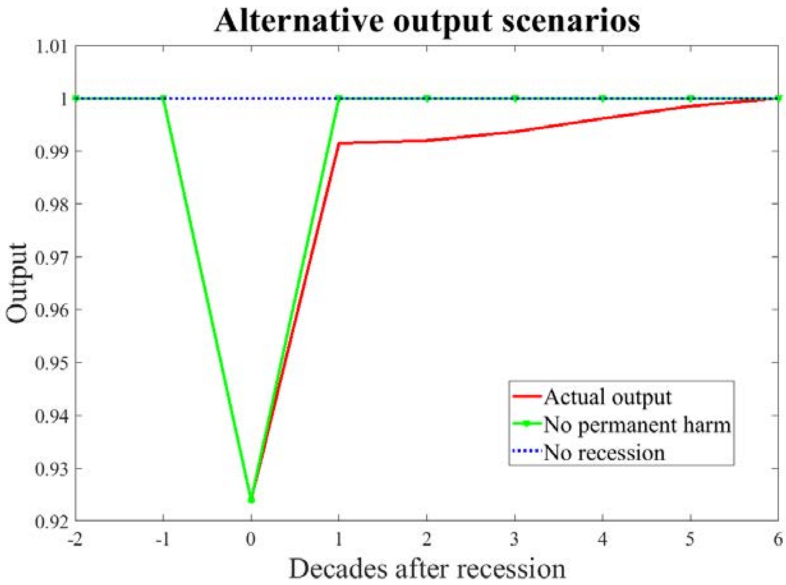

Figure 6: Recession long-term effects on output in the model

Under this modelization the permanent harm of the economy is removed without altering the effects of the recession on the relative levels of human capital among cohorts or their portfolio choices. We denote with $\tilde{\delta}^i_{t_0}$ the welfare losses computed according to Definition 1. Then $\delta^i_{t_0} - \tilde{\delta}^i_{t_0}$, i.e. the difference in one period consumption equivalent between the baseline scenario and the counterfactual scenario, is the loss coming from the long-term fall in output. Results are shown in Table 4. Firstly, welfare losses in this counterfactual scenario are lower, $\forall i \ \delta^i_{t_0} \geq \tilde{\delta}^i_{t_0}$, since all cohorts enjoy at least the same amount of consumption in the counterfactual scenario. What is more welfare losses of the oldest cohort arising from this channel are obviously zero, they are not alive in periods after recession and therefore this channel is not hurting their consumption. Finally the losses are decreasing in age, $\forall j > i \ \forall i \ \left(\delta^j_{t_0} - \tilde{\delta}^j_{t_0}\right) < \left(\delta^i_{t_0} - \tilde{\delta}^i_{t_0}\right)$, because the younger the agent is when the recession hits the longer she will live in a below potential economy. Nonetheless, in relative terms (last column of Table 4) this channel is more relevant for middle-aged cohorts (between 30 and 49) than the youngest cohort.

Temporary and permanent effects of unemployment

We now explore the relevance of labour market and its associated permanent and temporary losses for cohorts' welfare. The rise in unemployment determines a fall in income today but also a decrease of tomorrow labour income since the amount of effective labour provided by individuals depends on past occupational status. An analytical assessment of the losses coming from unemployment can be done by looking at the

Age spent in recession	$\tilde{\delta}^i_{t_0}$	$\delta^i_{t_0}$	$\delta^i_{t_0} - \tilde{\delta}^i_{t_0}$	In percentage of total WL
20-29	19.43%	22.78%	3.35%	14.71%
30-39	7.60%	9.52%	1.92%	20.18%
40-49	7.79%	9.62%	1.83%	19.02%
50-59	8.01%	8.89%	0.87%	9.80%
60-69	8.11%	8.39%	0.29%	3.40%
70+	8.14%	8.14%	0%	0%

Table 4: Decomposition of welfare losses generated by the permanent harm to output.

effect of the recession on the human wealth of households of different cohorts. Define the human wealth of cohort of age i at time t (HW^i_t) as the sum of discounted labour income from today onward:

$$HW^i_t = W_t l_t + \sum_{j=1}^{T-i} \frac{W^{i+j}_{t+j} l^{i+j}_{t+j}}{\prod_{k=0}^{j-1} R_{t+k}} \tag{12}$$

Use Δ to denote the difference between the value of a variable when the good aggregate state realizes (ω_H) and its counterpart when the recession state appears (ω_L)[13], a recursive formulation of equation (12) can be used to make a distinction between losses on today labour income and losses on tomorrow labour income:

$$\Delta HW^i_t = \Delta \left[l_t W_t \right] + \Delta HW^{i+1}_{t+1} \tag{13}$$

Equation (13) distinguishes between temporary and permanent losses of human wealth in the recession. The first term represents the loss in earnings arising today and it is the product of the decrease in labour wage and the decrease in labour supply, the second term entails the permanent losses. Table 5 contains the decomposition of human wealth losses as the sum of today and future losses. The table shows that the younger cohorts are the most affected both from temporary and permanent losses from unemployment.

Temporary losses are higher for younger workers as a result of higher unemployment. The calibration of the model, in line with empirical data, mechanically determines an higher fall of today earnings for younger workers.

Permanent losses are also higher for younger cohorts as the results of three different channels: the labour market is more tight for young workers during the recession thus increasing the human capital loss, they are living a crucial phase for human capital accumulation and and they have a longer residual working life.

[13]For a generic variable $x(\omega)$, $\Delta x := x(\omega_H) - x(\omega_L)$.

The first channel is highlighted in employment-population ratios observed in the data (see Figure 1). The second channel is a result of the calibration of $\left(\chi^i\right)_{i=1}^T$: the first cohort is one of those with the highest χ^i. This means that the age between 20 and 29 is crucial to cumulate human capital by experience and therefore time spent in unemployed during this period determines an higher loss in future earnings as compared with other cohorts[14]. Finally, the younger the cohort the longer the residul working life and therefore a given loss in human capital will reduce labour income for a longer time determining a greater loss in human wealth.

Age spent in recession	$\frac{\Delta\left[l_t W_t\right]}{l_t W_t}$	$\frac{\Delta HW_{t+1}^{i+1}}{HW_{t+1}^{i+1}}$	$\frac{\Delta HW_t^i}{HW_t^i}$
20-29	-13.25%	-22.31%	-17.06%
30-39	-9.71%	-21.08%	-13.45%
40-49	-9.53%	-20.03%	-12.58%
50-59	-7.81%	-18.85%	-10.00%
60-69	-3.11%	-16.46%	-5.13%
70+	-1.55%	0%	-1.55%

Table 5: Decomposition of human wealth losses.

Financial markets and benefits from leveraging

While earnings losses and unemployment are known to hit harder younger cohorts, Kiyotaki et al. (2011) argued that young households with almost zero wealth during a crisis have the opportunity of buying "underpriced" risky assets (housing and equity) by leveraging through cheap credit. What is more Glover et al. (2020) have shown that this channel should have been particularly relevant during the Great Recession where the fall in asset prices have been much deeper than the fall in output. The model endogenously determines a portfolio composition of different cohorts in line with the one observed in the data: younger cohorts have an overwhelming exposition towards risky assets while they have negative position (i.e. they borrow) through the risk-free asset. Indeed, younger cohorts have a longer time horizon ahead and therefore are more willing to bear the risk of high losses in the short-run since they will be able to recoup them in the subsequent periods when the economy will exit recession. On the contrary, households at the final stage of their life would be forced to sell their risky assets even if underpriced suffering losses that won't be able to smooth in subsequent periods. [15]

[14]From Table 2 the only χ^i greater in terms of magnitude is the one the cohort in its fifties.

[15]This feature of an OLG model with aggregate uncertainty and complete markets has been deeply discussed in Glover et al. (2020).

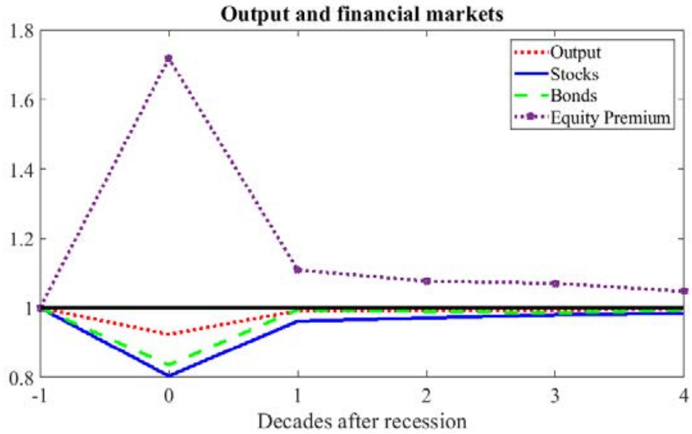

Figure 7: Output, stock and bond prices across the recession.

In Figure 7 the behaviour of stocks and bonds is compared with the output. The fall in the price of stocks is higher than the fall in the price of bonds and stocks remains relatively cheaper than bonds for some periods. This divergence in prices is equivalent to an increase in equity premium that is determined by an increase in the risk-aversion of households in the economy. Indeed, by definition the equity premium is the difference between the risky expected return of stocks and the risk-free rate in the economy:

$$EP_t = \frac{\mathbb{E}_t \left[d_{t+1} + p_{t+1} \right]}{p_t} - R_t$$

The recession determines a spike in the equity premium by around 70%, it falls abruptly in the period after the crisis, above its steady state level, and then slowly declines. The increase in the equity premium can be explained by the human capital loss suffered by all households that determines a permanent loss of wealth. Indeed due to CRRA preferences their risk-aversion increases.[16]

The increase in equity premium by itself could increase the gains from the financial channel pointed out in the previous literature. Indeed the widening gap between the cost of equity and the cost of debt should favor younger households. Nonetheless, the increase in risk-aversion is not equally spread among all the living cohorts. The highest loss in human wealth is indeed suffered by the youngest cohort that becomes the most risk-averse partially offsetting the endogenous "risk-loving" typical of their life-cycle phase.

