



# Reverse Stress Testing

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# Outline

- ① Motivation: Automated stress scenario design
- ② Modeling Fire Sales
- ③ Reverse Stress Testing and Scenario Design
- ④ Empirical Application to European Banks
- ⑤ Conclusion

# Automated stress scenario design

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# Automated stress scenario design

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- What type of scenario could lead to a “worst-case” contagion in terms of fire sales?
- Which banks (or other institutions) may become key channels of contagion in a stress scenario?
- Is the current financial system particularly vulnerable to specific “classes/families” of scenarios?

# The literature is burgeoning

## **Stress testing and policy:**

(Baudino et al., 2018)  
(Bookstaber et al., 2013)  
(Bookstaber et al., 2014)  
(Henry et al., 2013), (Dees et al., 2017)  
(Aymanns et al., 2018)  
(Aikman et al., 2019)

## **Contagion:**

(Covi et al., 2019), (Battiston et al., 2016)  
(Baptista et al., 2016), (Hüser, 2015)  
(Calimani et al., 2017), (Coen et al., 2019)  
(Cont and Schaanning, 2016),  
(Cont et al., 2019), (Bardoscia et al., 2019)  
(Brinkhoff et al., 2018)

## **Monitoring and portfolio overlaps:**

(Abad et al., 2017)  
(Cont and Wagalath, 2016)  
(Guo et al., 2015)  
(Caccioli et al., 2015)  
(Cont and Schaanning, 2019)

## **Scenario design:**

(Glasserman et al., 2015)  
(Breuer and Summer, 2017)  
(Breuer et al., 2009)  
(Bassanin et al., 2019)

**Vast** literature - very incomplete overview!

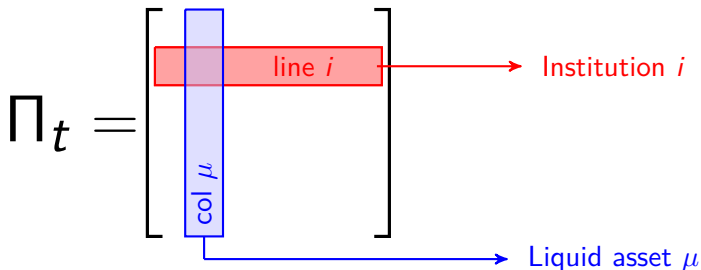
## Our model relies on EBA 2016 stress test data

$N = 51$  institutions  $i \in [N]$

$M = 93$  liquid asset classes  $\mu \in [M]$

$K = 89$  illiquid asset classes  $k \in [K]$

$\Pi_t \equiv [\Pi_t^{i,\mu}]_{i,\mu}$  :  $N$ -by- $M$  matrix  
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marketable assets of institutions at time  $t$ :

- Corporate bonds
- Sovereign exposures
- Securitized exposures



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illiquid assets of institutions at time  $t$ :

- Residential mortgage exposures
- Commercial real estate exposures
- Retail exposures
- Defaulted exposures
- Residual exposures
- Marketable asset holdings beyond market depth

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collecting illiquid assets of institutions at time  $t$

$C_t \equiv [C_t^i]_i$   $N$ -vector: Tier 1 capital of institutions

Data source: [European Banking Authority \(EBA\)](#)

## Any institution must satisfy a leverage constraint

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The *leverage ratio* of  $i$  at  $t$  is

$$\frac{\text{All Assets of } i}{\text{Capital of } i} = \frac{\sum_{\mu} \Pi_t^{i,\mu} + \sum_k \Theta_t^{i,k}}{C_t^i}$$

and should be kept smaller than  $\lambda_{\max} := 33$  (Basel III).

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③ These fire sales have an **impact on the price** of the liquidated assets. Hence a **further** loss, even for institutions not exposed to the initial shock, but holding assets that others liquidated.



