

Illiquidity, Closure Policies and the Role of Lender of Last Resort

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The views expressed here are the authors' and do not necessarily reflect those of the ECB or the Eurosystem.

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Questions:

How does Central Bank (CB) collateral policy affect bank's investment and cash buffer decisions?

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- How does CB collateral policy affect bank's dividend payments and equity issuance decisions?
- What are the benefits and costs of CB collateral policy for equity and debt holders?
- What are the trade-offs involved in equity issuance versus asset liquidation decisions in managing liquidity and default?

1. Develop a dynamic model of a bank with and without a central bank (counterfactual to serve as a benchmark)

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Key Frictions:

- Dynamic loan portfolio adjustments are costly
- Equity issuance is costly
- Bank closures by resolution authorities are costly

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2. Linking to empirical evidence on bank's borrowing capacity and the use of ECB's liquidity

Effects of CB on cash buffers & loan portfolio

 ► Cash buffers → Banks hold lower cash buffers (relative to the counterfactual)

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Effects of CB on cash buffers & loan portfolio

- ► Cash buffers → Banks hold lower cash buffers (relative to the counterfactual)
- Loan portfolio
 - Higher loan portfolio investments in "bad states" (relative to the counterfactual) Mendicino, Jasova and Supera (2018) provide evidence for Portugal

2. Reduces the expected costs of asset liquidations

Effects of CB on equity holders & debt holders

Equity holders

- Banks issue equity less often in the presence of CB
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- ▶ Debt holders → decreases credit spreads of private debt of banks

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- ECB individual balance sheet data (IBSI)
 - around 250 monetary financial institutions (MFIs) from 17 countries

- Around 70% of euro area banking sector
- ► Sample: 2008 2017 (monthly frequency)

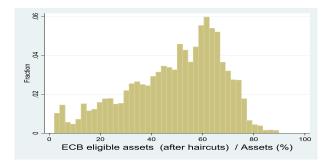
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- Objective: identify the eligible IBSI asset categories and assign ECB haircuts
- Our approach:
 - Debt securities: security data holdings (SHS) and ECB eligibility list
 - Loans: ECB eligibility criteria

 \blacktriangleright Significant cross-sectional variation of $1-\Theta$ in the euro area banking sector



$$1 - \Theta = \frac{\text{Eligible assets (after haircuts)}}{\text{Assets}}$$

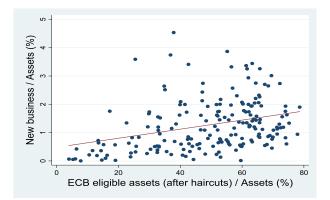


Figure: New loan business vs Borrowing Capacity with ECB

 Banks with a higher borrowing capacity who borrow from ECB invest more in new loans/investments

Related strands of literature

- Role of Lender of Last Resort (LOLR) Bagehot (1873), Bhattacharya and Gale (1987), Goodhart (1999,IF), Freixas, Parigi and Rochet (2000,JMCB), Rochet and Vives (2004,JEEA), Repullo (2005,IJCB), Bindseil (2014), Acharya and Tuckman (2014,IMFer), Nyborg (2016), Santos and Suarez (2016)
- ECB collateral and liquidity policies Brunetti, di Filippo and Harris (2011,RFS), Acharya and Steffen (2015,JFE), Drechsler, Drechsel, Marques-Ibanez and Schnabl (2016,JF), Garcia De Andoain, Heider, Hoerova and Manganelli (2016,JFI), Corradin and Rodriguez (2016), Carpinelli and Crosignani (2017), Crosignani, Faria-e-Castro and Fonseca (2017)
- ► Capital structure and State-contingent liquidity Leland (1994, JF), Asvanunt, Broadie and Sundaresan (2011, IJTAF), Bolton, Chen, Wang (2011, JF), De Nicolo, Gamba and Lucchetta (2014, RFS), Hugonnier and Morellec (2017, JFE).