By definition the amount of equity held by the i-cohort in period t is $s_t^i \varphi_t^i$ and the amount of bonds is $s_t^i \left(1 - \varphi_t^i\right)$. The solid lines in left and central panels of Figure 8 represent respectively for $j = 0, \ldots, 4$ $\frac{s_{t=j}^{1+j} \varphi_{t=j}^{1+j}}{s_{SS}^{1+j} \varphi_{SS}^{1+j}}$ and $\frac{s_{t=j}^{1+j} \left(1 - \varphi_{t=j}^{1+j}\right)}{s_{SS}^{1+j} \varphi_{SS}^{1+j}}$, i.e. the evolution of the portfolio of the younger cohort when the recession hits

[16] Households whose preferences are represented by a CRRA utility function increase their risk-aversion as their level of wealth decreases.

relative to a cohort that does not experience the recession. Firstly, $\forall j = 0, \dots, 4 \; \frac{s^{1+j}_{t=j}\left(1-\varphi^{1+j}_{t=j}\right)}{s^{1+j}_{SS}\varphi^{1+j}_{SS}} \leq 1$, i.e. the younger cohort born during a recession during his life will never own more risky assets than a counterfactual cohort born in normal times. What is more $\frac{s^{1}_{t=0}\varphi^{1}_{t=0}}{s^{1}_{SS}\varphi^{1}_{SS}} < 1$, i.e. when the recession hits and credit is relatively cheap the younger cohort becomes less indebted than a cohort born in normal times.

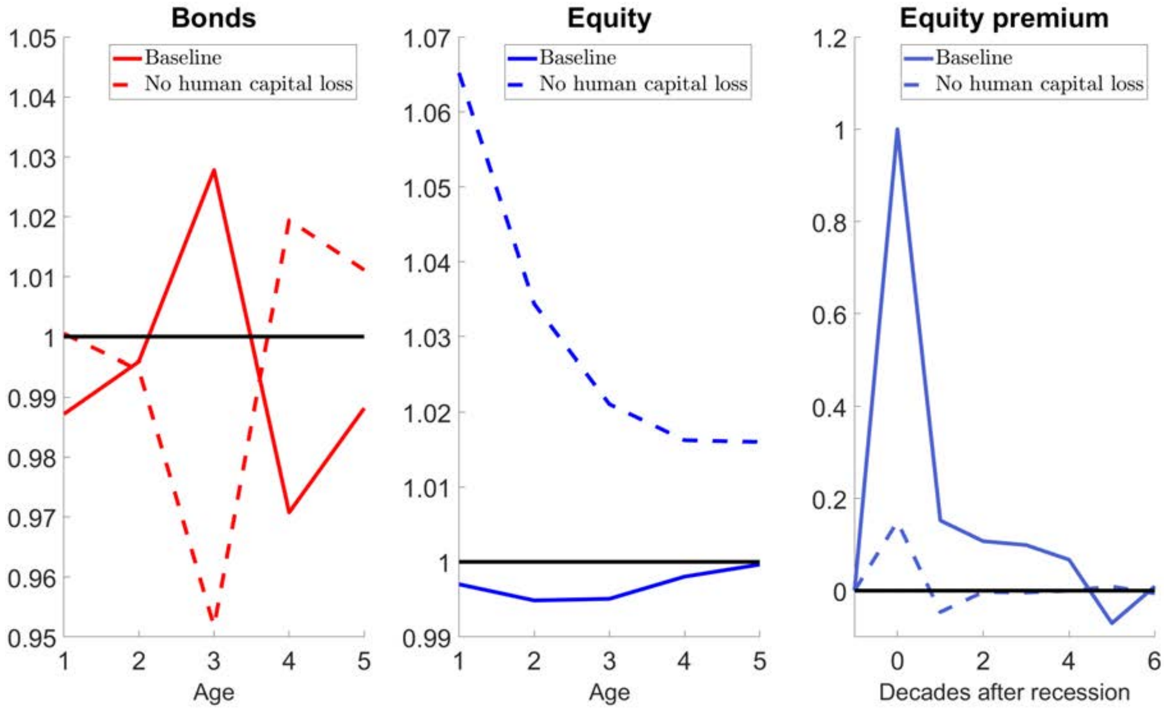

Figure 8: Relative portfolio composition of households in their 20s during the Great Recession across their life cycle with and without permanent income losses from unemploymen.

The quantitative model can then be used to make an assessment on the role of permanent losses in income compared to transitory ones in this risk-taking behaviour. We define $\forall j = 0, \dots, 4 \; \frac{\hat{s}^{1+j}_{t=j}\hat{\varphi}^{1+j}_{t=j}}{\hat{s}^{1+j}_{SS}\hat{\varphi}^{1+j}_{SS}}$ and $\forall j = 0, \dots, 4 \; \frac{\hat{s}^{1+j}_{t=j}\left(1-\hat{\varphi}^{1+j}_{t=j}\right)}{\hat{s}^{1+j}_{SS}\hat{\varphi}^{1+j}_{SS}}$ the ratios computed in a counterfactual world where the recession determines only a fall in today's labour income, i.e. the same ratios as before but computed from a model where $\forall i \; \chi^i = 0$ and keeping unaltered the others parameters.[17]

[17] More precisely we also change $\hat{h}$ and $\left(\varepsilon^i\right)^T_{i=1}$ to match the labour income profile in steady state but the target of the calibration is not changed. Notice that this counterfactual analysis is almost equivalent to the endogenous portfolio analysis in Glover et al. (2020).

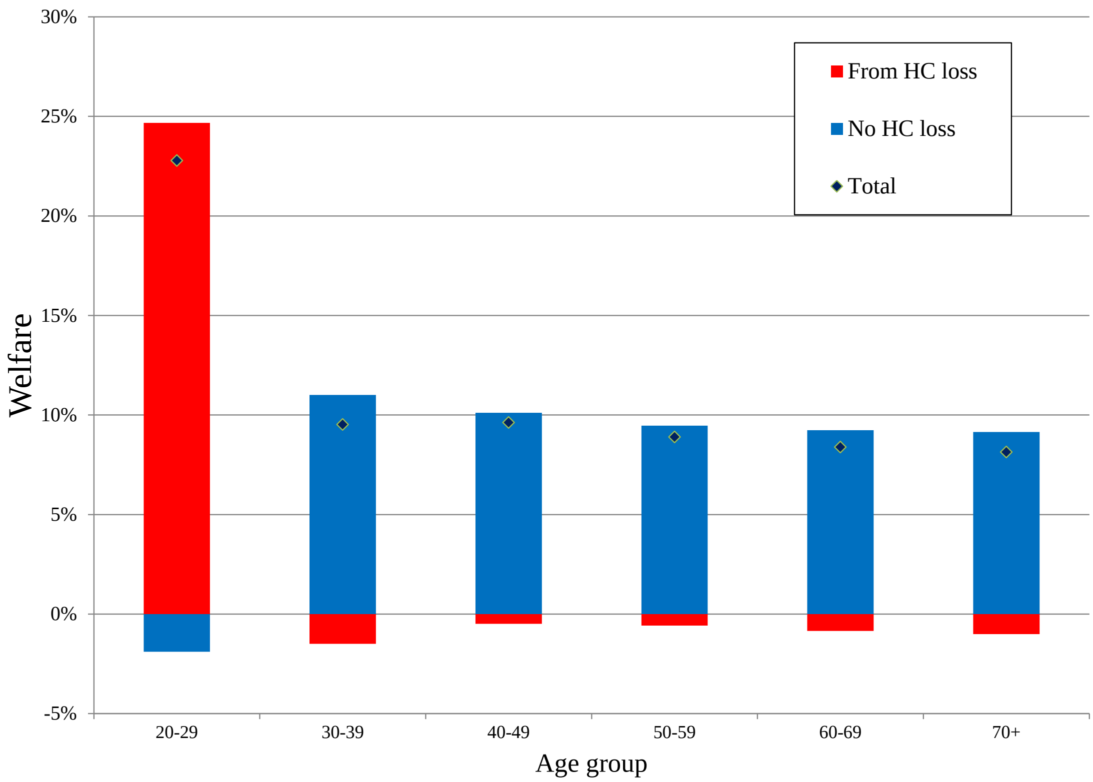

Figure 9: welfare loss generated by the Great Recession in today consumption by age group

The introduction of human capital from the recession has two effects on risk-taking behaviour in financial markets that pushes welfare gains in opposite directions. On the one hand the permanent income loss for all living households increases the equity-premium during the recession raising the potential gains of the young cohort. From the right panel of Figure 8 we can see that the increase in the risk premium is five times greater in magnitude when human capital losses are included in the model and that it remains at an higher level for several periods after the shock. On the other hand, it makes the younger cohort relatively poorer and therefore relatively more risk-averse, reducing their leverage and their risk-exposure, as it is evident from left and central panels of Figure 8, thus dampening the gains in financial markets identified by Kiyotaki et al. (2011). As it can be seen from Figure 9 quantitatively the second effect prevails and the introduction of human capital reverses the potential gains of younger households.

