# Monetary Policy and Heterogeneity: An Analytical Framework 

Florin O. Bilbiie

University of Lausanne \& CEPR
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## Motivation

- 2008 Great Expansion—stabilization policies (mon\&fisc)
-     + inequality-redistribution (i.a. Bernanke, Yellen, Draghi)
- Micro data in macro + solving HA models
- Krusell Smith, Den Haan, Reiter; Mean-Field Games (Lasry Lions), Many others
- Overwhelming evidence:
- aggregate Euler eqn?: Hall 88 Cambell Mankiw to Yogo, Vissing-Jorgensen, Canzoneri Cumby Diba, Bilbiie Straub
- high fraction with zero net worth: Wolff, SCF (Bricker et al)
- Important recent empirical work
- consumption-income: Johnson, Parker, Souleles; Misra Surico; Surico Trezzi
- liquidity constraints \& MPC, wealthy hand-to-mouth: Kaplan Violante; Cloyne Ferreira Surico; Gorea Midrigan


## HA NK

## $\mathrm{HA}_{\text {мк }}$ ${ }_{\text {HANK }}$

## | Quantitative Analytical

| TANK |  | Bilbiie 2008 JET (2004) |
| :--- | :---: | :---: |
|  |  |  |
| HANK |  | $\underset{\chi}{2}$ |

Key channel: $\chi \gtrless 1 \sim$ Cyclical Inequality
Core: individual income elasticity to aggregate (cycle), as determined by profits \& their (re)distribution link

## This Paper: a-HANK

1. representation of several quantitative-HANK channels (prequel: The New Keynesian Cross (JME): one channel)
2. "analytical" fits purpose: closed-form, full-blown NK

## Using a-HANK: Closed-form

1. Determinacy with IR rules and No-Puzzle (iff) conditions.

- HANK Taylor Principle \& Sargent-Wallace

2. Catch-22:
no-Puzzles $\chi<1 \quad$ Amplif. (multipliers) $\chi>1$
3. Solutions (ways out):

- add (better be!) distinct HANK channel: cyclical risk
- Wicksellian rule (price-level targeting) in HANK

4. Optimal policy (closed-form)
5. Liquidity Traps: Redo 1-4

## Literature: TANK 2000s

- Campbell Mankiw 90s: estimate elasticity of intertemporal substitution from aggregate Euler
- Bilbiie 08, analytical: aggregate demand and monetary policy; profits and redistribution; optimal monetary policy
- Galí Lopez-Salido Vallés 07, quantitative: physical capital or not; fiscal multipliers, determinacy, numerical; Mankiw 00: fiscal policy in TA-RBC; Bilbiie Straub 04 fiscal multipliers TANK analytical+dist. tax; Bilbiie Meier Muller 08: estimated fiscal TANK; Colciago; Ascari, Colciago and Rossi (sticky wages); Eser; Farhi Werning 16: currency unions
- different (related) TA (borrower-saver): Iacoviello 05; Eggertsson Krugman; Curdia Woodford; Nistico; Monacelli Perotti; Bilbiie Monacelli Perotti;
- Bilbiie Straub: estimated TANK (Bayesian and GMM)
- Debortoli Galí 17: compare TANK with HANK


## Literature: HANK 2010s

- Monetary policy: Kaplan, Moll, Violante (KMV): monetary policy transmission, direct-indirect effects, liquid assets; McKay Nakamura Steinsson (MNS): FG power; Guerrieri Lorenzoni: deep LT-deleveraging recessions; Auclert; Auclert Rognlie: transmission channel; Bayer Luetticke Pham-Dao Tjaden; Luetticke: liquidity, portfolio composition;
- Endogenous unemployment implications: Ravn Sterk; Den Haan Rendahl Riegler; Gornemann Kuester Nakajima; McKay Reis; Challe Matheron Ragot Rubio
- Fiscal: Oh Reis; Hagedorn Manovskii Mitman; Ferrière Navarro; Auclert Rognlie Straub
- Analytical HANKs: Acharya Dogra (isolate cyclical-risk channel); Broer, Hansen, Krusell, Oberg (virtues of sticky wages); Ravn Sterk (add analytical SAM, transmission and puzzles); Werning (cyclical risk\&liquidity); Bilbiie Ragot (endogenous liquidity-money); this