Margins, Collateral and Liquidity

Bartolini, et.al (RFS, 2010), Garleanu and Pedersen (2009), Ashcraft, Garleanu, Pedersen (2010,NBER), Geanakoplos (2009, 2013), Risk Topography Systemic Risk and Macro Modeling (2014), Woodford (2015)

Model

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Bank's balance sheet

Assets

• A - Assets' portfolio - loans and securities (hit by shocks)

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• $W > 0 \rightarrow Cash$

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Liabilities

- ▶ *P*_D Risk-free deposits (deposit insurance is implicit)
- P Risky short-term external debt (endogenous credit spreads) → rollover risk (subject to premature freezes)

- $W < 0 \rightarrow CB$ liquidity (illiquidity)
- *E* Equity (or raise equity) (**illiquidity**)

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Bank does not hold cash and borrow from CB at the same time $% \left({{{\rm{CB}}} \right)$

Implicit CB objective: mitigate asset sales in "bad states"

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Implicit CB objective: mitigate asset sales in "bad states"

▷ Θ is average haircut on eligible assets as a percentage of all assets (eligible and ineligible)

- $r + s^{CB}$ is borrowing rate against eligible collateral
- CB debt is senior to bank's existing debt

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We calibrate the model to the mean observed borrowing capacity $1-\Theta:$ $\approx 50\%$

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• Closure: $\frac{\text{Equity Book Value}}{\text{Assets}} < 5\%$

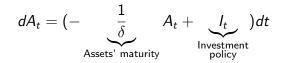
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- how much to invest or disinvest in the assets' portfolio
- how much cash to retain
- how much CB liquidity to draw or equity to raise
- ► **State variables:** *W* (cash) and *A* (Assets' portfolio)
- Closure: $\frac{\text{Equity Book Value}}{\text{Assets}} < 5\%$
- Scenarios:
 - 1. Counterfactual (No CB): Costly issuance of equity (W = 0)
 - 2. **CB liquidity:** (W < 0) vs Costly issuance of equity

State variables

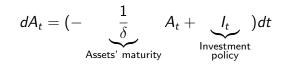
1. Assets' portfolio A



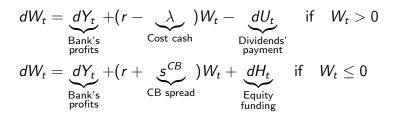
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State variables

1. Assets' portfolio A



2. Cash W



$\mathsf{Cash}\ W$

► Revenues' shock X

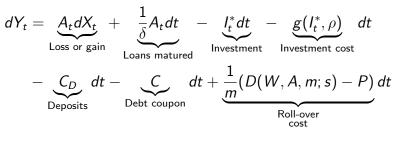
$$dX_{t} = \mu_{X}dt + \sigma_{X,1}\rho \underbrace{dZ_{1,t}}_{\text{Systematic}} + \sigma_{X,2}\sqrt{1-\rho^{2}} \underbrace{dZ_{2,t}}_{\text{Idiosyncratic}}$$

Cash W

Revenues' shock X

$$dX_t = \mu_X dt + \sigma_{X,1} \rho \underbrace{dZ_{1,t}}_{\text{Systematic}} + \sigma_{X,2} \sqrt{1 - \rho^2} \underbrace{dZ_{2,t}}_{\text{Idiosyncratic}}$$

Bank's profits



where D(W, A, m; s) market value of debt (*m* maturity)

Model's results

Parameters

-

Va	ria	ble

Symbol Value

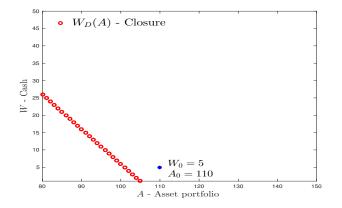
Face value deposits	P_D	70
Face value external risky debt	P	30
Years of reimbursed loan	δ	10
Years to maturity debt	т	1
Mean Ioan-shock	μ_{X}	7.2%
Volatility of systematic loan shock	$\sigma_{X,1}$	7.5%
Volatility of idiosyncratic loan shock	$\sigma_{X,2}$	2.5%
Central bank haircut	θ	50%
Central Bank penalty rate	s ^{CB}	0.50%
Liquidation value	1	70%
Marginal cost of equity issuance	γ	6%
Fixed cost of equity issuance	Φ	1%