## Modeling the price impact

Following shock  $\epsilon \in [0, 1]^K$  at time  $t$ , institution  $i$  liquidates a portion  $\Gamma_t^{i,\mu}(\epsilon) \in [0, 1]$  of its liquid asset  $\mu$ .

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For instance,  $\Psi_\mu(q) = 1 - q/D_\mu$  satisfies (1)-(2)-(3).

# The effect of a shock on a portfolio

For the asset  $\mu$  of Institution  $j$ :

$$\Pi_t^{j,\mu} = \underbrace{\Pi_{t-1}^{j,\mu}}_{\text{Previous value}} \overbrace{\left(1 - \Gamma_t^{j,\mu}(\epsilon)\right)}^{\text{Non-liquidated assets}} \underbrace{\Psi_\mu \left(\sum_i \Pi_{t-1}^{i,\mu} \Gamma_t^{i,\mu}(\epsilon)\right)}_{\text{Price impact on remaining holdings}}$$



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Hence, the loss is, in addition to the initial  $[\Theta_t \epsilon]_j$ :

$$\underbrace{\sum_\mu \Pi_{t-1}^{j,\mu}}_{\text{Previous value}} - \underbrace{\sum_\mu \Pi_t^{j,\mu}}_{\text{Current value}} - \underbrace{\sum_\mu \Pi_{t-1}^{j,\mu} \Gamma_t^{j,\mu}(\epsilon) \Psi_\mu \left(\sum_i \Pi_{t-1}^{i,\mu} \Gamma_t^{i,\mu}(\epsilon)\right)}_{\text{Fire sales revenue}}$$

$$= \sum_\mu \Pi_{t-1}^{j,\mu} - \sum_\mu \Pi_t^{j,\mu} \Psi_\mu \left(\sum_i \Pi_{t-1}^{i,\mu} \Gamma_t^{i,\mu}(\epsilon)\right)$$

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$$\text{All Assets of } j \text{ at } t = \sum_{\mu} \Pi_t^{j,\mu} + [\Theta_{t-1}(1 - \epsilon)]_j.$$

$$\text{Capital of } j \text{ at } t = C_t^j - [\Theta_{t-1}\epsilon]_j - \text{Fire Sales Loss of } j \text{ at } t.$$

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In principle, an institution needs to guess the quantities traded by other institutions to get  $\sum_{i \neq j} \Pi_{t-1}^{i,\mu} \Gamma_t^{i,\mu}(\epsilon)$  correctly.

If Institution  $j$  doesn't know better,

it can assume that  $\sum_{i \neq j} \Pi_{t-1}^{i,\mu} \Gamma_t^{i,\mu}(\epsilon) = 0$ .

Surprisingly, we can solve this problem  
*very* efficiently up to 3% accuracy

If the price impact is linear:  $\Psi_{\mu}(q) = 1 - q/D_{\mu}$ ,  
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If the price impact is concave:

- The loss is a convex function of  $\Gamma^j$
- The leverage constraint is convex

We developed a very efficient method to compute the optimal  $\Gamma^j$   
with a provable accuracy and guaranteed speed.

# Reverse Stress Testing and Scenario Design



## Looking for the worst-case scenario

Find the stress scenario(s)  $\epsilon \in [0, 1]^K$ , that

- generate(s) the worst total fire-sales loss,
- under the assumption that banks react optimally,

Also, the initial shock **should not be “too severe”** ,  
and **should make economic sense (historically consistent)** .

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$$\max \sum_{j=1}^N \text{Loss}_j(\Gamma^j(\epsilon))$$

$$\text{s.t. } 0 \leq \epsilon \leq 1$$

$$\sum_{i=1}^N \sum_{\nu=1}^K \Theta^{i,\nu} \epsilon_{\nu} \leq L_{\max}$$

$$\underline{\epsilon}_{\nu} \leq \epsilon_{\nu} \leq \overline{\epsilon}_{\nu}$$

some “historical constraint”

## Worst-case scenarios that are historically meaningful

Let  $\Sigma_{\Theta}$  be the covariance matrix of the 89 illiquid assets' returns.