## Literature: NK Analysis

- Determinacy: RA(NK) Benhabib Farmer; Leeper; Woodford; Cochrane; Benhabib Schmitt-Grohé Uribe; Dupor; Lubik Schorfheide; etc.; TANK: Galí Lopez-Salido Vallés (quant.); Bilbiie (analytical); Ascari et al;
- FG puzzle (Del Negro, Giannoni, Patterson): perfect information/rational expectations: sticky info (Kiley; Carlstrom et al); eductive stability Garcia-Schmidt Woodford ( $+6=$ Farhi-Werning); dispersed information Wiederholt; Andrade et al; sparsity (Gabaix); imperfect common knowledge (Angeletos Lian); ; fiscalist: Cochrane (FTPL+long-term debt); OLG: Del Negro, Giannoni, Patterson; peg interest on reserves: Diba Loisel; wealth-in-utility:
Canzoneri Diba, Michaillat Saez; heterogeneity+incomplete markets: MNS, KMV; Ravn Sterk; Acharya Dogra; Hagedorn
- Optimal policy in TANKs (Bilbiie 08, Ascari et al; Nistico; Curdia Woodford) and HANKs: Bhandari Evans Golosov Sargent; Challe; McKay Reis; Nuno Thomas; Bilbiie Ragot


## Analytical (a-)HANK Model (Ingredients)

- Two states: constrained hand-to-mouth $H$ and unconstrained "savers" S
- switch exogenously (idiosyncratic uncertainty).
- Insurance:
- full within type (after idiosyncratic uncertainty revealed)
- limited across types.
- Two assets and liquidity:
- bonds are liquid (can be used to self-insure, before idiosyncratic uncertainty is revealed)
- stocks are illiquid (cannot ——",
- Bond trading
- equilibrium liquidity
- or not (most analytical HANK): "Bondless limit"


## Self-insurance and Asset Market

- separable $U^{j}\left(C^{j}, N^{j}\right) ; \sigma^{-1} \equiv-u_{C C}^{j} C^{j} / u_{C}^{j} ; \varphi \equiv u_{N N}^{j} N^{j} / U_{N}^{j}$
- shocks $\mathrm{S} \leftrightarrows \mathrm{H}, \mathrm{p}(S \mid S)=s ; \mathbf{p}(H \mid H)=h ; H$ mass:

$$
\lambda=\frac{1-s}{2-s-h}
$$

- $\mathrm{S} \xrightarrow{1-\mathrm{s}} \mathrm{H}$ bonds liquid, stock illiquid (temporarily)
- self-insurance w/ bonds (priced even when not traded):

$$
\left(C_{t}^{S}\right)^{-\frac{1}{\sigma}}=\beta E_{t}\left\{\left(1+r_{t}\right)\left[s\left(C_{t+1}^{S}\right)^{-\frac{1}{\sigma}}+(1-s)\left(C_{t+1}^{H}\right)^{-\frac{1}{\sigma}}\right]\right\}
$$

- H Euler with inequality (constrained); with no liquidity:

$$
C_{t}^{H}=Y_{t}^{H}
$$

- "wealthy" H


## HA-1: Income Processes

- Autocorrelation ( $>0$ if $s \geq 1-h$ )

$$
\operatorname{corr}\left(Y_{t+1}^{j}, Y_{t}^{j}\right)=s+h-1=1-\frac{1-s}{\lambda}
$$

- Conditional variance ( ~ income risk):

$$
\operatorname{var}\left(Y_{t+1}^{S} \mid Y_{t}^{S}\right)=s(1-s)\left(Y_{t+1}^{S}-Y_{t+1}^{H}\right)^{2}
$$

- Conditional skewness and kurtosis:

$$
\begin{aligned}
\operatorname{skew}\left(Y_{t+1}^{S} \mid Y_{t}^{S}\right) & =\frac{1-2 s}{\sqrt{s(1-s)}} \\
\operatorname{kurt}\left(Y_{t+1}^{S} \mid Y_{t}^{S}\right) & =\frac{1}{s(1-s)}-3
\end{aligned}
$$

(Basically Rouwenhorst with 2 states.)

- Benchmark: approximate model $\sim$ long-run symmetric steady-state $Y^{S}=Y^{H}$
$\rightarrow$ risk acyclical


## Rest $=$ TANK (Bilbiie 2008 version)

- Core distinction: hold assets ( $\rightarrow$ profits) or not ©hil
- $H$ ( $\lambda$ fraction) consume all their income

$$
c_{t}^{H}=w_{t}+n_{t}^{H}+\frac{\tau^{D}}{\lambda} d_{t}
$$

- Equilibrium (2 lines of algebra):

$$
\begin{aligned}
c_{t}^{H} & =\chi y_{t} \\
\text { key } & : \chi \equiv 1+\underbrace{\varphi}_{\text {labor mkt. }} \times \underbrace{\left(1-\frac{\tau^{D}}{\lambda}\right)}_{\text {fiscal redistrib. }} \gtrless 1 \\
c_{t}^{S} & =\frac{1-\lambda \chi}{1-\lambda} y_{t}
\end{aligned}
$$