There are two last remarks that can be done on the financial markets and the associated gains from the recession.

Firstly, notice that the introduction of human capital accumulation makes the model able to replicate under fully rational expectations the empirical evidence on investment behaviour of younger households during the Great Recession described in the literature as the result of extrapolative expectations.[18]

Secondly, the driver of financial gains in previous literature is the existence of an higher than usual equity premium when entering the economy in the absence of any future income loss that can alter the risk attitude. From right panel of Figure 8 it is possible to see that when human capital is introduced the risk premium is higher than usual also in the period after the recession. Therefore the newborn cohort in the first period after the recession can indeed exploit the previously identified channel of buying risky assets through cheap credit and in the end have a welfare gain (given that they did not incur into the human capital loss).

A robustness check on the chosen measure of welfare loss is performed in Appendix E.

5 Empirical analysis

5.1 Household level data

In this section we compare the main implications of the model with micro-level evidence on consumption and portfolio choice to test the goodness of the model. We use three datasets: two for consumption data and one for portfolio data. For consumption data the two main datasets available for US are the Consumer Expenditure Survey (CE) and the Panel Survey of Income Dynamics (PSID) while for portfolio data we use the Survey of Consumer Finances (SCF).

The CE is a nationwide household survey conducted by the U.S. Bureau of Labor Statistics (BLS) on American households expenditures. It is the only federal government survey that provides information on the complete range of consumers' expenditures as well as their incomes and demographic characteristics. BLS publishes 12-month estimates of consumer expenditures twice a year with the estimates summarized by various income levels and household characteristics.

The PSID is a household panel survey that began in 1968 to study the dynamics of income and poverty. To this end, the original sample contained two independent sub-samples: an over-sample of roughly 2000 poor families selected from the Survey of Economic Opportunities (SEO) and a nationally representative

[18] Malmendier and Nagel (2011) show that individuals who have experienced low stock market returns throughout their lives so far report lower willingness to take financial risk, are less likely to participate in the stock market, invest a lower fraction of their liquid assets in stocks if they participate, and are more pessimistic about future stock returns.

sample of roughly 3000 families designed by the Survey Research Center (SRC) at the University of Michigan. Survey waves are annual from 1968 to 1997, and biennial since then. In the PSID data on consumption are not as detailed and complete as those of the CE. Nonetheless some specific extra-questions have been proven to be effective in matching consumption data in CE [19]. The main advantage of the PSID with respect to CE is its panel structure that can be exploited to follow an household across the recession.

The SCF is by far the best source of micro level data on household-level assets and liabilities for the United States. It is conducted every three years by the Board of Governors of the Federal Reserve System and collects detailed information on income and assets. With respect to assets the survey is particularly detailed, it contains information on financial and non-financial assets, debts and capital gains. The survey has two parts: a standard random sample of US households, and a second sample that focuses on wealthy households, identified on the basis of tax returns.

5.2 Comparing model implications with data

Consumption data

The model predicts that the share of total consumption of the generation that enters the model during a recession compared with the share of the young generation before the recession comes is lower by -0.15% or more formally that $\Delta\left(\frac{c^1}{\sum_i c^i}\right) = -0.15$[20]. Given the reference years for the calibration of the steady state and the crisis, ideally one would like to check whether $\Delta c^1_{2010-2007} = \frac{c^1_{2010}}{\sum_i c^i_{2010}} - \frac{c^1_{2007}}{\sum_i c^i_{2007}} = -0.15$ where c^1_t is the average consumption of households between 20 and 29, i.e. $c^1 = \frac{\sum_{j\in[20,29]} c^j}{\sum_{j\in[20,29]} \mathbf{1}(j\in[20,29])}$ and $\sum_i c^i_{2010}$ is the sum of average consumption of each 10 years cohort.

The age groups in which households are grouped in CE do not match exactly with those used in the model: the youngest group of households in the calibration is 20-29 and it (partially) overlaps with the two subgroups 18-24 and 25-34. Under the assumption that the relative dimension of cohorts is similar across years, that is reasonable given the small time span, the data available for the two subgroups can be averaged to get the change in consumption share of the 18-34 age-group. Estimate of the change in consumption share for the cohort 18-24 and the one recoupled for the cohort 18-34 across three different time spans are displayed in Table 6. The change in consumption share has the sign predicted by the quantitative model for all the time spans considered for both subgroups used as a comparison. What is more the magnitude of the

[19] Andreski et al. (2014)

[20] Where Δ is used as defined in footnote 13.

change is in line or greater than the one predicted by the model suggesting that the losses estimated in this paper may be a lower bound.

Age group	$\Delta c^1_{2008-2007}$	$\Delta c^1_{2009-2007}$	$\Delta c^1_{2010-2007}$
18-24	-0.17%	-0.34%	-0.37%
18-34	-0.09%	-0.26%	-0.11%

Table 6: Change in the share of consumption by age group. Source: CE

In the PSID we use the sample from the SRC together with the extra-questions on consumption and we create 4 years age brackets and we compute their mean (or median) of consumption in 2007. Then we compute the age group share of consumption over the sum of mean (or median) consumption of all age groups (thus assuming that all age groups are of equal size). Following the same households, exploiting the panel feature of the dataset, we compute the share of consumption in 2011 in the same way and we compare their share of consumption with those of the previous cohort of the same age in 2007. Results for the three cohorts closer to those the first cohort of the calibrated model are shown in Table 7. As in the CE, they are all negative as predicted but their magnitude is more than three times larger than model prediction pointing out that the losses through the quantitative model are likely a lower bound.

Age group	$\Delta c^1_{2011-2007}$ (mean)	$\Delta c^1_{2011-2007}$ (median)
18-21	-0.43%	-0.54%
22-25	-0.71%	-0.90%
26-29	-0.46%	-0.62%

Table 7: Change in consumption share of households across the Great Recession. Source: Panel Survey of Income Dynamics

Portfolio data

As it has been extensively discussed, households that becomes economically active during a downturn suffer a huge loss in their human wealth that makes them relatively more risk-averse than other cohorts. As a result they hold less risky assets than household born in normal times and they take less debt, therefore not exploiting the potential gains of the financial channel identified in Kiyotaki et al. (2011).

Our quantitative model predicts that the cohort that enters the model during the recession will have 4.2% less of risky assets in his portfolio with respect to a cohort that enters in normal times. We look at the empirical counterpart of the model estimates in SCF.

In Table 8 mean and median of main portfolio datas for households where the head is between 20 and 29 and with a positive net worth are reported. Risky assets are computed as the sum of of housing and equity directly or indirectly held through mutual funds.

As predicted, households in their twenties reduced the share of risky asset in their portfolios by around -10% between 2007 and 2010 when looking at the median value. The change in the mean is in the same direction, but with a greater magnitude.[21] The leverage, measured as the ratio between total debt and total assets, also points to a reduction in debt exposure of younger households, in line with model predictions.

Portofolio data display trends in line with model predictions and of greater magnitude signalling, as in the case of consumption, that welfare losses estimated by the model should be considered a lower bound.

Year	2007		2010		2013	
Variable	Mean	Median	Mean	Median	Mean	Median
Total value of debt held by household, 2013 dollars	59,986	6,961	48,339	5,251	39,051	3,700
Ratio of total debt to total assets	0.338	0.261	0.348	0.247	0.305	0.173
Total net worth of household, 2013 dollars	120,800	14,304	58,541	13,825	75,329	15,700
Total amount of risky assets of household, 2013 dollars	92,799	1,235	62,785	214.3	67,534	700
Share of risky assets in the portfolio	2.162	0.209	1.941	0.108	1.440	0.088

Table 8: 20-29 cohort portfolios across the Great Recession. Source: SCF

6 Conclusions

In this paper we develop an OLG model to analyze the welfare losses of different age groups during the Great Recession taking into account both the financial disruption, that affected more the older generations, and the losses associated with unemployment, that inflicted a greater damage to younger generations. Differently from previous theoretical literature and in line with empirical evidence we include in our model the possibility of long-term losses associated with unemployment through a lower accumulation of human capital that determines a lower productivity in the subsequent periods. Allowing for these extra channel we

[21] The interpretation of the change in the mean is more complex because the amount of risky assets is greater than total wealth because the amount of safe assets owned is negative.

find that the losses in human wealth suffered by households that become economically active during the downturn are greater than losses on financial wealth of older cohorts. In particular we show that, as a consequence of their increased risk aversion, the financial gains for younger households previously identified by Glover et al. (2020) are small and not enough to compensate them for the loss in human wealth. Welfare losses in households in their twenties in consumption equivalent are more than twofold those of any other living cohort.

Model implications on consumption and portfolio choice are validated using moments from micro-level data. Both CE and PSID consumption data provide evidence of a smaller consumption share going to younger households, supporting the model results. SCF data on portfolio confirm the deleveraging of the youngest cohort. Empirical moments on both consumption and portfolio choice go in the same direction of those estimated from the model but are larger than predicted, signalling that welfare losses estimated in this paper are likely a lower bound for the actual ones.

In the future the model could be extended to keep into account the intra-cohort heterogeneity that Hur (2018) has shown to be extremely relevant when looking at younger cohorts welfare loss.

The paper has two main policy implications.

Firstly, the quantitative analysis points out that welfare losses arising from employment fall dominate those arising from assets' markets collapse. Then, for a utilitarian social planner policy intervention should be focused on restoring employment level, and in particular youth employment, for two main reasons: it reduces the welfare loss of those cohorts that are most damaged and it reduces the welfare loss of all cohorts minimizing the loss on potential growth that affects welfare also to those cohorts that are already out of the labour market.

