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Caterina Mendicino, Kalin Nikolov,
Juan Rubio-Ramirez, Javier Suarez,
Dominik Supera

Twin defaults and bank capital requirements

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Abstract

We examine optimal capital requirements in a quantitative general equilibrium model with banks exposed to non-diversifiable borrower default risk. Contrary to standard models of bank default risk, our framework captures the limited upside but significant downside risk of loan portfolio returns (Nagel and Purnanandam, 2020). This helps to reproduce the frequency and severity of *twin defaults*: simultaneously high firm and bank failures. Hence, the optimal bank capital requirement, which trades off a lower frequency of twin defaults against restricting credit provision, is 5pp higher than under standard default risk models which underestimate the impact of borrower default on bank solvency.

Keywords: Financial Intermediation, Macroprudential Policy, Default Risk, Bank Assets Returns.

JEL codes: G01, G28, E44

Non-technical Summary

Our paper examines the way in which the solvency of borrowers and banks interact and affect financial and macroeconomic stability. The main cause of bank losses and equity declines is the realization of abnormally high default rates among bank borrowers. Quantifying this crisis transmission channel is essential for a macroprudential calibration of bank capital requirements, which are the main policy tool to protect the economy against bank insolvencies.

We build a structural model of bank default risk, embed it into an otherwise standard quantitative macroeconomic framework and estimate its parameters on Euro Area macro, financial and banking data. A crucial innovation of our approach is to explicitly model bank assets as loans which are subject to default. This feature generates returns on banks' loan portfolios which are asymmetric and feature limited upside potential but significant downside risk. Intuitively, the asymmetry arises due to the fact that loan returns are capped by the promised interest rate but losses are potentially unlimited and depend on the default rate as well as the recovery value of collateral assets. This implies that banks can remain healthy up to a point but their solvency may deteriorate rapidly if economic conditions become sufficiently adverse. In the end, the economy enters a financial crisis (which we term a Twin Default Crisis) in which both bank and firm default are highly elevated and GDP growth falls sharply.

Our ability to capture the asymmetric distribution of bank loan returns not only adds realism but also allows us to replicate key aspects of the relationship between firm and bank defaults and the macroeconomy, including the striking non-linearities observed in the data. We show using quantile regressions on Euro Area data that bank and corporate default risk are more highly correlated in bad times when the risk of bank failure is already elevated. In addition, bank default and GDP growth are more highly correlated when GDP growth is already low. The model is able to capture both of these non-linearities in the data due to

a combination of a non-linear solution method and a framework that features asymmetric loan portfolio returns. The model is able to generate rare but severe crisis events with a frequency and severity which is close to that observed in the data.

Having built a model of financial crises, we use it to analyse quantitatively the optimal level of bank capital requirements which keep the banking system safe without imposing unduly large output costs. We show that under high capital requirements of 16 per cent (double the Basel II minimum) crises almost disappear because the banking sector is much more robust to economic shocks. This is beneficial because crises are costly and severe events which reduce welfare very significantly when they occur. However, higher capital requirements also entail costs because they lead to elevated loan interest rates and lower investment and output. Our welfare analysis suggests that a capital requirement of approximately 15 per cent optimally trades off the costs and benefits of increasing banks capitalization.

The model implies optimal capital requirements which are around 5 percentage points higher compared to others in the literature, demonstrating the importance of capturing the non-linearities and linkages between borrower and bank defaults. Our non-linear solution method and framework which captures loan return asymmetries is able to generate bank equity returns with much fatter tails. This implies that more capital is needed to make banks safe.

1. Introduction

More than a decade after the 2008-2009 financial crisis, the optimal level of bank capital requirements still remains an open question. Bank capital is considered the best way to protect individual banks and the aggregate economy against the risk of bank insolvencies. When bank capital ratios are low, abnormally high default rates among bank borrowers lead to sharp declines in bank net worth and increases in bank failures. The resulting fall in bank lending further amplifies the real and financial implications of credit losses. Thus, many academics and policy-makers have made the case for significantly higher capital requirements (see e.g. Admati and Hellwig, 2013; The Federal Reserve Bank of Minneapolis, 2017). However, when banks' capacity to raise equity is limited, lowering the frequency of severe bank insolvencies may come at the cost of restricting bank credit provision in normal times (see e.g. Calomiris, 2013). Quantifying this trade-off is crucial for the assessment of optimal capital requirements and requires a framework that captures well the behavior of the economy in normal times as well as the frequency and severity of twin defaults – i.e. episodes of simultaneously high levels of borrower and bank defaults.

This paper studies this important trade-off in a quantitative macro-banking model whose main distinguishing feature is to account for the special structure of bank asset risk (see Nagel and Purnanandam, 2020). Specifically, in our model banks hold portfolios of risky loans whose risk of default is not fully diversifiable at the bank level. As a result, bank solvency problems arise endogenously from high default rates among bank borrowers. Such features allow the model to replicate the high and positive correlation between borrower and bank defaults observed in the data and capture the behavior of the economy not only in normal times but also during periods of twin defaults. While rare, such episodes involve very large deadweight losses due to the simultaneous occurrence of high default rates among both banks and their borrowers. This exacerbates the contraction in economic activity and the welfare losses associated with bank insolvencies.[1] Hence, for the same level of bank insolvencies, our model implies optimal capital requirements that are five percentage points higher than under alternative specifications of bank asset returns which overlook the impact of borrowers' default on banks' default.

As noted by Gornall and Strebulaev (2018) and Nagel and Purnanandam (2020) in a partial equilibrium setup, capturing bank default risk dynamics requires a structural model of bank asset returns. We model banks' portfolios of loans subject to non-diversifiable default risk. The distribution of the returns of these portfolios differs from the log-normal distribution of asset returns assumed in standard models of default since Merton (1974) (which we call the Merton-type model). In our framework, loans yield bounded repayments when they perform but may entail significant losses when banks' borrowers default, so the portfolio returns exhibit limited upside potential but significant downside risk. Importantly, the asymmetry in bank asset returns arises endogenously in our model as loan performance is the main driver of bank insolvencies. We show that in our general equilibrium set up this is essential to reproduce the high and positive correlation between borrower and bank defaults and generate the frequency and severity of twin default crises observed in the data.

Existing macro-banking papers on the optimal level of capital requirements typically instead assume that banks earn equity-like returns with unlimited upside, like in the Merton-type model. Some of the models abstract from the default of bank borrowers and assume that banks invest directly in productive capital (e.g. Van Den Heuvel, 2008; Begenau and Landvoigt, 2017; Begenau, 2020).[2] Others adopt a "double-decker" framework where banks explicitly provide defaultable loans to firms (e.g., Clerc et al., 2015; Mendicino et al., 2018, 2020; Elenev, Landvoigt and Nieuwerburgh, 2020). However, for tractability, these models

[1] As in the costly state verification model (Townsend, 1979) adopted by the financial accelerator literature (e.g. Bernanke and Gertler, 1989), defaults in our model entail deadweight bankruptcy costs.

[2] This approach is similar to the one adopted in seminal macro-banking models (e.g., Gertler and Kiyotaki, 2010; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014) whose focus is neither on bank default risk nor the optimal level of capital requirements.

assume that bank solvency risk arises from idiosyncratic shocks to bank revenues which are unrelated to the performance of the underlying loans.[3] We show that, for this reason, standard Merton-type models of bank default underestimate the correlation between firm and bank defaults and the frequency of twin defaults observed in the data. Hence, in these frameworks bank insolvencies are associated to remarkably lower deadweight losses and contractions in economic activity than in our model. This biases downward the net benefits of higher capital requirements, and, thus, underestimates their optimal level.[4]

In our quantitative framework banks extend loans to firms using insured deposits and equity (own net worth) and are subject to regulatory capital requirements. Firms produce the final good using capital and labor and pay for their inputs of production partly using external financing in the form of bank loans. Both firms and banks operate under limited liability and can default on their debt obligations. As in Baron, Verner and Xiong (2021), bank equity declines are the key driver of bank solvency crises in our model.[5] Banks are exposed to default risk because firms' performance is affected by shocks which are not fully diversifiable at the bank level. Specifically, we assume that credit markets are segmented into islands: a bank can only grant loans to a continuum of firms on a given island.[6] Each firm in the island is exposed to both firm- and island-idiosyncratic productivity shocks. Banks can diversify away firm-idiosyncratic shocks by lending to all firms in the island. But island-idiosyncratic shocks affect all firms operating in the island in the same way and, hence, are not-diversifiable at the bank level.[7] Thus island risk generates heterogeneity in banks' asset

[3]A single risk factor specification (Vasicek, 2002) or aggregate shocks could also generate bank default risk. In the absence of idiosyncratic shocks to bank revenues, these approaches, however, would not produce heterogeneity in default outcomes across banks.

[4]The comparison is based on a calibrated Merton-type variant of our model. Specifically, we assume that the default risk of banks comes from exogenous disturbances that directly hit banks' loan returns. Importantly, the average probability of bank default and its standard deviation is the same in both models.

[5]Using historical data, Baron, Verner and Xiong (2021) find bank equity losses to predict subsequent contractions in bank credit and aggregate economic activity. Their evidence clearly shows that while panics are an amplification mechanism, they are not necessary for banking crisis to have severe economic consequences. Our analysis therefore abstracts from the complications associated with the modelling of panics.

[6]In our model, the segmentation does not apply to any other market, including the funding of banks.

[7]Our assumption on the exposure of banks to non-diversifiable risk is consistent with the evidence in

returns and default outcomes.

The asset returns of individual banks depend on the island-idiosyncratic shock in a highly non-linear manner. In islands with high realizations of this shock, a large fraction of borrowers repay the contractual amount. In islands with low realizations, more borrowers default and banks make significant losses. Thus, asset returns of individual banks are characterized by limited upside risk but significant downside risk. While idiosyncratic shocks are assumed to be log-normally distributed, individual bank banks' asset returns endogenously feature highly left-skewed and asymmetric returns.

After building a macro-banking model of default risk, we ensure that it reproduces relevant features of the data, including the positive correlation between bank and firm defaults and the frequency and severity of twin default episodes. To generate aggregate fluctuations in macroeconomic and financial variables, the model includes aggregate shocks: total factor productivity (TFP) shocks, as well as firm- and island-risk shocks. Firm- and island-risk shocks affect the variance of the idiosyncratic shocks to firms and islands, respectively, and resemble the risk and uncertainty shocks commonly used in the literature (see Bloom, 2009; Christiano, Motto and Rostagno, 2014). In our model, they are crucial to generate fluctuations in firm and bank defaults.

We estimate the model parameters using the generalized method of moments, targeting a large set of unconditional moments in macro, banking and financial euro area (EA) data over the period 1992-2016. To capture the non-linearity intrinsic in the returns on bank loans in a tractable way, we use a higher order perturbation solution method. Our model matches well the targeted mean and standard deviation of firm and bank defaults, as well as the positive correlation that these rates exhibit in the data. In contrast, as mentioned above, the standard Merton-type model of bank default risk commonly used in the literature underestimates the correlation between the default rates of banks and their borrowers.

Galaasen et al. (2020), which using matched bank-firm data for Norway show that idiosyncratic borrower risk is an economically significant source of non-diversifiable risk affecting banks' loan portfolio returns.

We also validate the performance of the model in terms of empirical moments describing the relationship between firm and bank defaults and GDP growth not targeted in the estimation. In the data, the overall positive correlation between the two default rates hides substantial non-linearity in their co-movement. Quantile regressions clearly show that the sensitivity of bank default to firm default is higher in the upper quantiles of bank default. Once bank default risk is already very high, its sensitivity to an increase in borrowers' default is higher than in good times. In addition, there is a strong negative link between GDP growth and bank default at lower quantiles of GDP growth, consistent with the importance of financing conditions as a determinant of the economy's downside risk (Adrian, Boyarchenko and Giannone, 2019). Contrary to the Merton-type approach, our model can mimic these non-linearities well thanks to the non-linear structure of bank asset returns, which enables it to reproduce the frequency and severity of the twin default episodes and the associated macroeconomic outcomes.

In addition to helping match the data, the structural link between the solvency of firms and banks constitutes a powerful amplification mechanism which allows the model to generate twin default episodes without the need for large exogenous aggregate shocks. In fact, these episodes are the result of sequences of small negative island-risk shocks that become increasingly amplified as the probability of bank failure grows. Intuitively, the non-linearity in bank asset returns implies that, once banks have a high risk of failure, the marginal impact of additional credit losses on banks' solvency and the macroeconomy is much larger than in normal times.

After validating the quantitative implications of the model, we turn to the assessment of the optimal level of capital requirements. The rationale for bank capital requirements in our setup stems from the presence of safety net guarantees for banks.[8] Banks' outside funding comes from insured deposits which pay an interest rate that is independent of banks' leverage choices. This gives banks with limited-liability an incentive to under-price borrower

[8] See Kareken and Wallace (1978) for an early reference.

risk, as they do not internalize the effects of their individual choices on the social costs of their failures. In addition, they also neglect their impact on the aggregate dynamics of bank equity, which is key to determine the lending capacity of the whole banking sector and, hence, the dynamics of the real economy. Thus, the model combines conventional micro- and macro-prudential rationales for regulatory capital requirements.

Higher bank capital requirements limit bank risk taking incentives and make the banking sector more resilient to credit losses. This reduces the probability of twin defaults and, hence, the negative impact of high firm and bank defaults on welfare. However, higher capital requirements also imply a higher average cost of funding for banks, which translates into higher average borrowing costs for firms and lower average equilibrium levels of credit. Assessing the optimal level of the capital requirements that maximizes social welfare requires quantifying this trade-off.

In our estimated model, a fifteen percent bank capital requirement brings the probability of twin defaults close to zero and maximizes social welfare. This is about five percentage points higher than the optimal level of capital requirements implied by the Merton-type model of bank default risk, which underestimates the probability of twin defaults. While in the Merton-type model firm default is not the main driver of bank default, in our framework bank insolvencies are always accompanied by high levels of defaults among banks' borrowers. Hence, bank default events are significantly more severe in our model compared to the Merton-type framework. For the same level of bank insolvencies, our model predicts larger costs for the society, as the economy experiences deadweight default losses and equity declines not only for banks but also for firms. This result underscores the importance of modelling bank default risk in a structural way. Failing to generate the right frequency and severity of twin defaults understates the costs associated with bank default and, hence, biases downwards the net benefits of higher capital requirements.

Related literature This paper contributes to several strands of the macro-finance literature. First, from a modeling perspective, we contribute to the macro-banking literature by capturing the link between borrowers' and banks' default in a structural and yet tractable way. In an important departure from the standard financial accelerator literature (e.g., Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997; Jermann and Quadrini, 2012), we assume that banks are not a veil and are subject to default risk. We share with earlier papers the assumption that the returns on the firms' productive projects are log-normally distributed, as in the classical Merton model of corporate default (Merton, 1974). But, in line with Gornall and Strebulaev (2018) and Nagel and Purnanandam (2020), the returns on the portfolio of defaultable loans feature limited upside but unlimited downside risk. This appears endogenously in our model due to the incidence of borrower default risk which is non-diversifiable at bank level (island setup). This natural but non-trivial extension of the standard framework is what crucially distinguishes our model also from other models in which banks directly hold productive assets (e.g. Gertler and Kiyotaki, 2010; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Piazzesi, Rogers and Schneider, 2019) as well as from standard double-decker models of bank default risk. [9]

The tractability of the Merton-type approach to bank default risk is useful when solving large models which include, for instance, different types of intermediaries (e.g., Begenau and Landvoigt, 2017) or loans (e.g., Mendicino et al., 2018), long-term debt (e.g., Jermann, 2019; Elenev, Landvoigt and Nieuwerburgh, 2020), liquidity interventions (e.g., Gete and Melkadze, 2020) and monetary policy (e.g., Mendicino et al., 2020).[10] However, our structural approach is better suited to understand the normative and positive implications of credit losses as the main driver of bank insolvencies.

[9] Gete (2018), Rampini and Viswanathan (2019), Ferrante (2019) and Villacorta (2020), among others, develop double-decker models of banks' and borrowers' net-worth which, however, abstract from bank default.

[10] In these models, banks are only exposed to aggregate risk and the ex-post heterogeneity in bank asset returns arises from shocks that affect directly the aggregate returns on the loan portfolio of the bank and not the performance of the individual loan/borrower. We share with this earlier literature the focus on banking crises without panics (Baron, Verner and Xiong (2021)).

Second, capturing the special structure of bank asset risk, and hence the frequency and severity of twin defaults, allows us to properly account for the social costs of bank insolvencies. This implies substantially higher optimal capital requirements than in the literature that provides a quantitative macroeconomic assessment of bank capital requirements (e.g. Van Den Heuvel, 2008; Clerc et al., 2015; Begenau, 2020; Corbae and D'Erasmo, 2019; Davydiuk, 2019; Mendicino et al., 2018, 2020; Elenev, Landvoigt and Nieuwerburgh, 2020). Regardless of the differences in the underlying frictions, a common result in this strand of the literature is that the optimal level of capital requirements is only a couple of percentage points different from baseline pre-crisis levels.[11]

Third, our structural approach to bank asset risk also contributes to the understanding of how financial vulnerabilities lead to downside risks to GDP. Consistent with recent evidence on the link between financial vulnerabilities and downside risks to GDP (e.g. Adrian, Boyarchenko and Giannone, 2019), we show that bank default risk is a strong determinant of the economy's downside risk. In our model, when the risk of bank insolvencies is high, small shocks to banks' non-diversifiable risk have a magnified negative impact on aggregate macroeconomic outcomes.

Finally, our focus on the non-linearities due to the special structure of bank asset risk and its impact on bank default risk adds a complementary perspective to the literature that emphasizes other non-linear aspects of financial crises. Aspects analyzed by prior work include asset price feedback loops (He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014), occasionally binding constraints (Mendoza, 2010; Benigno et al., 2013; Bianchi, 2016), bank panics (Gertler, Kiyotaki and Prestipino, 2019), liquidity problems (Bigio, 2015; De Fiore, Hoerova and Uhlig, 2018), systemic risk (Martinez-Miera and Suarez, 2014), time varying risk-premia (Coimbra and Rey, 2019) and sovereign defaults (Arellano, 2008; Bocola, 2016).

[11] The majority of such papers suggests gains from higher capital requirements. An exception is Elenev, Landvoigt and Nieuwerburgh (2020) whose results point to an optimal level of capital requirements one percentage point below the baseline pre-crisis level.

2. The Model

Household. The model economy is populated by a representative household that provides consumption insurance to three types of members: workers, entrepreneurs and bankers. Workers supply labor to the production sector and transfer their wage income back to the family. Entrepreneurs and bankers provide equity to entrepreneurial firms and banks, respectively.[12]

Entrepreneurial firms and banks. Entrepreneurial firms produce the final good and pay for the inputs of production in advance. Banks grant loans to firms. Both entrepreneurial firms and banks live for one period, issue equities among, respectively, entrepreneurs and bankers (both with limited net worth) and obtain external financing by issuing non-recourse non-contingent debt in the form of bank loans and deposits, respectively. They operate under limited liability and default when their terminal asset value is lower than their debt obligations. In the case of default, their lenders take possession of their assets at a cost equal to a proportion $\mu_{\varkappa}$ of assets (with $\varkappa$=e,b). Non-defaulted entrepreneurial firms and banks pay their terminal net worth to entrepreneurs and bankers, respectively. Explicit safety net guarantees for banks are modelled in the form of insured deposits.