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Let  $H$  be the 14-dimensional subspace spanned by these eigenvectors.

We require for  $\epsilon$  to be at a Euclidean distance of 0.05 from  $H$ .

That is, we want  $\langle u^k, \epsilon \rangle \leq 0.05$   
for all eigenvectors  $u^k$  of  $\Sigma_{\Theta}$ ,  $15 \leq k \leq K$ .

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$$\langle u^k, \epsilon \rangle \leq 0.05 \quad \text{for } k_{\min} \leq k \leq K$$

We have a convex *maximization* problem over a polyhedron

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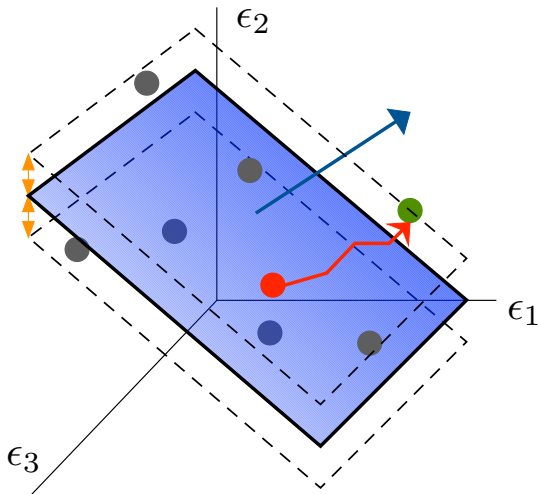
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- We can have multiple local maximums.
- We find a collection of local maximums by a multiple starting points gradient ascent method.
- We have to take full advantage of the simplicity of the constraints set (projections are cheap)
- We **critically** needed an efficient method for evaluating  $\text{Loss}_j(\Gamma^j(\epsilon))$  and  $\partial \text{Loss}_j(\Gamma^j(\epsilon)) / \partial \epsilon$ .



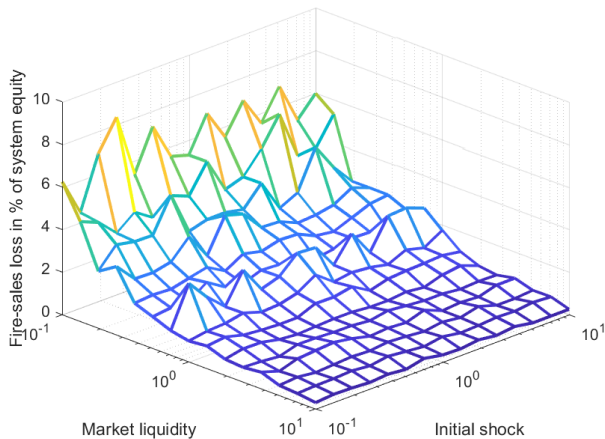
# Systematic algorithmic exploration of “scenario space”



**Figure:** Intuitive visualization of our algorithmic approach.

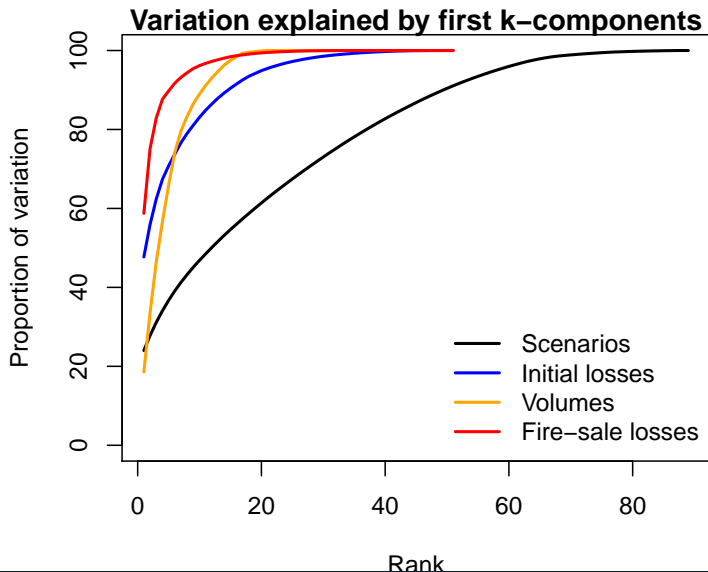
# Empirical Application to European Banks