- When the bank is solvent but runs of cash W
 - $1. \ \mbox{Liquidation:}$ the bank is liquidated
 - 2. Equity funding: the bank can issue equity incurring a cost

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- When the bank is solvent but runs of cash W
 - $1. \ \mbox{Liquidation:}$ the bank is liquidated
 - 2. Equity funding: the bank can issue equity incurring a cost
- Bank holds cash to avoid liquidation or costly issuance of equity

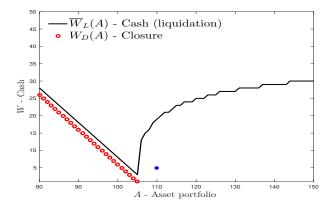
Counterfactual: Map (I)



Closure by resolution authorities when

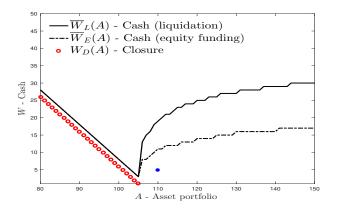
$$\frac{\text{Equity Book Value}}{\text{Assets}} = \frac{\max(A + W - P_D - P, 0)}{A + W} \le 5\%$$

Counterfactual: Map (II)



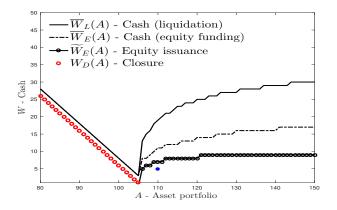
- Liquidation: the bank cannot issue equity when A > 105 and W = 0
- Bank's cash buffer decisions as a function of the state of the economy

Counterfactual: Map (III)



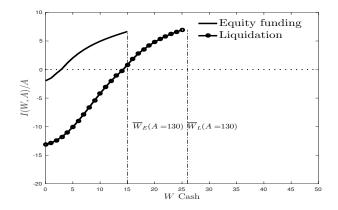
Equity funding: the bank holds less cash balance when can issue equity

Counterfactual: Map (IV)



Equity issuance: the bank issues more equity at higher asset levels to alleviate the illiquidity problem

Counterfactual: Investment



- ▶ When the bank runs of out cash → asset sales (negative intestment)
- Larger asset sales when the cost of issuing equity is extremely high (liquidation)

Two constraints:

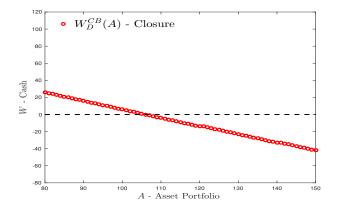
 $1. \ {\rm Bank's}$ borrowing capacity with CB is

$$0 > \underbrace{W(A)}_{\mathsf{CB \ liquidity}} \geq -(1 - \underbrace{\Theta}_{\mathsf{Haircut}})A$$

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2. Bank has to be solvent

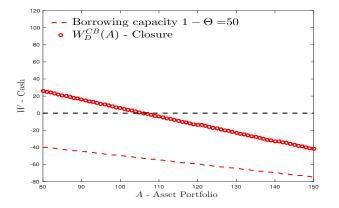
CB: Map with $\Theta = 50\%$ (I)



Closure by resolution authorities when

$$\frac{\text{Equity Book Value}}{\text{Assets}} = \frac{\max(A + W - P_D - P, 0)}{A} \le 5\%$$

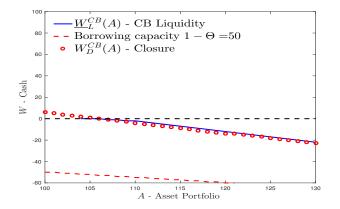
CB: Map with $\Theta = 50\%$ (II)