Extra income effect $w \uparrow \rightarrow d \downarrow$ keystone: profits (\#45-60) ${ }^{1}$

$$
\text { Cyclical Inequality: } y_{t}^{S}-y_{t}^{H} \equiv(1-\chi) \frac{1}{1-\lambda} y_{t}
$$

[^0]
## Analytical HANK: Aggregate Euler-IS

- Aggregate, replace $c_{t}^{j} \longrightarrow c_{t}^{S}=s E_{t} c_{t+1}^{S}+(1-s) E_{t} c_{t+1}^{H}-\sigma r_{t}$

$$
c_{t}=\underbrace{\left[1+(\chi-1) \frac{1-s}{1-\lambda \chi}\right]}_{\equiv \delta} E_{t} c_{t+1}-\underbrace{\sigma \frac{1-\lambda}{1-\lambda \chi}}_{\text {TANK }} r_{t}
$$

"Discounting" $\quad \delta<1$ iff PI $\chi<1$ "Compounding" $\delta>1$ iff CI $\chi>1$

- complementarity $\frac{\partial^{2} \delta}{\partial \lambda \partial(1-s)} \sim \chi-1 \lessgtr 0$
- Exact same $\chi$-channel as TANK, but intertemporal
- Acyclical idiosyncratic income risk (approx $\sim Y^{H}=Y^{S}$ )


## HA-2: The New Keynesian Cross

$$
\text { PE: } c_{t}=[1-\beta(1-\lambda \chi)] y_{t}-(1-\lambda) \beta \sigma r_{t}+\beta \delta(1-\lambda \chi) E_{t} c_{t+1}
$$

Total effect $\Omega$ Indirect-effect share $\omega$

|  | ("multiplier") | ("aggregate MPC") |
| :--- | :--- | :--- |
| TANK | $\frac{\sigma}{1-p} \frac{1-\lambda}{1-\lambda \chi}$ | $\frac{1-\beta(1-\lambda \chi)}{1-\beta p(1-\lambda \chi)}$ |
| a-HANK | $\frac{\sigma}{1-\delta p} \frac{1-\lambda}{1-\lambda \chi}$ | $\frac{1-\beta(1-\lambda \chi)}{1-\delta \beta p(1-\lambda \chi)}$ |

The New Kenesian Cross

$$
c_{t}=\omega \hat{y}_{t}-(1-\omega) \Omega r_{t}+(1-\omega)(M-1) g_{t}
$$


aggreg. MPC $\omega \equiv \lambda \times 1 \times \chi+(1-\lambda) \times(1-\beta) \times \frac{1-\lambda \chi}{1-\lambda}$

## The New Kenesian Cross (in a-HANK)




Calibrate $\operatorname{KMV} \Omega / \Omega_{r a n k}=1.5, \omega=.8: \quad \chi=1.42 ; \lambda=.37 ; 1-s=.04$



## Calibrating Simple to Match Complicated

Table 1: Approximating HANK

| HANK: Equilibrium objects |  |  |  |  | Implied parameters |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\frac{\Omega}{\Omega^{*}}$ | $\omega$ | $\frac{\Omega_{1}^{F}}{\Omega^{*}}$ | $\frac{\Omega_{20}^{F}}{\Omega^{*}}$ |  | $\lambda$ | $1-s$ |
| Kaplan et al | 1.5 | . 8 | - | - | $\begin{aligned} & 1.48 \\ & 1.48 \end{aligned}$ | $\begin{aligned} & .41 \\ & .37 \end{aligned}$ | $\begin{aligned} & 0 \text { (TANK) } \\ & .04 \end{aligned}$ |
| McKay et al | - | - | . 8 | . 4 |  | $\overline{.} 21$ | $\begin{aligned} & 0 \text { (TANK) } \\ & .04 \end{aligned}$ |

Paper: other HANKs (Goremann et al, Debortoli Gali, Hagedorn et al, Auclert et al)

## HA-3: iMPCs in a-HANK (w / liquidity)

- Auclert Rognlie Straub; Hagedorn Manovskii Mitman
- fiscal policy
- most compelling critique of TANK ... not of a-HANK!
- better still: $\chi$ helps match data (Fagereng Holm Natvik)


