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On the effectiveness of loan-to-value regulation in a multiconstraint framework

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Abstract

Models in the infinite horizon macro-housing literature often assume that borrowers are constrained exclusively by the loan-to-value (LTV) ratio. Motivated by the Swedish micro-data, I explore an alternative arrangement where borrowers are constrained by the feasibility of repayment, but choose a house of maximum permissible size conditional on the LTV restriction. While stricter LTV limits are often considered as a measure to tackle the rise in household indebtedness, I find that policy designed to lower the maximum permissible LTV ratio may actually leave the debt-to-GDP ratio unchanged and increase housing prices in equilibrium if borrowers are bound by two constraints at the same time. In a model with occasionally binding constraints, I show that also for the analysis of the short-run effects of different policies, the consideration of multiple constraints, possibly binding at the same time, is important. The effectiveness of LTV as a measure to tackle the rise in indebtedness has to be reassessed and is likely lower than previously shown.

Keywords: borrowing constraints, household indebtedness, macroprudential policy, housing prices, loan-to-value ratio, debt-service-to-income ratio
JEL-Classification: E32, E44, E58, R21

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1. Introduction

Which macroprudential measures are most effective in addressing household indebtedness? Empirical studies studying this question face the challenge of the coexistence and comovement of multiple measures in the same country at the same time, which makes measuring the effect of one single policy difficult. This identification difficulty makes a strong case for studying the impact of these measures in a structural model in which different channels can be shut down, and thus, separated. Among popular macroprudential tools, many theoretical papers advocate the use of stricter loan-to-value (LTV) policy as a very effective measure reducing household indebtedness, which lowers house prices as well (see Chen and Columba, 2016 and Finocchiaro, Jonsson, Nilsson, and Strid, 2016 for Sweden, Alpanda, Cateau, and Meh, 2014 for Canada), but they do mostly so in the models where the LTV constraint is the only constraint imposed on borrowers, following Iacoviello (2005). This may overstate the effectiveness of LTV limits in affecting the debt in real economies, where multiple constraints are applied to borrowers and may interact.

In this paper, using a simple real business cycle model with short-term debt, and its New-Keynesian extension with long-term debt, I argue that in a framework where both LTV limits and debt repayment limits (debt service to income ratio - DSTI) are imposed on borrowers, tighter LTV regulation may have no effect on household indebtedness ratios (defined as debt to GDP or debt to income) and may actually lead to an increase in housing prices in equilibrium. This happens if borrowers are both at their LTV limit and at the DSTI limit at the same time. Bindingness of two borrowing constraints imposes a direct relation between borrowers' labor income and the value of their housing stock implying a constant debt to GDP ratio for different LTV ratios, equal to the DSTI limit. Thus, changing LTV will not affect debt to GDP ratios, while changing DSTI will. Under a realistic distribution of borrowers across different constraints, in equilibrium, the effectiveness of LTV in influencing debt-to-GDP ratios is greatly reduced. Apart from analyzing long-run implications of different macroprudential policies, in order to study business cycle implications of two possibly binding constraints, I consider a model with occasionally binding constraints with four regimes allowing for different combinations of slack and binding constraints. Even if one of the constraints is slack in a given period, its presence and potential future bindingness affects the behaviour of indebted households. Hence, the interaction of the LTV and DSTI constraints is important for the dynamics of indebtedness ratio and housing prices in the economy.

While the effect of LTV regulation has been extensively studied in the literature, the interaction between the LTV constraint and the DSTI constraint has not gained much attention so far, apart from a recent study of Greenwald (2016).[1] However, their coexistence is fairly common

[1] Admittedly, the heterogeneous agents literature studying household borrowing in an overlapping generations

both in advanced and emerging economies and increasingly many countries are considering implementing DSTI measures along with existing LTV measures, given that it has been found that sound debt repayment ratios contribute to financial stability and reduce banks' portfolio risk (see Dietsch and Welter-Nicol, 2014). In some countries, like Canada or Estonia, regulation explicitly sets the upper limit on the LTV and DSTI ratios, in other, like Sweden or France, it is the banking practice to look at both components while deciding on the loan application.