Secondly, the cohorts that entered the labour market during the Great Recession suffered a huge welfare loss from the uninsurable shock of being born during a major downturn. This very specific market incompleteness calls for a redistributive policy action of a social planner in favor of younger households. Since results of quantitative model suggests that households entering the economy after the crisis will have a net welfare gain from the same uninsurable shock, financing the above mentioned policy intervention through public debt repaid by future taxes on next generation would be optimal from a utilitarian point of view.

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Appendix

A The algorithm

Given that there are complete markets, we can solve an equivalent problem in which instead of equity and bonds there are Arrow-Debreu securities. Denote with μ_i the Negishi weight associated to the i-cohort and with $q(\eta', \boldsymbol{\mu})$ the price of the Arrow-Debreu security that pays 1 when the exogenous state η' realizes and the Negishi weights in the previous period were $\boldsymbol{\mu}$. In the model there are no investments (K is in fixed supply) and therefore the whole output is consumed by the cohorts, i.e.

$$Y(\eta) = \sum_{i \in I} c_i(\eta, \boldsymbol{\mu}) = \sum_{i \in I} \mu_i(\eta, \boldsymbol{\mu}) Y(\eta) \tag{14}$$

If we normalize the weights to $\sum_i \mu_i = 1$ then the fraction of output assigned to consumption of i-cohort is $c_i(\eta, \boldsymbol{\mu}) = \mu_i(\eta, \boldsymbol{\mu}) Y(\eta)$. Then, given that there is perfect aggregate risk sharing among those cohorts that are alive when financial markets open we have:

$$\begin{aligned}
\forall i \in \{1, \ldots, T-1\}: \qquad (c_i)^{-\sigma} &= \beta_i \frac{1}{q(\eta, \boldsymbol{\mu}, \eta')} \left(c'_{i+1}\right)^{-\sigma} \\
\left(\mu'_{i+1} Y(\eta')\right)^{\sigma} &= \beta_i \frac{1}{q(\eta, \boldsymbol{\mu}, \eta')} \left(\mu_i Y(\eta)\right)^{\sigma} \\
\mu'_{i+1} Y(\eta') &= \left[\beta_i \frac{1}{q(\eta, \boldsymbol{\mu}, \eta')}\right]^{\frac{1}{\sigma}} \mu_i Y(\eta) \\
\mu'_{i+1} Y(\eta') &= \beta_i^{\frac{1}{\sigma}} \left[\frac{1}{q(\eta, \boldsymbol{\mu}, \eta')}\right]^{\frac{1}{\sigma}} \mu_i Y(\eta)
\end{aligned} \tag{15}$$

Then, using the fact that $\sum_{i=1}^{T-1} \mu'_{i+1} = 1 - \mu'_1\left(\eta', \boldsymbol{\mu}\right)$ we have:

$$\begin{aligned}
\sum_{i=1}^{T-1} \mu'_{i+1} Y(\eta') &= \sum_{i=1}^{T-1} \left[\beta_i^{\frac{1}{\sigma}} \left[\frac{1}{q(\eta, \boldsymbol{\mu}, \eta')}\right]^{\frac{1}{\sigma}} \mu_i Y(\eta)\right] \\
Y(\eta') \left[1 - \mu'_1\left(\eta', \boldsymbol{\mu}\right)\right] &= \sum_{i=1}^{T-1} \left[\beta_i^{\frac{1}{\sigma}} \mu_i\right] \frac{Y(\eta)}{\left[q(\eta, \boldsymbol{\mu}, \eta')\right]^{\frac{1}{\sigma}}}
\end{aligned} \tag{16}$$

Taking the ratio between equation 15 and equation 16 we get the following expression:

$$\begin{aligned}
\frac{\mu_{i+1}^{'}Y\left(\eta'\right)}{Y\left(\eta'\right)\left[1-\mu_{1}^{'}\left(\eta',\boldsymbol{\mu}\right)\right]} &= \frac{\beta_i^{\frac{1}{\sigma}}\left[\frac{1}{q(\eta,\boldsymbol{\mu},\eta')}\right]^{\frac{1}{\sigma}}\mu_i Y\left(\eta\right)}{\sum_{i=1}^{T-1}\left[\beta_i^{\frac{1}{\sigma}}\mu_i\right]\frac{Y(\eta)}{\left[q(\eta,\boldsymbol{\mu},\eta')\right]^{\frac{1}{\sigma}}}} \\
\frac{\mu_{i+1}^{'}}{\left[1-\mu_{1}^{'}\left(\eta',\boldsymbol{\mu}\right)\right]} &= \frac{\beta_i^{\frac{1}{\sigma}}\left[\frac{1}{q(\eta,\boldsymbol{\mu},\eta')}\right]^{\frac{1}{\sigma}}\left[q\left(\eta,\boldsymbol{\mu},\eta'\right)\right]^{\frac{1}{\sigma}}\mu_i}{\sum_{i=1}^{T-1}\left[\beta_i^{\frac{1}{\sigma}}\mu_i\right]} \\
\mu_{i+1}^{'} &= \beta_i^{\frac{1}{\sigma}}\frac{\left[1-\mu_{1}^{'}\left(\eta^{'},\boldsymbol{\mu}\right)\right]}{\sum_{i=1}^{T-1}\left[\beta_i^{\frac{1}{\sigma}}\mu_i\right]}\mu_i
\end{aligned}$$

Since it holds for a generic i-cohort that is alive in the next period (i.e. $\forall i\in\{1,\ldots,T-1\}$) we have:

$$\begin{aligned}
\forall i\in\{1,\ldots,T-1\}:\ \mu_{i+1}^{'} &= \beta_i^{\frac{1}{\sigma}}\frac{\left[1-\mu_{1}^{'}\left(\eta^{'},\boldsymbol{\mu}\right)\right]}{\sum_{i=1}^{T-1}\left[\beta_i^{\frac{1}{\sigma}}\mu_i\right]}\mu_i \\
\mu_i^{'} &= \Gamma_i\left(\eta^{'},\boldsymbol{\mu}\right)
\end{aligned} \tag{17}$$

Rearranging:

$$\forall i\in\{1,\ldots,T-1\}:\ \frac{\mu_{i+1}^{'}}{\mu_i}=\beta_i^{\frac{1}{\sigma}}\frac{\left[1-\mu_{1}^{'}\left(\eta^{'},\boldsymbol{\mu}\right)\right]}{\sum_{i=1}^{T-1}\left[\beta_i^{\frac{1}{\sigma}}\mu_i\right]} \tag{18}$$

Under complete markets agents provide themselves insurance against idiosyncratic shocks and they share the same stochastic discount factor, that is:

$$\forall i\in\{1,\ldots,T-1\}:\ \pi\left(\eta^{'}|\eta\right)\beta_i c_{i+1}^{-\sigma} = q\left(\eta,\boldsymbol{\mu},\eta'\right)c_i^{-\sigma}$$

Then the price of an Arrow-Debreu security that pays 1 in the state $\eta^{'}$ given the current :

$$\begin{aligned}
q\left(\eta,\boldsymbol{\mu},\eta'\right) &= \pi\left(\eta^{'}|\eta\right)\frac{\beta_i c_{i+1}^{-\sigma}}{c_i^{-\sigma}}\\
q\left(\eta,\boldsymbol{\mu},\eta'\right) &= \pi\left(\eta^{'}|\eta\right)\frac{\beta_i\left[\mu_{i+1}^{'}Y\left(\eta^{'}\right)\right]^{-\sigma}}{\left[\mu_i Y\left(\eta\right)\right]^{-\sigma}}\\
q\left(\eta,\boldsymbol{\mu},\eta'\right) &= \pi\left(\eta^{'}|\eta\right)\beta_i\left(\frac{\mu_{i+1}^{'}}{\mu_i}\right)^{-\sigma}\left(\frac{Y\left(\eta^{'}\right)}{Y\left(\eta\right)}\right)^{-\sigma}
\end{aligned}$$

Now, using equation (18):

$$\begin{aligned}
q\left(\eta,\boldsymbol{\mu},\eta'\right) &= \pi\left(\eta^{'}|\eta\right)\beta_i\left[\beta_i^{\frac{1}{\sigma}}\frac{1-\mu_1^{'}\left(\eta^{'},\boldsymbol{\mu}\right)}{\sum_{i=1}^{T-1}\left(\beta_i^{\frac{1}{\sigma}}\mu_i\right)}\right]^{-\sigma}\left(\frac{Y\left(\eta^{'}\right)}{Y\left(\eta\right)}\right)^{-\sigma}\\
q\left(\eta,\boldsymbol{\mu},\eta'\right) &= \pi\left(\eta^{'}|\eta\right)\left[\frac{1-\mu_1^{'}\left(\eta^{'},\boldsymbol{\mu}\right)}{\sum_{i=1}^{T-1}\left(\beta_i^{\frac{1}{\sigma}}\mu_i\right)}\right]^{-\sigma}\left(\frac{Y\left(\eta^{'}\right)}{Y\left(\eta\right)}\right)^{-\sigma} \qquad (19)
\end{aligned}$$

The whole numerical algorithm is based on equation (19). Following the Brumm and Kubler (2013) define the intra-period excess consumption of each cohort in the following way:

$$\Psi_T\left(\eta,\boldsymbol{\mu}\right) = c_T\left(\eta,\boldsymbol{\mu}\right)-w_I\left(\eta\right) \qquad (20)$$

$$\forall i\in\{1,\ldots,T-1\}:\ \Psi_i\left(\eta,\boldsymbol{\mu}\right) = c_i\left(\eta,\boldsymbol{\mu}\right)-w_i\left(\eta\right)+\sum_{\eta^{'}}q\left(\eta,\boldsymbol{\mu},\eta'\right)\Psi_{i+1}\left(\eta^{'},\boldsymbol{\mu}\right) \qquad (21)$$

Proposition 1 Given the laws of motions for the distribution of consumption $\forall i\in\{2,\ldots,T\}$ defined by equation (17) and the consumption sharing rule in equation (14), the allocation defined by $\mu_1^{'}\left(\eta^{'},\boldsymbol{\mu}\right)$ is a competitive equilibrium if and only if $\forall\eta\in|\omega|^T,\forall\boldsymbol{\mu}\in S^T:\ \Psi_1\left(\eta,\boldsymbol{\mu}\right)=0$.