## HA-3: iMPCs in a-HANK (w / liquidity)

- Proposition: iMPCs $\frac{d c_{t}}{d \hat{y}_{T}}$

$$
\begin{aligned}
& <T: \frac{1-\lambda \chi}{s} \frac{\delta-\beta x_{b}}{1-\beta x_{b}^{2}}\left(\beta x_{b}\right)^{T-t}\left(1-x_{b}+x_{b}\left(1-\beta x_{b}\right)\left(\beta x_{b}^{2}\right)^{t}\right) \\
& =\mathrm{T}: 1-\frac{1-\lambda \chi}{s} \beta x_{b}-\left(\delta-\beta x_{b}\right) x_{b} \frac{1-\lambda \chi}{s}\left(1-\beta x_{b}\right) \frac{1-\left(\beta x_{b}^{2}\right)^{T}}{1-\beta x_{b}^{2}} \\
& >T: \frac{1-\lambda \chi}{s} \frac{1-\beta x_{b}}{1-\beta x_{b}^{2}} x_{b}^{t-T}\left(1-x_{b} \delta+x_{b}\left(\delta-\beta x_{b}\right)\left(\beta x_{b}^{2}\right)^{T}\right)
\end{aligned}
$$

$x_{b}(s, \lambda)<1$ the stable root of asset-accumulation equation

- increasing with $\chi$ when $t<T$, decreasing when $t \geq T>0$ (given $d c_{0} / d \hat{y}_{0}$ ).


## iMPCs in a-HANK


$\chi=1$ (dot-dash); TANK (red dash); $\chi>1$ thick and $<1$ thin

## Recap: a-HANK as HANK projection

1. Idiosyncratic uncertainty, income processes

- persistence, risk, skewness, kurtosis, etc.

2. NK Cross:

- cyclical inequality $\chi$, Euler discounting/compounding $\delta$

3. iK Cross:

- iMPCs (with liquidity)

Remainder: zero-liquidity limit (passive-Ricardian FP, zero SS debt)

## The Analytical-HANK 3-Equation Model

$$
\begin{aligned}
c_{t} & =\delta E_{t} c_{t+1}-\sigma \frac{1-\lambda}{1-\lambda \chi}\left(i_{t}-E_{t} \pi_{t+1}-\rho_{t}\right) \\
& : \quad\left(\text { with } \delta \equiv 1+(\chi-1) \frac{1-s}{1-\lambda \chi}\right) \\
\pi_{t} & =\kappa c_{t}+\beta E_{t} \pi_{t+1} \\
i_{t} & =\rho_{t}+i_{t}^{*}+\phi \pi_{t}
\end{aligned}
$$

- (here $\pi_{t}=\kappa c_{t}$ simple closed forms, paper NKPC)
- Boil down to one (!) equation $\lambda, \chi, \delta \longrightarrow \mathcal{V}$ ["new"]


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## The HANK Taylor Priciple

$$
\begin{aligned}
c_{t} & =V E_{t} c_{t+1}+\text { shocks } \\
\text { News on } \mathrm{AD} & : \quad v \equiv \frac{\delta+\kappa \sigma \frac{1-\lambda}{1-\lambda \chi}}{1+\phi \kappa \sigma \frac{1-\lambda}{1-\lambda \chi}}
\end{aligned}
$$

- $\exists$ ! REE (local determinacy) with $\lambda<\chi^{-1}$ :

$$
v<1 \Leftrightarrow \phi>1+\frac{\delta-1}{\kappa \sigma \frac{1-\lambda}{1-\lambda \chi}} .
$$

- Taylor principle $\phi>1$ sufficient if:

$$
\delta \leq 1 \longrightarrow \chi \leq 1
$$

## The HANK Taylor Priciple




CI: $\chi>1$

## HANK and Sargent-Wallace

- Determinacy with peg $\phi=0$

$$
c_{t}=v_{0} E_{t} c_{t+1}+\text { shocks }
$$

news on AD under peg

$$
v_{0}=\delta+\kappa \sigma \frac{1-\lambda}{1-\lambda \chi}<1
$$

## HANK Cures FG Puzzle IFF:

## HANK AD-Discounting $\gg$ RANK AS-Compounding

$$
v_{0}=\delta+\kappa \sigma \frac{1-\lambda}{1-\lambda \chi}<1
$$

- $\delta<1$ necessary not sufficient (complementarity):

$$
1-s>0 \text { and } \chi<1-\sigma \kappa \frac{1-\lambda}{1-s}<1
$$

Proof
$c_{t}=v_{0} E_{t} c_{t+1}-\sigma \frac{1-\lambda}{1-\lambda \chi} i_{t}^{*}=v_{0}^{\bar{T}} E_{t} c_{t+\bar{T}}-\sigma \frac{1-\lambda}{1-\lambda \chi} E_{t} \sum_{j=0}^{\bar{T}-1} \nu_{0}^{j} i_{t+j}^{*}$
FG: cut at time $t+T, \frac{\partial c_{t}}{\partial\left(-i_{t+T}^{*}\right)}=\sigma \frac{1-\lambda}{1-\lambda \chi} v_{0}^{T}$