If many borrowers in a given economy in addition to the LTV constraint are bound also by the DSTI constraint, lowering LTV is a very ineffective policy, if the aim is to reduce debt to income or debt to GDP. In the extreme scenario in which all borrowers are both at their LTV and at their DSTI limits, stricter LTV policy not only does not influence debt ratios at all, but it also drives house prices up in the equilibrium. Ceteris paribus, if the debt level is determined by a DSTI limit and one unit of housing can pledge less collateral, its value has to increase if it has to collateralize the same amount of debt. A similar mechanism is described in an example in a recent paper by Greenwald (2016). It presents a model for the U.S. in which new borrowing is determined by an LTV and a payment to income constraint. Borrowers switch between being bound by each of constraints and in equilibrium, only one constraint is binding for a given type of borrower. This is different from the setup presented in this paper. First, I consider a model with DSTI and LTV constraints and show that in equilibrium both of them can bind at the same time. Second, I simulate an economy with occasionally binding constraints. As a result of shocks hitting the economy, either one or both considered constraints may become slack and the dynamic model simulations take into account the existence of four possible regimes in which: both constraints bind, only the DSTI constraint binds, only the LTV constraint binds, or neither the DSTI, nor the LTV constraint bind. As Guerrieri and Iacoviello (2017) show, taking into account the occasionally binding constraints has crucial implications for considering possible asymmetries arising during the business cycles. However, while Guerrieri and Iacoviello (2017) show it for an LTV-only model in the presence of very large housing preference shocks, I demonstrate that in the model with multiple occasionally binding constraints, sizable asymmetries arise even in the presence of relatively small shocks.

The mechanism described in this paper is crucial for the analysis of macroprudential policies in countries with multiple constraints. It may be relevant even for countries without an established DSTI limit, if borrowers, aside of the banks, impose such a limit on themselves. There is evidence for the euro area and U.S. that the debt service to income ratios are approximately stable in the long run (see European Central Bank, 2005; BIS, 2017; Federal Reserve Board, 2017). Obviously, the extent to which the mechanism presented in this paper will be relevant

setup (see e.g. Iacoviello and Pavan, 2013 and Hull, 2015) takes into account the coexistence of two constraints. However, the interaction of constraints is not explicitly studied in these papers. In the infinite horizon setup, Gelain, Lansing, and Mendicino (2013) consider an example with a combined borrowing constraint, where different weights are attached to the LTV and to the loan-to-income assessment.

for a given country can be only assessed using the micro-data with detailed information on the constraints faced by individuals. Having access to the Swedish data, I am able to make such an assessment for Sweden, showing that the share of constrained borrowers that are constrained both by LTV and DSTI constraint is non-negligible. These are predominantly young households, most likely to be relatively more constrained than an average borrower due to a short saving history and hence, potentially low downpayment, and a relatively lower wage income. First-time borrowers often maximize their loan amount as given by the DSTI limit, and scrounge money from parents and relatives for the minimum downpayment. Moreover, in the broader perspective, the dynamic implications of multiple possibly binding constraints become important in events of severe crises, when both house prices go down and the income of the households suffers due to an increased unemployment rate. The Swedish 1990s banking crisis is an example of such an environment, as is the recent Great Recession in the U.S.

While my paper mostly refers to the theoretical macroeconomic literature in the infinite horizon setup that has predominantly neglected the existence of DSTI constraint so far, it is important to note the existence of empirical studies that focus on establishing links between different macroprudential measures and changes in them and credit. Some of them focus on DTI and LTV limits (see Cerutti, Claessens, and Laeven, 2017 and Crowe, Dell'Ariccia, Igan, and Rabanal, 2013) and show that there is a mixed empirical evidence of longer run effectiveness of macroprudential measures, less than the one suggested by theoretical contributions, which may be due to interaction between particular constraints that was mostly absent in the theoretical frameworks so far. DSTI limits seem to be more prevalent than DTI ratios. Akinci and Olmstead-Rumsey (2017) perform panel regressions for 57 countries in years 2000-2013 to assess the effectiveness of different measures (including DSTI) in affecting credit growth and house price appreciation. They note that the LTV and DSTI caps are often used simultaneously, which may be a problem for identification of their separate effects in empirical work and makes a strong case for studying the impact of these measures in a structural model in which different channels can be shut down, and thus, separated. Akinci and Olmstead-Rumsey (2017) conclude that DSTI caps have a stronger effect on housing credit than the LTV caps, which supports my theoretical results.