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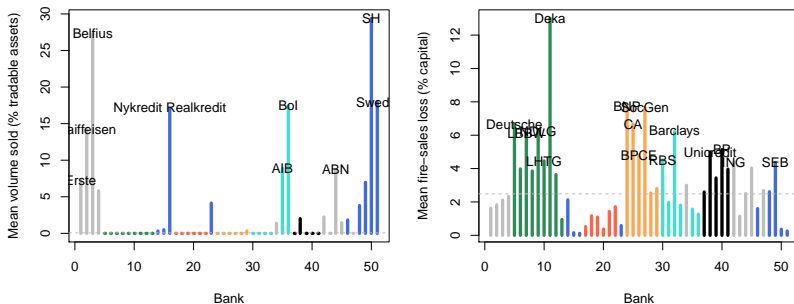


**Figure:** Fire-sales losses as function of price impact and initial shock size.

# An Anna Karenina principle of stress test scenarios

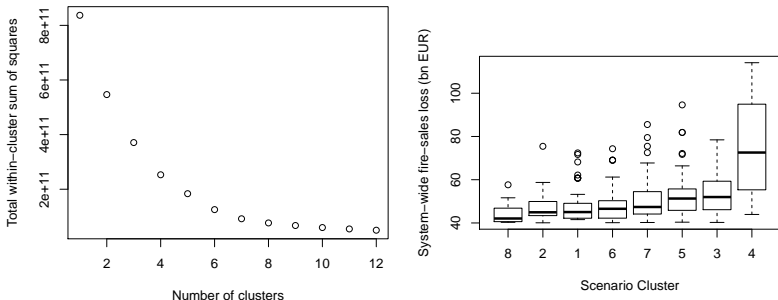


# Mean losses and sales



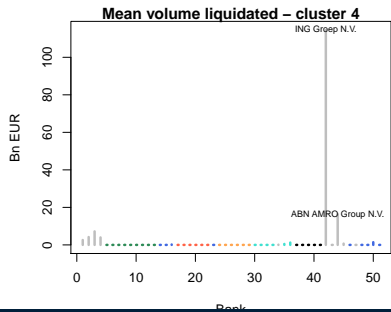
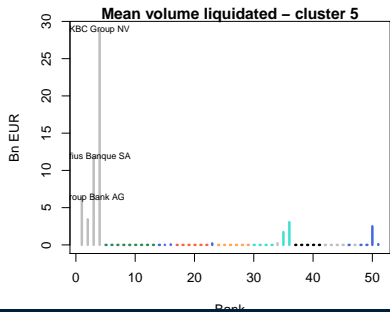
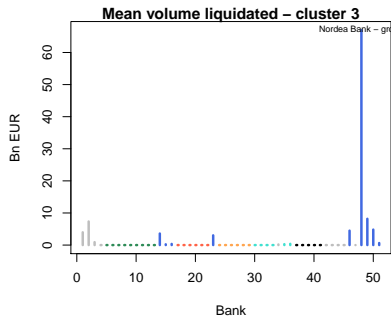
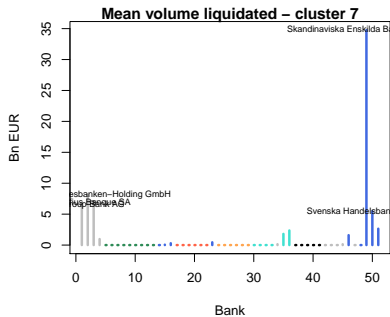
**Figure:** Left: Mean volume liquidated, right: mean fire-sales loss.

