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Mitigating the forward guidance puzzle: inattention, credibility, finite planning horizons and learning

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Abstract

This paper develops a simple, consistent methodology for generating empirically realistic forward guidance simulations using existing macroeconomic models by modifying expectations about policy announcements. The main advantage of our method lies in the exact preservation of all other shock transmissions. We describe four scenarios regarding how agents incorporate information about future interest rate announcements: "inattention", "credibility", "finite planning horizon", and "learning". The methodology consists of describing a single loading matrix that augments the equilibrium decision rules and can be applied to any standard DSGE, including large-scale policy-institution models. Finally, we provide conditions under which the forward guidance puzzle is resolved.

Keywords: Monetary policy, Expectations, Unconventional Policy

JEL Classifications: C63, E32, E52

Non-technical Summary

Since the 2007-08 global financial crisis, central banks across the developed world have struggled with weak aggregate demand and short-term nominal interest rates at historically low levels. In the face of these challenges, central banks have started to make increased use of interest rate forward guidance – that is, announcements to keep future short-term nominal interest rates lower for longer than expected by economic agents. Standard macroeconomic models predict unrealistically powerful effects of forward guidance announcements – a problem called the Forward Guidance Puzzle.

This paper provides a practical and discreet methodology for generating empirically realistic forward guidance experiments in otherwise standard macroeconomic models. We propose four scenarios to dampen the Forward Guidance Puzzle: "inattention", "credibility", "finite planning horizon", and "learning".

The method relies on a transparent modification of agents' forecasts of the monetary policy "news states", which is recorded in a single lower-triangular matrix. This modification guarantees that the solution to the model is not changed in any other way, therefore keeping the transmission of all other shocks exactly the same as in the non-modified case. This is especially beneficial in case of large-scale policy-institution models that exhibit shock transmissions which have undergone significant vetting.

We explore four lower-triangular matrices (henceforth, *scenarios*) that embody the above four scenarios. These scenarios (loosely) coincide with several of the more rigorous resolutions of the forward guidance puzzle in the literature using small-scale models. Inattention is when a fraction of agents are inattentive to central bank announcements about the future. Credibility is when a decaying fraction of agents believe promises of the central bank further out in the future. Finite planning horizon is when agents dismiss all announcements past a certain horizon. Finally, learning is when a small fraction of agents initially incorporate forward guidance announcements in their expectations, but as the horizon at which announcements are made increases, a growing fraction begin to incorporate the forward guidance announcements into their expectations.

The modifications are presented in a simple small-scale model that captures the problem of the Forward Guidance Puzzle under full attention. The scenarios are able to dampen the FG Puzzle under various parameterisations. An extended medium-scale model, which includes a modification that is informed by empirical evidence, resolves the FG Puzzle. This model does not exhibit the counterintuitive reversals pointed out by Carlstrom et al. (2015), where the increase of a peg by one quarter switches the effects from highly expansionary to highly contractionary.

1 Introduction

Since the 2007-08 global financial crisis, central banks across the developed world have struggled with weak aggregate demand and short-term nominal interest rates stuck at historically low levels. In the face of these challenges, central banks have used forward guidance—that is, announcements to keep future short-term nominal interest rates lower-for-longer in the future. Standard DSGE models predict unrealistically powerful effects of forward guidance announcements—a problem first articulated by Del Negro et al. (2012) as the forward guidance puzzle.

Many insightful and elegant resolutions to the puzzle have since been proposed. However, many of them are not able to provide quantitative policy advice. This is because they fall into one of two general problems. Problem 1: The proposed resolution requires a substantial reworking of the microfounded building blocks of the existing model. For example, Del Negro et al. (2012)'s proposed resolution is an overlapping generations (OLG) structure, thus doing away with the infinitely-lived representative agent. In so doing, however, the modeler needs to take a stance on fiscal policy, since the model also becomes non-Ricardian as a consequence of the OLG assumption. Problem 2: The proposed resolution requires non-linear solution techniques. For example, McKay et al. (2016) propose modeling heterogeneous households with uninsurable income risk. Solving the model non-linearly, however, constrains the scope of the model to be small-scale. Other resolutions, like sticky information al a Kiley (2016), substantially change the transmission of standard monetary policy and all other shocks. After all, with an inertial Taylor rule, the power of an unanticipated monetary policy shock is also largely the result of agents beliefs that rates will return to normal only gradually. Sticky information discounts how forward looking agents are, thus, not only dampening forward guidance, but dampening the effectiveness of monetary policy in general.

Faced with these challenges, we propose a *practical* and *discreet* methodology for generating *empirically realistic* forward guidance experiments using existing models. Realistic, because the method can be easily calibrated to the growing number of empirical studies of the power of forward guidance. Practical, because, it is an addition to any standard (linearized) structural model, and discreet because it does not alter the predictions of the model to other standard shocks.[1] Our implicit assumption, therefore, is that a policy institution's current model is the

[1] On request, a `Dynare` add-on to replicate the results in this paper will be provided by the authors.

best model for simulating standard monetary policy, aggregate demand, technology shocks etc., because it has been rigorously estimated and its out-of-sample properties rigorously tested. And so, we do not want to alter this. The only dimension along which we want to alter the predictions of the model is the dimension that these models do poorly on—forward guidance.

Our method relies on modifying agents' forecasts of the monetary policy "news states" (that is, the exogenous state variables that carry the announcement from about the future to the present). This modification is recorded in a single lower-triangular matrix, thus providing full transparency and control over how the simulation is conducted. We explore four lower-triangular matrices (henceforth, *scenarios*) that we call "inattention", "credibility", "finite planning horizon", "learning". These labels (loosely) coincide with several of the more rigorous resolutions of the forward guidance puzzle in the literature using small-scale models. Inattention is when a fraction $1 - \alpha$ of agents are inattentive to central bank announcements about the future. Credibility is when a decaying fraction of agents believe promises of the central bank further out in the future. That is, a fraction α believe promises about the next quarter, α^2 believe promises about 2-quarters ahead, and so on. Finite planning horizon is when agents dismiss all announcements past horizon N. Finally, learning is when a small fraction of agents initially incorporate forward guidance announcements in their expectations, but as the horizon at which announcements are made increases, a growing fraction begin to incorporate the forward guidance announcements into their expectations. We parameterize this learning process with a 2-parameter logistic function (that controls initial beliefs and the speed of learning, respectively).

The rest of the paper proceeds as follows. Section 2 describes the methodology. Section 3 provides illustrative results from a small model, including conditions under which the forward guidance puzzle is resolved. Section 4 provides results from the Smets and Wouters (2007) model. Section 5 concludes.

2 Methodology

2.1 Setup

The model we are interested can be represented in the following form

$$\tilde{A}y_t = B\mathbb{E}_t^* y_{t+1} + Cy_{t-1} + Di_t + Eu_t, \tag{1}$$

$$i_t = Fy_t + Gm_t, \tag{2}$$

$$m_t = Mm_{t-1} + N\varepsilon_t, \tag{3}$$

$$u_t, \varepsilon_t \sim iid\,(0,1)\,,$$

where (1) is the non-policy block of equilibrium conditions, (2) is the monetary policy reaction function, and (3) governs the evolution of forward guidance promises (the "news states"). The rest of the notation is as follows: y_t is an $\eta \times 1$ vector of endogenous state variables, i_t is the nominal short-term interest rate (the policy instrument of the central bank), u_t is an $\eta_u \times 1$ vector of shocks, and ε_t is a $T \times 1$ vector of anticipated monetary policy shocks, where T is the maximum horizon at which the central bank makes promises.[2] $\tilde{A}$, B, C, D, E, F, G, M, and N are coefficient matrices of appropriate dimension. $\mathbb{E}_t^*(\cdot)$ is an expectations operator, with the * denoting that the expectations are not necessarily full-information rational expectations with respect to the news states, (3).