Borrowing capacity with CB

$$-(1- \underbrace{\Theta}_{\mathsf{Haircut}})A$$

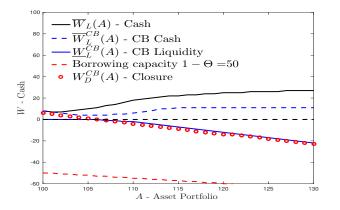
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Borrowing capacity with CB

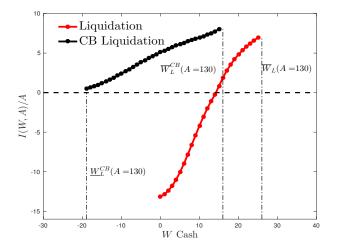
$$0 > \underbrace{W(A)}_{CB \text{ liquidity}} \geq -(1 - \underbrace{\Theta}_{Haircut})A$$

CB: Map with $\Theta = 50\%$ (IV)



Lower cash buffers in the presence of CB

CB: Investment



Higher loan portfolio investments in "bad states" (relative to the counterfactual) ◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへぐ

CB: Effects of CB policies (I)

Table:
$$W_0 = 5$$
, $A_0 = 110$, $P = 70$, $P_D = 30$ and $\overline{E} = \max(W_0 + A_0 - P - P_D, 0) = 15$

	(1)	(2)	(3)	(4)
	Θ	$E(W_0, A_0)$	P_D	$E\left[\frac{I^*(W_0,A_0)}{A_0}\right]$
			Spread	
	(%)		(%)	(%)
Liquidation				
	100	15.48	11.94	-5.08
	50	19.01	6.23	15.29
Equity financing				
	100	21.93	1.07	16.45
	50	26.22	0.52	17.39

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CB: Effects of CB policies (II)

CB policies generally lead to

1. Greater investment in new loans and lower asset liquidations

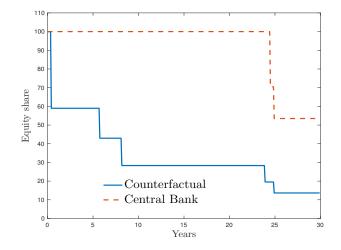
- 2. Larger equity values
- 3. Lower credit spreads
- 4. Lower cash buffers than would be desirable

CB: Effects of CB policies (II)

CB policies generally lead to

- 1. Greater investment in new loans and lower asset liquidations
- 2. Larger equity values
- 3. Lower credit spreads
- 4. Lower cash buffers than would be desirable
- What about incentives of the bank to issue equity or cut dividends?
 - Monte carlo simulation of the model (50,000 paths for 30 years)

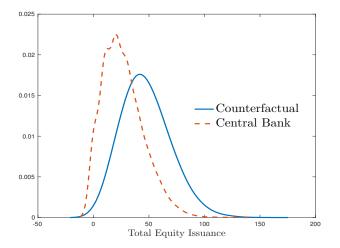
CB: Equity share of insider equity holders



Possible path: In the presence of CB less dilution of the stakes of the incumbent equity holders

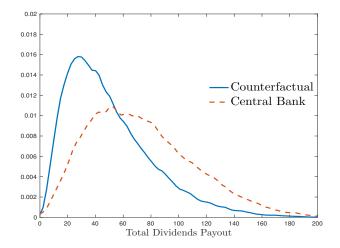
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CB: Distribution of equity issuance after 30 years



When the bank has access to the CB liquidity, fewer issuances of equity (\rightarrow region shifts to the left)

CB: Distribution of dividends payout of incumbent equity holders



When the bank has access to the CB liquidity, higher dividend payouts to current equity holders

Policy experiments (I)

Table: $W_0 = 5$, $A_0 = 110$, P = 70, $P_D = 30$ and $\overline{E} = \max(W_0 + A_0 - P - P_D, 0) = 15$