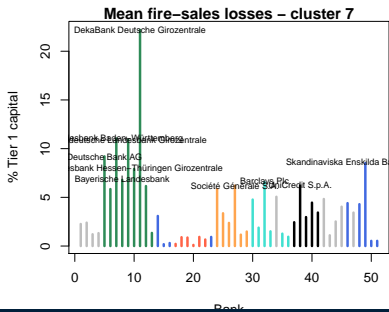
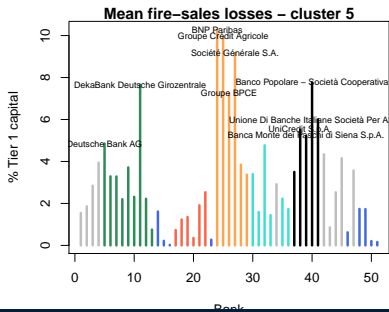
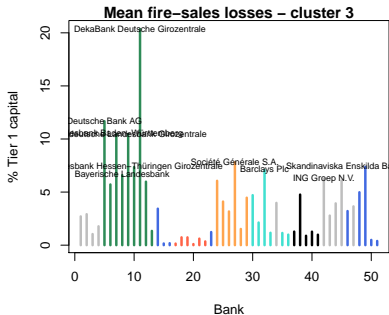
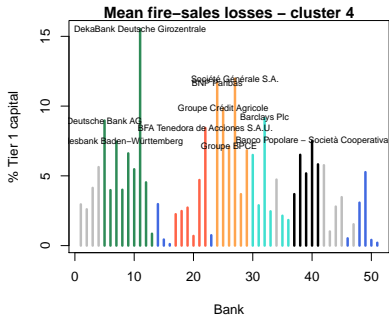
# Clustering analysis



**Figure:** Clustering analysis unveils 8 “scenario” clusters.

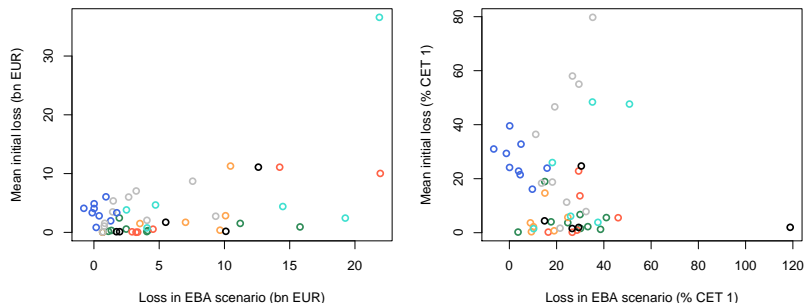
Next two slides show the *volume of liquidations* and the *fire-sales losses* in the four worst scenarios respectively.







## Scenario design - targeting vulnerabilities



**Figure: Preliminary results:** Comparing the losses in the EBA scenario to the average initial loss across the worst case scenarios

# Conclusion

## Conclusions (preliminary). What we did.

- We have introduced a computational approach to search **systematically** for scenarios that exploit the vulnerabilities of current portfolio holdings.
- The methodology allows to work **rapidly** through thousands of scenarios and identify the relevant scenarios and banks.

## Conclusions (preliminary). What we found.

- An Anna Karenina principle of scenario design:  
*All stressful scenarios stress the **same set of banks**,  
each stressful scenario is stressful in its own way.*  
→ This suggests that regulators may wish to focus  
on identifying *vulnerable institutions*,  
rather than plausible scenarios.

## Conclusions (preliminary). What we found.

- An Anna Karenina principle of scenario design:  
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→ This suggests that regulators may wish to focus  
on identifying *vulnerable institutions*,  
rather than plausible scenarios.
- EBA 2016 scenario does not seem to have targeted  
the banks that were most vulnerable to drive contagion losses  
(according to this methodology and metric).
- Implications for micro- and macroprudential stress testing.

Thank you!

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