Island setup. There exist a continuum of measure one of islands. In each island there is a continuum of measure one of entrepreneurial firms and a representative bank. Entrepreneurial firms are subject to both firm- and island-idiosyncratic shocks, whose realizations affect their terminal net worth. Banks cannot lend across islands. So, each representative bank diversifies its lending across entrepreneurial firms in its island but not across

[12] The focus of our paper on bank lending to firms is consistent with the important role of EA banks in lending to non-financial corporations (NFCs) and the importance of NFC defaults as drivers of credit losses in Europe (EBA, 2018). Our model could be adapted to consider the case in which bank borrowers are households to finance house purchases with mortgages. However, such a setup would be less relevant in the EA since the recourse nature of most European mortgages makes the default rates of these loans very low even in bad times.

islands.[13] The bank's terminal net worth therefore depends on the realization of the island-idiosyncratic shock.

The island-based market segmentation only applies to the credit market. All factors of production as well as the final output are freely mobile across islands. Deposit and equity funding are also not island specific. Without loss of generality, each firm (bank) receives an identical amount of equity from entrepreneurs (bankers).[14] Given that all firms (banks) are identical ex-ante they all receive the same loan (deposit) amount.

2.1 The Household

In each period some workers become either entrepreneurs (e) or bankers (b) and the same measure of entrepreneurs and bankers retire and become workers again.[15] At the beginning of each period entrepreneurs (bankers) receive payments from last period entrepreneurial firms (banks) and stay active with probability $\theta_{\varkappa}$ (with $\varkappa$=e,b) or retire otherwise. Upon retirement entrepreneurs (bankers) transfer any accumulated net worth to the household. At the same time, a mass $(1-\theta_{\varkappa})$ of workers become entrepreneurs (bankers). The new entrepreneurs (bankers) receive aggregate endowments $\iota_{\varkappa,t}$ from the household. Bankers pay lump-sum taxes to the deposit guarantee scheme (DGS) so as to cover the losses on the insured deposits in banks that defaulted in the previous period.

The household chooses consumption, C_t, hours worked, H_t, and insured bank deposits,

[13] Imperfect diversification can be interpreted as the result of credit market segmentation and/or specialization. In Europe, banks operate largely within national borders and many specialize in lending to particular industries and sectors (Guiso, Sapienza and Zingales, 2004; De Bonis, Pozzolo and Stacchini, 2011; Behr and Schmidt, 2016; De Jonghe et al., 2020). Geographic and sectoral specialization is also a feature of US small and medium-sized banks (Deyoung et al., 2015; Regehr and Sengupta, 2016) and banks in Peru (Paravisini, Rappoport and Schnabl, 2015).

[14] This is also equal to the aggregate wealth of entrepreneurs (bankers).

[15] This assumption guarantees that entrepreneurs and bankers do not accumulate enough net worth such that entrepreneurial firms and banks can be entirely financed with internal funds (see Gertler and Kiyotaki, 2010).

D_t, to maximize the present discounted value of utility

$$\mathbb{E}_t \sum_{s=t}^{\infty} \left(\beta^s \log \left(C_s \right) - \frac{\varphi}{1+\eta} H_s^{1+\eta} \right)$$

subject to the budget constraint

$$C_t + D_t = w_t H_t + R_{d,t-1} D_{t-1} + \Upsilon_t + \Xi_t \tag{1}$$

where η is the inverse of the Frisch elasticity of labor supply, φ is the weight of labor supply in the utility of households, w_t is the real hourly wage and $R_{d,t-1}$ is the gross rate of deposits. In addition Υ_t are aggregate net transfers from entrepreneurs and bankers to households (including both the aggregate initial transfers ($\iota_{\varkappa,t}$, $\varkappa$=e,b) and the accumulated net worth upon retirement), and Ξ_t is profits from the capital producing firms that households own. We are interested in a symmetric equilibrium, hence we assume that the household invests its deposits symmetrically in all the (symmetric) banks in the economy. All the variables in the problem of the household represent aggregate variables.

2.2 Entrepreneurs and Bankers

As the problems of entrepreneurs ($\varkappa$=e) and bankers ($\varkappa$=b) are identical, we outline them together in general terms in this section. Let $V_{\varkappa,t}\left(n_{\varkappa,t}(i)\right)$ be the value of being an entrepreneur (banker) $i \in (0,1)$ with net worth $n_{\varkappa,t}(i)$ at period t from the perspective of the household to which she belongs. Every period, entrepreneur (banker) i decides how much of her net worth, $n_{\varkappa,t}(i)$, to invest in a portfolio of equity of the continuum of entrepreneurial firms (banks) living in period t, $eq_{\varkappa,t}(i)$, and how much to pay back to the household in the form of dividends, $dv_{\varkappa,t}(i)$, to maximize

$$V_{\varkappa,t}\left(n_{\varkappa,t}(i)\right) = \max_{eq_{\varkappa,t}(i), dv_{\varkappa,t}(i)} \left\{ dv_{\varkappa,t}(i) + \mathbb{E}_t \Lambda_{t+1} \left[\left(1-\theta_{\varkappa}\right) n_{\varkappa,t+1}(i) + \theta_{\varkappa} V_{\varkappa,t+1}\left(n_{\varkappa,t+1}(i)\right) \right] \right\} \tag{2}$$

subject to

$$
\begin{aligned}
& eq_{\varkappa,t}(i) + dv_{\varkappa,t}(i) = n_{\varkappa,t}(i), \\
& n_{\varkappa,t+1}(i)) = \theta_{\varkappa}\rho_{\varkappa,t+1}eq_{\varkappa,t}(i), \text{ and} \\
& dv_{\varkappa,t}(i) \geq 0.
\end{aligned}
\tag{3}
$$

From the point of view of the entrepreneur (banker) maximizing the value function described above, $\rho_{\varkappa,t+1}$ is an exogenous random variable. In equilibrium, $\rho_{\varkappa,t+1}$ equals the gross rate of return of the portfolio of entrepreneurial (banker) equity. A detailed expression for the equilibrium value of $\rho_{\varkappa,t+1}$ is provided below. We are interested in a symmetric equilibrium. Hence, we assume that each entrepreneur (banker) invests symmetrically in all the (symmetric) entrepreneurial firms (banks) of the economy. The constraint $dv_{\varkappa,t}(i) \geq 0$ reflects the fact that entrepreneurs (bankers) can freely pay positive dividends back to the household but the household cannot provide further net worth to the entrepreneurs (bankers). All the variables in the problem of the entrepreneur (banker) represent per capita variables.[16]

As in Gertler and Kiyotaki (2010), we guess that the value function of being an entrepreneur (banker) is linear in her net worth, $V_{\varkappa,t}\left(n_{\varkappa,t}(i)\right) = \nu_{\varkappa,t}n_{\varkappa,t}(i)$, where $\nu_{\varkappa,t}$ is the shadow value of one unit of entrepreneurial (banker) net worth.[17] Then we can write the

[16]The rate of return $\rho_{\varkappa,t+1}$ is a function of time t endogenous aggregate state variables and time $t+1$ exogenous aggregate state variables (i.e., aggregate shocks). Hence, the value function $V_{\varkappa,t}$ is not only a function of the individual state variable, $n_{\varkappa,t}(i)$ but also of the aggregate state variables. For this reason the individual entrepreneur (banker) needs to guess rules to forecast the aggregate state variables. In equilibrium, those rules need to be coherent with behavior. To simplify notation, we only describe the dependence of the value function with respect to the individual state variable.

[17]We need an index i for individual entrepreneurs (bankers) because each of them has been an entrepreneur (banker) for a different length of time and has therefore accumulated a different level of net worth. However, since individual entrepreneur (banker) value functions and policy functions are linear in own net worth, the distribution of entrepreneurial (banker) wealth is irrelevant for aggregate outcomes. As we will see below, aggregate investment and credit demand depend only on the aggregate wealth of entrepreneurs (bankers).

Bellman Equation (2) as

$$\nu_{\varkappa,t} n_{\varkappa,t}(i) = \max_{eq_{\varkappa,t}(i), dv_{\varkappa,t}(i)} \left[dv_{\varkappa,t}(i) + \mathbb{E}_t \Lambda_{t+1} \left(1 - \theta_{\varkappa} + \theta_{\varkappa} \nu_{\varkappa,t+1}\right) n_{\varkappa,t+1}(i) \right]. \tag{4}$$

We guess and later verify that, in the proximity of the steady state, $\nu_{\varkappa,t} \geq 1$. From the envelope theorem $dv_{\varkappa,t}(i) = 0$ whenever $\nu_{\varkappa,t} > 1$. Under our parameter values $\nu_{\varkappa,t} = 1$ with a probability close to zero. As a result, we impose $dv_{\varkappa,t}(i) = 0$ such that the Bellman equation (4) reduces to

$$\nu_{\varkappa,t} = \mathbb{E}_t \Lambda_{t+1} \left(1 - \theta_{\varkappa} + \theta_{\varkappa} \nu_{\varkappa,t+1}\right) \rho_{\varkappa,t+1} \tag{5}$$

and the evolution of an entrepreneur's (banker's) net worth is $n_{\varkappa,t+1}(i) = \rho_{\varkappa,t+1} n_{\varkappa,t}(i)$.

We assume that continuing entrepreneurs (bankers) cannot raise additional outside equity from the household. This creates an aggregate shortage of entrepreneurial (banker) equity, which keeps the risk-adjusted expected return to entrepreneurial (banker) equity greater than the risk-free rate in equilibrium. As a result, the shadow value of funds in the hands of entrepreneurs (bankers), $\nu_{\varkappa,t}$, is greater than unity (that is, if the household were able to transfer more funds to entrepreneurs (bankers), it would do so). The entrepreneur (banker) therefore retains all her net worth and pays no dividends until she retires. Equation (5) defines the entrepreneurs' (bankers') stochastic discount factor for later use as $\Lambda_{\varkappa,t+1} = \Lambda_{t+1} \left(1 - \theta_{\varkappa} + \theta_{\varkappa} \nu_{\varkappa,t+1}\right)$, where $\Lambda_{t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t}$ is the household's stochastic discount factor and λ_t is the Lagrange multiplier of the budget constraint of the household problem.

2.3 Entrepreneurial firms

Entrepreneurial firms active in period t produce the final good, y_{t+1}, using labor, h_t, and capital, k_t

$$y_{t+1} = A_{t+1} k_t^{\alpha} h_t^{1-\alpha}, \tag{6}$$

where A_t is an aggregate TFP shock. At the beginning of the period, entrepreneurial firms buy capital from capital producers at price q_t. In period t, they pay $w_t h_t + q_t k_t$ using equity from entrepreneurs, $EQ_{e,t}$, and loans from the bank in their island, $B_{f,t}$, with gross loan interest rate $R_{f,t}$. At the beginning of period $t+1$, the final good is produced and sold to the households and the depreciated capital, $(1-\delta)\,k_t$, is sold back to the capital producers at price q_{t+1}. Although entrepreneurial firms can only borrow from island-specific banks, final goods, labor, and capital can move freely across islands.[18]

The idiosyncratic shocks. Entrepreneurial firms active in period t face a firm-idiosyncratic shock, ω_i, and an island-idiosyncratic shock, ω_j to the terminal value of their assets (output plus the market value of undepreciated capital).[19] We assume that ω_i and ω_j are log-normally distributed, $\log(\omega_\vartheta) \sim \mathcal{N}\left(-\frac{\sigma^2_{\omega_\vartheta,t+1}}{2}, \sigma^2_{\omega_\vartheta,t+1}\right)$ for $\vartheta = i, j$. The standard deviation of both idiosyncratic shocks is time-varying and subject to persistent aggregate shocks whose law of motion will be introduced below. We denote the CDFs of ω_i and ω_j by $F_{i,t+1}$ and $F_{j,t+1}$, respectively. The subscript $t+1$ captures the dependence of the CDFs on the aggregate risk shocks.

Terminal net worth of an entrepreneurial firm. An entrepreneurial firm i living on an island j borrows from bank j and default if its terminal assets values is insufficient to pay back its loan, $R_{f,t}B_{f,t}$, in full

$$\Pi_{i,j,t+1}\left(\omega_i, \omega_j\right) = \omega_i \omega_j \left[q_{t+1}\left(1-\delta\right)k_t + y_{t+1}\right] - R_{f,t}B_{f,t} < 0. \tag{7}$$

and the entrepreneurial firm defaults on its loan when $\Pi_{i,j,t+1}\left(\omega_i, \omega_j\right) < 0$.

[18]Since entrepreneurial firms have constant returns, the scale of an individual firm is indeterminate. Without loss of generality, we assume that there is a continuum of measure one of firms in each island.
[19]These shocks are independent across firms and across time and they are realized at the same time.

Thus, entrepreneurial firms can default both for aggregate (e.g. large movements in TFP, A_{t+1} or in the price of capital, q_{t+1}) and idiosyncratic reasons. While the two idiosyncratic shocks are indistinguishable from the point of view of the individual firm, they have very different implications for the bank lending to the firms on a given island. The ω_i is idiosyncratic across the continuum of firms operating in an island and, thus, can be diversified by the banks lending in each island. Instead, the ω_j is idiosyncratic across islands and hits all the firms within an island in the same way. As banks cannot lend across islands, this shock is non-diversifiable by the bank lending within an island.

From Equation (7), it is useful to define the threshold value for the firm-idiosyncratic shock ω_i below which entrepreneurial firms experiencing an island-idiosyncratic shock ω_j default is

$$\bar{\omega}_{t+1}(\omega_j) = \frac{R_{f,t}B_{f,t}}{\omega_j\left(q_{t+1}(1-\delta)k_t + y_{t+1}\right)}. \tag{8}$$

A low realization of the island-idiosyncratic shock increases the threshold at which firms default on their bank. This implies that a larger fraction of firms default.Thus, the default rate on the portfolio of the bank on island j depends on the realization of the island-idiosyncratic shock ω_j in the island. This is a key source of default risk for banks in our framework.

Entrepreneurial firms choose capital, hours worked, the loan amount, and the gross loan rate to maximize the net present value of the entrepreneurs equity stake conditional on not defaulting

$$\max_{k_t,h_t,B_{f,t},R_{f,t}} \mathbb{E}_t\Lambda_{e,t+1}\left(\int_0^\infty\int_0^\infty \max\left[\Pi_{i,j,t+1}(\omega_i,\omega_j)\ ,0\right] dF_{i,t+1}(\omega_i)\, dF_{j,t+1}(\omega_j)\right)$$

subject to

$$B_{f,t} + EQ_{e,t} = w_t h_t + q_t k_t, \text{ and} \tag{9}$$

$$\mathbb{E}_t\Lambda_{b,t+1}\Pi_{b,t+1} \geq \nu_{b,t}\phi B_{f,t}, \tag{10}$$

where Equation (9) is the entrepreneurial firm's budget constraint. Equation (10) is the bankers' participation constraint, which determines the interest rate $R_{f,t}$ at which a bank is willing to lend such that the expected discounted bank profits are sufficient to compensate for the cost of equity required to provide the loan. As explained and fully specified in detail below, $\Lambda_{b,t+1}$ is bankers' stochastic discount factor, $\Pi_{b,t+1}$ is the payoff that bankers would receive from equity invested across banks that provide the corresponding bank loans to all the firms operating in their island (that will be defined below), $\nu_{b,t}$ is the shadow value of bankers' net worth, and ϕ is the regulatory capital requirement that determines the fraction of the loan amount $B_{f,t}$ that must be funded with equity at period t. Thus, the participation constraint of the bank reflects the competitive pricing of the loans that banks are willing to offer for different leverage and productive choices by entrepreneurial firms.

Finally, the gross rate of return on the portfolio of equity of an entrepreneur that symmetrically invests in all entrepreneurial firms is $\rho_{e,t+1} = \frac{\Pi_{f,t+1}}{EQ_{e,t}}$.

2.4 Banks

As entrepreneurial firms, banks are active for a single period. In period t banks use equity $EQ_{b,t}$ from bankers and deposits d_t from households in order to provide loans $b_{f,t}$ to entrepreneurial firms operating in their island. Hence, they face the following balance-sheet constraint

$$b_{f,t} = EQ_{b,t} + d_t. \tag{11}$$

We assume that banks invest symmetrically in all the (symmetric) entrepreneurial firms in their island. Banks also face the following regulatory capital constraint

$$EQ_{b,t} \geq \phi b_{f,t} \tag{12}$$

where ϕ is the capital requirement on loans. As explained below, in equilibrium, the capital constraint is always binding because funding the bank with deposits is always cheaper than funding it with equity.

Gross rate of return on assets of the bank in island j. Banks operate under constant returns to scale; hence, their individual loan supply is perfectly elastic as long as the loan rate and the decisions of the borrowing firm satisfy bankers' participation constraint. This constraint plays the role of the zero profit condition in standard production theory and it stipulates that the loan must guarantee the bankers the equilibrium expected rate of return on banker equity. Because the bank is a levered institution with the possibility to default at time $t+1$, the expected equity return also includes the value of limited liability, which allows shareholders to avoid negative returns. In the remainder of this section, we cover the steps needed to derive a detailed expression for bankers' payoffs.

We start by computing the terminal asset value of the representative bank living in island j. At the beginning of period $t+1$, firm- and island-idiosyncratic shocks, ω_i and ω_j, hit the entrepreneurial firms living at period t. As derived in Equation (8), conditional on the island-idiosyncratic shock ω_j, an entrepreneurial firm pays back its loan in full when it experiences a firm-idiosyncratic shock no lower than $\bar{\omega}_{t+1}(\omega_j)$. Entrepreneurial firms with firm-idiosyncratic shocks smaller than $\bar{\omega}_{t+1}(\omega_j)$ default on their loans and the bank only recovers a fraction $1-\mu_f$ of the terminal value of the entrepreneurial firm's assets. Hence, the gross rate of return on assets of the bank in island j is

$$\tilde{R}_{f,t+1}(\omega_j) = \frac{(1-\mu_f)\,\omega_j\left[q_{t+1}(1-\delta)\,k_t + y_{t+1}\right]}{b_{f,t}} \int_0^{\bar{\omega}_{t+1}(\omega_j)} \omega_i dF_{i,t+1}(\omega_i) + R_{f,t} \int_{\bar{\omega}_{t+1}(\omega_j)}^{\infty} dF_{i,t+1}(\omega_i)\,. \tag{13}$$

where the first (second) part of Equation 13 represents the repayment per unit of loan from non-defaulting (defaulting) entrepreneurial firms in an island experiencing an island-idiosyncratic shock ω_j.