Proof. The economy is isomorphic to an Arrow-Debreu economy, therefore an allocation is a competitive equilibrium if:

- Agents maximize their utility given the price of securities (equation (17) is derived using the FOCs of the households' problem);

- The aggregate resource constraint is satisfied ((14));
- The agent's budget constraints are satisfied at any age and financial wealth at the beginning of life is 0 (it is guaranteed by the set of equations (20)-(21)).

□

Notice that $\boldsymbol{\mu} \in S^T$ and therefore, WLOG, it can be projected in $\mathbb{R}_+^{T-1}$.

The aim of the numerical computation is to define the function $\mu_1^{'}\left(\eta^{'}, \boldsymbol{\mu}\right)$. Indeed, given $\mu_1^{'}\left(\eta^{'}, \boldsymbol{\mu}\right)$ and $\boldsymbol{\mu}$, the shares of consumption of the other cohorts (i.e. $\mu_i^{'}$ for $i = 2, \ldots T$) can be computed using equation (17). To characterize the function $\mu_1^{'}\left(\eta^{'}, \boldsymbol{\mu}\right)$ over the state space we use the following procedure:

1. Create the state space grid: generate a Smolyak grid of $\{\mu_i\}_{i=1}^{T-1}$ (a $T-1$ dimensional vector) for each exogenous state $\eta \in |\omega|^T$;
2. Create a guess of $\mu_1^{'}\left(\eta^{'}, \boldsymbol{\mu}\right)$ over the entire state space grid and an interpolating function $\hat{\mu}_1\left(\eta^{'}, \boldsymbol{\mu}\right)$ (Chebyshev polynomial) over the whole state space;
3. For any point of the grid, construct the feasible consumption histories, the price of the securities and the implied budget excesses for a cohort born in that state using the guess $\hat{\mu}_1\left(\eta^{'}, \boldsymbol{\mu}\right)$;
4. Use the fact that in a competitive equilibrium period-0 budget excess must be 0, i.e. $\Psi_1\left(\eta^{'}, \boldsymbol{\mu}\right) = 0$, to get an equation containing $\hat{\mu}_1\left(\eta^{'}, \boldsymbol{\mu}\right)$;
5. Repeat steps 3 and 4 for all points in the state-space grid and use a non-linear solver to get a new guess for $\hat{\mu}_1\left(\eta^{'}, \boldsymbol{\mu}\right)$
6. Repeat steps from 3 to 5 until the difference between guesses is small enough.

B Calibration

B.1 Financial market

The parameters to calibrate are α (the capital share of the production function), p (the steady state price of equity), q (the price of the bond), B (the amount of bonds in the economy). The empirical moments to match in the data are: the aggregate share of risky assets in households' net worth (which we denote with $\overline{\lambda}$), the aggregate wealth to labour income ratio ($\overline{W}$) and the gross interest rate ($\overline{R}$). Thus, the steady state relationships are:

$$\overline{\lambda} = \frac{p}{p+qB} \tag{22}$$

$$\overline{W} = \frac{p+qB}{1-\alpha} \tag{23}$$

In a deterministic world the two assets must satisfy the non-arbitrage condition. Therefore given the gross interest rate $\overline{R}$ the two following equations must hold:

$$\overline{R} = \frac{1}{q} \tag{24}$$

$$\overline{R} = \frac{p+\alpha Y+qB-B}{p} \tag{25}$$

Then, the calibration is done using four equations (Equation 22, Equation 23, Equation 24, Equation 25) and four unknowns. The closed form solution of the four unknowns is:

$$\begin{aligned} p &= \overline{\lambda}\frac{\overline{W}}{1+\left(\overline{R}-1\right)\overline{W}} \\ q &= \frac{1}{\overline{R}} \\ B &= \frac{\left(1-\overline{\lambda}\right)\overline{RW}}{1+\left(\overline{R}-1\right)\overline{W}} \\ \alpha &= \frac{\left(\overline{R}-1\right)\overline{W}}{1+\left(\overline{R}-1\right)\overline{W}} \end{aligned}$$

The empirical moments $\overline{\lambda}$ and $\overline{W}$ are taken from 2007 SCF. They are computed as the average across age-group averages because we want to replicate the distribution across cohorts that are assumed to be of equal size. And then:

$$\overline{\lambda} = 0.918$$
$$\overline{W} = 0.788$$

The apparently low level of the wealth-to-labour income ratio reflects the fact that we am considering a 10-years period and therefore at the denominator is the cumulated labour income over the same time span.

The gross real return in the economy, $\overline{R}$ is computed as a weighted average return across asset classes. Piazzesi et al. (2007) estimate a real return on safe assets of 0.75% and on risky assets of 4.75% (which is an average between housing 2.52% and stocks of 6.94%). Given the 10-year period the implied $\overline{R}$ is:

$$\overline{R} = \overline{\lambda}\,(1.0475)^{10} + \left(1 - \overline{\lambda}\right)(1.0075)^{10} = 1.5485$$

The implied values are $\alpha = 0.3017$, $B = 0.0699$, $p = 0.6458$ and $q = 0.5050$. The value of the capital share is consistent with calibrations coming from estimates on micro data.

Labor market

The $\left(\varphi^i\left(\omega\right)\right)_{i=1}^{T}$ represent the probability that an agent of the i-cohort is employed given the state of the economy and therefore they are set to match the cohort-specific employment-to-population ratios in 2007 Q2 and in 2010 Q2.

The Bureau of Labor Statistics does not provide employment to population data for all the age brackets. For those that are missing I compute the data using the absolute number of employed (available for 5 years brackets) and population estimate by cohorts taken from Census Bureau. Data for the reference periods and the calibration of $\left(\varphi^i\left(\omega\right)\right)_{i=1}^{T}$ are shown in Table 9.

Age group	2007:Q2	2010:Q2	Change	$\varphi^i\left(\omega_H\right)$	$\varphi^i\left(\omega_L\right)$
20-29	73.89 %	66.97 %	-6.92 %	0.7389	0.6697
30-39	80.30 %	75.75 %	-4.55 %	0.8030	0.7575
40-49	81.07 %	76.62 %	-4.45 %	0.8107	0.7662
50-59	74.12 %	71.39 %	-2.73 %	0.7412	0.7139
60-69	42.12 %	42.42%	0.30 %	0.4212	0.4242
70+	9.93 %	10.22%	0.29 %	0.0993	0.1022

Table 9: Employment-to-population ratio anc calibration of $\left(\varphi^i\left(\omega\right)\right)_{i=1}^{T}$. *Source: Bureau of Labor Statistics and Census Bureau*

B.2 Human capital accumulation

To make some examples and to clarify the role of χ^i, effects on human capital profiles of one period spent in unemployment for an household between 20 and 29 and for an household between 40 and 49 are plotted in Figure 10 left and right panel respectively.

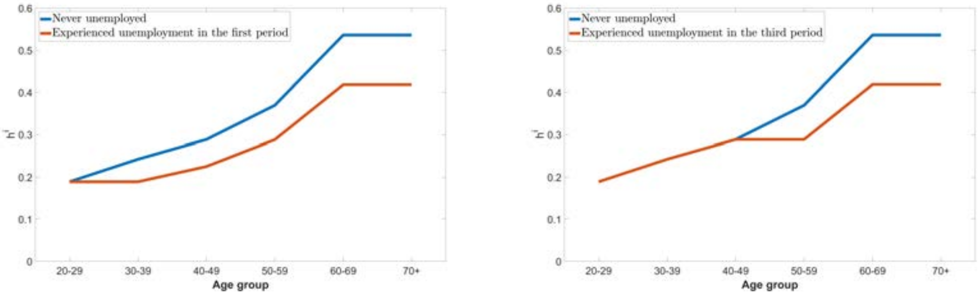

Figure 10: Comparison between human capital profile of a worker that is always employed and a worker that is unemployed in his twenties (left panel) or in his fourties (right panel)

Since human capital increases labour effectiveness and households of the same cohort have the same probability of being employed in the future, the wedge between the two profiles of human capital accumulation reflects in a wedge between the profiles of income.

Therefore, the calibration strategy of $\left(\chi^i\right)_{i=1}^{T}$ is the following: PDV of labour income losses implied by unemployment have to match those estimated by Davis and von Wachter (2011). Therefore, using equation (26) and filling L^i with estimate on the effect on labour income from layoff for the specific cohort it is possible to uniquely recouple χ^i. Estimate on the fall in PDV of labour income and calibration of $\left(\chi^i\right)_{i=1}^{T}$ can be found in Table 10.

Some remarks on the source of the micro-estimates and the assumption needed in this calibration are needed.