The key vector of interest for us is m_t which keeps track of the evolution of anticipated monetary policy shocks. It has the following form

$$m_t \equiv \left[[m_{00t}, \ldots, m_{0Tt}]\,, [m_{11t}, \ldots, m_{1Tt}]\,, \ldots, [m_{T-1T-1t}, m_{T-1Tt}]\,, [m_{TTt}]\right]'\,, \tag{4}$$

with dimensions $\mathbb{T} \times 1$, where $\mathbb{T} \equiv (T+2)(T+1)/2$. The notation works as follows: m_{00t} is the standard unanticipated monetary policy shock. m_{11t}, in contrast, is the announcement of a policy shock that will occur one-period ahead. m_{01t} is the arrival of the shock that was announced one-period ahead, and so on. Thus, $m_{00t} = \varepsilon_{0t}$ and $m_{11t} = \varepsilon_{1t}$ are iid shocks, while

[2] In principle, T can be arbitrarily large. For expositional purposes we restrict ourselves to announcements up to $T = 12$ quarters ahead.

$m_{01t} = m_{11t-1}$. Generalizing this gives the structure of G, M, and N as follows

$$\underset{1\times\mathbb{T}}{G} = \begin{bmatrix} \underset{1\times(T+1)}{\mathbf{1}} & \underset{1\times(T+1)T/2}{\mathbf{0}} \end{bmatrix}, \tag{5}$$

$$\underset{\mathbb{T}\times\mathbb{T}}{M} = \left[\begin{array}{ccccc} \underset{1\times(T+1)}{\mathbf{0}} & \mathbf{0} & \mathbf{0} & \cdots & 0 \\ \mathbf{0} & \underset{T\times T}{\mathbf{I}} & \mathbf{0} & & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & & 0 \\ \mathbf{0} & \mathbf{0} & \underset{(T-1)\times(T-1)}{\mathbf{I}} & & \mathbf{0} \\ \hline \vdots & & & \ddots & \vdots \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & & \underset{1\times 1}{1} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & 0 \end{array}\right], \quad \underset{\mathbb{T}\times T}{N} = \left[\begin{array}{cccccc} 1 & 0 & 0 & \cdots & 0 & 0 \\ \underset{T\times 1}{\mathbf{0}} & \mathbf{0} & \mathbf{0} & & \mathbf{0} & \mathbf{0} \\ \hline 0 & 1 & 0 & & 0 & 0 \\ \underset{(T-1)\times 1}{\mathbf{0}} & \mathbf{0} & \mathbf{0} & & \mathbf{0} & \mathbf{0} \\ \hline \vdots & & & \ddots & & \vdots \\ \hline 0 & 0 & 0 & & 1 & 0 \\ \underset{1\times 1}{0} & 0 & 0 & & 0 & 0 \\ \hline 0 & 0 & 0 & \cdots & 0 & 1 \end{array}\right]. \tag{6}$$

Note that only the variables m_{0nt} for $n = 1, \ldots, T$ enter the monetary policy reaction function. These are when the previously announced promises are actually implemented. Note also that M and N are both sparse matrices. In M, the 1s capture the transmission of future announcements to the present. In N, the 1s relate the m_{nnt} variables for $n = 1, \ldots, T$ to the appropriate shock, ε_{nt}.

2.2 Expectations formation

Agents in the model need to form expectations of the evolution of the forward guidance states. In particular, they need to evaluate $\mathbb{E}_t^* m_{t+1}$. The full-information, rational expectations solution forecast is given by $\mathbb{E}_t m_{t+1} = M m_t$. Instead, we assume that the agents form the following forecasts

$$\mathbb{E}_t^* m_{t+1} = (Z \circ M)\, m_t, \tag{7}$$

where Z is also of dimension $\mathbb{T} \times \mathbb{T}$ and $\circ$ denotes element-by-element multiplication. Since this is an element-by-element operation, only $(T + 1)\, T/2$ will be nonzero, corresponding to the location of the 1s in M. Thus, we use the following compact notation, denoted $\mathbb{Z}$, which maps into the sparse matrix Z. In particular, the $\mathbb{Z}$ notation is given by

$$\mathbb{Z} = \left| \begin{array}{cccc} z_{11} & & & \\ z_{12} & z_{22} & & \\ \vdots & \vdots & \ddots & \\ z_{1T} & z_{2T} & \cdots & z_{TT} \\ \hline \end{array} \right. \tag{8}$$

and maps into Z as follows

$$Z = \left[\begin{array}{cccccccccccccc} 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & z_{11} & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & & 0 \\ \vdots & & \vdots & 0 & z_{12} & & \vdots & \vdots & & & \vdots & & \vdots \\ \vdots & & \vdots & \vdots & & \ddots & 0 & \vdots & & & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & z_{1T} & 0 & \cdots & \cdots & 0 & & 0 \\ \hline 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & \cdots & 0 & & 0 \\ 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & z_{22} & 0 & \cdots & 0 & & 0 \\ \vdots & & \vdots & \vdots & & & \vdots & 0 & z_{23} & & \vdots & & \vdots \\ \vdots & & \vdots & \vdots & & & \vdots & \vdots & & \ddots & 0 & & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & z_{2T} & & 0 \\ \hline \vdots & & & & & & & & & & & \ddots & \vdots \\ \hline 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & \cdots & 0 & & 0 \\ 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & \cdots & 0 & & z_{TT} \\ \hline 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & \cdots & 0 & \cdots & 0 \end{array} \right]. \tag{9}$$

In the non-modified case, all elements of $\mathbb{Z}$ are 1. At the other extreme, when all elements of $\mathbb{Z} = 0$, agents react to all forward guidance announcements as if they were unanticipated shocks when they arrive. We are interested in four intermediate scenarios we call "inattention", "credibility", "finite planning horizon" and "learning", as follows:

Scenario I (Inattention): Under inattention, a fraction $1 - \alpha$ of agents are inattentive to forward guidance announcements of the central bank, with $\alpha \in [0,1]$. In this case

$$\mathbb{Z}^{I} = \left| \begin{array}{llll} \alpha & & & \\ \alpha & \alpha & & \\ \vdots & \vdots & \ddots & \\ \alpha & \alpha & \cdots & \alpha \end{array} \right. \tag{10}$$

Scenario II (Credibility): Under credibility, a decaying fraction of agents believe forward guidance announcements at further horizons. In particular, a fraction $\alpha \in [0,1]$ believe announcements 1-quarter ahead, a fraction α^2 believe announcments 2-quarters ahead etc. In this case

$$\mathbb{Z}^{II} = \left| \begin{array}{llll} \alpha & & & \\ \alpha & \alpha^2 & & \\ \vdots & \vdots & \ddots & \\ \alpha & \alpha^2 & \cdots & \alpha^T \end{array} \right. \tag{11}$$

Scenario III (Finite planning horizon): Under a finite planning horizon, all agents dismiss announcements that are more than N periods ahead. Columns 1 to N of $\mathbb{Z}$ will contains 1s and the rest will contain 0s. For example, if agents dismiss all announcements more than 1-quarter ahead ($N = 1$), we get

$$\mathbb{Z}^{III} = \left| \begin{array}{llll} 1 & & & \\ 1 & 0 & & \\ \vdots & \vdots & \ddots & \\ 1 & 0 & \cdots & 0 \end{array} \right. \tag{12}$$

Scenario IV (Learning): Under learning, a large fraction of agents initially dismiss forward guidance announcements, but this fraction falls as time passes and the central bank is able to show its commitment to its promises.

$$\mathbb{Z}^{IV} = \begin{bmatrix} \frac{1}{1+e^{-\beta_1(1-\beta_2)}} & & & \\ \frac{1}{1+e^{-\beta_1(2-\beta_2)}} & \frac{1}{1+e^{-\beta_1(1-\beta_2)}} & & \\ \vdots & \vdots & \ddots & \\ \frac{1}{1+e^{-\beta_1(T-\beta_2)}} & \cdots & & \frac{1}{1+e^{-\beta_1(1-\beta_2)}} \end{bmatrix} \tag{13}$$

The learning scenario is parameterized by a 2-parameter logistic function where β_1 controls the speed of learning, while β_2 controls the initial beliefs.