Following Bernanke, Gertler and Gilchrist (1999) it is useful to define

$$\Gamma_{i,t+1}\left(\bar{\omega}_{t+1}\left(\omega_j\right)\right)=\int_0^{\bar{\omega}_{t+1}(\omega_j)}\omega_i dF_{i,t+1}\left(\omega_i\right)+\bar{\omega}_{t+1}\left(\omega_j\right)\int_{\bar{\omega}_{t+1}(\omega_j)}^{\infty}dF_{i,t+1}\left(\omega_i\right)$$

and

$$G_{i,t+1}\left(\bar{\omega}_{t+1}\left(\omega_j\right)\right)=\int_0^{\bar{\omega}_{t+1}(\omega_j)}\omega_i dF_{i,t+1}\left(\omega_i\right).$$

Then Equation (13) can be rewritten more compactly as

$$\tilde{R}_{f,t+1}\left(\omega_j\right)=\left[\Gamma_{i,t+1}\left(\bar{\omega}_{t+1}\left(\omega_j\right)\right)-\mu_f G_{i,t+1}\left(\bar{\omega}_{t+1}\left(\omega_j\right)\right)\right]\frac{\omega_j\left[q_{t+1}\left(1-\delta\right)k_t+y_{t+1}\right]}{b_{f,t}}. \quad (14)$$

Yet again, it is worth noting that the return of the bank in island j is a function of the island-idiosyncratic shock ω_j. The mechanism works through the impact of ω_j on the default rate of entrepreneurial firms on island j as well as on the recovery value of the assets of defaulted entrepreneurial firms.

Equation (14) shows that the bank's loan portfolio return is a non-linear function of ω_j and hence is not log-normal even if ω_j itself is. Thus our model departs from the standard Merton approach (Merton, 1974) where it is assumed that bank asset returns follow a log-normal process.[20] Our choice complicates the model solution with respect to the Merton's approach. More details will be provided in Section 3.1.

We will analyze extensively the implications of this feature of our model in Section 4.. Most importantly, our approach captures the limited upside of loan payoffs together with the significant downside risks posed by borrower defaults. As we will see this enables our model to capture important features of the data, including the frequency and severity of twin default crises.[21]

[20]Island-idiosyncratic shocks play at bank level the same role as the so-called "single risk factor" in the partial equilibrium model of Vasicek (2002). See also (Gordy, 2003). In contrast to direct implementations of such framework (e.g., Repullo and Suarez, 2013) our continuum of islands (and, hence, risk factors) allows us to capture ex post heterogeneity in bank performance.

[21] We will also show that, in contrast, a reduced-form version of bank default risk (Merton-type model)fails

Terminal net worth of a bank. Banks default on their deposits if their loan returns at the beginning of period $t+1$, $\tilde{R}_{f,t+1}(\omega_j) b_{f,t}$, are insufficient to pay the promised repayment, $R_{d,t}d_t$, in full

$$\Pi_{b,t+1}(\omega_j) = \tilde{R}_{f,t+1}(\omega_j) b_{f,t} - R_{d,t}d_t < 0. \tag{15}$$

It is possible to show that the returns on the bank loan portfolio, $\tilde{R}_{f,t+1}(\omega_j)$, is a highly non linear function of ω_j. See Section 3.1.

Gross rate of return on the portfolio of bank equity. From Equation (15), it is useful to define a threshold value for the island-specific shock ω_j below which the bank in island j defaults. This is implicitly done in the next equation

$$\tilde{R}_{f,t+1}(\bar{\omega}_{j,t+1}) b_{f,t} - R_{d,t}d_t = 0. \tag{16}$$

Equation (16) implies that banks' failure rate at the beginning of period $t+1$ is $F_{j,t+1}(\bar{\omega}_{j,t+1})$. Thus, the aggregate payoffs of a portfolio containing the equity of all banks are then

$$\Pi_{b,t+1} = \int_{\bar{\omega}_{j,t+1}}^{\infty} \tilde{R}_{f,t+1}(\omega_j) b_{f,t} dF_{j,t+1}(\omega_j) - R_{d,t}d_t (1 - F_{j,t+1}(\bar{\omega}_{j,t+1})) \tag{17}$$

and the gross rate of return on the portfolio of equity of a banker that symmetrically invests in all banks is

$$\rho_{b,t+1} = \frac{\Pi_{b,t+1}}{EQ_{b,t}} = \frac{\Pi_{b,t+1}}{\phi b_{f,t}}. \tag{18}$$

2.5 Capital Production

Capital producers combine the final good, I_t, with the last period capital goods, K_{t-1}, in order to produce new capital goods that competitively sell to entrepreneurial firms at price

to reproduce them and consequently also generates lower prescriptions for the optimal level of capital requirements than our model.

q_t. Capital producers face adjustment costs, $S\left(\frac{I_{k,t}}{K_{t-1}}\right)$, as in Jermann (1998).[22] The law of motion of the capital stock can be written as

$$K_t = (1-\delta) K_{t-1} + S\left(\frac{I_t}{K_{t-1}}\right) K_{t-1}. \tag{19}$$

where δ is the capital depreciation rate.

2.6 Deposit Guarantee Scheme

The deposit guarantee scheme (DGS) charges lump-sum taxes to bankers to ex-post balance its budget. The DGS has to balance its budget period-by-period. Hence, the total lump sum tax T_t imposed on bankers to balance the agency's budget is

$$T_t = F_{j,t}(\bar{\omega}_{j,t}) R_{d,t-1} d_{t-1} - \frac{1-\mu_j}{1-\phi} d_{t-1} \int_0^{\bar{\omega}_{j,t}} \tilde{R}_{f,t}(\omega_j)\, dF_{j,t}(\omega_j), \tag{20}$$

which uses the fact that in equilibrium $d_t = (1-\phi) b_{f,t}$.

2.7 Aggregate Shocks

We assume the following AR(1) law of motion for the TFP shock

$$log(A_{t+1}) = \rho_A log(A_t) + \sigma_A \epsilon_{A,t+1}, \tag{21}$$

where $\epsilon_{A,t+1}$ is normally distributed with mean zero and variance one.

The standard deviation of the distribution of each idiosyncratic shock is time-varying

[22] The adjustment costs take the functional form: $S\left(\frac{I_{k,t}}{K_{t-1}}\right) = \frac{a_{k,1}}{1-\frac{1}{\psi_k}}\left(\frac{I_t}{K_{t-1}}\right)^{1-\frac{1}{\psi_k}} + a_{k,2}$, where $a_{k,1}$ and $a_{k,2}$ are chosen to guarantee that in the steady state the investment-to-capital ratio is equal to the depreciation rate and $S'(I_t/K_{t-1})$ equals one.

and evolves as an AR(1) process

$$\log\left(\frac{\sigma_{\omega_\vartheta,t+1}}{\bar{\sigma}_{\omega_\vartheta}}\right) = \rho_{\sigma_i}\log\left(\frac{\sigma_{\omega_\vartheta,t}}{\bar{\sigma}_{\omega_\vartheta}}\right) + \sigma_\vartheta\epsilon_{\omega_\vartheta,t+1} \tag{22}$$

for $\vartheta = i, j$, where $\epsilon_{\omega_\vartheta,t+1}$ is normally distributed with mean zero and variance one.[23] These shocks resemble the risk and uncertainty shocks commonly used in the literature (Bloom, 2009; Christiano, Motto and Rostagno, 2014). We will refer to them as firm- and island-risk shocks. In the next sections we will show that these shocks are important source of aggregate risk in the model and will be vital to generate fluctuations in firm and bank defaults.

2.8 Aggregation, Market Clearing and Equilibrium

Aggregation and market clearing conditions as well as the exhaustive list of equilibrium conditions of the model are reported in Internet Appendix B.

3. Solution, Estimation and Model Validation

In this section we present the solution of the model, the estimation of the parameters and the model validation results.

3.1 Solving the Model

We solve the system of stochastic difference equations implied by the equilibrium conditions using a pruned state-space system for the third-order approximation around the steady state as defined in Andreasen, Fernandez-Villaverde and Rubio-Ramirez (2017). This approach eliminates explosive sample paths and greatly facilitates inference. In particular, it ensures the existence of unconditional moments. This enables us to estimate the parameters of the

[23] This specification is similar to the one adopted in Christiano, Motto and Rostagno (2014).

model by applying the generalized method of moments (GMM).

In order to use perturbation methods to approximate the solution to the model, we need to compute the aggregate loan returns that banks generate conditional on not defaulting, defined here as $R_{p,t+1}$. These returns are the first term in Equation (17), namely,

$$R_{p,t+1} \equiv \int_{\bar{\omega}_{j,t+1}}^{\infty} \tilde{R}_{f,t+1}(\omega_j)\, dF_{j,t+1}(\omega_j)\,. \tag{23}$$

As already mentioned in the previous section, the bank's loan return $\tilde{R}_{f,t+1}(\omega_j)$ is not log-normally distributed because ω_j enters non-linearly in its definition. This introduces a complication: the integral in Equation (23) as well as its derivatives cannot be written as an explicit function of the state variables. We overcome this challenge by (i) splitting this integral into the sum of integrals taken over smaller intervals, and (ii) computing a series of quadratic Taylor approximations of $\tilde{R}_{f,t+1}(\omega_j)$ around a mid-point of each interval. Because the powers of log-normally distributed variables are themselves log-normally distributed, the quadratic approximation to the bank profit function is itself approximately log-normally distributed and the expected profits as well as its derivatives can be computed as explicit functions of the state variables.[24] This approach is tractable and highly accurate. More details are provided in Internet Appendix 3.1.

3.2 Model Estimation

The estimation of the model follows a two-step procedure. First, prior to the estimation procedure, some parameters are set to commonly used values in the literature. Second, we estimate the rest of the parameters using quarterly euro area (EA) data between 1992:Q1 and 2016:Q4.

First Step. Since we use quarterly data, the discount factor of the households, β, is set

[24] The state variables of the model are $\mathbf{w_t} = \left(D_t, K_t, H_t, N_{e,t}, N_{b,t}, q_t, w_t, R_{f,t}, R_{d,t}, A_{t-1}, \sigma_{\omega_j,t-1}, \sigma_{\omega_i,t-1}\right)$.

Table 1: Estimated Parameters

Par.	Description	Value	Par.	Description	Value
χ_b	Bankers' endowment	0.6520	χ_e	Entrepreneurs' endowment	0.5435
$\bar{\sigma}_{\omega_i}$	Mean firm-risk shock	0.3447	$\bar{\sigma}_{\omega_j}$	Mean island-risk shock	0.2625
σ_A	Std TFP shock	0.0044	ρ_A	Persistence TFP shock	0.9804
σ_i	Std firm-risk shock	0.0729	ρ_{σ_i}	Persistence firm-risk shock	0.9141
σ_j	Std island-risk shock	0.0936	ρ_{σ_j}	Persistence island-risk shock	0.7539
ψ_k	Capital adjustment cost	1.9942			

Notes: The reader should note that σ_i is not the standard deviation of firm-risk shock, which is $\frac{\sigma_i}{\sqrt{1-\rho^2_{\sigma_i}}}$. The same applies for the standard deviation of the island-risk shock.

to 0.995, the Frisch elasticity of labor supply, η, to one, the value of capital depreciation, δ, to 0.025, and the capital-share parameter of the production function, α, to 0.30. The cost parameters μ_f and μ_j are all set equal to 0.30.[25] The capital requirement level, ϕ, is set to be 0.08 which was the regulatory minimum in the Basel II regime. We set both θ_e and θ_b to 0.975, implying that bankers and entrepreneurs remain active for ten years on average. Finally, the labor utility parameter, φ, which only affects the scale of the economy, is normalized to one.

Second Step. We estimate the parameters summarized in Table 1 by targeting a number of macroeconomic, financial and banking moments.[26] We target the standard deviations of GDP, investment and consumption growth, the mean ratio of corporate loans to GDP (B_t/GDP_t in the model) along with the standard deviation of loan growth, the mean and standard deviation of the loan spread ($R_{f,t} - R_t$ in the model), and the mean

[25] Similar values of firm default costs are used, among others, in Carlstrom and Fuerst (1997), who refer to the evidence in Alderson and Betker (1995), where estimated liquidation costs are as high as 36 percent of asset value. Among non-listed bank-dependent firms, these costs can be expected to be larger than those among the highly levered, publicly traded US corporations studied in Andrade and Kaplan (1998), where estimated financial distress costs fall in the range of 10 percent to 23 percent. Our choice of 30 percent is consistent with the large foreclosure, reorganization and liquidation costs found in some of the countries analyzed by Djankov et al. (2008). The value of the bank default cost parameter we pick is also in line with Granja, Matvos and Seru (2017) who find that the average FDIC loss from selling a failed bank is 28% of assets.

[26] Internet Appendix A describes the data counterpart of these model variables.

and standard deviation of ROE of banks ($\rho_{b,t}$ in the model).[27] Additionally, we also target the mean and standard deviation of the conditional expectation of entrepreneurial firm and bank default rates along with the unconditional correlation between the two default probabilities. The conditional expectation of entrepreneurial firm defaults is defined as $DF_t = \mathbb{E}_t \left(\int_0^{\infty} \int_0^{\bar{\omega}_{t+1}(\omega_j)} dF_{i,t+1}(\omega_i)\, dF_{j,t+1}(\omega_j) \right)$, while the conditional expectation of bank defaults is $DB_t = \mathbb{E}_t \left(\int_0^{\bar{\omega}_{j,t+1}} dF_{j,t+1}(\omega_j) \right)$.

Table 2 shows that our model matches the data targets reasonably well. Importantly, the model is able to reproduce the positive unconditional correlation between firm and bank default (0.64 in the data versus 0.76 in the model).[28] Matching this correlation turns out be of first-order importance when drawing conclusions about optimal bank capital requirements.

Table 2: Targeted Moments: Baseline Model

Variable	Data	Model	Variable	Data	Model
Std GDP growth	0.6877	0.6217	Std Cons. growth	0.5617	0.4912
Mean Loans/GDP	2.442	2.6386	Std Loan growth	1.1965	0.5936
Mean Loan spread	1.2443	1.0058	Std Loan spread	0.6828	0.7535
Mean Firm default	2.6469	2.2497	Std Firm default	1.0989	2.4384
Mean Bank default	0.6646	0.5860	Std Bank default	0.8438	1.2320
Mean ROE banks	6.4154	6.4652	Std ROE banks	4.1273	3.7971
Corr (FD & BD)	0.6421	0.7648	Std Inv. growth	1.3908	2.0106

Notes: Interest rates, equity returns, default rates, and spreads are reported in annualized percentage points. The standard deviations (Std) of GDP growth, Investment (Inv), and Loan growth are reported in quarterly percentage points.

3.3 Model Validation

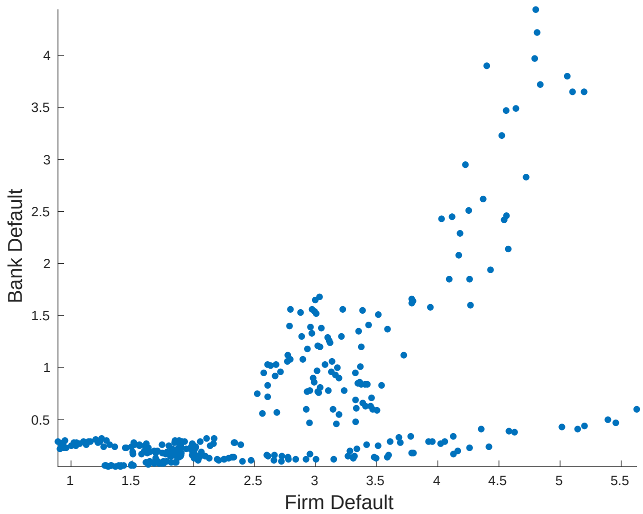
As shown in Table 2, the model is able to match the unconditional moments related to defaults and macroeconomic variables targeted in the calibration. In this section we perform

[27] To avoid the impact of the resource costs of default on the measurement of output, we define GDP_t as $GDP_t = C_t + I_t$.

[28] A similar degree of correlation can be observed in US data.

model validation by comparing the model's implications for important untargeted conditional moments of firm and bank defaults and GDP growth. This is a relevant step, since the assessment of the benefits and costs of higher capital requirements hinges upon the ability of the model to match key features of the data, including the frequency and severity of bank insolvency crises.

Figure 1: Firm and Bank Default Rates



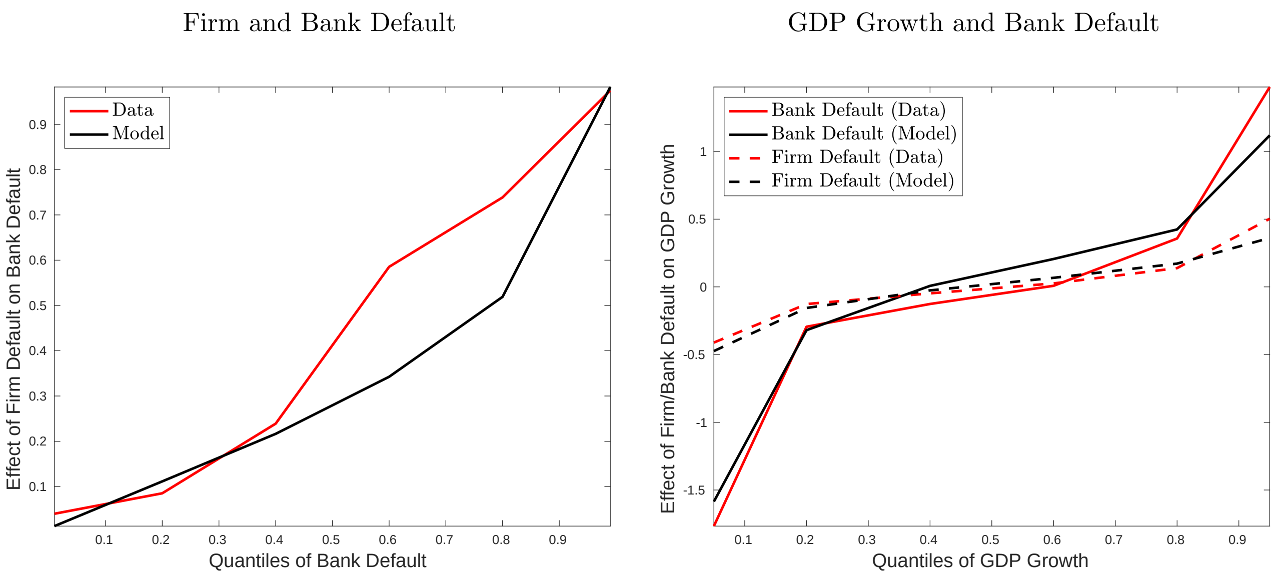
Scatter plot of Moody's cross-sectional average of expected default frequencies (EDFs) within one year for the 1992:M1 to 2016:M12 (monthly frequency) sample of firms (non-financial corporations) and banks in the EA; series in percent.

3.3.1 Defaults and economic performance in the data

Firm and bank defaults are positively correlated, as successfully matched in the estimation. However, as Figure 1 reveals, the overall positive correlation between the two default rates hides substantial non-linearity in their co-movement. The figure displays a scatter plot of the average expected default frequencies (EDFs) of firms and banks in the euro area (EA) over the period 1992-2016.[29]

Broadly speaking, one can identify three different regimes in the relationship between firm and bank default. In the most frequent regime, the default rates of both firms and banks are low. In another regime, the firm default rate is high but the bank default rate is modest. The last regime is one in which the default rates of both firms and banks are elevated.[30] We deem the EDFs of firms and banks to be "high" when they are above their respective 90th percentile in the data.

Figure 2: Quantile Regression: Data vs Model



Notes: The left panel of this figure presents coefficients ζ_τ from the quantile regression in Equation (24). The right panel of this figure presents coefficients β_τ from the quantile regression in Equation (25). Both equations are estimated on EA data (1992-2016) and on simulated data from the baseline model.