## Amplification without Puzzles: Catch-22?

- HANK multiplier (balanced-budget uniform $t_{t}=g_{t}$ ): ${ }^{2}$

$$
\frac{\partial c_{t}}{\partial g_{t}} \sim \underbrace{\zeta(\chi-1) \frac{\lambda\left(1-\mu_{g}\right)+(1-s) \mu_{g}}{1-\lambda \chi}}_{\text {TANK + HANK AD }}-\underbrace{\kappa \zeta \sigma \frac{1-\lambda}{1-\lambda \chi}\left(\phi-\mu_{g}\right)}_{\text {RANK AS }}
$$

(times $\left.\frac{1}{1-v \mu_{g}}\left(1+\phi \kappa \sigma \frac{1-\lambda}{1-\lambda \chi}\right)^{-1}\right)$

- The New Keynesian Cross gets going iff cross

$$
\chi>1
$$

Underlies amplification-HANKs (Kaplan Moll Violante, etc.)

- $\rightarrow$ Catch-22

$$
{ }^{2} \text { Income effect } \zeta \equiv\left(1+\varphi^{-1} \sigma^{-1}\right)^{-1}
$$

## Cyclical Risk: a Different Channel

- (Ravn Sterk; Challe et al; Werning; Acharya Dogra)
$-s^{\prime}\left(Y_{t+1}\right) \gtrless 0 \rightarrow$ pro-(counter-)cyclical risk (NB: $\lambda$ invariant)
- Aggregate Euler-IS $\left(\Gamma=Y^{S} / Y^{H} \geq 1\right)$ :

$$
\begin{gathered}
c_{t}=(\delta+\eta) E_{t} c_{t+1}-\sigma \frac{1-\lambda}{1-\lambda \chi} r_{t} \\
\eta \equiv \frac{s_{Y} Y}{1-s}\left(1-\Gamma^{-1 / \sigma}\right)(1-\tilde{s}) \sigma \frac{1-\lambda}{1-\lambda \chi}
\end{gathered}
$$

- Similar equilibrium implications:
- pro-(counter-)cyclical $\rightarrow$ discounting (compounding)
- Different economic mechanism ("precautionary saving")
- earlier on risk and aggregate Euler with incompleteness: Krueger and Lustig 2010


## Solution to Catch-22? Cyclical Inequality vs Risk

- Yes and No
- No-Catch-22 (Amplification without Puzzles) iff

$$
\begin{aligned}
& \text { Countercyclical Inequality }: \\
& \text { Procyclical (enough) Risk }: \\
& \hline<1-\delta<0
\end{aligned}
$$

- Flip side: everything worse if risk countercyclical too
- Policy options?


## HANK: Virtues of a Wicksellian Rule

- Proposition: Even with Amplification ++ (both inequality and risk countercyclical): ${ }^{3}$
$i_{t}=i_{t}^{*}+\phi_{p} p_{t}$ with $\phi_{p}>0$ (Woodford \& Giannoni in RANK)

1. $\exists$ ! REE (local determinacy) with $\lambda<\chi^{-1}$
2. no FG puzzle

- Intuition: PID control-bygones are not bygones;

Hagedorn, other ways out
${ }^{3}$ Corrolary: also in RANK!

## Optimal Policy in a-HANK

- Aggregate welfare $2^{\text {nd }}$ order, Ramsey problem becomes (Woodford 2003 RANK, Bilbiie 2008 TANK):
- approx. around first-best, perfect-insurance y

$$
\begin{gathered}
\min _{\left\{c_{t}, \pi_{t}\right\}} \frac{1}{2} E_{0} \sum_{t=0}^{\infty} \beta^{t}\left\{\alpha y_{t}^{2}+\pi_{t}^{2}\right\}, \\
\alpha \equiv \frac{\sigma^{-1}+\varphi}{\psi}[\underbrace{1}_{\text {RANK }}+\underbrace{\varphi^{-1} \sigma^{-1} \frac{\lambda}{1-\lambda}(\chi-1)^{2}}_{\text {HANK-inequality }}]
\end{gathered}
$$

- more $\mathrm{H} \rightarrow$ less $\pi$ stabilization
- key: profits, $\pi$ like a distortionary tax
- more $\pi$ volatility under optimal policy (discretion, commitment)
- makes large difference in: ...