Notice that in scenarios I-III, the elements of the same column of $\mathbb{Z}$ contain the same loading. In contrast, in scenario IV, the diagonals contain the same loadings. Finally, note that all elements of $\mathbb{Z}$ are bounded by the $[0, 1]$ interval. If all elements of $\mathbb{Z}$ are equal to 1, then we get the standard full-information rational expectations solution. If instead all element of $\mathbb{Z}$ are equal to 0 then every announcement arrives as if it were a completely unanticipated shock. Thus, all our simulation results will be bounded between these two extreme outcomes.

2.3 Solving the model

Except for the assumption we make regarding the forecasting of m_{t+1}, the method for solving the equilibrium decision rules is standard.[3] However, the loading coefficients on m_t in the equilibrium decision rules take a particular form that is of interest for understanding the forward guidance puzzle and its potential resolution.

First, we postulate the minimum state variable decision rule as

$$y_t = H_y y_{t-1} + H m_t + H_u u_t, \tag{14}$$

where H_y, H, and H_u are the unknown loading coefficients that we wish to solve for. The matrix of loading coefficients of m_t is denoted by H and has the following form

$$\underset{\eta\times\mathbb{T}}{H} \equiv [H_{00:T}, H_{11:T}, \ldots, H_{TT:T}], \tag{15}$$

[3]See Sims (2002); Uhlig (1998).

where $H_{jj:T}$ has dimensions $\eta \times (1 + T - j)$. That is, in a single endogenous variable model, $H_{00:T}$ contains $[h_{00}, h_{01}, \ldots, h_{0T}]$ and $H_{01:T}$ contains $[h_{11}, h_{12}, \ldots, h_{1T}]$, etc. Substituting the postulated decision rule into the equilibrium conditions, (1), gives

$$A\left(H_y y_{t-1} + Hm_t + H_u u_t\right) = BH_y\left(H_y y_{t-1} + Hm_t + H_u u_t\right) + BH\mathbb{E}_t^*\left(m_{t+1}\right)$$
$$+ Cy_{t-1} + DGm_t + Eu_t,$$

where $A \equiv \tilde{A} - DF$. Substituting for our forecasting rule, (7), gives

$$A\left(H_y y_{t-1} + Hm_t + H_u u_t\right) = BH_y\left(H_y y_{t-1} + Hm_t + H_u u_t\right) + BH\left(Z \circ M\right) m_t$$
$$+ Cy_{t-1} + DGm_t + Eu_t.$$

This equation imposes a set of identifying restrictions that allow us to solve for the unknown loading coefficients, H_y, H, and H_u. In particular, H_y and H_u solve

$$AH_y = BH_y^2 + C, \tag{16}$$

$$AH_u = BH_y H_u + E, \tag{17}$$

respectively. The first implicitly defines H_y as the solution of a matrix-quadratic system of equations. The second is a linear-matrix system and solves H_u, given H_y. Notice though that neither of these two sets of loading coefficients are dependent on Z. In that sense, the response of the model to the non-policy shocks, u_t, are completely unaffected by our assumption about how agents forecast the propagation of forward guidance announcements.

Proposition 1 *The decision rule loading coefficients, H_y and H_u, (and thus the behavior of the model to non-policy shocks) are independent of Z (our assumptions regarding agents reactions to forward guidance announcements).*

We are interested in H, which solves

$$\left(A - BH_y\right) H = BH\left(Z \circ M\right) + DG. \tag{18}$$

Given the form of G and $Z \circ M$ (see (5), (6), and (9)), we can solve this in a block-recursive manner. First, to solve for the loading coefficients on $m_{00:Tt} \equiv [m_{00t}, \ldots, m_{0Tt}]$, the above equation simplifies

$$AH_{00:T} = BH_y H_{00:T} + D \times \underset{1\times(T+1)}{\text{ones}}, \tag{19}$$

giving

$$H_{00:T} = (A - BH_y)^{-1} D \times \underset{1\times(T+1)}{\text{ones}}. \tag{20}$$

Note that the $(i,j)^{th}$ element of $H_{00:T-}$ is the response of the i^{th} variable in y_t to a change in m_{0jt} and that the value of the $(i,j)^{th}$ and the $(i,k)^{th}$ element are equal for all $j,k = 0,...,T$. Next, it follows that

$$H_{jj:T} = (A - BH_y)^{-1} BH_{j-1j:T} \times \text{diag}\left(\underset{1\times(T+1-j)}{[z_{jj}, \cdots, z_{jT}]}\right) \quad \text{for all} \quad j = 1, \ldots, T. \tag{21}$$

Thus, $H_{11:T}$ is a function of $H_{01:T}$, and $H_{22:T}$ is a function of $H_{12:T}$ etc. For Scenario I, II, and III, $z_{jj} = \cdots = z_{jT}$, which means that the elements of each row of $H_{jj:T}$ are identical. That is, the marginal effect of a policy shock is the same independent of when the shock was announced.

Since the above notation is rather cumbersome, consider a simple example with $\eta = 1$ and $T = 3$. In this case, (18) becomes

$$(a - bh_y)\begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{11} \\ h_{12} \\ h_{22} \end{bmatrix}' = b\begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{11} \\ h_{12} \\ h_{22} \end{bmatrix}' \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & z_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & z_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & z_{22} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + d\,[1,1,1,0,0,0]\,.$$

where the lower case a, b, d and h_y denote that the coefficient matrices are scalars in this simple example. Rearranging, this can be written as

$$(a - bh_y)\,[[h_{00}, h_{01}, h_{02}]\,, [h_{11}, h_{12}]\,, h_{22}] = [[0,0,0]\,, b\,[z_{11}h_{01}, z_{12}h_{02}]\,, bz_{22}h_{12}] + [d\,[1,1,1]\,, [0,0]\,, 0]\,,$$

or, in recursive form as follows

$$[h_{00}, h_{01}, h_{02}] = \frac{d}{(a - bh_y)}[1,1,1],$$
$$[h_{11}, h_{12}] = \frac{b}{(a - bh_y)}[z_{01}h_{01}, z_{02}h_{02}],$$
$$h_{22} = \frac{b}{(a - bh_y)} z_{12}h_{12}.$$

2.4 Dampening the effects of forward guidance

The recursive form in the previous section means that we do not need to resolve the original model. Rather, let us define H^* as the H when all elements of $\mathbb{Z}$ are set to 1. That is, the decision rule coefficients absent any of the modifications that we apply in this paper. It is then possible to write the new augmented H as follows

$$H = \underset{\eta\times 1}{\text{ones}} \times \underset{1\times\mathbb{T}}{Z^*} \times H^*, \tag{22}$$

where Z^* takes the following form

$$Z^* \equiv \left[[z^*_{00}, \ldots, z^*_{0T}], [z^*_{11}, \ldots, z^*_{1T}], \ldots, [z^*_{T-1T-1}, z^*_{T-1T}], [z^*_{TT}]\right]', \tag{23}$$

or, using the compact triangular-form notation, $\mathbb{Z}^*$, as follows

$$\mathbb{Z}^* = \begin{array}{|lllll} z^*_{00} & & & & \\ z^*_{01} & z^*_{11} & & & \\ z^*_{02} & z^*_{12} & z^*_{22} & & \\ \vdots & \vdots & \vdots & \ddots & \\ z^*_{0T} & z^*_{1T} & z^*_{2T} & \cdots & z^*_{TT} \\ \hline \end{array} \tag{24}$$

Notice that (24) is larger than $\mathbb{Z}$, with an additional first column. The elements in the first column are generically always equal to 1. The relation between the elements of $\mathbb{Z}$ and $\mathbb{Z}^*$ are given as follows

$$\mathbb{Z}^* = \left| \begin{array}{lllll} (1) & & & & \\ (1) & (z_{11}) & & & \\ (1) & (z_{12}) & (z_{12}z_{22}) & & \\ \vdots & \vdots & \vdots & \ddots & \\ (1) & (z_{1T}) & (z_{1T}z_{2T}) & \cdots & (z_{1T} \times \cdots \times z_{TT}) \end{array} \right. \tag{25}$$

From this we can understand how assumptions about Z impact the decision rule loading coefficients, H.