Firstly, the age groups used for micro-estimates do not perfectly overlap with those of the model, since the difference is of one year we use them without any adjustment. What is more there are no available estimates for the last age group of the model: we set the parameter to zero thus setting no long-term losses from unemployment experienced in the sixties. Given the small amount of labour income in the last period of life (the employment ratio is around 10% in both states, see Table 9) the assumption is not crucial for the results.

Secondly, there are several papers that try to estimate long term losses in labour income from unemployment. we choose those from the work of Davis and von Wachter (2011) since they are disaggregated by age and by state of the economy when the layoff happen. They are estimated using US data, while Jarosch (2014) finds losses of similar magnitude on German data.

Finally, we use a conservative calibration that reduces at the minimum the long-term losses from unemployment and therefore the overall losses should be considered a lower bound. Indeed, the representative agent of the cohort will suffer a loss proportional to the change in the employment ratio between 2007 Q2 and 2010 Q2 but some households were fired and in the meanwhile they found another job, therefore the change in the employment ratio is the lowest possible loss from lay-offs. What is more, according to the estimate of Davis and von Wachter (2011) the loss is different depending on the state of the economy. In our model the parameter χ_i is not state dependent and we calibrate it to match the losses during a recession. As a result the losses with respect to a counterfactual cohort will be reduced.

An overall calibration test of the set of parameters can be found in Appendix D.1.

As explained, their calibration is taken from the micro estimates of Davis and von Wachter (2011).

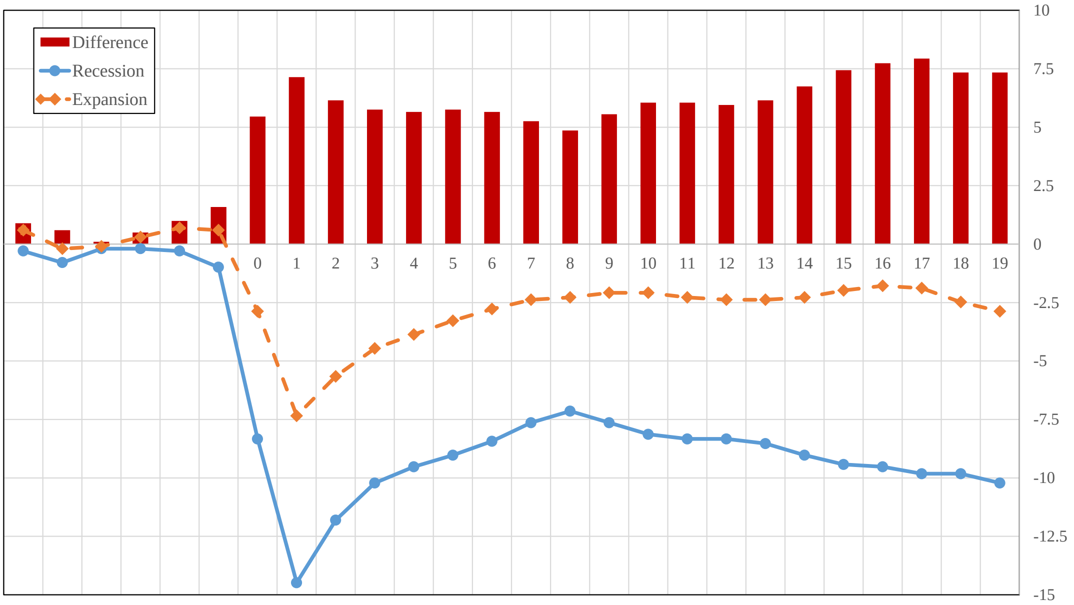

Figure 11: Annual earnings losses at displacement during a recession (blue continuous line) or an expansion (orange dashed line) as computed by Davis and von Wachter (2011) for households at age 21-30. The difference between the two lines is represented by the red bars.

Age of layoff (during recessions, with $u \geq 8\%$)	Fall in PDV of labour income (Davis and von Wachter (2011) estimate)	χ^i
21-30	22.0%	0.2821
31-40	16.0%	0.1905
41-50	21.9%	0.2804
51-60	31.1%	0.4514
61-70	N.A.	0

Table 10: Effects on future earnings of being fired during a mass layoff and calibration of χ^i

B.3 Cohort specific productivity

$\left(\varphi^i\right)_{i=1}^T$ and $\left(\chi^i\right)_{i=1}^T$, as calibrated before, are able to replicate with a good degree of approximation the gross labour income across households of different age that we observe in SCF data in 2007. Nonetheless, the $\left(\varepsilon^i\right)_{i=1}^T$ are needed to match labour income net of taxes and Social Security and Retirement transfers, particularly relevant for older cohorts.

The introduction of a government with a fiscal policy while increasing the burden of numerical computation, thus not improve the fit of households labour income[22].

Firstly, consider that the calibration of $(\varepsilon_i)_{i=1}^T$ in the model has an impact on the effective labour supply and therefore is deeply interconnected with the calibration of the initial level of human capital $\overline{h}$. Then, given that the targets of the calibration are the mean of cohort labour income net of transfers in 2007, therefore T moments, and there are $T+1$ parameters to be calibrated there is one degree of freedom. Indeed, for any initial level of human capital is possible to find a "neutral" value of ε^i, i.e. a $\overline{\varepsilon}$ equal for all cohorts and such that the sum of wages of different cohorts is not altered. More formally, $\forall \overline{h} \in \mathbb{R}_+ \; \exists \overline{\varepsilon}$ such that $\forall i \; \varepsilon^i = \overline{\varepsilon}$ and $\left(\sum_{i=1}^{I} \varphi^i \left(\omega_H\right) \overline{\varepsilon} h^i \left(\eta^{SS}\right)\right) = (1-\alpha) Y \left(\eta^{SS}\right)$. we calibrate $\overline{h}$ in order to get $\overline{\varepsilon} = 1$, i.e. to get a neutral value of ε^i equal to 1 and making the interpretation of their calibration simpler.

Then $\left(\varepsilon^i\right)_{i=1}^T$ can be seen as a parsimonious representation of the Social Security system in the following way: if $\varepsilon^i > \overline{\varepsilon}$, i.e. if the i-cohort productivity is higher than the neutral one, it is as if the cohort is receiving a transfer from the government, At the opposite if $\varepsilon^i < \overline{\varepsilon}$ it is as if the cohort is paying a tax (non distortionary since labour supply is inelastic).

Three implications that must be discussed:

- The government pay actual pensioners using taxes of actual workers: this is consistent with the Pay-

[22] In Appendix D.2 a complete fiscal policy with a government budget is introduced as a calibration test on ε^i withouth improving the fit of the model

As-You-Go (PAYG) structure of the Social Security system;

- A lower h^i reduces the amount of transfers received from older households. h^i in this model records the working history of the cohort. The pension paid by Social Security system is, nowadays, directly related to earnings during the working life. Then, this feature of the modelization is consistent with the US system;

- A lower level of output reduces the value of actual transfers, nonetheless this modelization does not force period by period budget equilibrium of the government. It can be justified with budgetary pressure: the government has to reduce already defined benefits. Nonetheless, since the model does not have to satisfy a period-by-period budget constraint this modelization has a better match of 2010 data on income (see Appendix D.2).

Then, we calibrate $(\varepsilon_i)_{i=1}^T$ to match the wage profile in 2007. Results are shown in Table 11. As a comparison we put also the vector of $(\tilde{\varepsilon_i})_{i=1}^T$, that would match 2007 data on gross labour income (net of government transfers).

As expected the ε^i are increasing with age and for the mid-cohorts are close to 1. What is more $\varepsilon^1 < 1$, i.e. the first cohort has a net income that is lower than its wage and ε^6 is much higher than 1, i.e. the oldest cohort rely heavily on social security transfers.

Cohort	ε^i	$\tilde{\varepsilon}^i$ (gross labour income)
1	0.6778	0.7803
2	0.9068	1.0372
3	0.9730	1.1000
4	1.0237	1.1306
5	1.0638	0.9112
6	2.1809	0.7048

Table 11: Calibration of $(\varepsilon^i)_{i=1}^T$

B.4 Aggregate shock

The level of $z(\omega_H)$ is normalized to 1 while $z(\omega_L)$ is calibrated to match the fall in output coming from the Great Recession. We do not match the actual fall in output (5.4% between the NBER recession dates) but the fall from the pre-crisis trend since the model is stabilized. we start from the work of (Hall, 2014) that estimates a trend between 1990-2007 for all the components of output growth and makes projections on 2010. It estimates a fall in GDP by 12.4% composed by:

- TFP: 3.1%
- Capital contribution: 2.1%
- Population: 0.8%
- labour Force participation: 1.2% (of which 0.9% from ageing)
- Unemployment: 3.5%
- labour Quality: -0.6%
- Hours per week: 1.6%
- Business fraction: 0.7%

We take away from the calibration the fall coming from decreasing population (0.8%) and from ageing labour (0.9%) (they relate to long-term processes on which the paper is not focused and that are not related to the Great Recession). Thus the implied fall is 10.7% of which 5.4% is related to labour and therefore is endogenously modeled. Then we impute to $z(\omega_L)$ the residual fall of 5.3%.

On the labour side, given ω_L, there is a fall in output automatically implied by the worsening condition of the labour market. In particular $\varphi(\omega_L)$ implies a fall in employment by 3.0%. This is roughly consistent with the estimate above, indeed the fall in employment rate is given by the sum of labour Force participation and unemployment (4.7%) but 0.9% comes from ageing of the labour force. The residual discrepancy (0.8%) comes from the assumption of equal size of the cohorts[23]. There is no intensive margin in the model and therefore the change in hours is not endogenous. The (positive) contribution of labour quality is -0.6% also in the model (without considering the effect of ε^i, -1.0% considering them), an additional evidence of the good calibration of $\left(\chi^i\right)_{i=1}^T$.