## Liquidity Traps with a-HANK

- use Bilbiie 2016 (Optimal Forward Guidance, AEJ-Macro) simple closed-form, FG "state"
- first closed-form optimal policy in RANK-LT ( $\sim$ Ramsey) + "simple rule FG" (how long should CB $i=0$ )
- Extend Eggertsson-Woodford to three states:

$$
\begin{gathered}
\mathrm{P}(\mathrm{~L} \rightarrow \mathrm{~F})=(1-z) q, \mathrm{P}(\mathrm{~F} \rightarrow \mathrm{~F})=q ; \mathrm{E}(\mathrm{FG} \text { duration })=1 /(1-q) \\
E_{t} c_{t+1}=z c_{L}+(1-z) q c_{F}+(1-z)(1-q) 0
\end{gathered}
$$

- Equilibrium $d c_{F} / d q>0 ; d c_{L} / d q>0$

$$
\begin{aligned}
& c_{F}=\frac{1}{1-q v_{0}} \sigma \frac{1-\lambda}{1-\lambda \chi} \rho \\
& c_{L}=\frac{1-z}{1-z v_{0}} \frac{q v_{0}}{1-q v_{0}} \sigma \frac{1-\lambda}{1-\lambda \chi} \rho+\frac{\sigma}{1-z v_{0}} \frac{1-\lambda}{1-\lambda \chi} \rho_{L}
\end{aligned}
$$

## FG: Dampening and Amplification

$c_{L}$ (thick) and $c_{F}$ (thin): RANK, TANK and iid-HANK


PI: $\chi<1$

$\mathrm{CI}: \chi>1$

## FG Power and Puzzle

- FG power:

$$
\mathcal{P}_{F G} \equiv \frac{d c_{L}}{d q}=\left(\frac{1}{1-q v_{0}}\right)^{2} \frac{(1-z) v_{0} \sigma \frac{1-\lambda}{1-\lambda \chi}}{1-z v_{0}} \rho .
$$

- $\chi>1: \partial \mathcal{P}_{F G} / \partial \lambda>0 ; \partial \mathcal{P}_{F G} / \partial(1-s)>0$
- Corollary:

FG puzzle: $\frac{\partial \mathcal{P}_{F G}}{\partial z} \geq 0$ ruled out iff

$$
v_{0}<1
$$

## FG Puzzle: Resolution or Aggravation?

RANK, TANK and iid-HANK; $q=0.5 \lambda=0.1$



PI: $\chi<1$

## Optimal Policy in LT (and the Dark Side of FG Power)

- E(PDV(Welfare)) w/ Markov chain:

$$
W=\frac{1}{1-\beta z} \frac{1}{2}\left[c_{L}^{2}+\omega(q) c_{F}^{2}\right]
$$

- $\omega(q)=\frac{1-\beta z+\beta(1-z) q}{1-\beta q}, \omega^{\prime}(q)>0$ : the longer in $F$, the larger the total welfare cost.
- $\min _{q} W$ s.t. equilibrium $c_{F}$ and $c_{L}$

$$
c_{L} \frac{d c_{L}}{d q}+\omega(q) c_{F} \frac{d c_{F}}{d q}+\frac{1}{2} \frac{d \omega(q)}{d q} c_{F}^{2}=0
$$

- simple case: closed-form $q^{*}$ (paper)


## Optimal FG duration



## Convergence



## Convergence

- Paramount policy inputs: $\chi_{,}(\lambda, 1-s, \eta)=$ ?
- inequality ( $\chi$ ): Heathcote Perri Violante HPV-2010
- worryingly, countercyclical: Patterson (2019)
- ... risk too! Storesletten et al; Guvenen et al
$-\longrightarrow$ Urgency

1. Estimate quantitative model with all channels (disentangle)
2. Optimal policy (recent numerical advances)

## No-Puzzle Threshold Redistribution



Redistribution threshold $\tau_{\text {min }}^{D}$ in TANK $1-s \rightarrow 0$ (dash); $1-s=0.04$ (solid); iid HANK $1-s=\lambda$ (dots).