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Base case	19.01	6.23	15.29
$s^{CB}=1.5\%$ ($\mathbf{0.5\%}$)	17.87	7.95	15.02
$\Theta=80\%$ (50%)	18.89	6.62	14.57
Closure = 6%(5 %)	17.63	6.75	12.95

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$\Theta=80\%$ (50%)	23.69	0.54	17.27
Closure = 6%(5%)	22.12	0.59	14.95

 Equity values, credit spreads and expected investment not very sensitive to CB policy changes

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- Equity values, credit spreads and expected investment not very sensitive to CB policy changes
- ► ↑ capital requirements decreases expected investment

 We provide a dynamic framework for bank's cash buffer and asset size decisions with and without a CB

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- CB policies reduce the incentives of the bank to issue equity or cut dividends
- We provide an empirical measure of borrowing capacity of banks at the CB liquidity facilities

Thanks!

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Additional slides

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$1-\Theta$: ECB collateral and liquidity

ECB accepts

- debt securities (conditional on eligibility criteria)
- some type of loans

Table: October 2008 - Dec. 2016 - * Over collateral ** Over assets

	Mean	St. Dev.	P10	P90
Collateral (euro mil.)	5,985	11,656	0	17,115
Sov. Debt Securities* (%)	17.58	27.84	0.00	57.66
Other Debt Securities* (%)	67.45	33.63	1.46	100.00
Loans* (%)	14.97	26.06	0.00	53.06
Implied ECB Haircut (%)	8.97	55.06	2.10	19.62
Liquidity (euro mil.)	2,101	5,202	0	6,308
Dummy Liquidity	47.92	49.96	0.00	100.00
Liquidity** (%)	2.21	3.92	0.00	7.57
Liquidity* (%)	21.58	30.45	0.00	74.80

$1 - \Theta$: Debt securities (I)

Data (quarterly):

- 1. ECB eligibility: ECB haircut at ISIN-level
- 2. Securities Holding Statistics: quarterly ISIN-level holdings by investor sector (including MFIs) at country level (i.e. ISIN-level holdings of german banking sector)
- ▶ Merge (1) + (2) :
 - Identify whether an ISIN is eligible
 - Compute securities holdings shares of each country MFI sector split in (IBSI balance sheet items):
 - Issuer sector (Sovereign, MFIs, ...)
 - Issuer geographical location (domestic, euro area, other euro area)

$1 - \Theta$: Debt securities (II)

Table: ECB Eligibility and Haircuts (numbers in %)

IBSI categories		ECB Eligible			ECB Haircut θ	
Issuer	Mean	P10	P90	Mean	P10	P90
Sovereign						
Domestic	97.86	94.19	100.00	2.80	1.81	3.25
Other Euro Area	94.69	90.15	99.57	3.41	2.10	4.66
Extra Euro Area	28.29	3.53	51.32	3.79	2.29	5.67
Non MFI						
Domestic	56.37	16.13	80.65	10.61	5.11	21.30
Other Euro Area	68.32	46.19	83.81	9.30	5.55	13.68
Extra Euro Area	27.13	9.96	38.82	9.90	5.12	17.90
MFI						
Domestic	74.10	47.20	90.82	8.16	5.26	11.86
Other Euro Area	72.04	50.61	87.83	8.13	5.84	10.85
Extra Euro Area	50.05	26.09	75.62	6.49	4.48	8.40

 $\Theta_{i,j,t} = \mathsf{Eligible}_{i,j,t} \times \theta_{i,j,t} + (1 - \mathsf{Eligible}_{i,j,t}) \times 100\%$

MFI sector country *i*, security debt type *j* and quarter t_{\pm} , t_{\pm}

$1 - \Theta$: Loans (I)

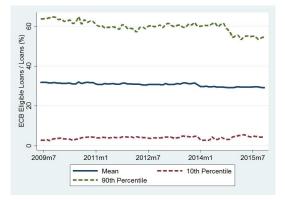
Rely on ECB eligibility criteria for loans:

- Compute (weighted) average ECB haircut for the item loans in IBSI
- ► ECB eligible loans easily identifiable on IBSI loan items:
 - Issuer: public sector, non-financial corporations (non-MFIs), international and supranational institutions
 - Place of establishment: euro area
 - Currency: euro
- We cannot use other eligibility criteria
 - Credit assessment: ECB haircuts depends on on the credit quality

Minimum size of loans (i.e. euro 500,000)

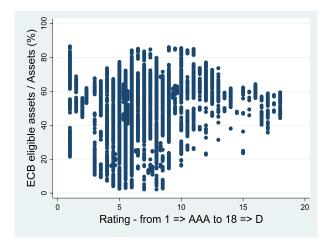
$1 - \Theta$: Loans (II)

Significant cross-sectional variation of ECB eligible loans



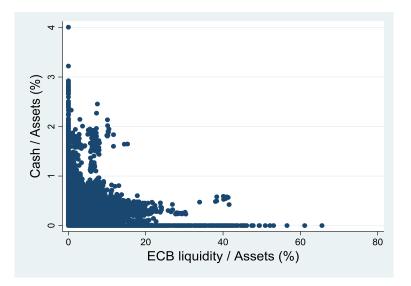
 $\Theta_{j,t} = \text{Eligible}_{j,t} \times \theta_{j,t} + (1 - \text{Eligible}_{j,t}) \times 100\%$ "Upper bound" loan type j and quarter t but Eligible_{j,t} = 0 or 100%

Is $1 - \Theta$ related to bank's risk? Not really



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Model assumption: Cash vs ECB liquidity



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Model: Debt structure

- Stationary debt structure (C, P, m) as in Leland (1998, JF)
 - ▶ aggregate face value *P* and coupon payment *C*
 - each bond has maturity m
 - debt expirations are uniformly spread across time over $(t, t + dt) \frac{1}{m} dt$ fraction of the bonds matures
- ▶ Maturing bonds are replaced with bonds with (*C*, *P*, *m*)
- Over (t, t + dt) the debt cash flow is (from equity holders' point view)

$$-C+\frac{1}{m}\left[D(W,A,m;s)-P\right]$$

D(W, A, m; s) market value of new bond (s spread)
when bond price drops, equity holders face losses

Dividend's payout and equity funding

Dividends' payout

$$\underbrace{E(W, A)}_{\text{Equity value}} = E(\overline{W}(A), A) + \underbrace{(W - \overline{W}(A))}_{\text{Cash distribution}} \qquad W > \overline{W}(A) > 0$$

where $\overline{W}(A)$ is the payout boundary

Equity funding: *M* liquidity amount after equity issuance

$$E(0, A) = E(M, A) - (1 + \underbrace{\gamma}_{\text{Marginal cost}})M \text{ no CB}$$
$$E(\underline{W}(A), A) = E(\underline{W}(A) + M, A) - (1 + \gamma)M \text{ with CB}$$

where $\underline{W}(A) < 0$ is the CB boundary

Numerical approach

Iterative procedure for a sequence of fixed boundary problems:

- 1. At time T
 - Equity and debt value correspond to book values
 - The bank does not hold or borrow cash
- 2. At time T 1 the inaction region $(\overline{W}(A), \underline{W}(A))$ is relaxed
 - 2.1 Jointly solve the PDEs associated with the equity E(W, A) and debt D(W, A) subject to the boundary conditions at $(\overline{W}(A) \text{ and } \underline{W}(A))$ (FD method)
 - 2.2 Compute the optimal investment policy I^*
 - 2.3 Boundary update procedure verify whether smooth pasting conditions at W(A) and W(A) are satisfied
 2.3.1 No: update the inaction region and solve 2.1 with 2.2
 2.3.2 Yes: convergence
- Move backward and repeat the procedure 2 for any time t to get a sequence of regions of inaction