Scenario I: Under the inattention scenario, all elements of $\mathbb{Z}^I$ equal α, which translates into $(\mathbb{Z}^*)^I$ as follows

$$(\mathbb{Z}^*)^I = \left| \begin{array}{lllll} 1 & & & & \\ 1 & \alpha & & & \\ 1 & \alpha & \alpha^2 & & \\ \vdots & \vdots & \vdots & \ddots & \\ 1 & \alpha & \alpha^2 & \cdots & \alpha^T \end{array} \right. \tag{26}$$

Thus, even though a fixed fraction of agents are inattentive to the forward guidance announcements, the loading coefficients on news states far out in the future are discounted at a higher rate than near news states.

Scenario II: Consequently, under the credibility scenario, the dampening of loading coefficients on news states far out in the future is even more rapid. In particular,

$$(\mathbb{Z}^*)^{II} = \left| \begin{array}{lllll} 1 & & & & \\ 1 & \alpha & & & \\ 1 & \alpha & \alpha^3 & & \\ \vdots & \vdots & \vdots & \ddots & \\ 1 & \alpha & \alpha^3 & \cdots & \alpha^{(T+1)T/2} \end{array} \right. \tag{27}$$

For scenario III (planning horizon), the elements of $\mathbb{Z}^*$ simply reflects $\mathbb{Z}$ with elements either zero or one. For scenario IV (learning), the loadings in $\mathbb{Z}^*$ are growing as you move South, but shrinking as you move South-East and East through the triangle.

3 Results

We first demonstrate the methodology from Section 2 in a benchmark model before proceeding to a medium- and large-scale new-Keynesian model.

3.1 Benchmark model

In this section we consider a simple model consisting of only an IS equation (with habits in consumption) and a Taylor rule responding to the output gap, with inflation fixed at zero. Thus, (1) and (2) are as follows

$$x_t = (1-\omega)\,\mathbb{E}_t^* x_{t+1} + \omega x_{t-1} - i_t, \tag{28}$$

$$i_t = \phi x_t + \sum_{j=0}^{T} m_{0jt}, \tag{29}$$

where x_t is the output gap. Table 1 summarizes the parameterization of the model.

Table 1: Parameterization

Parameters	ω	ϕ
Values	0.5	1.5

Table 2 summarizes and Figure 1 depicts graphically the parameterization of Z that we will use in the experiments below. In Scenario I we consider the two extreme calibrations (full-attention and full-inattention, Case 1 and Case 3, respectively) in addition to an intermediate case in which half of all agents are attentive to forward guidance announcements. In Scenario II, the intermediate case is one in which the agents that believe the forward guidance announcement decay at a rate of 0.5. Scenario III considers the possibility that agents dismiss all announcements above an horizon of 1 and 2 quarters, respectively. Finally, scenario IV presents a situation in which the initial beliefs about forward guidance are close to zero, but grow as the peg is maintained over time. With the blue line ($\beta = (10, 5)$) agents almost completely dismiss forward guidance for the first 5-periods before learning to be almost fully attentive to them thereafter. The red line ($\beta = (1, 5)$) is one in which agents learn more steadily, incorporating 50% of forward guidance announcements by period 5 are almost full attentive by period 10. In contrast, the yellow line ($\beta = (1, 10)$) is one in which the learning process is much slower.

Table 2: Z parameters

Scenario	Parameters	Case 1	Case 2	Case 3
I	α	1	0.5	0
II	α	1	0.5	0
III	N	2	1	0
IV	(β_1, β_2)	$(10, 5)$	$(1, 5)$	$(1, 10)$

Figure 2 shows the response of the output gap x_t to a 1 ppt unanticipated monetary policy tightening at horizons 0 to 10. The top row shows the full-attention responses while the bottom row shows the case where agents pay zero attention to forward guidance announcements. Notice that the responses look near identical. For the 10 period-ahead shock, the initial response is essentially zero and the peak response is approximately -0.4%, for both $\alpha = 0$ and $\alpha = 1$. The only difference comes in the few quarters leading up to the announced shock. In the full attention model, x_t already begins to fall, albeit by a modest amount (0.08% in quarter 9), while in the full inattention model, there is not response.

Figure 3 shows the responses to pegging the interest rate 400 basis points below the steady state for 1 to 10 quarters(which corresponds to a peg close to the zero lower bound in case of a nominal steady state interest rate of 4%). In each case, the steady state interest rate is 4%. In this case, the difference in responses between the three values of α (1, 0.5, and 0) is stark. In particular, with full-attention, the peak response of the output gap x_t to a 10 quarter peg is 32%, while under full-inattention, it is only 2.3%. Thus, the responses to individual unanticipated responses as in Figure 2 can be misleading with regards to predicting the effect of an interest rate peg.

Figure 1: Visual representation of Z

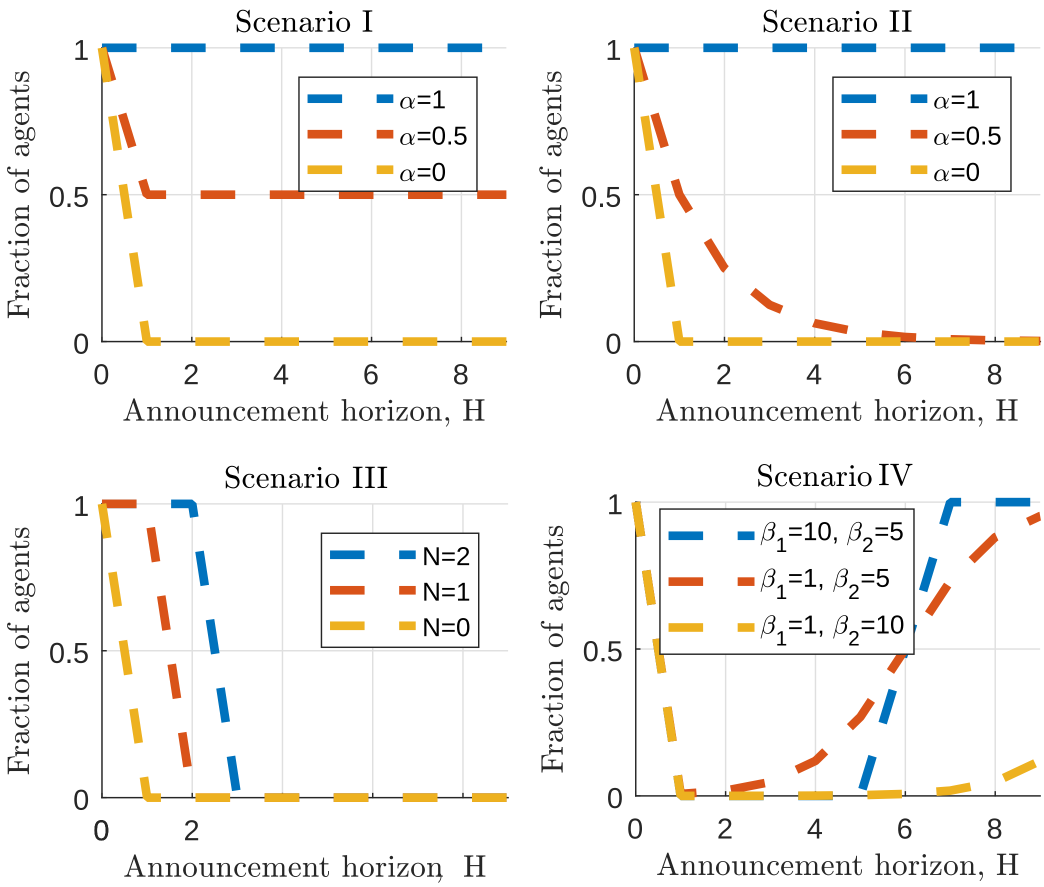
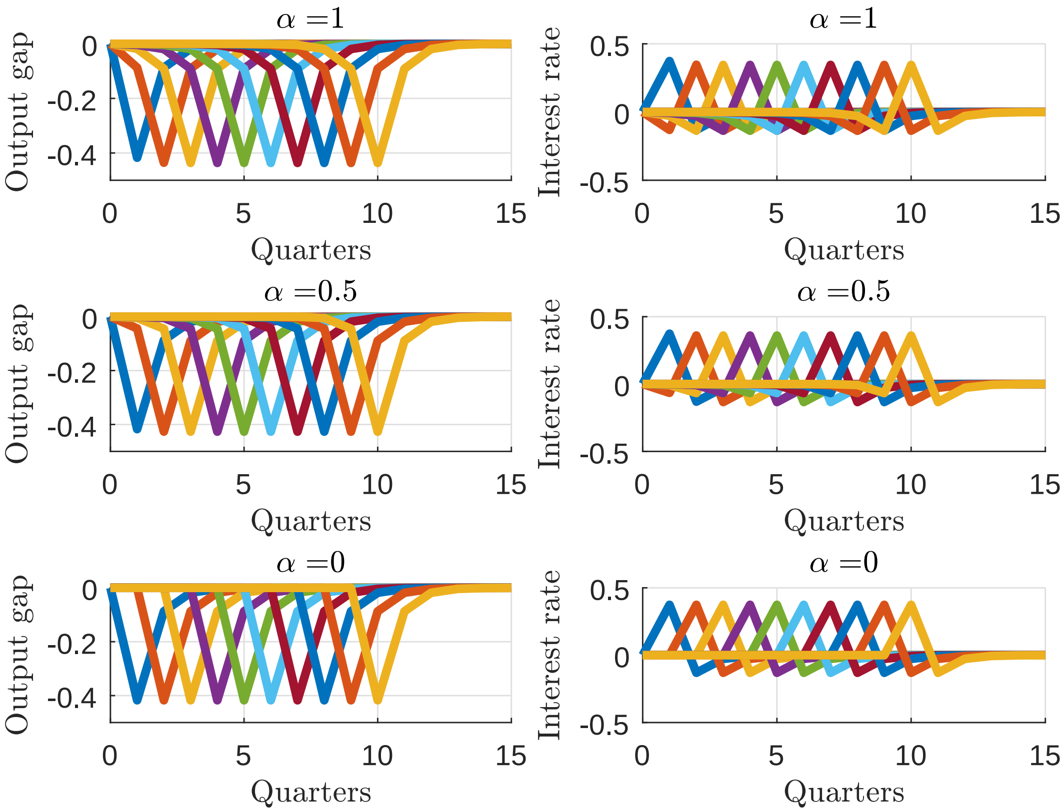