Another way of representing the state-contingent relationship between firm and bank default risk is through quantile regressions of the following form:

[29] Each dot represents a monthly average of the corresponding probabilities of default over one year. The underlying EDFs are estimates provided by Moody's. See Internet Appendix A for more details on the data.

[30] The same pattern can be observed in other countries, including the US.

$$\text{BankDef}_t(\tau) = \zeta_\tau \text{FirmDef}_t, \tag{24}$$

where FirmDef_t is firm EDF and BankDef_t is bank EDF.

The left panel of Figure 2 (red line) plots the quantile regression coefficients ζ_τ in Equation (24). The non-linearity in the relationship between the two defaults is clearly visible and highly statistically significant. At higher levels of bank default risk, the coefficient obtained by regressing bank on firm defaults is higher. The quantile regression coefficients indicate that the correlation between firm and bank default is state dependent and increasing with the bank default rate.[31]

Table 3: Average Quarterly GDP Growth

	High Firm Def.	Twin Defaults
Euro Area	-0.0466	-0.5842
Germany	-0.2550	-0.6690
France	-0.0718	-0.6605
Italy	-0.0242	-0.5471
Netherlands	-0.5043	-2.1904
Belgium	-0.3645	-0.4051
US	-0.0781	-0.9790

Notes: First column refers to periods of high firm defaults and low bank defaults, whereas the second column uses periods of twin defaults. GDP growth rates (demeaned) are reported in quarterly rates. Sample: EA 1992Q1-2016Q4, US: 1940:Q1-2016:Q4.

Next, we explore the relationship between aggregate economic activity and firm and bank defaults, respectively. A simple way to analyze this relationship is to look at GDP growth during the different firm and bank default regimes discussed above. As documented in Table 3, the growth rates of GDP in the EA, the US and a number of European countries are below normal when firm default is high but much lower when firm and bank defaults are both high. This is consistent with standard definitions of a systemic financial crisis and the large bank

[31] The variance of firm and bank defaults is roughly constant across bank default quantiles.

default rates and output losses associated with them (see, e.g., Laeven and Valencia, 2013).[32]

We investigate the relationship between firm and bank defaults and GDP growth using quantile regressions of the following form

$$\Delta y_t(\tau) = \beta_\tau \text{Def}_{t-1} + \gamma_\tau \Delta y_{t-1}, \tag{25}$$

where Def_{t-1} can either be FirmDef_{t-1} or BankDef_{t-1} and Δy_t represents GDP growth.

This exercise is similar in spirit to the one performed in Adrian, Boyarchenko and Giannone (2019) who run a quantile regression of GDP growth on lagged GDP growth and an index of financial conditions using US data. Firm and bank defaults are the main proxies for financial conditions in our framework. Hence, we regress GDP growth on the lagged GDP growth and the lagged level of default (Def_{t-1}) of either firms or banks. The right panel of Figure 2 plots the coefficients for either firm (the dashed red lines) or bank (the solid red lines) default in the corresponding quantile regressions estimated on EA data. The results highlight three key features of the non-linear relationship between defaults and real activity. First, the link between both defaults and economic growth is weak for GDP growth quantiles close to the median. This suggests that defaults (whether bank or firm) have only a weak correlation with GDP growth in normal times.

Second, the negative relationship between bank default and GDP growth becomes quantitatively more negative for the bottom quantiles. Increases in bank defaults have a larger (negative) impact on GDP growth when the economy is already in a recession (i.e. at the bottom quantile for GDP growth).

Third, the above relationship does not hold for firm default. In sharp contrast to the non-linear pattern between bank default and economic activity, the impact of corporate defaults on GDP growth is small and flat across all GDP growth quantiles. Thus, Figure 2 (right panel) clearly shows that it is the risk of bank failures that is driving the deterioration in

[32] Average growth rates have been demeaned using the unconditional mean of GDP growth for each country.

macroeconomic performance during periods of twin defaults identified in Table 3. This link between bank default and economic performance during twin default crises will explain the importance of capital regulation in mitigating the downside risk to the real economy.

3.3.2 Defaults and economic performance in the model

Section 3.3.1 established a number of important data facts regarding the state-dependent co-movement between default rates and GDP growth. We learned that the marginal impact of corporate failures on bank solvency is stronger when banks are weaker. We saw that twin defaults are associated with deeper recessions. Finally, our results established that the correlation of bank (but not firm) defaults with real activity is higher in recessions. We now test the performance of the model in reproducing these important empirical regularities not targeted in the estimation.

In the previous section, we used a 90th percentile-based criterion to identify the low default, high firm default and twin defaults regimes in the EA data. Here, we use DF_t and DB_t as the model counterparts for firm and bank EDFs, respectively, and we employ the same criterion to split the model-simulated time series into the three regimes.

Table 4 compares the model-simulated (Baseline Model) and EA data (Data) averages for firm default, bank default and GDP growth within each regime. The baseline model does a good job in reproducing these untargeted conditional moments. First, it reproduces remarkably well the frequency of the three default regimes. Second, it reproduces the same ranking observed in the data in terms of the drop in GDP growth in the three regimes. The twin default regime features by far the worst GDP growth realizations, whereas the high firm default regime features a relatively mild recession despite the fact that firms' default rates are very similar across these two regimes.[33]

[33] By definition, in the data there is a fourth regime where the bank default rate is above the 90th percentile but firm default is below the 90th percentile. The model also performs well in matching this additional regime. Even though the average firm default in this regime is below 90th percentile, it remains at an elevated level (on average at the 85th percentile in the model).

Table 4: Three Default Regimes

	Frequency	GDP growth	Bank default	Firm default
	Low Default Regime			
Data	86.0%	0.0923	0.4346	2.3480
Baseline Model	85.9%	0.0588	0.2388	1.5154
Merton-type Model	82.1%	0.0522	0.3351	1.7118
1st Order App.	87.3%	0.0318	0.3130	1.7113
Higher Cap. Req.	91.5%	0.0293	0.0453	1.5406
	High Firm Default Regime			
Data	4.0%	-0.0466	0.4033	4.8500
Baseline Model	4.0%	-0.1344	0.9270	6.8463
Merton-type Model	7.5%	-0.1887	0.3980	7.4093
1st Order App.	1.0%	-0.1556	1.4211	5.5756
Higher Cap. Req.	8.3%	-0.3079	0.3387	7.693
	Twin Defaults Regime			
Data	7.0%	-0.8189	3.0224	4.6076
Baseline Model	5.9%	-0.5737	4.0225	8.6266
Merton-type Model	1.1%	-0.5399	3.2624	7.7483
1st Order App.	4.1%	-0.2735	2.4816	6.0247
Higher Cap. Req.	0.1%	-0.8990	2.2445	11.833

Notes: This table compares the model and data averages for firm default, bank default and GDP growth within three default regimes for the EA data and the simulated data from different models. The baseline model corresponds to a capital requirement set to 8 percent ($\phi = 0.08$) and solved with third-order perturbation. Merton-type Model corresponds to the model in which the Merton-type specification of bank asset returns is adopted. 1st Order App. corresponds to the model solved with first-order perturbation methods. Higher Cap. Req. corresponds to the model with a capital requirement set to 15 percent ($\phi = 0.15$). Twin Defaults episodes are defined as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. High Firm Default are episodes with firm default above the 90th percentile and bank default below the 90th percentile. In Low Default episodes, both bank and firm default are below the 90th percentile. The default thresholds used to define the three regimes in the Merton-type model and the 1st Order App. model are the ones determined by the baseline model. Model results are based on 1,000,000 simulations. GDP growth is demeaned.

In the previous section we also used quantile regressions to characterize the non-linear relationships among the two default series and GDP growth. The black lines in Figure 2 show that our model can replicate both quantile regressions remarkably well.[34] The model is qualitatively and quantitatively consistent with the key facts identified in our description of the quantile regressions on EA data. The correlation between firm and bank default is higher when banks are more fragile and their probability of default is high. During times of average GDP growth, neither firm nor bank defaults affect economic performance in a significant manner. Bank (but not firm) defaults have a large and negative impact on GDP growth when the economy is already in recession.

Both the island-idiosyncratic and island-risk shocks are vital in generating realistic conditional and unconditional correlation patterns between firm and bank defaults and economic activity. When the non-diversifiable risk is constant (no island-risk shocks), the relationship between the two defaults and bank default and GDP growth is significantly weakened. Hence, the model cannot reproduce well the non-linearities observed in the data.

When non-diversifiable risk is absent (no island-idiosyncratic shocks), banks do not default in our calibrated model.[35] In the absence of island-idiosyncratic shocks, banks are only exposed to aggregate shocks and their net worth evolves ex post in a fully symmetric manner. Bank default could only occur as a result of implausibly large aggregate shocks that, would, thus, happen with a very low probability. Additionally, this would imply that either all banks default at the same time or none does which would be counterfactual.

Given the non-linearity in bank asset returns with respect to non-diversifiable borrower risk, a crucial element for the ability of our model to reproduce the non-linearities observed in the data is the use of a higher-order solution method. Table 4 also reports the performance of the linear approximated version of the model (1st Order App.) in terms of untargeted

[34] Regression coefficients for the model are obtained using simulations of the model for 100,000 periods. As before, we use DF_t and DB_t as the model counterparts for firm and bank EFDs, respectively.

[35] Internet Appendix D (section 4.1 - 4.2) explores the importance of each of these shocks in detail.

conditional moments in the three default regimes. The frequency of the twin defaults regime reproduced by the linearized modelis somewhat lower than the one observed in the data and in our baseline model. In addition, it underestimates (overestimates) the severity of the twin defaults (high firm default) regime in terms of GDP growth.[36]

Overall, our model reproduces well the importance of financial vulnerabilities as determinants of the economy's downside risk (Adrian, Boyarchenko and Giannone, 2019). In particular, it reproduces the fact that a deterioration in bank default risk corresponds to an increase in the downside risk to GDP growth, consistently with what observed in EA data. In the next section, we will show that the reason why our model can replicate these non-linearities observed in the data is because it features a non-linear structure of bank asset returns. This is essential for the model to generate the right frequency and severity of the twin default episodes and the associated macroeconomic outcomes.

4. Understanding the Model: Bank Asset Returns

Modelling bank portfolios as consisting of defaultable loans introduces an important non-linearity into bank asset returns and hence into bank default realizations. In what follows we first show that this is crucial for our model to be able to replicate well the non-linearities observed in the data as well as the frequency and severity of the twin default episodes and the associated macroeconomic outcomes.[37] Next, we confirm the result of Nagel and Purnanandam (2020) and Gornall and Strebulaev (2018), who show that a reduced-form approach to bank default risk that uses a Merton-type formulation – in which bank asset returns have a log normal distribution – cannot capture the downward skewness in bank loan

[36]Both the linear and the second-order model clearly fail to match the non-linearities found in the data.Internet Appendix D (section 4.3) provides further details on the role of the higher-order solution method.

[37]The results shown in section 3.3.2 show that a third-order solution is sufficient to capture such non-linearities in an accurate manner. See also Intenet Appendix D (section 4.1).

portfolio returns.[38] Finally, we show that in our general equilibrium set up, the reduced-form Merton approach also fails to reproduce the frequency and severity of twin defaults and associated macroeconomic outcomes.

Bank asset returns in our model. A distinguishing feature of our model is the structural approach to loan default risk whereby banks fail only when a significant fraction of the borrowers in their imperfectly diversified loan portfolios default. This modelling strategy is appropriate for bank asset returns. Indeed, even if the banks are financing underlying projects with log-normal returns, their payoff structure is downwardly skewed. This is because financial intermediaries hold portfolios of loans with asymmetrically distributed payoffs.[39] If borrowers repay, they repay a fixed contractual amount. If they default, the recovery value on loans is limited to a fraction of firms' asset values.

In the top left panel of Figure 3 we report the gross loan returns of the representative bank of island j, $\tilde{R}_{f,t+1}(\omega_j)$ as a function of the island-idiosyncratic shock, ω_j. In the top right panel, we depict the distribution of $\tilde{R}_{f,t+1}(\omega_j)$.[40] In the top right panel, we depict the distribution of $\tilde{R}_{f,t+1}(\omega_j)$. The top left panel of Figure 3 clearly shows that bank asset returns are highly non-linear in the island-idiosyncratic shock (ω_j). When ω_j is very high, all borrowers repay and the bank receives the promised repayment, including interest, from all its borrowers. But the upside is limited for the lender as is naturally the case under a standard debt contract. However, the presence of default creates downside risk for the bank. As the island idiosyncratic shock takes lower and lower values, the fraction of defaulting firms in the island increases and bank asset returns decline in a highly non-linear fashion.

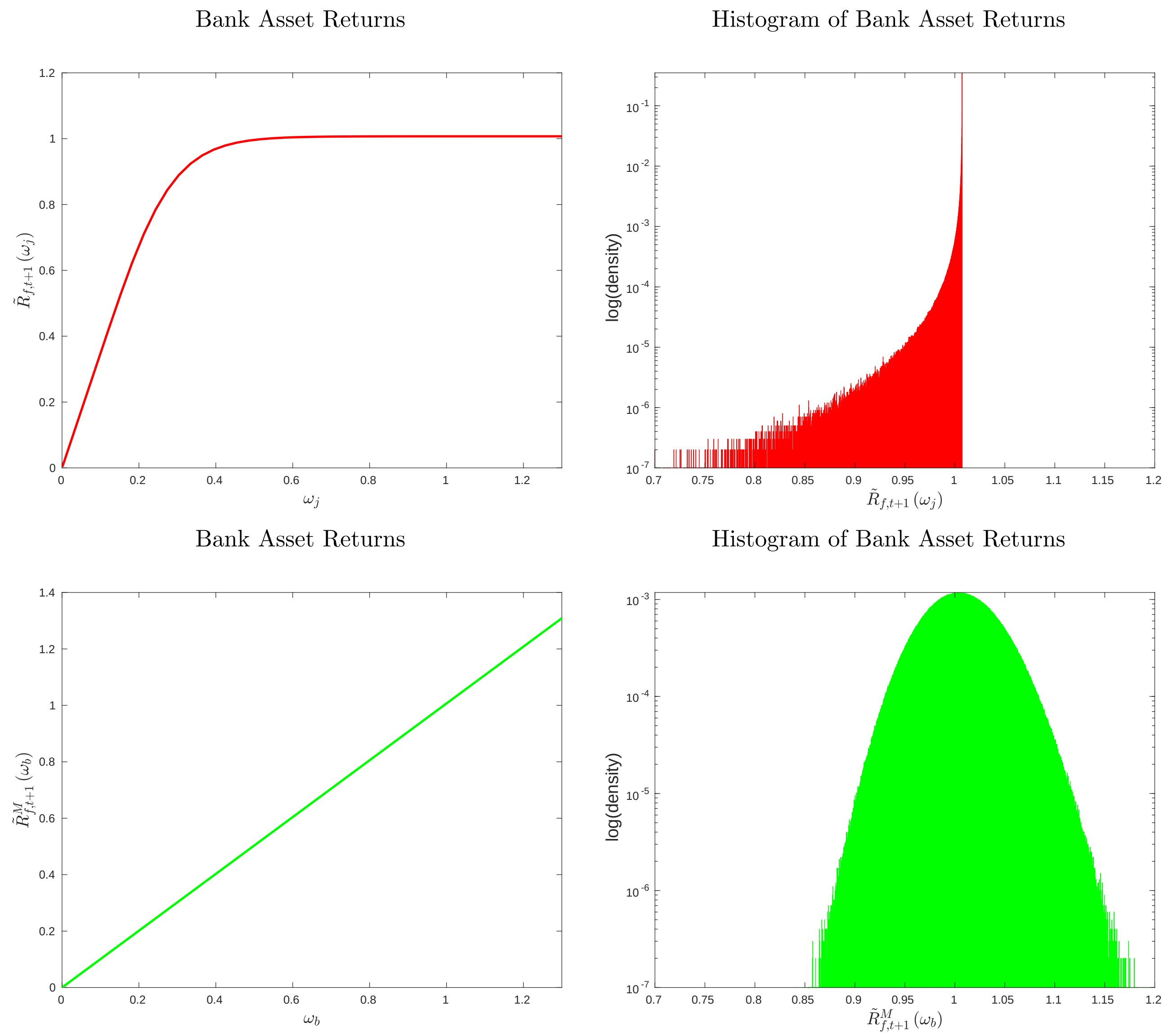
The top right panel of Figure 3 shows the distribution of $\tilde{R}_{f,t+1}(\omega_j)$, which is clearly not log-normal, despite the fact that the idiosyncratic shocks are assumed to be log-normally

[38] Baron, Verner and Xiong (2021) document that at the start of banking crises, the distribution of bank equity returns is considerably more left-skewed than that of non-financial equity returns.

[39] Note that we assume that all shocks are symmetrically distributed, as standard in the literature.

[40] For these figures, we have fixed $q_{t+1}, k_t, y_{t+1}, b_{f,t}, R_{d,t}, d_t$ to their steady-state values obtained with the parameter values described in Section 3.2. We set $\sigma_{\omega_\vartheta,t+1}$ such that the expected bank default equals its targeted value from Table 2. We use 10,000,000 draws of ω_j to plot the histograms.

Figure 3: Bank Asset Returns: Baseline vs Merton-type Model



Notes: The top panels of this figure present bank asset returns as a function of the non-diversifiable island shock ω_j (left plot) and the histogram of bank asset returns (right plot) in the baseline model. The bottom panels of this figure present bank asset returns as a function of the bank-idiosyncratic shock ω_b (left plot) and the histogram of bank asset returns (right plot) in the Merton-type version of our model.

distributed. A large spike occurs when all borrowers repay. Bank asset returns are left-skewed with a fat left tail of low return realizations caused by firm defaults.

Comparison to the Merton-type model. Characterizing the asymmetric distribution of returns on bank loans in general equilibrium is a distinctive feature of our macro-banking framework. We now compare bank asset returns in our model with that embedded in the standard bank default risk approach, as commonly used in the literature. The latter considers banks with (ex-ante) perfectly diversified loan portfolios. As a result, the (ex-post) heterogeneity in bank asset returns comes from shocks that directly affect the performance of banks' loan portfolios rather than the underlying borrowers. Hence, this makes loan returns and their implications for bank equity returns and bank failure similar to the classical Merton (1974) approach to corporate default.