## What (else) determines $\chi$ ?

- Fiscal redistribution:

$$
\chi \equiv 1+\varphi\left(1-\frac{\tau^{D}}{\lambda}\right)
$$

- "proper" progressive taxation: (Ferrière Navarro; Heathcote Storesletten Violante);
- crucial ingredient in all HANK: here spelled out transparently
- Sticky wages (HANK: Broer et al; TANK: Ascari et al), $f$ fixed

$$
\chi=(1+\varphi) \frac{1-f}{1-\lambda f} \lessgtr 1
$$

## Other ways out

- other puzzles with $\chi>1$
- add orthogonal ingredients: deviations from RE, PI, PCC; wealth in U , interest on reserves, etc.
- HA: Hagedorn (2018): a. positive B demand (BIU, HA), b. choose nominal $B, \mathrm{c}$. and d. commit to nominal T and to $\mathrm{i} \rightarrow P$
- "Amplification" with $\chi<1$
- note: "indirect effect" always there; multipliers with transfers (progressivity increase)
- wicksellian


Fig. 9. Percentiles of the household earnings distribution (CPS). Shaded areas are NBER recessions.
Source: Heathcote, Perri, Violante 2010

## Bilbiie (2008 JET) 4 contributions in nutshell

## 1. TANK analytical $\rightarrow$ income (profits') distribution key for AD\&MP


#### Abstract

Interest rate changes modify the intertemporal consumption and labor supply profile of asset holders, agents who smooth consumption by trading in asset markets. This affects the real wage and hence the demand of agents who have no asset holdings but merely consume their wage income. Variations in the real wage (marginal cost) lead to variations in profits and hence in the dividend incone of asset holders. These variations can either reinforce (if participation is not 'too' limited) or overturn the initial impact of interest rates on aggregate demand. The latter case occurs if the share of non-asset holders is high enough and/or and the elasticity of labor supply is low enough, for the potential variations in profit income offset the interest rate effects on the demand of asset holders. This is the main mechanism identified by this paper to change dramatically the effects of monetary policy as compared to a standard full-participation case whereby aggregate demand is completely driven by asset holders.

If participation is restricted below a certain threshold, the predictions are strengthened: as the share of non-asset holders increases, the link between interest rates and aggregate demand becomes stronger, and monetary policy is more effective; we label this case'standard aggregate demand logic', (SADI). However, when participation is restricted beyond a given threshold,


2. aggregate Euler-IS; key $c_{j}=\varepsilon(\lambda) * y$ (intuition Section 3.1) TANK

Straightforward algebraic manipulation of the equilibrium conditions in Table 1 allows the derivation of aggregate dynamics, similar to the standard, full-participation New Keynesian benchmark. We start by deriving the aggregate Euler equation, or 'IS' curve. To that end, we need to express consumption of asset holders (the only agents whose consumption obeys an Euler equation) in terms of aggregate consumption/output. Since hours of non-asset holders are constant $n_{H, t}=O,{ }^{13}$ their consumption tracks real wage, $c H, t=w_{t}$. Total labor supply (from the labor market clearing condition) is $n_{t}=[1-\lambda] n_{S, t}$. Using these last two expressions, asset lholders" labor supply equation, the production function and the goods market clearing condition into the definition of total consumption we find:

$$
\begin{equation*}
c_{s, t}=\delta y_{t}+(1+\mu)(1-\delta) a_{t}, \quad \text { where } \delta \equiv 1-\varphi \frac{\lambda}{1-\lambda} \frac{1}{1+\mu} \tag{7}
\end{equation*}
$$

Substituting (7) into the Euler equation of asset holders we find the aggregate Euler equation, or 'IS curve' :

$$
\begin{equation*}
y_{t}=E_{t} y_{t+1}-\delta^{-1}\left[\gamma_{t}-E_{t} \pi_{t+1}\right]+(1+\mu)\left(1-\delta^{-1}\right)\left[a_{t}-E_{t} a_{t+1}\right] \tag{8}
\end{equation*}
$$

Direct inspection of ( 8 ) suggests the impact that LAMP has on the dynamics of a standard business cycle model through modifying the elasticity of aggregate demand to real interest rates

$$
w_{t}=\chi y_{t}-\varphi a_{t}, \quad \text { where } \chi \equiv 1+\varphi /(1+\mu) \geqslant 1 \geqslant \delta
$$