Figure 2: Scenario I (Inattention): Anticipated MP shock IRFs



For the "Credibility" scenario (II), the IRFs and the responses to the ZLB scenarios are very similar to the "Inattention" scenarios (shown in Appendix Figure A.1). This is because it is the response of x_t, one-quarter before an announced shock that is the key to determining the power of forward guidance.

Figure 3: Scenario I (Inattention): ZLB simulations

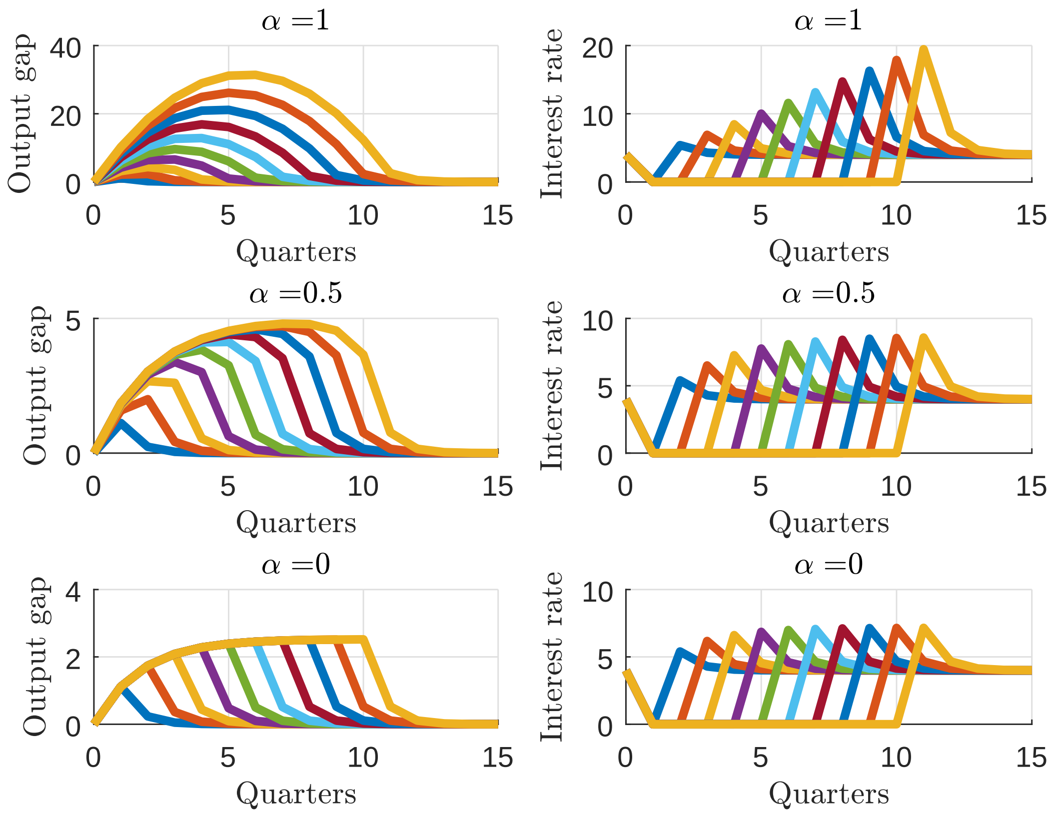
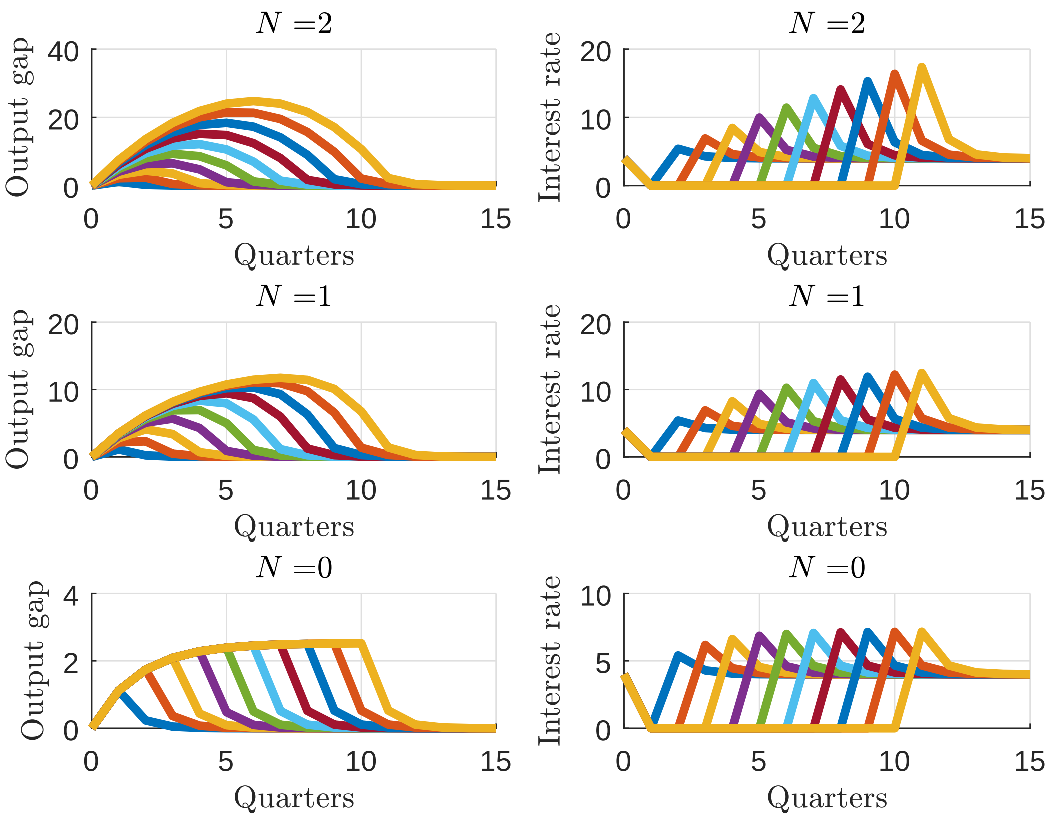


Figure 4 plots simulations using the "Planning horizon" scenario (III). When $N = 2$, agents are fully attentive to announcements 2-quarters ahead but completely dismiss announcements further in the future. Remarkably, this assumption generates responses that are of the same order of magnitude as the full-attention model. In particular, a 10-quarter ZLB announcement generates a peak output gap of 24% despite the fact that agents dismiss all announcements apart from those being realized in the next two periods.