To create a Merton-type version of our model that is in line with formulations in the the existing macro-banking literature, we modify Equation (14) in two ways. First, we remove the impact of the island-idiosyncratic shocks by setting them to unity at all times $\omega_j = 1$. This is equivalent to assuming that banks are perfectly diversified across borrowers. Second, in order to introduce ex-post heterogeneity in bank default outcomes, we include a log-normally distributed bank-idiosyncratic shock to bank revenues ω_b. The loan portfolio returns under this specification are determined by

$$\tilde{R}^{M}_{f,t+1}\left(\omega_b\right) = \left[\Gamma_{i,t+1}\left(\bar{\omega}_{t+1}\left(1\right)\right) - \mu_f G_{i,t+1}\left(\bar{\omega}_{t+1}\left(1\right)\right)\right] \frac{\omega_b\left[q_{t+1}\left(1-\delta\right)k_t + y_{t+1}\right]}{b_{f,t}}. \tag{26}$$

Identically to the island-idiosyncratic shock, the standard deviation of the distribution of the bank-idiosyncratic shock, ω_b, is also time-varying and evolves as in Equation 22. We keep the rest of the model identical.[41]

In the bottom left panel of Figure 3 we report the gross loan portfolio returns under

[41] We estimate the parameters of the Merton-type version of our model to match the set of moments presented in Table 2.

the Merton-type formulation, $\tilde{R}^M_{f,t+1}(\omega_b)$, as a function of the idiosyncratic shock to bank loan revenues and in the bottom right plot we depict the distribution of $\tilde{R}^M_{f,t+1}(\omega_b)$. The figures show that the Merton-type model produces bank asset returns that are linear in the exogenous bank-idiosyncratic shock to loan revenues. Thus, banks experience upside and downside shocks in a fully symmetric fashion. Since ω_b is log-normal, $\tilde{R}^M_{f,t+1}(\omega_b)$ is log-normal too. Banks in the Merton-type model are exposed to less risk due to a smaller left tail. This will turn out to be crucial to understand the differences across models regarding the level of capital requirement needed to make banks safe.

Characterizing bank asset returns in an accurate manner is essential when studying the relationship between firm and bank defaults. The Merton-type model fails to reproduce the non-linearity in the relationship between firm and bank defaults along several dimensions. The top right panel of Figure 4 presents a scatter plot of firm and bank defaults implied by the Merton-type model, which uses $\tilde{R}^M_{f,t+1}(\omega_b)$ instead of $\tilde{R}_{f,t+1}(\omega_j)$. Contrary to what implied by our model (top left panel), the standard Merton-type representation of bank asset returns implies that banks can default also when borrowers repay in full. Thus, the correlation between firm and bank failures is zero in such a variant of our model. In contrast, our mechanism treats the two dfaults as intimately linked, endogenously generating an empirically realistic relationship between them.

Another way to examine the relationship between firm and bank defaults is through quantile regressions. The bottom panels of Figure 4 compare the quantile regression coefficients for Equations (24) and (25) in our model (black line) to those obtained from its Merton-type variant (blue line). For completeness, we also include the estimated coefficient using EA data (red line). Yet again, in the Merton-type model the relationship between firm and bank defaults is close to zero at all quantiles of bank default. The standard approach to bank default risk also fails to match the relationship between GDP growth and bank default at both the top and the bottom quantiles of GDP growth.

Failing to capture the non-linear relationship between firm and bank default has impor-

Figure 4: Scatter Plots and Quantile Regressions: Baseline vs Merton-type Model

Notes: The top panels display the scatter plot of firm and bank default produced with the baseline model (top left plot) and the Merton-type version of our model (top right). The bottom left panel presents coefficients ζ_τ from the quantile regression in Equation (24), whereas the bottom right panel presents coefficients β_τ from the quantile regression in Equation (25) for the Merton-type model (blue line), the data (red line) and the baseline model (black line).

tant implications for the frequency and severity of the twin defaults regime. In Table 4 we compare the performance of our baseline model with the Merton-type model in terms of the untargeted conditional moments in the three default regimes. First, the frequency of the twin defaults regime implied by the Merton-type model is significantly lower than the one observed in the data and in our baseline model. Second, the Merton approach underestimates (overestimates) the severity of the twin defaults (high firm default) regime in terms of GDP growth.

In Section 6. we will show that the Merton-type variant of our model, by underestimating the frequency and severity of twin defaults, also implies a lower optimal level of capital requirements compared to our baseline model.

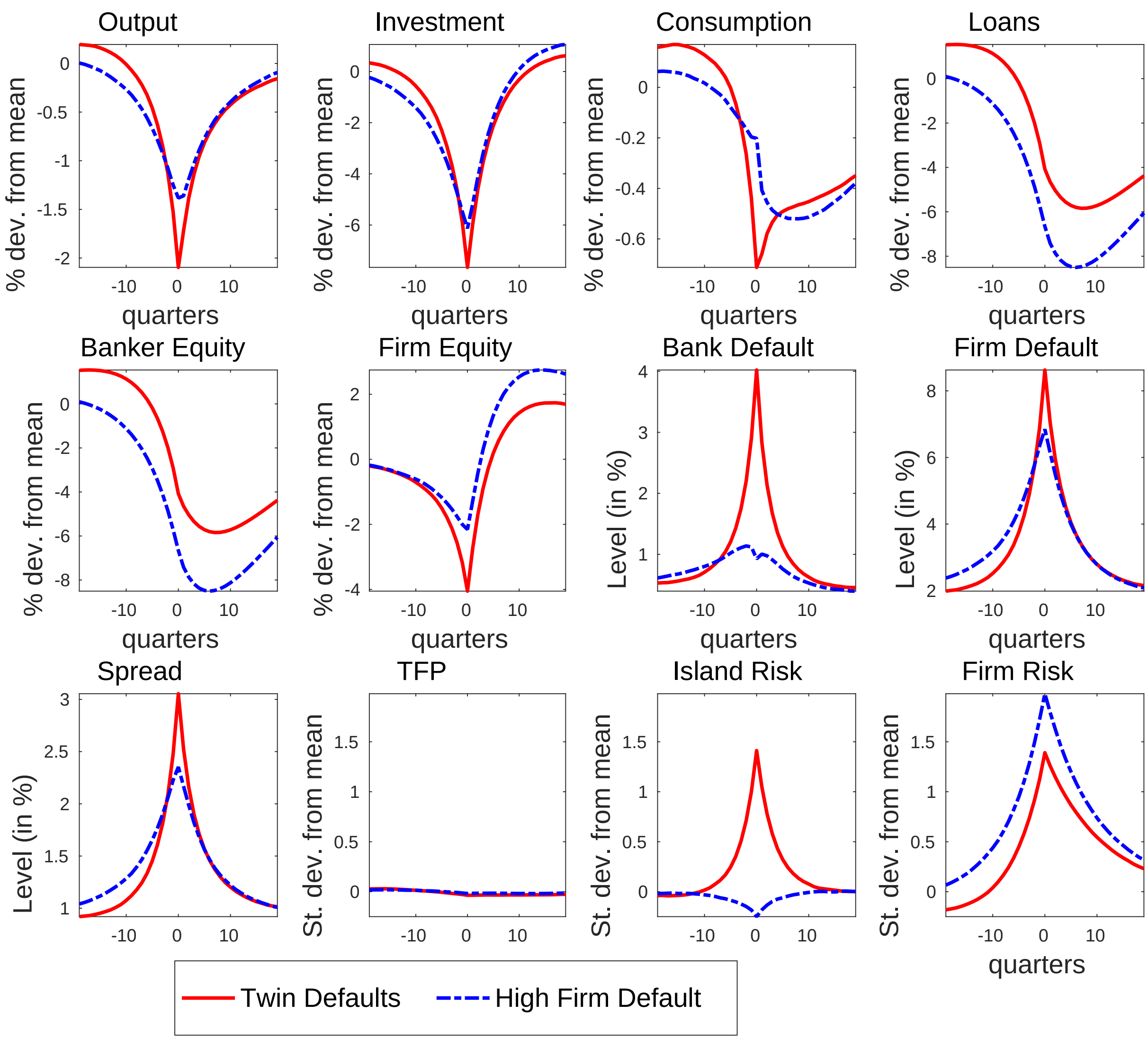
5. The Anatomy of Twin Default Crises

After validating the quantitative implications of our framework, we are well equipped to understand the factors that engender twin default crises in our model. An appealing feature of our setup is that episodes of simultaneously high firm and bank defaults appear as a result of sequences of small negative island-risk shocks that become increasingly amplified as the probability of bank failure grows. Intuitively, the non-linearity in bank asset returns implies that, once banks have a high risk of failure, the marginal impact of additional credit losses on banks' solvency and the macroeconomy is much larger than in normal times. When the probability of twin defaults is high, small shocks can have severe consequences. Indeed, for levels of firm and bank defaults already high, an island-risk shock of limited size disproportionately increases bank default and leads to a large drop in output.

Figure 5 shows the average path leading to high firm default (blue line) and twin defaults (red line) regimes.[42] Two facts are noteworthy. First, the model implies that twin default

[42]The figure is generated by simulating the model for 1,000,000 periods, identifying periods in which defaults are above the 90th percentile and then computing the average realizations of shocks and endogenous variables for twenty periods before and after the crisis periods.

Figure 5: Paths to Crises



Notes: This figure shows the average path leading to high firm default (blue dashed line) and twin defaults (red solid line) regimes. The figure is generated by simulating the model for 1,000,000 periods, identifying periods in which defaults are above the 90th percentile and then computing the average realizations of shocks and endogenous variables for twenty periods before and after the crisis periods. We define twin defaults as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. High firm default periods are those where firm default is above the 90th percentile and bank default is below the 90th percentile. TFP, Island Risk and Firm Risk represent the level of A_t, $\frac{\sigma_{\omega_j,t+1}}{\bar{\sigma}_{\omega_j}}$ and $\frac{\sigma_{\omega_i,t+1}}{\bar{\sigma}_{\omega_i}}$ in their respective standard deviation units.

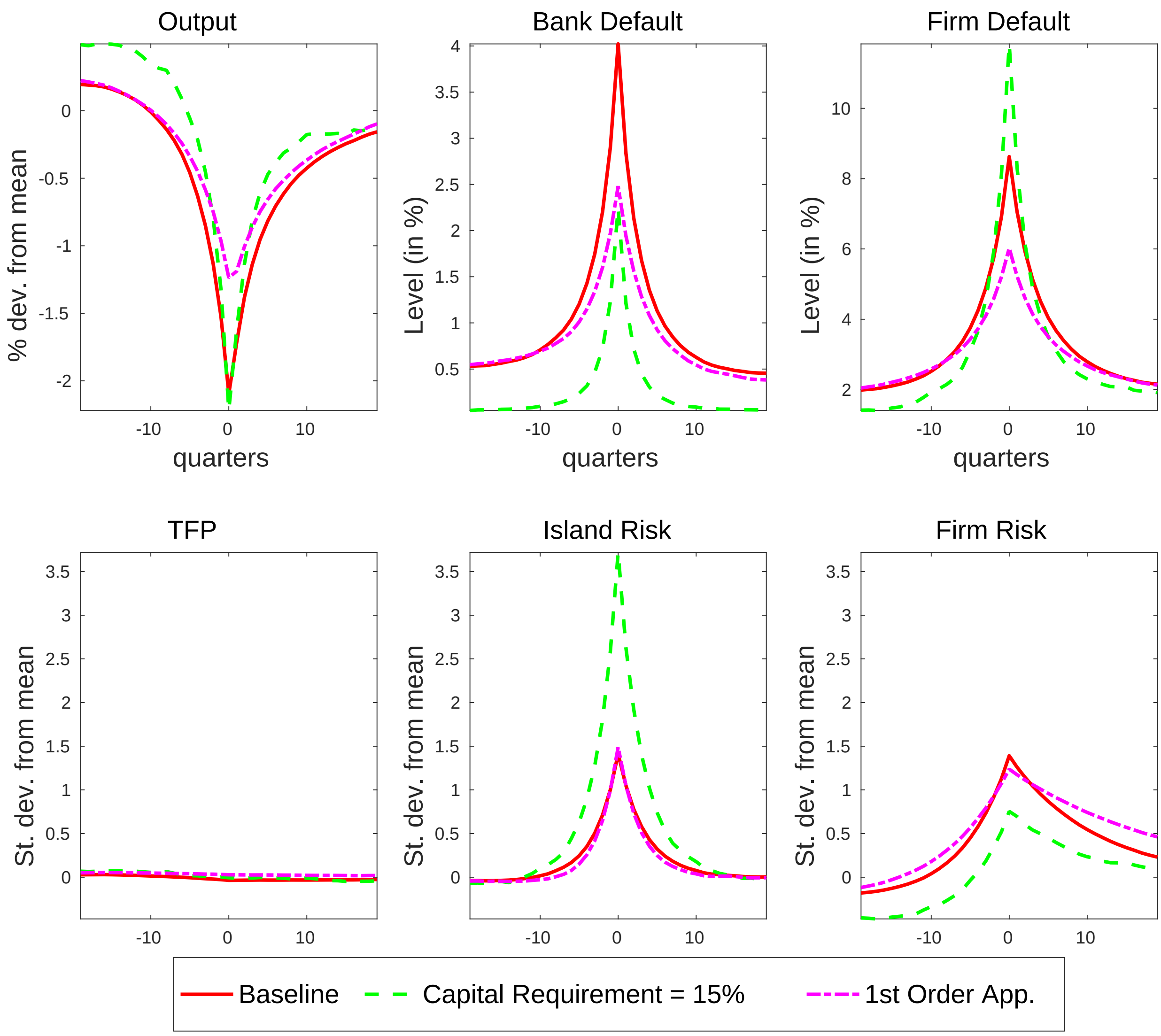
episodes generate output falls that are twice as large compared to high firm default events. Second, the model captures remarkably well the evolution of bank defaults for both regimes. Bank defaults rise above 4 percent during twin defaults, which is very close to what we observe in the EA data during the recent financial crisis. In contrast, bank failures remain below 1 percent in the episodes of high firm defaults. Both cases are very close to the evidence reported in Table 4. The declines in output, investment and lending are more pronounced in the case of twin defaults when compared with high firm defaults.

The model is also consistent with the empirical finding that financial crises tend to be preceded by above average economic activity and lending (e.g. Schularick and Taylor, 2012; Jorda, Schularick and Taylor, 2016). In the twin defaults regime, output, bank lending, bank capital and consumption reach a cyclical peak around 8-10 quarters before the crisis event and fall sharply as it approaches. In our model, the pre-crisis boom is mainly driven by good (below average) realizations of the firm-risk shock. As a result, corporate leverage is elevated, making firms (and hence banks) more vulnerable to subsequent adverse shocks.

The figure also shows that, on average, TFP remains broadly unchanged in both types of episodes. The two risk shocks instead play an important role. The rise in the firm-risk shocks plays a role in both high firm defaults and twin defaults, while the rise in the island-risk shocks plays a key role in generating twin default crises.

Finally, the model does not need very large risk shocks in order to generate a twin default crisis. These episodes occur following a sequence of small and positive risk shocks that accumulate into a 1.5 standard deviation increase. The island-risk shocks are crucial to generate bank defaults and, therefore, twin default crises. Firm-risk shocks by themselves can only create high firm default events. The fact that our baseline model does not need very large risk shocks explains why it can match the frequency of the twin defaults regime in the data. In the next section we will show that increasing bank capital requirements makes banks more resilient to credit losses induced by island-risk shocks and thus reduces the frequency of such twin default crises.

Figure 6: Path to Twin Defaults in Different Scenarios



Notes: This figure shows the average path leading to a twin default episode under different model assumptions. Baseline (red solid line) corresponds to our baseline model with the capital requirement sets to 8 percent ($\phi = 0.08$) and solved with third order perturbation methods. Capital Requirement = 15% (green dashed line) corresponds to the model with capital requirement sets to 15 percent ($\phi = 0.15$). 1st Order App. (pink dashed-dotted line) corresponds to the model solved with a first-order perturbation method. The figure is generated by simulating the model for 1,000,000 periods, identifying periods of twin defaults and then computing the average realizations of shocks and endogenous variables for twenty periods before and after the crisis periods. We define a twin default episode as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. The 90th percentile default thresholds used to define the three regimes in the three models are always the ones determined by the baseline model. TFP, Island Risk and Firm Risk represent the level of A_t, $\frac{\sigma_{\omega_{j,t+1}}}{\bar{\sigma}_{\omega_j}}$ and $\frac{\sigma_{\omega_{i,t+1}}}{\bar{\sigma}_{\omega_i}}$ in their respective standard deviation units.

In addition to matching the data, the microfounded link between the solvency of banks and firms also introduces a powerful amplification mechanism in the model which allows it to generate crises episodes without the need for large exogenous aggregate shocks. The importance of the non-linearities introduced by the structural modelling of bank asset returns is underlined by the fact that, if solved to a first-order approximation, the model only generates twin default crises if hit by implausibly large realizations of the island-risk shock. The dotted-dashed line in Figure 6 shows that, in the first-order approximation, island-risk shocks need to increase by 3 standard deviations rather than only 1.5 standard deviations in the baseline.[43] Nevertheless, despite the large shocks, the first-order approximation cannot generate a realistic increase in the probability of bank default. This is consistent with the fact that, as shown in Table 4, the linear model can only produce twin defaults with a 4 percent probability, while the frequency implied by the baseline model is very close to the 7 percent observed in the data.

The strong non-linear effects of island-risk shocks can also be demonstrated by means of generalized impulse response (GIRFs) functions as in Andreasen, Fernandez-Villaverde and Rubio-Ramirez (2017), which are presented in Internet Appendix D (Section 4.4).[44] The model solved with a third-order approximation is able to amplify island-risk shocks during crisis times more strongly than during normal times. In our model, once the economy finds itself in a situation of high firm default, it becomes very vulnerable to additional island-risk shocks. The economy "accelerates" into a twin default event as the impact of additional island-risk shocks grows. This internal propagation helps the model generate twin default crises without the need for implausible huge shocks.[45]

[43] As was the case in Table 4, the thresholds used to define the three regimes are always the ones determined by the baseline model.

[44] Details on how to compute both conditional and unconditional GIRFs can be found in Internet Appendix C (Section 3.2).

[45] The results in the Internet Appendix D (Section 4.4) clearly show the island-risk shocks has a much larger impact when conditioning on either a twin defaults or a high firm default episode. In contrast, the GIRFs conditional only on high firm default shows much less amplification than when we condition on a twin defaults episode.

6. Implications for Capital Requirements

After documenting the quantitative performance of our model and analyzing how twin defaults arise, this section addresses the main question of the paper. What is the level of capital requirements that optimally trades off a lower frequency of crises with a more limited provision of credit to the economy?

The rationale for capital requirements in this model is related to the presence of safety net guarantees and to externalities associated with the cost of bank failures and the disruption of bank lending during twin default crises. The presence of safety net guarantees modelled in the form of insured deposits, makes the interest rate on deposit funding independent from banks' leverage choices. This provides an incentive for banks to under-price their borrowers' risk. Further, banks operate under limited liability and do not internalize the social cost of their failures and the effects of their choices on the bank equity returns and, hence, on the next period aggregate lending capacity of the whole banking sector.[46] All this provides a clear rational for the macroprudential calibration of bank capital requirements.