[23]The fall by 3.0% is computed as the average of the fall (or the increase) in all the cohorts, in order to match the Hall estimates one should take into account that it is a weighted average.

C Mathematical appendix

C.1 χ_i calibration

Claim 1. *Define W_e^i the PDV of future earnings of an individual of the i-cohort that has been employed in the period, W_u^i the one of an unemployed and L^i the loss in PDV for the i-cohort. Then $\forall i$, χ_i is inversely proportional to the loss in PDV induced by unemployment at age i, that is:*

$$\frac{W_u^i\left(\underline{\chi}\right)}{W_e^i\left(\underline{\chi}\right)} = 1 - L^i = \frac{1}{1+\chi^i}$$

Proof. Start with the defintion definition $\forall i$:

$$\frac{W_u^i\left(\underline{\chi}\right)}{W_e^i\left(\underline{\chi}\right)} = 1 - L^i$$

Where $\underline{\chi}$ is the vector $\left[\chi^1 \dots \chi^I\right]$. Then we have:

$$\frac{W_u^i\left(\underline{\chi}\right)}{W_e^i\left(\underline{\chi}\right)} = \frac{\sum_{j=i+1}^{I} \frac{(1-\alpha)z\varepsilon^i \frac{h_u^j}{\mathfrak{L}^\alpha}}{(1+r)^{j-i}}}{\sum_{j=i+1}^{I} \frac{(1-\alpha)z\varepsilon^i \frac{h_e^j}{\mathfrak{L}^\alpha}}{(1+r)^{j-i}}}$$

Considering an atomistic worker his employment status does not affect the aggregate labour supply and therefore $\mathfrak{L}^\alpha$ is the same in the two scenarios. With a similar argument the interest rate r at which the two flows are discounted is the same.

As a result, using Equation 1, the previous expression can be reduced to:

$$\begin{aligned}
\frac{W_u^i\left(\underline{\chi}\right)}{W_e^i\left(\underline{\chi}\right)} &= \frac{\sum_{j=i+1}^{I} h_u^j}{\sum_{j=i+1}^{I} h_e^j} \\
&= \frac{\sum_{j=i+1}^{I} h_u^i \prod_{k=i}^{j} \left(1+\chi^k\right)}{\sum_{j=i+1}^{I} h_e^i \prod_{k=i}^{j} \left(1+\chi^k\right)}
\end{aligned}$$

And collecting the constant terms h_u^i and h_e^i

$$\begin{aligned}\frac{W_u^i\left(\underline{\chi}\right)}{W_e^i\left(\underline{\chi}\right)} &= \frac{h_u^i}{h_e^i}\frac{\left[\sum\limits_{j=i+1}^{I}\prod\limits_{k=i}^{j}\left(1+\chi^k\right)\right]}{\left[\sum\limits_{j=i+1}^{I}\prod\limits_{k=i}^{j}\left(1+\chi^k\right)\right]} \\ &= \frac{h_u^i}{h_e^i}\end{aligned}$$

Finally, using equation 1 and considering the employment status of the two workers the expression becomes:

$$\frac{W_u^i\left(\underline{\chi}\right)}{W_e^i\left(\underline{\chi}\right)} = \frac{h^{i-1}}{h^{i-1}\left(1+\chi^i\right)} = \frac{1}{1+\chi^i}$$

Then the relationship between L^i and χ^i is $\forall i$:

$$1 - L^i = \frac{1}{1+\chi^i} \tag{26}$$

□

D Calibration tests

D.1 Human capital accumulation parameters

The calibration of the χ^i is crucial in this estimation and it is based on micro-estimates, therefore we test extensively how well it matches related moments in aggregate data. Ideally, since $\left(h^i\right)_{i=1}^{T}$ capture the return from experience and employment in the life cycle and $\left(\varphi^i\right)_{i=1}^{T}$ capture the employment rate of the age group one would expect this two parameters to provide a good representation of the income life cycle profile observed in data without using the cohort specific parameter ε^i. Then $\overline{h}$ (the initial levelof human capital) is used as a normalization term for the aggregate level of wages[24].

Then suppose a series of ω_H (that is consistent with the modelization of the "steady state") then, the implied profile of $\left(h^i\right)_{i=1}^{T}$, the $\left(w^i\right)_{i=1}^{T}$ implied by the model and the ones observed in data[25] are shown in Table 12.

Age group	h^i	w^i (model)	w^i (data, 2007)
20-29	0.1889	0.0974	0.0760
30-39	0.2283	0.1280	0.1328
40-49	0.2632	0.1490	0.1639
50-59	0.3230	0.1672	0.1890
60-69	0.4311	0.1268	0.1155
70+	0.4311	0.0299	0.0211
Total		0.6983	0.6983

Table 12: Wage profile implied by model calibration and actual wage profile

From Figure 12 we can see that the implied wage profile is similar to the one observed in the data. The model seems to overestimate the wage of younger workers and to underestimate the one of middle-aged workers. This is consistent with theory and empirical studies on wage-productivity gap during working life, according to which, younger workers receive a wage which is below their productivity in exchange for a wage higher than their productivity later in life[26].

Another important concern may be on the role played by the employment-to-population rate in driving the good fit. To fully understand the determinants of the match between model and data of labour income, we "normalize" (dividing them by their maximum value among the cohorts) $\varphi^i\left(\omega_H\right)$, w^i in the data, w^i

[24]When considering the cohort specific productivity the role of $\overline{h}$ changes and it is discussed in the paragraph of $\left(\varepsilon^i\right)_{i=1}^{T}$ calibration.

[25]In this case we consider labour income the sum of wages and a fraction of $(1-\alpha)$ of business, farm and self-employment income.

[26]Seminal theoretical contribution from Lazear (1979).

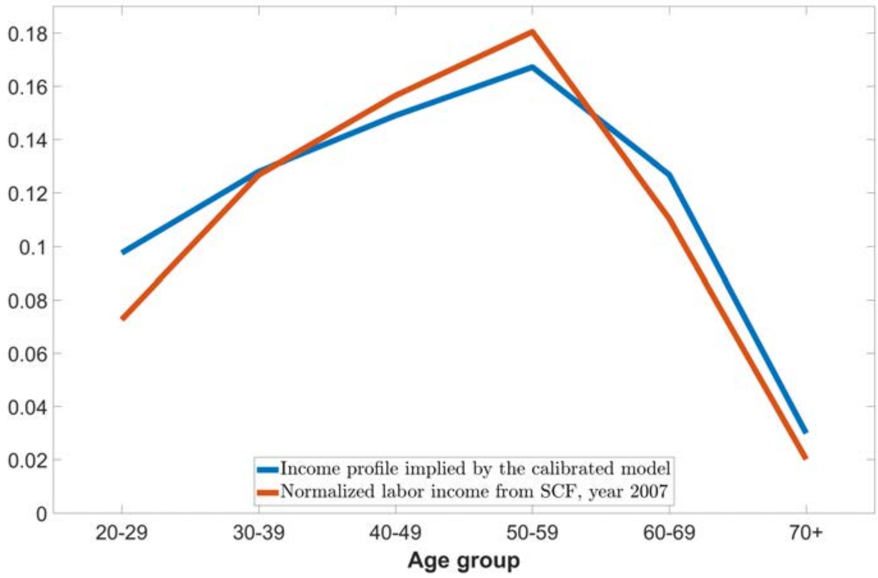

Figure 12: Wage profile in steady state

implied from the model and we make a comparison between the two series. Results are shown in Figure 13. As it can be seen, relative employment is the main driver for the high fall in the last two cohorts but the good match of the hump-shaped wage profile comes from the calibration of χ^i and from the implied h^i profile.

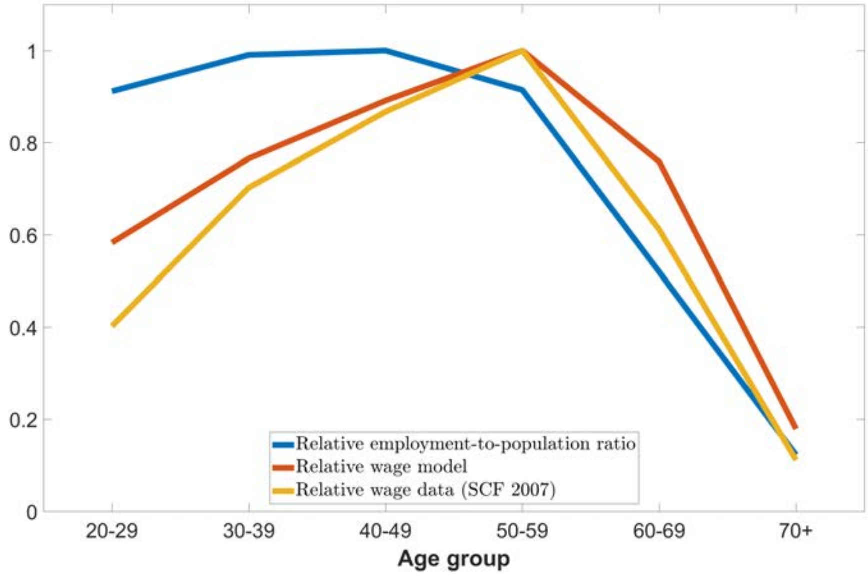

Figure 13: Relative wage implied from the model, relative wage in data and relative employment across age groups

D.2 Cohort specific productivity

Two different calibration tests are performed.