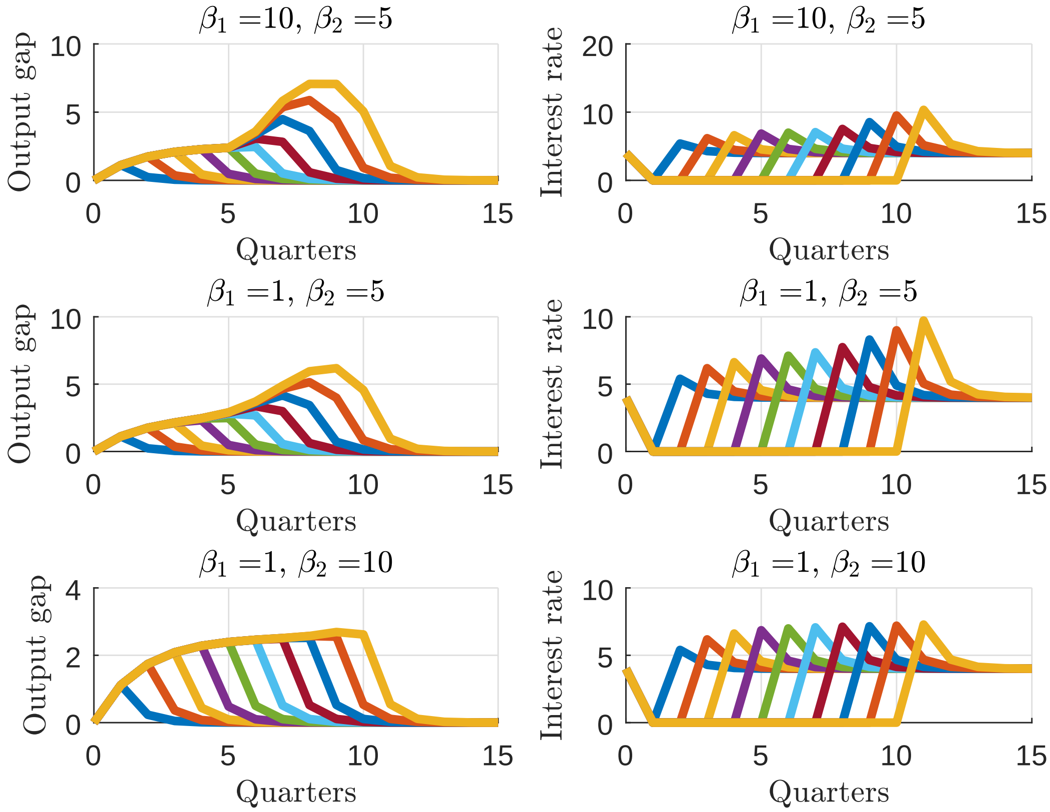
Finally, Figure 5 presents the "Learning" scenario (IV). In the top row, it takes approximately 5-periods of ZLB for agents to become convinced of the central bank's credibility. In period 5

Figure 4: Scenario III (Planning horizon): ZLB simulations



there is a large jump in the fraction of agents that deem the policy credible. Thus, the response of x_t begins modestly but accelerates after quarter 5. In contrast, in the middle row (with $(\beta_1, \beta_2) = (1, 5)$ the learning process in more constant.

Figure 5: Scenario IV (Learning): ZLB simulations



3.2 Resolving the forward guidance puzzle

It is instructive to provide a formal definition of the forward guidance puzzle and what constitutes a resolution of the puzzle. We thus provide the following definitions, where H denotes the length of the interest rate peg.

Definition 2 (Forward guidance puzzle:) *The forward guidance puzzle exists in a model if* $\lim_{H\to\infty} \max(x_t) = +\infty$.

Definition 3 (Resolution of the forward guidance puzzle:) *The forward guidance puzzle is resolved in a model if* $\lim_{H\to\infty} \max(x_t) = \bar{x}$*, where* $\bar{x}$ *is finite.*

A corollary of this definition of the forward guidance puzzle is that the marginal benefit of an additional period pegged far out in the future remains strictly positive. A resolution of the forward guidance puzzle is one in which this marginal benefit of an additional period pegged approaches zero as the forward guidance horizon tends to infinity.

The definitions are put into action in Figure 6. In the full attention model ($\alpha = 1$), the initial response of x_t (top-left panel) grows linearly while the maximum response (bottom-left panel) grows explosively. When $\alpha < 1$, the marginal contribution to the initial x_t response of an additional period pegged (top-right panel) is decaying and approaches zero, monotonically related to α. For $\alpha = 0.9$ the maximal x_t response (bottom-left panel) is initially convex before turning concave. That is, the marginal benefit of increasing the announcement from 2-quarters to 3-quarters is greater than extending the announcement from 1-quarter to 2-quarters. However, the marginal benefit of increasing from 7-quarters to 8-quarters is lower. Thus, we observe a hump shaped pattern in the green line of the bottom-right panel. In general, when $\alpha < 1$, the maximal output-gap curve changes from convex to concave, with the turning point increasing in α.

Figure 7 shows the same plots for the "Planning horizon" scenario (III). It again demonstrates that when agents are attentive to only 2-quarter-ahead announcements, then the responses are very close to the full-attention ($N \to \infty$ or equivalently $\alpha = 1$) experiment.

4 Extended model

The benchmark model in the previous section provided useful intuition. In this section, we demonstrate the results of this method in the Smets and Wouters (2007) model. We calibrate the model to the empirical evidence from Åhl (2017), shown in Appendix Figure A.2. In particular, we use the "Credibility" scenario (II) and derive $\alpha = 0.7$.

Figure 8 shows the ZLB experiments in the Smets and Wouters model. The middle panel shows

Figure 6: Marginal contribution of an additional ZLB period (Scenario I)