6.1 Optimal Capital Requirements

We first assess the implications of different values of the capital requirement on the mean of the ergodic distribution of selected variables for our baseline model.[47] Figure 7 shows that the imposition of higher capital requirements implies a trade-off between reducing the probability of twin default crises and maintaining the supply of bank credit. Higher capital requirements limit banks' risk taking incentives and make bank equity returns better protected against non diversifiable risk. When banks are less levered, non-diversifiable risk has a lower impact on banks' equity and this reduces banks' default, as well as, the correlation between firm

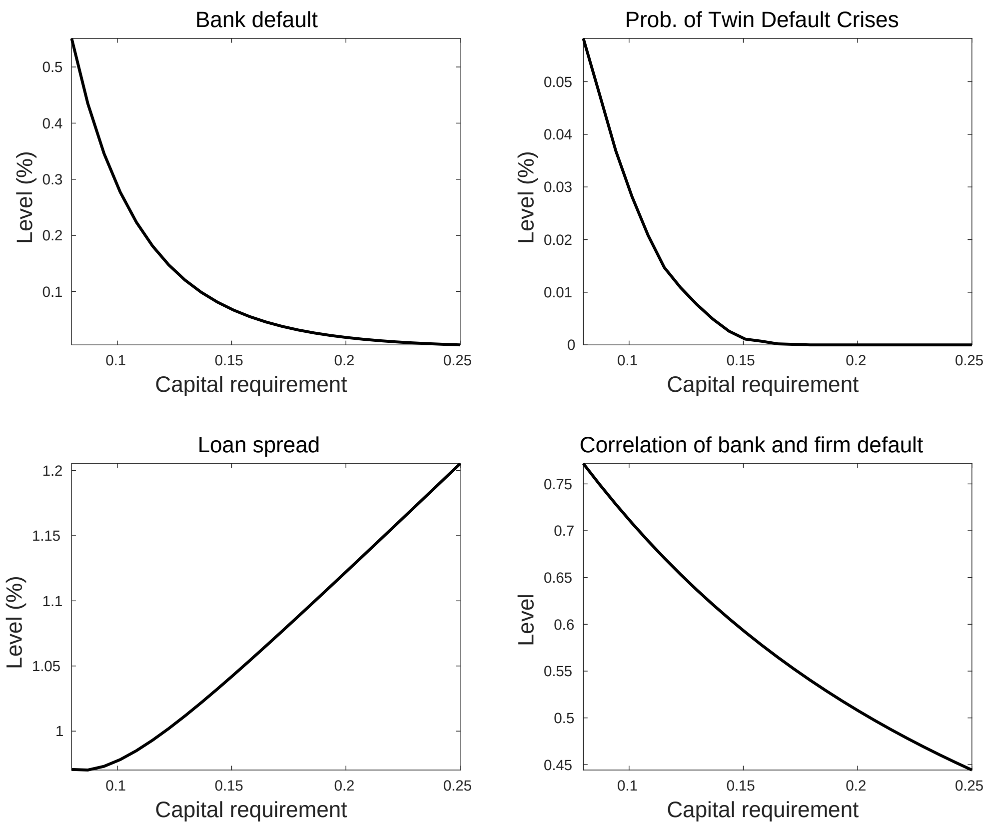
[46] In our model, the market segmentation does apply to the funding of banks. In every period the aggregate net worth of the whole banking sector is invest in a portfolio of equity of the continuum of banks.

[47] We consider changes in the capital requirement ϕ while keeping all other parameters unchanged.

and bank default. As a result, twin default crises become less frequent and deadweight losses associated with the costs of asset repossession decline. However, higher capital requirements are also costly for the economy. They increase the relative scarcity of bank net worth and the average cost of bank funding. This implies higher borrowing costs, reduced bank credit, and lower investment.

The trade-off is reflected in the overall effects of higher capital requirements on social welfare. The solid black line in Figure 8 reports the ergodic mean of household welfare as a function of the level of bank capital requirements. The optimal bank capital requirement is around 15 percent which is associated with welfare gains of approximately 0.1 percent in certainty equivalent consumption terms relative to the baseline model, wich feature bank capital requirement of 8 percent.

Figure 7: Comparative Statics with Respect to Capital Requirement Level

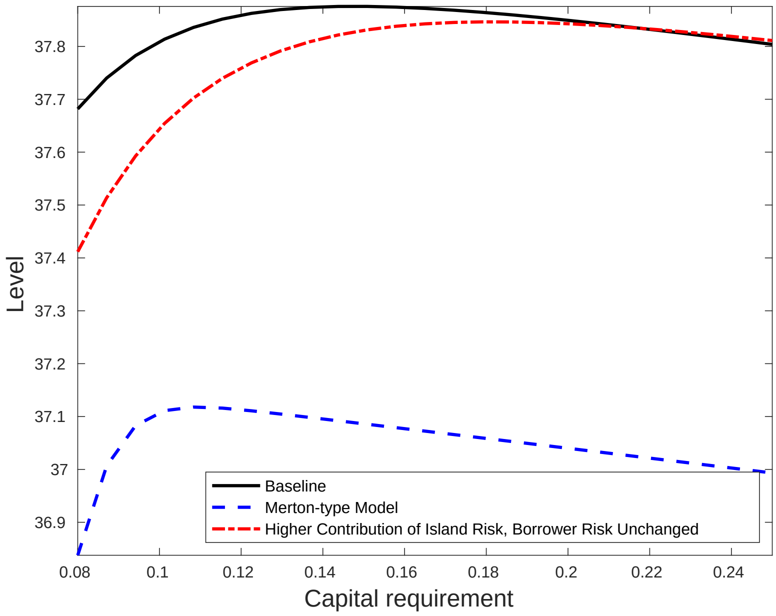


Notes: This figure shows the implications of different values of the capital requirement ϕ on the mean of the ergodic distribution of selected variables for our baseline model.

Starting from the 8 percent capital requirement, welfare first increases because the gains from the reduction in the probability of bank default outweigh the losses from imposing higher funding costs on banks. At the optimum, the probability of bank default is below 0.1 percent and further reductions in bank failures have a limited impact on welfare. For a capital requirement above 15 percent, the negative effect of elevated borrowing costs for firms dominates and welfare declines.

Imposing such a capital requirement is welfare-improving because of the high costs associated with twin default crises. In order to understand the implications of higher capital

Figure 8: Welfare Effects of the Capital Requirement in Different Scenarios



Notes: This figure reports the ergodic mean of household welfare as a function of the level of bank capital requirements in different scenarios. Baseline (black solid line) corresponds to our baseline model. Merton-type model (blue dashed line) corresponds to the model in which the Merton-type specification of bank asset returns is adopted. Higher Contribution of Island Risk, Borrower Risk Unchanged (red dashed-dotted line) corresponds to the model in which we increase the average standard deviation of the island-idiosyncratic shock and reduce the average standard deviation of the firm-idiosyncratic shock while keeping the probability of firm default unchanged.

requirements on the insurgence of twin default crisis, we can turn back to Figure 6 and compare the baseline path to crisis with a bank capital requirements of 8 percent with the path implied by the model under the optimal capital requirement level (Higher Cap. Req).[48]

Figure 6 also clearly shows that the model with a bank capital requirement of 15 percent (dashed line) needs a much larger (3.5 standard deviation) increase in the island-risk shock in order to generate a twin default crisis than the model with a capital requirement of 8 percent (solid line). This is confirmed by Table 4 which describes the performance of the model in terms of untargeted conditional moments in the three default regimes also under the optimal level of capital requirements. The fact that with a 15 percent capital requirement level, the model requires larger shocks to generate twin default crises is consistent with the fact that at the optimal level of capital requirements, the economy experiences a much lower frequency of the twin defaults regime is dramatically lower.

6.2 The role of non-diversifiable Bank Risk

To gain some insights regarding the importance of properly quantifying the impact of borrower default risk on bank insolvencies, we consider two counterfactual experiments. In the first, firm default risk is assumed to be less diversifiable at the bank level than in the calibrated model. Hence, the link between the default of firms and banks is much stronger, implying a much higher probability of twin defaults. This is obtained by increasing the average standard deviation of the island-idiosyncratic shock and reducing the average standard deviation of the firm-idiosyncratic shock while keeping the probability of firm default unchanged.[49] This reduces the extent to which banks can diversify away firm default risk.

[48] As was the case in Table 4, the thresholds used to define the three regimes are always the ones determined by the baseline model.

[49] The firm default rate is the same as in the calibrated model. The only difference between the two versions of the model is the composition of diversifiable and non-diversifiable (firm- vs island-idiosyncratic) firm default risk for banks. The average standard deviation of the island-idiosyncratic shock is increased by 10 percent, whereas the average standard deviation of the firm-idiosyncratic shock is reduced by 6.3 percent. While the average probability of firm default remains equal to 2.25 percent, the probability of bank default

The red dashed line in Figure 8 shows social welfare as a function of the capital requirement level in this counterfactual scenario. As expected, when firm default risk is less diversifiable at the bank level, the optimal capital requirement needs to be substantially higher, i.e. close to 20 percent.

In the second experiment, we assume that all firm default risk is ex-ante diversifiable at the bank level (no island-idiosyncratic risk) and the default risk of banks comes from a an exogenous disturbance that directly hit the banks' loan returns. This is in line with the standard Merton-type model of bank default risk used in the previous literature. Figure 8 reports welfare as a function of the capital requirement also under this alternative specification (blue dashed line). Even though the probability of bank default is the same in both models, under the Merton-type formulation the optimal capital requirement is five percentage points lower, i.e. around 10 percent.

Importantly, we calibrate this Merton-type version of the model so as to ensure that the mean and standard deviation of bank default are the same as in our baseline model. However, since firm default is not the main driver of bank default in such version of the model, the probability of twin defaults drops to 1.1 percent, which is considerably lower than what is observed in the data and produced by our baseline model.

Table 5 reports the difference in key variables in the high vs low bank default episodes in each versions of the model. It clearly shows that in the Merton-type model high bank defaults are not accompanied by firm defaults and the deadweight losses associated with them. Therefore, the overall losses associated with bank default are lower, as reflected in the less sizable drop in economic activity. Hence, the Merton-type model features a much less substantial reduction in welfare (consumption equivalent) in the high bank default episodes relative to the low bank default ones. This explains why the standard model of bank default risk underestimates the welfare gains from increasing capital requirements compared to our

increases from 0.59 percent to 1.03 percent. The probability of twin default crises increases from 5.9 percent to 8.8 percent.

Table 5: High vs Low Bank Default Episodes in the Baseline vs Merton-type Models

	Baseline Model	Merton-type Model
Bank Default increase (pp)	3.011	2.985
Firm Default increase (pp)	4.940	0.317
Welfare drop (%)	-0.325	-0.052
Welfare drop (consumption equivalent terms)	-0.061	-0.009
Output drop (%)	-1.677	-0.512
Consumption drop (%)	-0.594	-0.312
Investment drop (%)	-6.006	-1.347
Bankers' equity drop	-2.061	-0.240
Entrepreneurs' equity drop (%)	-3.536	-0.831

Notes: This table presents the differences in endogenous variables between High and Low Bank Default episodes for the baseline and Merton-type models. High (Low) bank default are episodes with bank default above (below) the 90th percentile. Merton-type Model corresponds to the model in which the Merton-type specification of bank asset returns is adopted. The results are based on 1,000,000 simulations.

baseline model.[50]

Overall, our results show that capturing the special nature of bank asset returns and their implications for bank default risk is essential to provide accurate prescriptions on the optimal level of capital requirements. Indeed, microfounding the relationship between firm and bank defaults is crucial to reproduce the frequency and severity of twin defaults observed in the data, and, thus, properly account for the costs associated with bank insolvencies, and the net benefits of higher capital requirements.

7. Conclusions

The assessment of the benefits and costs of higher capital requirements requires a framework that adequately quantifies the trade off between a lower frequency of bank insolvency crises and a more limited provision of credit to the wider economy. Thus, it crucially hinges upon

[50] Internet Appendix Figure D3 compares the path to bank default episodes in our baseline model and in the Merton-type model. In line with the evidence presented in Table 5, this figure shows that bank default episodes in our baseline model are associated with a large increase in firm default and hence feature more severe output losses compared to the Merton-type model.

the ability of the model to match key features of the data, including the frequency and severity of twin defaults, i.e. episodes characterized by deep recessions and abnormally high default rates among both banks and their borrowers.

With this purpose in mind, we build a quantitative structural general equilibrium model of bank default risk in which bank solvency problems arise endogenously from high default rates among bank borrowers. Our paper represents the first quantitative exploration of the way bank borrowers' default translate into rare but severe episodes of bank insolvencies and the large output losses associated with them.

Microfounding the link between bank and firm solvency allows our framework to capture a very important aspect of bank loan portfolios: they deliver asymmetrically distributed payoffs which feature limited upside potential but significant downside risk due to borrowers defaults. This feature allows our model to reproduce the non-linearities associated with firm and bank defaults and macroeconomic outcomes observed in the data. Thus, our model captures well the behavior of the economy not only in normal times but also in twin defaults.

We show that our model implies higher optimal capital requirements than standard Merton-type models of bank default risk, which neglect or underestimate the impact of borrower default on bank solvency. Thus, our results suggest that a structural approach to bank default risk is crucial for the assessment of the net benefits of higher capital requirements.

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Supplementary Material – Internet Appendix

Twin Defaults and Bank Capital Requirements

Appendix A Data

- Investment: Gross Fixed Capital Formation, Millions of euros, Chain linked volume, Calendar and seasonally adjusted data, Reference year 1995, Source: the Area Wide Model (AWM) dataset.
- Gross Domestic Product (GDP): we define the GDP as the sum of Consumption and Investment.
- Loans: Outstanding amounts of loans at the end of quarter (stock) extended to non-financial corporations by Monetary and Financial Institution (MFIs) in EA, Source: MFI Balance Sheet Items Statistics (BSI Statistics), Monetary and Financial Statistics (S/MFS), European Central Bank.
- Loan Spread: Spread between the composite interest rate on loans and the composite risk free rate. We compute this spread in two steps.
    1. Firstly, we compute the composite loan interest rate as the weighted average of interest rates at each maturity range (up to 1 year, 1-5 years, over 5 years).
    2. Secondly, we compute corresponding composite risk free rates that take into account the maturity breakdown of loans. The maturity-adjusted risk-free rate is the weighted average (with the same weights as in case of composite loan interest rate) of the following risk-free rates chosen for maturity ranges:
        - 3 month EURIBOR (up to 1 year).
        - German Bund 3 year yield (1-5 years).
        - German Bund 10 year yield (over 5 years for commercial loans).
        - German Bund 7 year yield (5-10 years for housing loans).
        - German Bund 20 year yield (over 10 years for housing loans).

    Source: MFI Interest Rate Statistics of the European Central Bank, Bloomberg.
- Return on equity of banks: Bank Equity Return (after tax), EA. Source: Global Financial Development, World Bank.

- Expected default of Banks: Asset weighted average of EDF within one year for the sample of banks in EA. The data comes on the monthly basis. We aggregate it to quarterly series by averaging the monthly series within a quarter. [51] Source: Moody's KMV.

- Expected default of non-financial firms: we compute it using Moody's EDF series for a sample of non-financial corporations in the EA. Since in the Moody's dataset we have an over-representation of large firms and under-representation of small and medium-sized enterprises (SMEs) compared to the loan portfolio of bank in the EA, we proceed in two steps.[52] Firstly, we construct two separate EDF indices: i) for SMEs, ii) for large firms.[53] Secondly, we build an aggregate default series for non-financial firms as a weighted average of EDF indices for SMEs and large firms. As weights we use the share of loans extended by banks in EA to SMEs and large firms respectively.[54] The data comes on the monthly basis. We aggregate it to quarterly series by averaging the monthly series within a quarter.

[51] See detailed EDF description on the Moody's webpage.

[52] We define SMEs as firms with average total assets below €43 m within the sample period in the database (as in the definition of the European Commission)

[53] EDF indices are constructed as asset weighted average of EDF within one year for the sample of non-financial firms within the size category

[54] We obtain the data on the share of SMEs loans in total loans from Financing SMEs and Entrepreneurs database of OECD.

Appendix B Model Details

2.1 First Order Conditions

Household. The household's problem yields the following FOCs with respect to consumption,

$$U_{C_t} = \lambda_t, \tag{27}$$

labor supply,

$$-U_{H_t} = w_t \lambda_t, \tag{28}$$

and demand for the portfolio of insured deposits,

$$1 = \mathbb{E}_t \left(\Lambda_{t+1} \right) R_{d,t}, \tag{29}$$

where λ_t is the Lagrange multiplier of the budget constraint and $\Lambda_{t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t}$ is the household's stochastic discount factor.

Entrepreneurial firm. The entrepreneurial firm's problem yields the following FOCs with respect to capital,

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial k_t} + \zeta_{f,t} q_t - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial k_t} = 0, \tag{30}$$

labor demand,

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial h_t} + \zeta_{f,t} w_t - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial h_t} = 0, \tag{31}$$

loans,

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial B_{f,t}} - \zeta_{f,t} - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial B_{f,t}} + \xi_{f,t} \nu_{b,t} \phi = 0, \tag{32}$$

and the gross loan rate,

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial R_{f,t}} - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial R_{f,t}} = 0, \tag{33}$$

where $\zeta_{f,t}$ is the Lagrange multiplier of the entrepreneurial firm's budget constraint, and $\xi_{f,t}$ is the Lagrange multiplier of the bankers' participation constraint.

Capital Producer. The FOC of the capital producer problem is

$$q_t = \left[S' \left(\frac{I_t}{K_{t-1}} \right) \right]^{-1}. \tag{34}$$

2.2 Model Aggregation and Market Clearing

In this subsection we describe model aggregation and market clearing conditions.