### 3.1. Intuition and the Labor market

How can an increase in interest rates become expansionary when asset market participation is restricted enough? To answer this question, it is useful to conduct a simple mental experiment whereby the monetary authority pursues a one-time discretionary increase in the interest rate $\varepsilon_{t}$, otherwise pursuing a policy that fully accommodates inflationary expectations, namely $r_{t}=$ $E_{x} \pi_{t+1}+\varepsilon_{t}$. In the standard, full-participation economy, an increase in interest rates leads to a fall in aggregate demand today. Asset holders are also willing to work more at a given real wage (labor supply shifts rightward), but labor demand shifts left because of sticky prices (not all the fall in demand can be accommodated via cutting prices). The new equilibrium is one with lower output, consumption, hours and real wage. Suppose now that we are in an economy with limited participation, but $\lambda<\lambda^{+}$either because participation is not restricted 'enough' or labor supply is not inelastic enough. The fall in real wage brought about by the intertemporal substitution of asset holders now means a further fall in demand, since non-asset holders merely consume their wage income. This generates a further shift in labor demand, so the new equilibrium is one with even lower (compared to the full-participation one) output, consumption, hours and real wage.

This effect could at first sight seem monotonic over the whole domain of $\lambda$ : the more restricted asset market participation, the stronger the contractionary effect on demand and hence on labor demand, and hence the more effective monetary policy. In order to understand why this is not the case, it is helpful to consider the additional distributional dimension introduced by limited asset market participation. The further demand effect that occurs because of non-asset holders has an effect on profits: both marginal cost (wage) and sales (output and hours) fall. The relative
size of these reductions (and the final effect on profits) depends on the relative mass of nonasset holders and on labor supply elasticity. In particular, if labor supply is inelastic enough

Note that such a wage-hours locus implies that the model generates a higher partial elasticity of hours to the real wage, and more so more negative $\delta$ is. Importantly, despite the potential decrease, in general equilibrium actual profits may not fall, precisely due to the negative income effect making asset holders willing to work more; for as a result of this effect hours will increase by more and marginal cost by less, preventing actual profits from falling. In fact, for certain combinations of parameters, shocks or policies our model would not imply countercyclical profits in equilibrium (or at least implies more procyclical profits than a standard full-participation model with countercyclical markups). This is an important point, since it is widely believed that profits are procyclical. ${ }^{23}$ It is also important to note that the negative income effect does not mean that

## ${ }^{25}$ See Section 7 of the working naner version $[71$ for a detailed discussion.

## Bilbiie (2008 JET) 4 contributions in nutshell - cont'd

3. fiscal redistribution key for AD amplification of MP; AD elasticity:

$$
c_{S, t}=\delta_{\tau} y_{t}, \quad \text { where } \delta_{\tau}=\frac{1}{1-\tau^{D}}\left[1-\tau^{D} \frac{\mu}{1+\mu}+\frac{\tau^{D}-\lambda}{1-\lambda} \frac{\varphi}{1+\mu}\right] .
$$

### 4.3. Redistribution restores Keynesian logic

The mechanism of all the previous results relies on the interaction between labor and asset markets, namely income effects on labor supply of asset holders from the return on shares. This hints to an obvious way to restore Keynesian logic relying on a specific fiscal policy rule that shuts off this channel: tax dividend income and redistribute proceedings as transfers to non-asset holders. We focus on the IADL case whereby in the absence of fiscal policy $\delta<0$. To make this point, consider the following simplified fiscal rule: profits are taxed at rate $\tau_{t}^{D}$ and the budget is balanced period-by-period, with total tax income $\tau_{t}^{D} D_{t}$ being distributed lump-sum to all nonasset holders. We focus on the case where profits are zero in steady state. The balanced-budget rule then is $\tau_{t}^{D} D_{t}=\lambda L_{H, t}$ which around the steady state (both profits and transfers are shares of
4. optimal policy in TANK Intro TANK

Proposition 4. If the steady state of the model in Section 3 is efficient the aggregate welfare function can be approximated by (ignoring terms independent of policy and terms of order higher than 2):

$$
\begin{align*}
& \mathbf{U}_{t}=-\frac{U_{C} C}{2} \frac{\varepsilon}{\psi} E_{t} \sum_{i=t}^{\infty}\left\{\alpha x_{t+i}^{2}+\pi_{t+i}^{2}\right\}  \tag{18}\\
& \alpha \equiv \frac{\varphi+\gamma}{1-\lambda}[1-\lambda(1-\gamma)(1+\varphi)] \frac{\psi}{\varepsilon}
\end{align*}
$$

## Caveat: given shock or given recession? (Nakata et al)


$q^{*}(\lambda)$ : fixed recession, $\Delta_{L}$ adjusts endogenously


[^0]:    ${ }^{1}$ Cyclicality? 1. conditional; 2. data?; 3. entry, variety, markups, profits: 15 years of Bilbiie Ghironi Melitz)