In the first one we impute the negative shock in the model (the calibration of the shock is discussed in the next subsection) and we compare the implied profile of labour income with the one observed in 2010 data (that is the year taken as a reference for the recession, the calibration of the exogenous state will be discussed in the next subsection). Results are shown in Table 13. The model does a good job in fitting the data.

Cohort	w^i (model)	w^i (data, normalized)	% difference
1	0.0548	0.0537	+1.96%
2	0.1002	0.0993	+0.83%
3	0.1253	0.1324	-5.61%
4	0.1508	0.1471	+2.48%
5	0.1243	0.1287	-3.61%
6	0.0613	0.0646	-5.38%

Table 13: Wage profiles after the shock

For an additional check we try to add a government to the model and we look at the implied profile of net labour income after the negative shock. The modelization is presented in appendix D.2. Results are shown in Figure 14: the differences are negligible and if anything the social security is exacerbating the

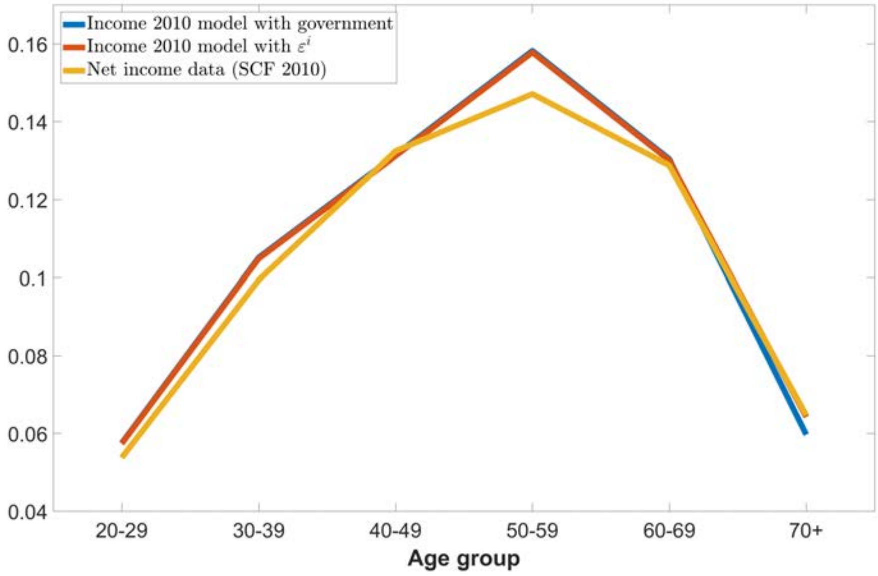

Figure 14: Wage profile after the shock as implied by the model with epsilon

under-performance for the older cohorts. This is due to the fact that the simple fiscal policy rule does not allow for intertemporal redistribution through government debt, given that the last cohort takes all fiscal residuals any change in employment determines a one to one fall of transfers to the last cohort.

Adding the social security system to the model The representation through the ε^i of the social security system simplifies the model giving satisfactory results. Indeed alternative, simple methods for representing the transfers do not perform significantly better. The simplest alternative method would be a system of taxes (and subsidies) on gross wages. Unless we add public debt and a more complex fiscal rule (that increases the state dimensionality thus making computation more complex), the government balance has to be in equilibrium period by period. Therefore, when calibrating the fiscal policy rule we do not have 6 degrees of freedom but only 5[27]. Then, we model a simple fiscal rule with proportional tax rates (subsidies if τ^i is negative) on the first five cohorts, residuals are used to pay sixth cohort. The government budget constraint is:

$$\sum_{i=1}^{5} \tau^i l^i w^i = t_6$$

where t_6 is a lump sum transfer to the sixth cohort.

In order to calibrate $\left(\tau^i\right)_{i=1}^{5}$ and t_6 we minimize the distance between data and model in 2007 under the

[27] We loose one degree of freedom adding the intra-temporal budget constraint of the government

government budget constraint, results are shown in Table 14.

Cohort	τ^i
1	0.32
2	0.09
3	0.03
4	-0.02
5	-0.06

Table 14: Cohort specific tax rates calibration

E Alternative welfare measures

As previously discussed welfare loss computation in OLG models is not standard. Then, we assess whether our results are dependent on the method used to compute losses. To do so we compute welfare losses using two alternative measures: *remaining lifetime consumption equivalent* and *ex-ante consumption equivalent*.

We define *ex-ante consumption equivalent* welfare loss the percentage increase in consumption in all states and in any period life that as a newborn he would require to be indifferent with the steady state consumption stream. More formally:

Definition 2 The *ex-ante consumption equivalent* welfare loss of the cohort born at time k and experiencing the recession at age i (i.e. the i-cohort when the recession comes) is the δ^i_{EA} such that:

$$\sum_{t=k}^{T+k}\left(\prod_{j=t-k}^{T}\beta_j\right)u\left(\left[c_t^{t+1-k}\left(\eta_t\right)\right]\left(1+\delta^i_{EA}\right)\right)=\sum_{t=k}^{T+k}\left(\prod_{j=t-k}^{T}\beta_j\right)u\left(c_t^{t+1-k}\left(\eta_{ss}\right)\right)$$

where $\eta_t = \left[(\omega_j)_{j=t-T+1}^{t}\right]$ and the sequence is such that:

$$\omega_t = \begin{cases} \omega_L & t = k-1+i \\ \omega_H & else \end{cases}$$

The consumption equivalent is "*ex-ante*" because it measures the cost in terms of consumption at the beginning of the life of the households, and makes him indifferent between living through a recession at age i or not living it before he enters the model. In a nutshell it takes the perspective of a newborn cohort and it identifies the worst period to leave in a recession from that perspective. The main advantage of this method for comparing households of different ages is that uses stream of consumption of the same length for all ages and all cohorts live through an equivalent amount of good and bad periods during their life cycle. At the same time the magnitude of the welfare loss from the negative shock on consumption depends on the calibration of betas and, in particular, the older cohorts have a lower cost in terms of consumption equivalent simply because it is more discounted.

An alternative method is the *remaining lifetime consumption equivalent*, used in OLG setup by Glover et al. (2020). Under this method welfare losses are measured in terms of consumption on remaining periods of life, i.e. as the percentage change in consumption (in all states and over all remaining periods of life) under a no recession scenario needed to make households indifferent between the current aggregate state being ω_H instead of ω_L.

Definition 3 The *remaining lifetime consumption equivalent* welfare loss of the cohort born at time k and experiencing the recession at age i (i.e. the i-cohort when the recession comes) is the δ^i_{RL} such that:

$$\sum_{t=k+i}^{T+k} \left(\prod_{j=t-k}^{T} \beta_j \right) u\left(\left[c_t^{t+1-k}(\eta_t) \right] (1+\delta^i_{RL}) \right) = \sum_{t=k+i}^{T+k} \left(\prod_{j=t-k}^{T} \beta_j \right) u\left(c_t^{t+1-k}(\eta_{ss}) \right)$$

where $\eta_t = \left[(\omega_j)_{j=t-T+1}^{t} \right]$ and the sequence is such that:

$$\omega_t = \begin{cases} \omega_L & t = k-1+i \\ \omega_H & else \end{cases}$$

The main disadvantage of this approach is that it treats equally consumption streams that are by construction different. Indeed, a newborn cohort has an higher residual life consumption (6 periods) if compared to the last generation (1 period) and therefore his losses will be diluted over a longer residual life by mechanical computation. On the one other hand the major fall in consumption arising from recession is in period 0 for all cohorts, therefore avoiding the discounting of older cohorts losses that the previous methods implied.

A comparison between the three computations is displayed in Table 15.

Age spent in recession	δ^i_{EA}	δ^i_{RL}	$\delta^i_{t_0}$
20-29	5.54%	5.54%	22.78%
30-39	2.46%	3.49%	9.52%
40-49	1.74%	3.99%	9.62%
50-59	1.27%	5.24%	8.89%
60-69	1.02%	7.15%	8.39%
70+	0.82%	8.11%	8.11%

Table 15: Alternative welfare measures.

Firstly $\forall i\ \delta^i_{t_0} \geq \delta^i_{RL} \geq \delta^i_{EA}$, i.e. for all cohorts the one-period consumption equivalent is equal or greater to the residual life consumption equivalent (equal for the older cohort and greater for all the others) that is equal or greater to the ex-ante consumption equivalent (equal for the younger and greater for all the others). The different magnitude of $\delta^i_{t_0}$ with respect to $\delta^i_{RL}, \delta^i_{EA}$ can be explained by the different horizons over which percentage compensation is distributed in the different computations. $\delta^i_{t_0}$ is the increase in consumption in one period as opposed to $\delta^i_{RL}, \delta^i_{EA}$ that are percentage compensations over more periods of consumption (but for the remaining life consumption of older cohort that is equivalent to the one period consumption since they die in the next period).

Secondly $\forall j > i \; \forall i \; \delta^j_{t_0} < \delta^i_{t_0}, \; \delta^j_{EA} < \delta^i_{EA}$, i.e. both in terms of ex-ante consumption and one period consumption the younger cohorts are the most hurt by the recession and the newborn cohort suffers a welfare loss that is double in magnitude with respect to other cohorts.

Losses are non-monotonic in age when looking at δ^i_{RL} but this hump-shape is obviously induced by the fact that the residual consumption is reduced with ages. Nonetheless, even with this methodology that tends to dilute losses of younger cohorts only the last two cohorts have greater losses than the newborn cohort.

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