Final good The clearing of the market for final good requires

$$Y_t = y_t, \tag{35}$$

where aggregate output Y_t equals household consumption, C_t, plus the investment in the production of new capital, I_t, plus the resources absorbed by the costs of repossessing assets from defaulting entrepreneurial firms and banks

$$Y_t = C_t + I_t + \Sigma_{b,t} + \Sigma_{e,t}, \tag{36}$$

where

$$\Sigma_{b,t} = \mu_j \int_0^{\bar{\omega}_{j,t}} \tilde{R}_{f,t}(\omega_j) B_{f,t-1} dF_{j,t}(\omega_j) \text{ and}$$

$$\Sigma_{e,t} = \mu_f \int_0^{\infty} \int_0^{\bar{\omega}_t(\omega_j)} \omega_i \omega_j \left[q_t (1-\delta) K_{t-1} + Y_t \right] dF_{i,t}(\omega_i) dF_{j,t}(\omega_j).$$

Labor The clearing of the labor market requires

$$H_t = h_t. \tag{37}$$

Physical capital The clearing of the market for physical capital requires

$$K_t = k_t. \tag{38}$$

Equity The clearing of the market for equity requires

$$EQ_{e,t} = \int_0^{\infty} eq_{e,t}(i) di \text{ and} \tag{39}$$

$$EQ_{b,t} = \int_0^{\infty} eq_{b,t}(j) dj. \tag{40}$$

Loans The clearing of the market for loans, requires

$$B_{f,t} = b_{f,t}. \tag{41}$$

Bank deposits The clearing of the market for bank deposits, requires

$$D_t = d_{f,t}. \tag{42}$$

Law of motion of capital Finally, the law of motion of capital is given by

$$K_t = (1-\delta) K_{t-1} + S\left(\frac{I_t}{K_{t-1}}\right) K_{t-1}. \tag{43}$$

Law of motion of entrepreneurs'aggregate net worth Let $N_{e,t}$ be the aggregate net wealth of entrepreneurs at period t. Then

$$N_{e,t} = \theta_e \rho_{e,t} N_{e,t-1} + \iota_{e,t}, \tag{44}$$

which reflects the retention of net worth by the fraction θ_e of non-retiring entrepreneurs, the aggregate endowment $\iota_{e,t}$ added by the entering entrepreneurs, and the fact that aggregate net wealth equals individual net wealth

$$N_{e,t} = \int_0^\infty n_{e,t}(i) di. \tag{45}$$

Law of motion of bankers aggregate net worth Let $N_{b,t}$ be the aggregate net wealth of bankers at period t. Then

$$N_{b,t} = \theta_b \rho_{b,t} N_{b,t-1} + \iota_{b,t} - T_t, \tag{46}$$

which reflects the retention of the aggregate net worth by the fraction θ_b of non-retiring bankers, the aggregate endowment $\iota_{b,t+1}$ received by the entering bankers, and the fact that aggregate net wealth equals individual net wealth

$$\int_0^\infty N_{b,t} = n_{b,t}(j) dj. \tag{47}$$

Endowments of entering entrepreneurs and bankers We model the aggregate endowment of entering entrepreneurs as a proportion χ_e of the aggregate net worth of the retiring entrepreneurs

$$\iota_{e,t} = \chi_e (1-\theta_e) \rho_{e,t} N_{e,t-1}. \tag{48}$$

Akin to the case of entrepreneurs, we model the aggregate endowment of entering bankers as a proportion χ_b of the aggregate net worth of the retiring bankers

$$\iota_{b,t} = \chi_b (1-\theta_b) \rho_{b,t} N_{b,t-1}. \tag{49}$$

Net transfers from entrepreneurs and bankers to the household Let the net transfers received by the household from entrepreneurs and bankers at period t be $\Upsilon_{e,t}$ and $\Upsilon_{b,t}$ respectively. Then, we have

$$\Upsilon_{e,t} = (1 - \theta_e)\, \rho_{e,t} N_{e,t-1} - \iota_{e,t}, \tag{50}$$

which reflects the the aggregate worth of the fraction $1 - \theta_e$ of retiring entrepreneurs minus the aggregate endowment $\iota_{e,t}$ added by the entering entrepreneurs. Equivalently, we also have that

$$\Upsilon_{b,t} = (1 - \theta_b)\, \rho_{b,t} N_{b,t-1} - \iota_{b,t}, \tag{51}$$

which reflects the the aggregate worth of the fraction $1 - \theta_e$ of retiring bankers minus the aggregate endowment $\iota_{b,t}$ added by the entering bankers. Thus, we have that the sum of net transfers from entrepreneurs and bankers to the household is

$$\Upsilon_t = (1 - \theta_e)\, \rho_{e,t} N_{e,t-1} - \iota_{e,t} + (1 - \theta_b)\, \rho_{b,t} N_{b,t-1} - \iota_{b,t}. \tag{52}$$

Profits from capital production Profits received by households from capital producing firms are

$$\Xi_t = q_t S\left(\frac{I_t}{K_{t-1}}\right) K_{t-1} - I_t. \tag{53}$$

2.3 Model Equilibrium Conditions

We provide the equilibrium conditions for our model. We begin with the equilibrium conditions related to the households, then entrepreneurs and entrepreneurial firms, then bankers and banks, then the capital production sector, and finally the market clearing conditions.

Household Using Equations (27) and (28) we obtain

$$-\frac{U_{H_t}}{U_{C_t}} = w_t, \tag{54}$$

Equation (29) is part of the equilibrium conditions. Hence, we have

$$1 = \mathbb{E}_t \Lambda_{t+1} R_{d,t}. \tag{55}$$

Entrepreneurs Equations (5) and stochastic discount factor of entrepreneurs give us

$$\nu_{e,t} = \mathbb{E}_t \Lambda_{e,t+1} \rho_{e,t+1}. \tag{56}$$

The elements of the law of motion of entrepreneurs'net worth reflected in Equations (44) and (48) are also part of the equilibrium. Hence, we have

$$N_{e,t+1} = \theta_e \rho_{e,t+1} N_{e,t} + \iota_{e,t+1} \text{ and} \tag{57}$$

$$\iota_{e,t+1} = \chi_e (1-\theta_e) \rho_{e,t+1} N_{e,t}. \tag{58}$$

Entrepreneurial Firm Equations from the entrepreneurial firms' problem are also part of the equilibrium conditions. Hence, we have

$$Y_{t+1} = A_{t+1} K_t^{\alpha} \left(H_t\right)^{(1-\alpha)}, \tag{59}$$

$$\Pi_{i,j,t+1}\left(\omega_i, \omega_j\right) = \omega_i \omega_j \left(q_{t+1}\left(1-\delta\right) K_t + Y_{t+1}\right) - R_{f,t} B_{f,t}, \tag{60}$$

$$\bar{\omega}_{t+1}\left(\omega_j\right) = \frac{R_{f,t} B_{f,t}}{\omega_j \left(q_{t+1}\left(1-\delta\right) K_t + Y_{t+1}\right)}, \tag{61}$$

$$\Pi_{f,t+1} = \int_0^{\infty} \int_{\bar{\omega}_{t+1}(\omega_j)}^{\infty} \Pi_{i,j,t+1}\left(\omega_i, \omega_j\right) dF_{i,t+1}\left(\omega_i\right) dF_{j,t+1}\left(\omega_j\right), \text{ and} \tag{62}$$

$$B_{f,t} + N_{e,t} = w_t H_t + q_t K_t, \tag{63}$$

$$\mathbb{E}_t \Lambda_{b,t+1} \Pi_{b,t+1} = \nu_{b,t} \phi B_{f,t}, \tag{64}$$

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial K_t} + \zeta_{f,t} q_t - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial K_t} = 0, \tag{65}$$

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial H_t} + \zeta_{f,t} w_t - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial H_t} = 0, \tag{66}$$

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial B_{f,t}} - \zeta_{f,t} - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial B_{f,t}} + \xi_{f,t} \nu_{b,t} \phi = 0, \tag{67}$$

$$\mathbb{E}_t \Lambda_{e,t+1} \frac{\partial \Pi_{f,t+1}}{\partial R_{f,t}} - \xi_{f,t} \mathbb{E}_t \Lambda_{b,t+1} \frac{\partial \Pi_{b,t+1}}{\partial R_{f,t}} = 0, \tag{68}$$

$$\rho_{e,t+1} = \frac{\Pi_{f,t+1}}{N_{e,t}}, \tag{69}$$

where we have also used the clearing of the market for the final good, labor, physical capital, and the entrepreneurial firms' equity and the fact that aggregate net wealth equals individual net wealth, i.e. Equations (35), (37), (38), (39), and (45) and the balance sheet of the entrepreneurs, i.e. Equation (3), together with $dv_{e,t} = 0$.[55]

[55]To simplify notation, we say that the derivative $\frac{\partial f}{\partial X}$ is evaluated at X, while the derivative $\frac{\partial f}{\partial x}$ is evaluated at x.

Bankers Equation (5) with $\varkappa = b$ gives us

$$\nu_{b,t} = \mathbb{E}_t \Lambda_{b,t+1} \rho_{b,t+1}. \tag{70}$$

The laws of motion of bankers net worth reflected in Equations (46) and (49) also part of the equilibrium. Hence, we have

$$N_{b,t+1} = \theta_b \rho_{b,t+1} N_{b,t} + \iota_{b,t+1} - T_t \text{ and} \tag{71}$$

$$\iota_{b,t+1} = \chi_b (1 - \theta_b) \rho_{b,t+1} N_{b,t}. \tag{72}$$

Banks Equations (11), (12), (14), (16), (17), and (18) from the banks' problem are also part of the equilibrium conditions. Hence, we have

$$B_{f,t} = N_{b,t} + D_t, \tag{73}$$

$$N_{b,t} = \phi B_{f,t}, \tag{74}$$

$$\tilde{R}_{f,t+1}(\omega_j) = \left[\Gamma_{i,t+1}(\bar{\omega}_{t+1}(\omega_j)) - \mu_f G_{i,t+1}(\bar{\omega}_{t+1}(\omega_j))\right] \frac{\omega_j \left[q_{t+1}(1-\delta) K_t + Y_{t+1}\right]}{B_{f,t}}, \tag{75}$$

$$\tilde{R}_{f,t+1}(\bar{\omega}_{j,t+1}) B_{f,t} - R_{d,t} D_t = 0, \tag{76}$$

$$\Pi_{b,t+1} = \int_{\bar{\omega}_{j,t+1}}^{\infty} \tilde{R}_{f,t+1}(\omega_j) B_{f,t} dF_{j,t+1}(\omega_j) - R_{d,t} D_t \left(1 - F_{j,t+1}(\bar{\omega}_{j,t+1})\right), \text{ and} \tag{77}$$

$$\rho_{b,t+1} = \frac{\Pi_{b,t+1}}{\phi B_{f,t}}, \tag{78}$$

where we have also used the clearing of the market for the final good, physical capital, and banks' equity and and the fact that aggregate net wealth equals individual net wealth, i.e. Equations (35), (38), (40), and (47) and the balance sheet of the bankers, i.e. Equation (3), together with $dv_{b,t} = 0$.

Capital production The evolution of capital is controlled by the FOC of the capital producer and the law of motion of capital, i.e. Equations (34) and (43)

$$q_t = \left[S'\left(\frac{I_t}{K_{t-1}}\right)\right]^{-1} \text{ and} \tag{79}$$

$$K_t = (1-\delta) K_{t-1} + S\left(\frac{I_t}{K_{t-1}}\right) K_{t-1}. \tag{80}$$

Deposit insurance costs By using $D_t = d_{f,t}$ we can write the above expression as in Equation (20)

$$T_t = \Omega_t D_{t-1}. \tag{81}$$

Market clearing The aggregate resource constraint Equation (36) can be written as,

$$Y_t = C_t + I_t + \Sigma_{b,t} + \Sigma_{e,t}, \tag{82}$$

where we have also used the clearing of the market for the final good, labor, and physical capital, i.e. Equations (35), (37), and (38).

Appendix C Methodological Details

3.1 Approximating Banks' Expected Profits

In order to use perturbation methods to approximate the solution to the model we need to compute bank's expected return on the loan portfolio (conditional on not defaulting), defined here as $R_{p,t+1}$, which is part of Equation (17) and is given by the integral defined in Equation (23).

We take $q_{t+1}, k_t, y_{t+1}, b_{f,t}, R_{d,t}, d_t$ as given and use the notation of $\tilde{R}_{f,t+1}$ to be the function of island shock, ω_j, only. From the analysis in Section 4., it should be clear that the bank's loan return $\tilde{R}_{f,t+1}(\omega_j)$ is not log-normally distributed. Mathematically, this is due to the fact that $\Gamma_{i,t+1}(\bar{\omega}_{t+1}(\omega_j))$ and $G_{i,t+1}(\bar{\omega}_{t+1}(\omega_j))$ which enter into $\tilde{R}_{f,t+1}(\omega_j)$ are both non-linear functions of ω_j. As a result of highly non-linear shape of $\tilde{R}_{f,t+1}(\omega_j)$, the integral in Equation (23) cannot be computed as explicit function of the state variables and perturbation methods cannot be applied. We overcome this challenge by (i) splitting this integral into the sum of integrals taken over smaller intervals, (ii) computing a series of quadratic Taylor approximations of $\tilde{R}_{f,t+1}(\omega_j)$ around a mid-point of each interval.

Formally, we split the domain of ω_j into N intervals of equal length defined on $N+1$ points x_k ranging from $x_1 = \bar{\omega}_{j,t+1}$ to $x_{N+1} = \omega_j^{\max}$ where the highest point $\omega_j^{\max}$ is chosen such that $\tilde{R}_{f,t+1}\left(\omega_j^{\max}\right) = R_{f,t}$ almost surely. Given those assumptions, $R_{p,t+1}$ is approximately given by:

$$R_{p,t+1} \approx \sum_{k=1}^{N} \left(\int_{x_k}^{x_{k+1}} \Theta^k(\omega_j)\, dF_{j,t+1}(\omega_j) \right) + \left[1 - F_{j,t+1}(x_{N+1})\right] R_{f,t} \tag{83}$$

where $\Theta^k(\omega_j)$ is a Taylor approximation of $\tilde{R}_{f,t+1}(\omega_j)$ around a point $\omega_j = \bar{x}_k \equiv \frac{x_{k+1}+x_k}{2}$ and is given by

$$\Theta^k(\omega_j) = \tilde{R}_{f,t+1}(\bar{x}_k) + \tilde{R}'_{f,t+1}(\bar{x}_k)(\omega_j - \bar{x}_k) + \frac{1}{2}\tilde{R}''_{f,t+1}(\bar{x}_k)(\omega_j - \bar{x}_k)^2 \tag{84}$$

All the derivatives of $\tilde{R}_{f,t+1}$ are with respect to ω_j and can be computed as an explicit functions of the state variables. Using the simplified expression for $\Theta^k(\omega_j)$ we can rewrite $\int_{x_k}^{x_{k+1}} \Theta^k(\omega_j)\, dF_{j,t+1}$ as follows:

$$\int_{x_k}^{x_{k+1}} \Theta^k(\omega_j)\, dF_{j,t+1} = Q_0(\bar{x}_k) + Q_1(\bar{x}_k) \int_{x_k}^{x_{k+1}} \omega_j dF_{j,t+1} + Q_2(\bar{x}_k) \int_{x_k}^{x_{k+1}} \omega_j^2 dF_{j,t+1} \tag{85}$$

where: $Q_i\left(\bar{x}_k\right)$ are just constants given by:

$$Q_0\left(\bar{x}_k\right)=\left[F_{j,t+1}\left(x_{k+1}\right)-F_{j,t+1}\left(x_k\right)\right]\left[\tilde{R}_{f,t+1}\left(\bar{x}_k\right)-\bar{x}_k\tilde{R}'_{f,t+1}\left(\bar{x}_k\right)+\frac{1}{2}\bar{x}_k^2\tilde{R}''_{f,t+1}\left(\bar{x}_k\right)\right],$$
$$Q_1\left(\bar{x}_k\right)=\left[F_{j,t+1}\left(x_{k+1}\right)-F_{j,t+1}\left(x_k\right)\right]\left[\tilde{R}'_{f,t+1}\left(\bar{x}_k\right)-\frac{1}{2}\bar{x}_k\tilde{R}''_{f,t+1}\left(\bar{x}_k\right)\right],$$
$$Q_0\left(\bar{x}_k\right)=\left[F_{j,t+1}\left(x_{k+1}\right)-F_{j,t+1}\left(x_k\right)\right]\left[\frac{1}{2}\tilde{R}''_{f,t+1}\left(\bar{x}_k\right)\right],$$

Given our assumption of log-normally distributed island shock, ω_j, we have expressions for $\int_{x_k}^{x_{k+1}}\omega_j dF_{j,t+1}$ and $\int_{x_k}^{x_{k+1}}\omega_j^2 dF_{j,t+1}$ as explicit functions of the state variables. Consequently, we can easily derive very accurate, the approximation of $R_{p,t+1}$ in Equation (83) as an explicit function of the state variables.

3.2 IRFs

Following Koop, Pesaran, and Potter (1996), the GIRF for any variable in the model var in period $t+l$ following a disturbance to the n^{th} shock of size ν_n in period $t+1$ is defined as

$$GIRF_{var}(l,\epsilon_{n,t+1}=\nu,\mathbf{w_t})=\mathbb{E}\left[var_{t+l}|\mathbf{w_t},\epsilon_{n,t+1}=\nu\right]-\mathbb{E}\left[var_{t+l}|\mathbf{w_t}\right], \tag{86}$$

where $\precapprox_t$ are the value of the state variables of the model at time t (The state variables of the model are $\mathbf{w_t}=\left(D_t,K_t,H_t,N_{e,t},N_{b,t},q_t,w_t,R_{f,t},R_{d,t},A_{t-1},\sigma_{\omega_j,t-1},\sigma_{\omega_i,t-1}\right)$) and $n\in\{A,\delta,i,j\}$. Hence, the GIRF depend on the value of the state variables when the shocks hits. For example,

$$GIRF_{\Delta\log Y_t}(4,\epsilon_{A,t+1}=-3,(1.1D_{ss},0.9K_{ss},\ldots,1.01\bar{\sigma}_{\omega_i}))$$

is the GIRF of GDP growth, $\Delta\log Y_t$, at period $t+4$, after a TFP shock of value -3 in period $t+1$, when D_t was 10 percent above the steady state, K_t was 10 percent below the steady state, $\ldots$, and $\sigma_{\omega_i,t}$ was one percent above steady state.

But GIRF defined in Equation (86) are conditioned on the value of the state variables when the shocks hits. In what follows, instead we want to compute GIRFs that are conditioned on the values of observables when the shocks hits. For example, we would condition on the expected default rate of firms, $ED_{f,t}$, to be above one percent at the time of the shock. In this case, we want to compute the following GIRF

$$GIRF_{var}(l,\epsilon_{n,t+1}=\nu,ED_{f,t}>0.01)=\int\mathbf{1}_{\{ED_{f,t}>0.01\}}\left(\mathbf{w_t}\right)GIRF_{var}(l,\epsilon_{n,t+1}=\nu,\mathbf{w_t})f\left(\mathbf{w_t}\right)d\mathbf{w_t}, \tag{87}$$

where $\mathbf{1}_{\{ED_{f,t}>0.01\}}\left(\mathbf{w_t}\right)$ takes a value equal to one if the state variables at time t are such that he expected default rate of firms is above one percent at time t and zero otherwise and where $f(\mathbf{w_t})$ is the unconditional density of the state variables. Of course Equation (87) needs to be computed by simulation.

Appendix D Additional Results

Our model with its endogenous connection between firm and bank solvency features a number of idiosyncratic and aggregate risk shocks that are important for the transmission of firm defaults to bank defaults and to the macroeconomy at large. In this section we investigate the importance of each of these shocks. We do this by removing them on an individual basis and then examining the extent to which this deteriorates the model's performance in replicating the quantile regressions in Section 3.1. We also show the importance of solving the model non-linearly by reporting the results that one would obtain by solving it using a first-order (instead of a third-order) approximation. Further, we also documents non-linearities in the transmission of shocks in the model using generalized impulse response functions (GIRFs).

4.1 Importance of the island-idiosyncratic and island-risk shocks

We start with the island-idiosyncratic and island-risk shocks. In our framework banks default when they experience abnormally low realizations of the island-idiosyncratic shock. Our model also allows aggregate fluctuations in the non-diversifiable (island) risk by means of island-risk shocks, i.e., shocks to the dispersion of the island-idiosyncratic risk. These shocks increase the probability of very low realizations of the island-idiosyncratic shocks, making banks more vulnerable.

The results of eliminating island-idiosyncratic and island-risk shocks are shown in the top panels of Figure D1. The figure presents the quantile regression coefficients for Equations (24) and (25) for the model without island-idiosyncratic shocks (green line), i.e., when the island-idiosyncratic shock is set to one, and without the island-risk shocks (blue line). The red and black lines correspond to our baseline model and the data, respectively.[56]

The figure shows that both island-idiosyncratic and island-risk shocks are vital in generating a realistic sensitivity of bank default to firm default and of GDP growth to bank defaults. In the model without island-idiosyncratic shocks, the quantile regression coefficients go to zero because banks are perfectly diversified and their loan portfolio returns are very stable. Firms continue to default because of the firm-idiosyncratic shocks but banks are diversified against these shocks. And while aggregate shocks induce some fluctuations in firm default, these are too small to make banks fail, since our banks' solvency is protected by their equity buffers. Thus, if the bank is fully diversified, bank defaults do not happen and cannot possibly affect GDP growth. The model without island-risk shocks shows that, although eliminating this shock does not lead to fully diversified banks, keeping the non-diversifiable risk (and hence the probability of bank default) low and relatively constant reduces the model's capability of the model to match the sensitivity of bank default to firm default and of GDP growth to bank default. Clearly, the model without island-risk shocks, although it

[56]Note that when we eliminate the island-idiosyncratic shocks, the island-risk shocks became irrelevant.

does a better job than the model without island-idiosyncratic shocks, fails to generate the state-dependent relationship between firm and bank defaults and economic activity that we see in the data.