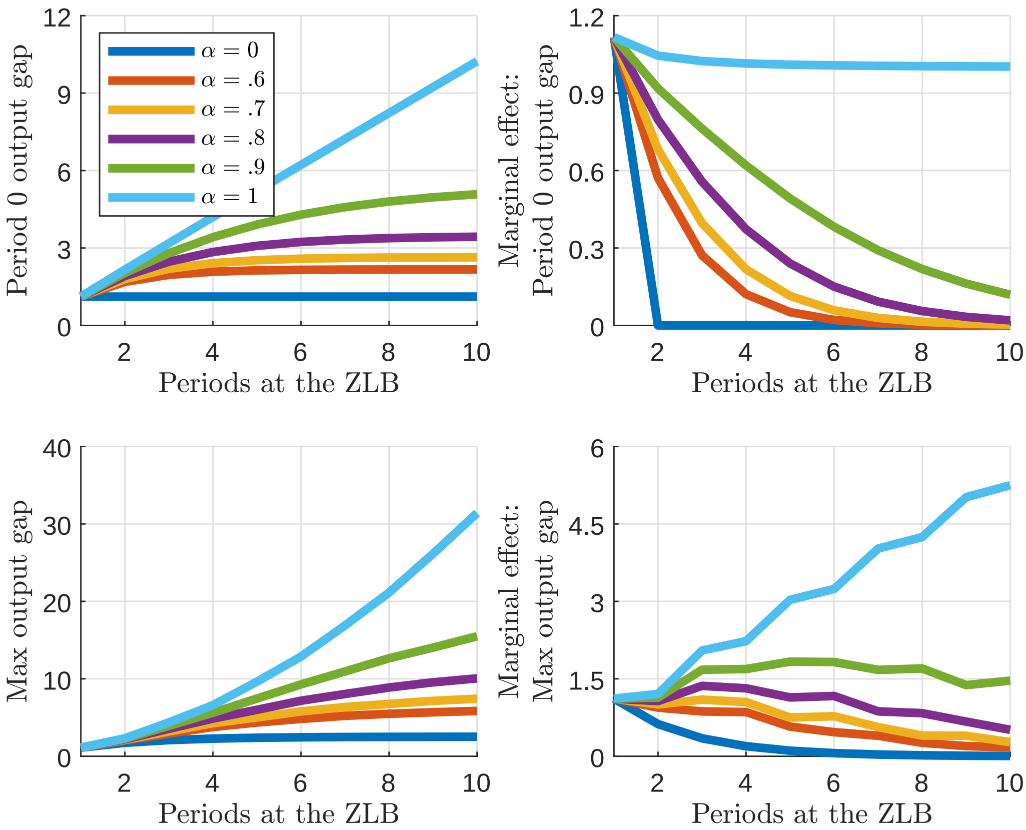
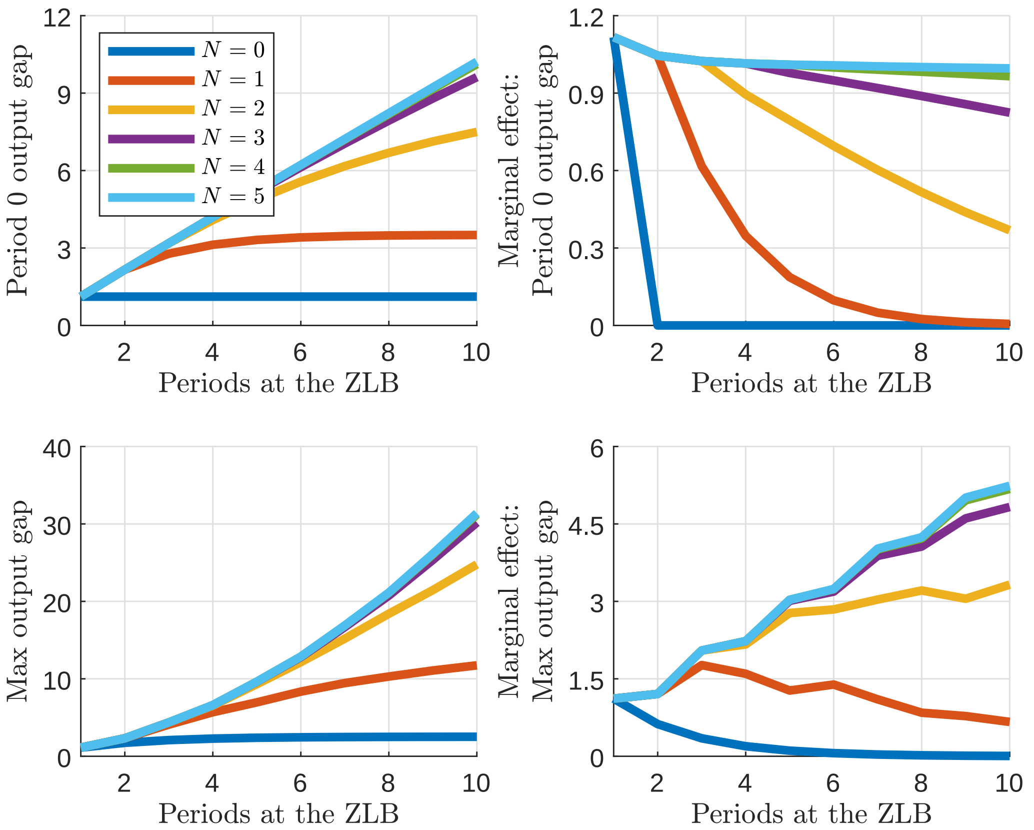


Figure 7: Marginal contribution of an additional ZLB period (Scenario III)



the responses uses our preferred calibration of $\alpha = 0.7$. This significantly dampens the effect of forward guidance relative to the full-attention model (in the top row).

Figure 8: Interest rate pegs in Smets & Wouters (Scenario II)

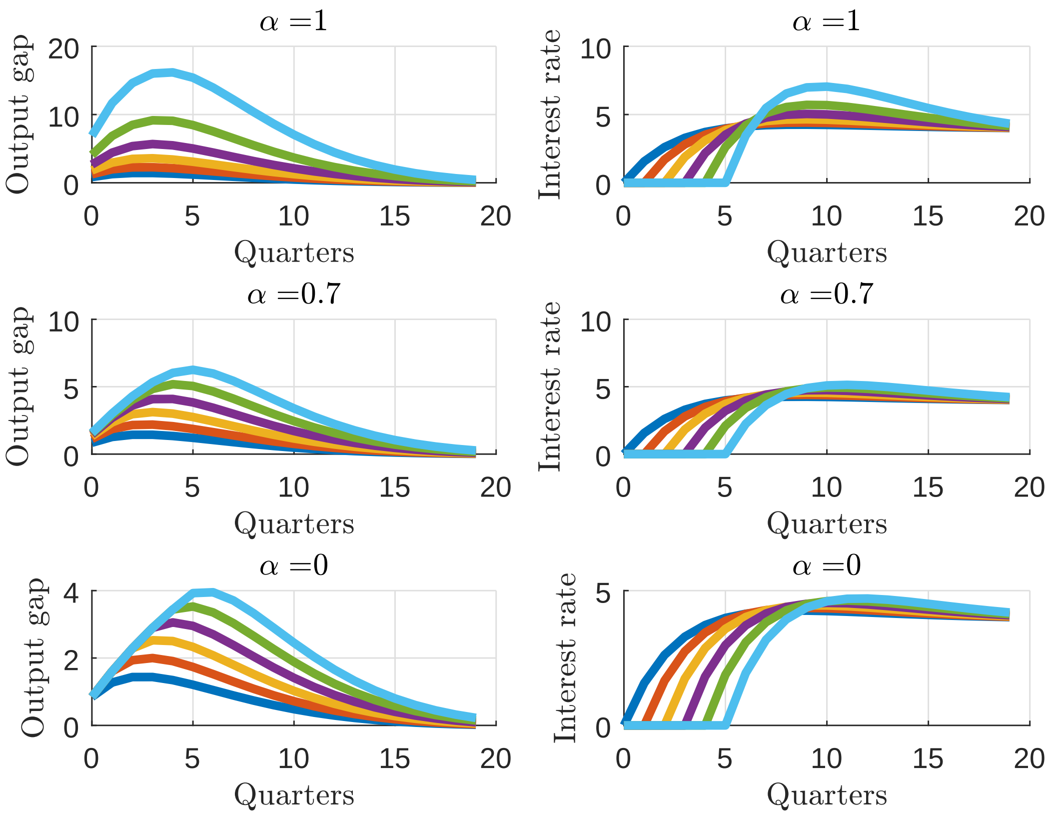
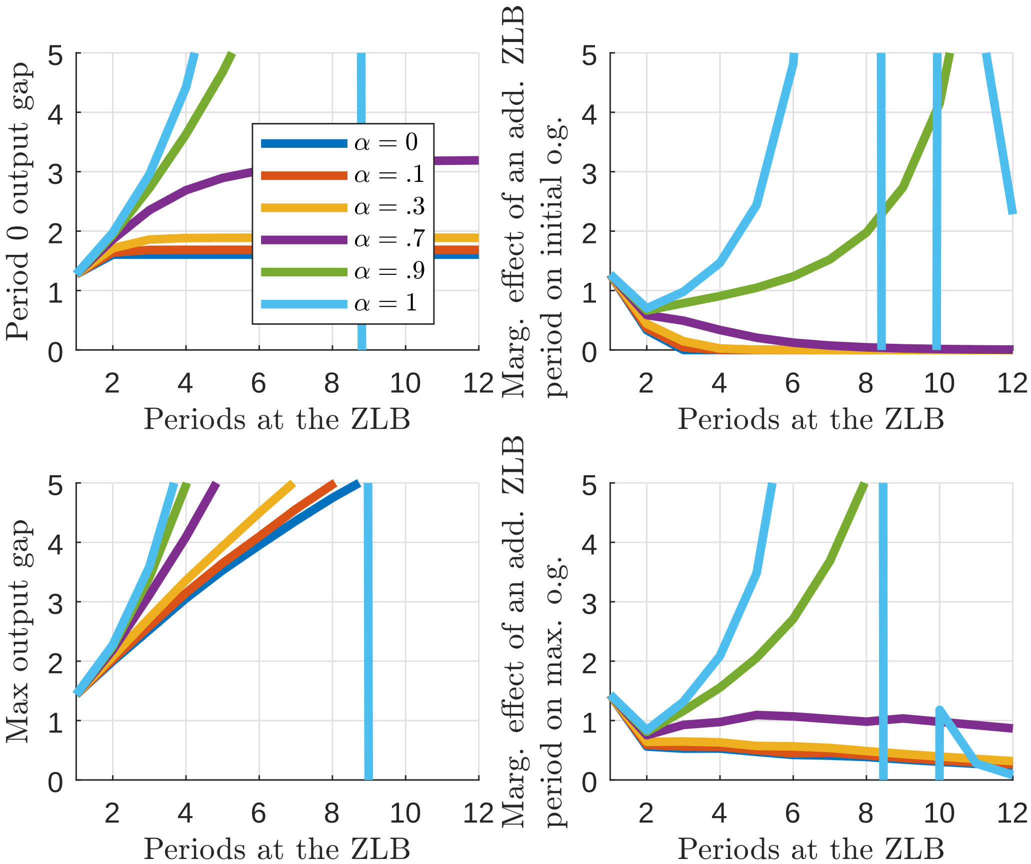


Figure 9 shows the effect of the peg at different horizons on the initial and maximal output gap and the marginal contribution is more informative. With $\alpha = 1$, we observe the asymptotes documented by Carlstrom et al. (2015): for a small change in the length of the interest rate peg from 8 to 9 periods, the effects of the peg switch from highly expansionary to highly contractionary. The effect reverses yet again when we keep the peg for an additional quarter.

With $\alpha = 0.9$, the marginal contribution of an additional period continues to grow exponentially. However, for our preferred calibration of $\alpha = 0.7$, marginal contribution to the initial output gap response is clearly falling (top-right panel). The marginal effect of the maximal output gap response appears to peak with the announcement of a 5 quarter peg (bottom-right) panel. The asymptotes observed in the full attention model are absent in any of the dampened scenarios.

Figure 9: Marginal contribution of an additional ZLB period in the Smets & Wouters model (Scenario II)



5 Conclusion

This paper provides a practical and discreet methodology for generating empirically realistic forward guidance experiments. We propose four scenarios to dampen the Forward Guidance Puzzle: "inattention", "credibility", "finite planning horizon", and "learning".

The method relies on a transparent modification of agents' forecasts of the monetary policy "news states", which is recorded in a single lower-triangular matrix. This modification guarantees that the solution to the model is not changed in any other way, therefore keeping the transmission of all other shocks exactly the same as in the non-modified case. This is especially beneficial in case of large-scale policy-institution models that exhibit shock transmissions which have undergone significant vetting.

The modifications are presented in a simple small-scale model that captures the problem of the Forward Guidance Puzzle under full attention. The scenarios are able to dampen the FG Puzzle under various parameterisations. An extended medium-scale model, which includes a modification that is informed by empirical evidence, resolves the FG Puzzle. This model does not exhibit the counterintuitive reversals pointed out by Carlstrom et al. (2015), where the increase of a peg by one quarter switches the effects from highly expansionary to highly contractionary.

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A Further derivations

Figure A.1: Scenario II (Credibility): ZLB simulations

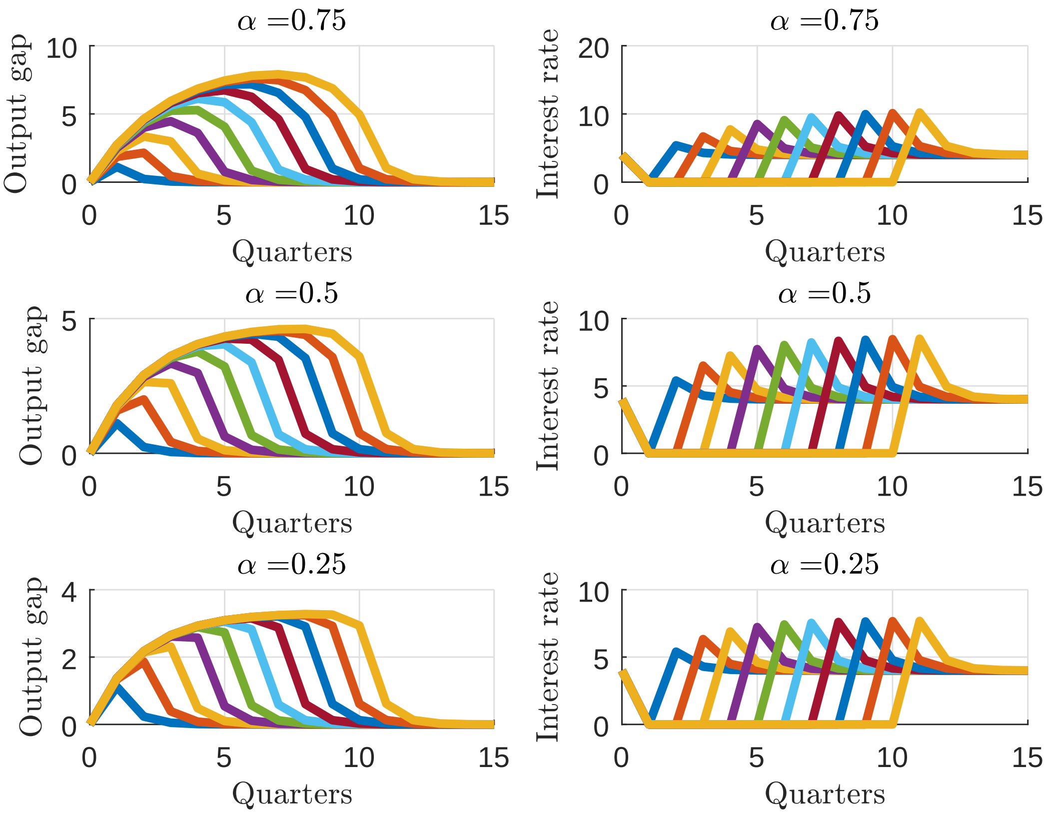
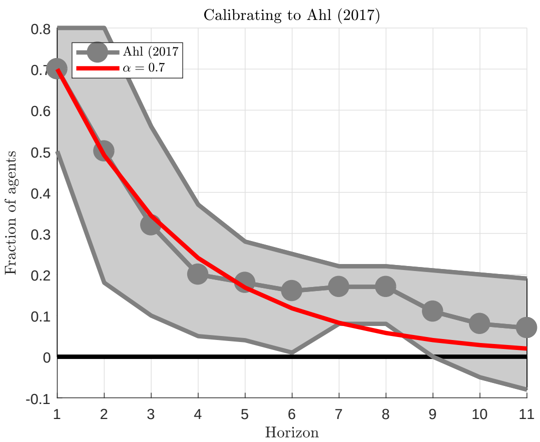


Figure A.2: Empirical evidence on α from Åhl (2017)



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