This experiment clearly indicates the importance of both island-idiosyncratic and island-risk shocks in generating realistic conditional and unconditional correlation patterns between firm and bank defaults and economic activity. When the non-diversifiable risk is constant (no island-risk shocks), bank defaults are rare, they are mostly unaffected by firm defaults, and they do not affect real economic activity. When non-diversifiable risk is absent (no island-idiosyncratic shocks), banks do not default.

4.2 Importance of the firm-idiosyncratic and firm-risk shocks

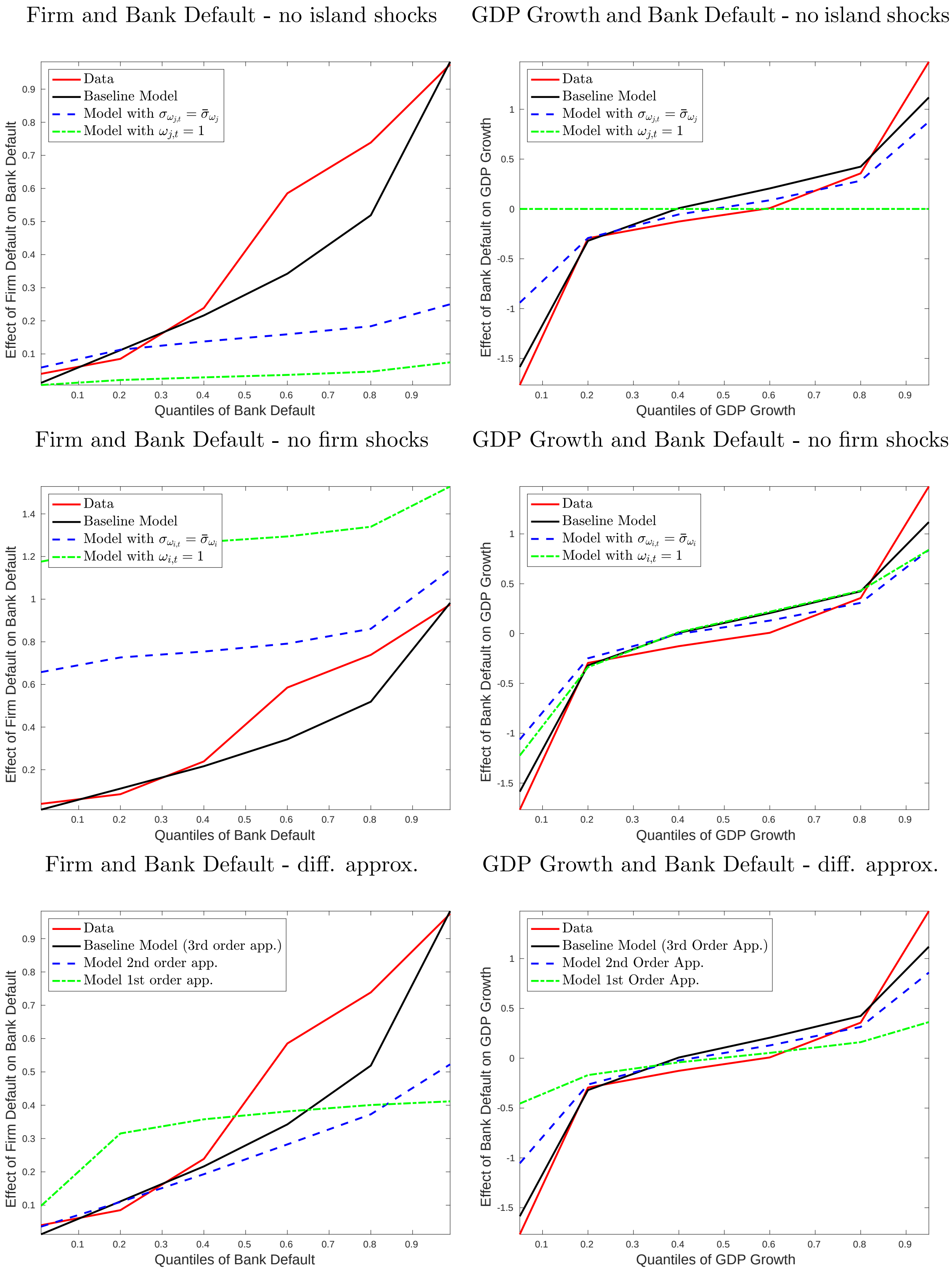
The other source of risk to firms in our model comes from firm-idiosyncratic and firm-risk shocks, i.e. shocks to the dispersion of the firm-idiosyncratic risk. These shocks capture risks to individual firms that are diversifiable at the individual bank level. The firm-risk shocks increase firm defaults but they affect different banks evenly rather than concentrating the bulk of losses on a few unlucky banks, as is the case for the island-risk shocks.

In this section we investigate how the model's ability to replicate the quantile regression coefficients for Equations (24) and (25) changes when we eliminate the firm-idiosyncratic and -risk shocks. The middle panels of Figure D1 show the results. This time the green line displays the quantile regression coefficients in the model where we set the firm-idiosyncratic shock equal to unity for all firms, while the blue line presents the results from the model where firm-risk shocks are shut down.

Both the green and blue lines display a relationship between firm and bank defaults. Intuitively, the green lines in the middle panels of Figure D1 correspond to an economy with fully non-diversified banks in which the defaults of banks and firms are almost perfectly correlated. This makes the sensitivity of bank default to firm default very large and rather constant over states. The impact of shutting down the firm-risk shocks is qualitatively similar to the elimination of the firm-idiosyncratic shocks but not as quantitatively large with respect to the quantile regression coefficients for Equation (24). The right middle panel shows that eliminating either firm-idiosyncratic or firm-risk shocks generates a state-dependent relationship between bank defaults and economic activity that is too weak compared both with the data and the implications of our baseline model.

This experiment clearly indicates the importance of both firm and island shocks in generating realistic conditional and unconditional correlation patterns between firm and bank defaults and economic activity. When we eliminate non-diversifiable risk (no island shocks), the conditional and unconditional correlation between firm and bank default is too small. Instead, when we eliminate diversifiable risk (no firm shocks), the conditional and unconditional correlation between firm and bank default is too large. In both instances the conditional and unconditional correlation between bank default and economic activity is too low.

Figure D1: Quantile Regressions: Key Model Features



Notes: The figure explores the importance of non-diversifiable risk (top panels), diversifiable risk (middle panels) and approximation order (bottom panels). The left column presents coefficients ζ_τ from the quantile regression in Equation (24), while the right column presents coefficients β_τ from the quantile regression in Equation (25).

4.3 Importance of the higher approximation order

Finally, we investigate the role of our solution method by comparing the quantile regressions implied by our baseline model (which is solved using third-order approximation) with the quantile regressions implied by first-order (green lines) or second-order (blue lines) approximate solutions.[57] The bottom panels of Figure D1 shows the results.

Both the linear and the second-order model clearly fail to match the non-linarities found in the data. They generate flat quantile regression coefficients in both panels. Intuitively, a model solved to first or second order works well in normal times but fails to generate the sharp and non-linear deterioration of economic and financial conditions in crises or recessions. In contrast, a third-order approximation captures the non-linearity in the co-movements of firm and bank defaults and economic activity.

We have already discussed in Section 4. that modelling bank portfolios as consisting of defaultable loans introduces an important non-linearity into bank asset returns and hence into bank default realizations. It is therefore natural that a non-linear solution method is needed to capture such non-linearities in an accurate manner. Our results show that a third-order solution is sufficient for this purpose.

4.4 Generalized Impulse Response Functions to an Island-risk Shock

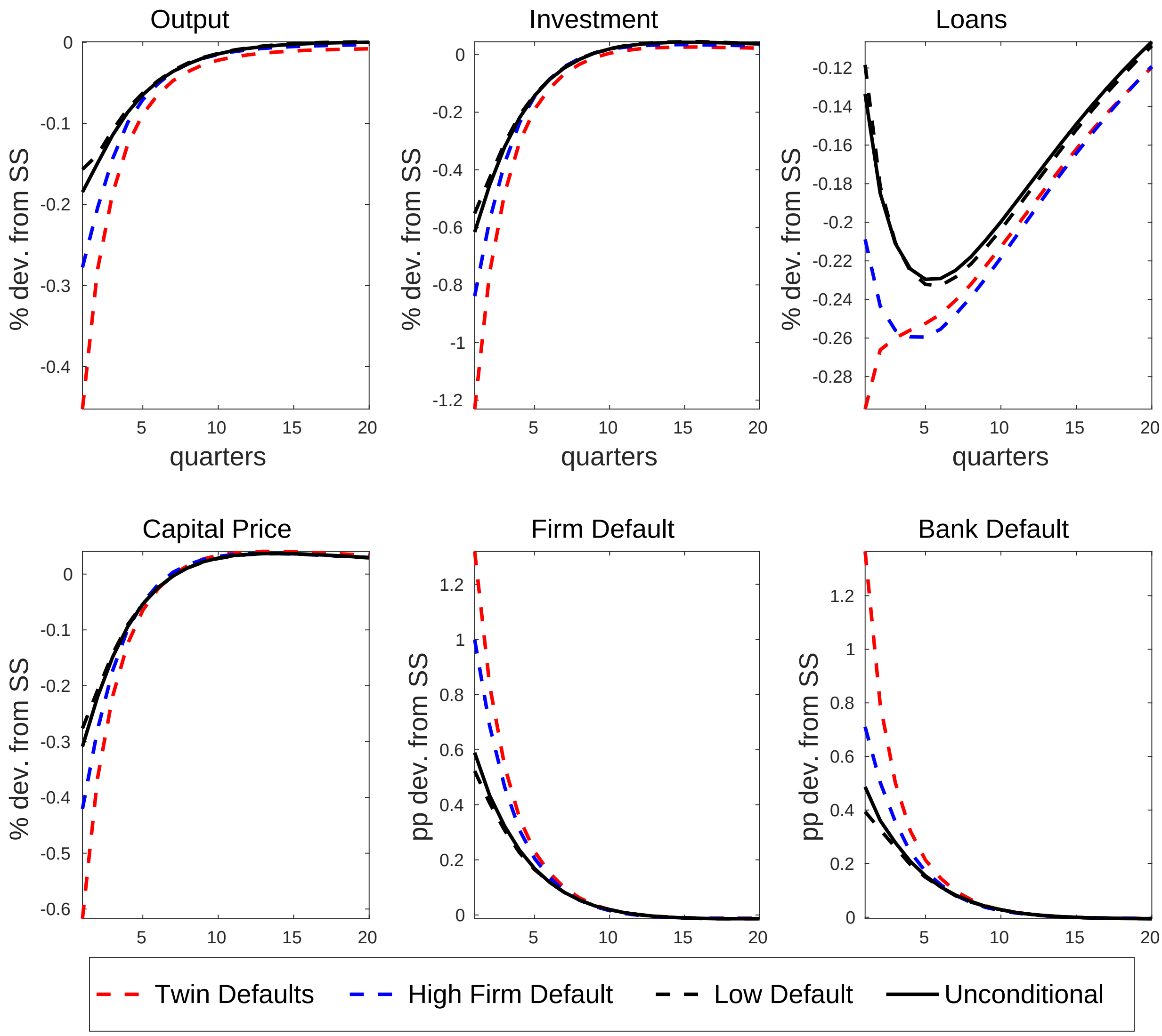
We now use the Generalized Impulse Response Functions (GIRFs) to show that the economy "accelerates" into a twin default event as the impact of additional island-risk shocks grows. This internal propagation helps the model generate twin default crises without the need for huge shocks. Figure D2 reports three sets of GIRFs to a one standard deviation island-risk shock. The solid line shows the unconditional GIRF, the blue dashed line shows the GIRF conditional on the economy being at a high firm default episode, and the red dashed line shows the GIRF conditional on the economy being in a twin default episode. Details on how to compute both conditional and unconditional GIRFs can be found in Appendix 3.2.

The shock has a much larger impact when conditioning on either a twin defaults or a high firm default episode.[58] The GDP drop is much larger than the effect in the unconditional GIRF. The same is true for the drop in investment and the price of capital and for the impact on firm and bank defaults. This shows how the model solved with a third-order approximation is able to amplify island-risk shocks during crisis times differently than during normal times. In our model, once the economy finds itself in a situation of high firm default, it becomes very vulnerable to island-risk shocks. The GIRFs conditional only on high firm default shows much less amplification than when we condition on a twin defaults episode.

[57] We estimate the parameters of the first- and second-order approximation versions of our model to match the set of moments presented in Table 2.

[58] It is important to note that the traditional linear IRFs are independent of the state of the economy.

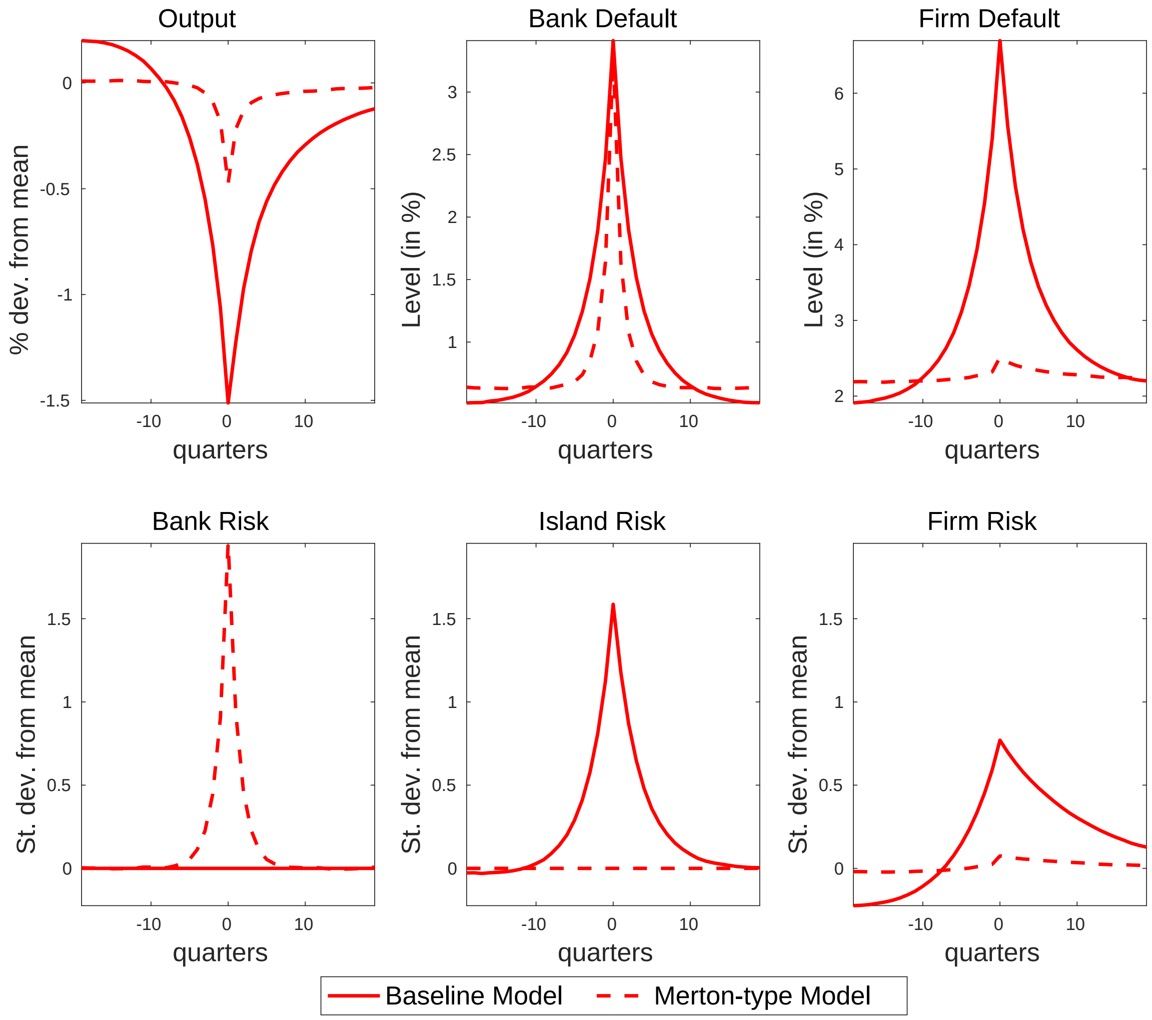
Figure D2: Conditional Impulse Response Functions: Island Risk Shock



Notes: This figure reports three sets of generalized impulse response functions (GIRFs) to a one standard deviation island-risk shock. The solid black line shows the unconditional GIRF, the black dashed line shows the GIRF conditional on the economy being in a low default episode, the blue dashed line shows the GIRF conditional on the economy being at a high firm default episode, and the red dashed line shows the GIRF conditional on the economy being in a twin defaults episode. We define a twin default s episode as the simultaneous occurrence of firm and bank default above their respective 90th percentiles. High firm default episodes are those where firm default is above the 90th percentile and bank default is below the 90th percentile. In the low default regime, both bank and firm default are below the 90th percentile. Details on how to compute both conditional and unconditional GIRFs can be found in Appendix 3.2.

4.5 Path to Twin Default and Bank Default episodes

Figure D3: Path to Bank Default events in Baseline and Merton-type models



Notes: This figure shows the average path leading to a bank default episodes under different model assumptions. Red solid line corresponds to our baseline model. Red dashed line corresponds to the Merton-type model. The figure is generated by simulating the model for 1,000,000 periods, identifying periods of twin defaults and then computing the average realizations of shocks and endogenous variables for twenty periods before and after the crisis periods. We define a bank default episode as the occurrence of bank default above its 90th percentiles. The 90th percentile bank default thresholds used to define the bank default regime in the two models are always the ones determined by the baseline model. Bank Risk, Island Risk and Firm Risk represent the level of $\frac{\sigma_{\omega_{b,t+1}}}{\bar{\sigma}_{\omega_b}}$, $\frac{\sigma_{\omega_{j,t+1}}}{\bar{\sigma}_{\omega_j}}$ and $\frac{\sigma_{\omega_{i,t+1}}}{\bar{\sigma}_{\omega_i}}$ in their respective standard deviation units.

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Caterina Mendicino
European Central Bank, Frankfurt am Main, Germany; email: caterina.mendicino1@ecb.europa.eu

Kalin Nikolov
European Central Bank, Frankfurt am Main, Germany; email: kalin.nikolov@ecb.europa.eu

Juan Rubio-Ramirez
Emory University and Federal Reserve Bank of Atlanta, Atlanta, Georgia, United States; email: juan.rubio-ramirez@emory.edu

Javier Suarez
Center for Monetary and Financial Studies, Madrid, Spain; CEPR, London, United Kingdom; email: suarez@cemfi.es

Dominik Supera
Wharton School, University of Pennsylvania, Philadelphia, Pennsylvania, United States; email: superad@wharton.upenn.edu

Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website www.ecb.europa.eu

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