In the following, in section 2, I present some empirical evidence with emphasis on Sweden, followed by the exposition of the basic idea of the paper in a simple real business cycle model with one-period debt in section 3. I compare the long-run and short-run implications of the model with two constraints to a model with LTV and DSTI constraint only. In order to be able to compare a broader range of macroprudential instruments in a realistic setup, in section 4, I extend the simple model with one-period debt to a New-Keynesian setup with long-term debt, where I show the equilibrium effects of changes in the LTV ratio, amortization rate, and DSTI ratio, as well as dynamic responses of economies to different shocks, comparing the multiple

Country	LTV-limit	DSTI-limit
Canada	95%	39-44%[a]
China	70%	50%
Cyprus	80%	35%
Estonia	85%	50%
Hong Kong	70%	50%
Hungary	80%[b]	10-60%
Israel	75%	50%
Korea	50-70%	50-60%
Lithuania	85%	40%
Netherlands	100%	10-38%[c]
Singapore	80%	60%
Slovenia	80%	50%

Notes: The table has been created using the information from Eesti Pank (2014), European Systemic Risk Board (2015), Bank of Slovenia (2016), European Systemic Risk Board (2017), International Monetary Fund, 2017.

[a] The limit for the gross and total debt service respectively.
[b] This is the limit for loans in HUF, for other currencies it is lower.
[c] The DSTI limit depends on the income.

Table 1: The contemporaneous usage of explixit LTV and DSTI limits in different countries

constraint model with a DSTI-only and LTV-only model.

2. Empirical evidence

The mechanism presented in this paper may be observed in countries, in which multiple simultaneously binding constraints are applied to loan applicants. In this paper, I focus on LTV and DSTI measures whose usage is highly correlated both in advanced and in emerging economies (Akinci and Olmstead-Rumsey, 2017).[2] Table 1 presents the regulatory limits set in advanced and emerging economies that implement both of these constraints at the same time.

Countries presented in Table 1 set explicit limits on the discussed ratios. In other countries, it is the banking practice to look at both components while deciding on the loan application: in Brasil, Colombia, Malaysia and Thailand banks tend to put more weight on the assessment of borrowers' financial capacity to settle loan installments rather than their LTV (de Carvalho, Castro, and Costa, 2014 and He, Nier, and Kang, 2016), in France, along with a downpayment requirement, banks use a so-called 33% rule under which the debt burden should not exceed

[2] Note that the DTI limit, present in some countries along with a LTV regulation, is very similar in spirit to a DSTI limit. However, the DSTI limit changes with changes in interest rates, while DTI not. In this paper, I focus on the DSTI limit.

one third of households' income (Dietsch and Welter-Nicol, 2014). Greenwald (2016) reports regulatory payment to income and LTV limits for the U.S. In some countries, banks lending practices follow the regulatory guidelines, that are, however, not formal requirements. This is the case of Sweden, where banks perform a discretionary income calculation along with the assessment of the downpayment. Such an assessment is also performed in Latvia, Poland, Romania, Slovakia and in the Czech Republic (see Fell, 2015, European Systemic Risk Board, 2016, European Systemic Risk Board, 2017).

2.1. The mortgage process in Sweden

In Sweden, the Swedish Financial Supervisory Authority (Finansinspektionen) issues general guidelines to banks regarding the mortgage lending. Since 1st October 2010, a guideline regarding the maximum LTV applies: *"When a firm grants a loan collateralised by a home the loan should be limited such that the loan-to-value ratio for the home does not exceed 85 per cent of the home's market value at the time the loan is granted. This limitation to the loan-to-value ratio should be established in the firm's credit instructions, where applicable."* (Finansinspektionen, 2010b). In 2004, a guideline about a DSTI-type of assessment for credit institutions and investment firms has been issued: *"The credit assessment should be based on information that provides an accurate picture of the credit applicant's financial status. It should include a sensitivity analysis of the credit applicant's repayment capacity and an assessment of the risk for a deterioration in the value of collateral."* (Finansinspektionen, 2004). Following this guidline, in the banking practice, the Swedish DSTI constraint takes the form of a 'discretionary income' limit (see Finansinspektionen, 2010a, Sveriges Riksbank, 2014 and Li and van Santen, 2017). Banks use the so-called KALP ("kvar att leva på" - the amount left to live on, also called discretionary income) calculation to establish the borrowing limit of a household. This calculation takes into account borrowers after-tax income, transfers, minimum consumption, housing expenses (including amortization and loan payments) and a stressed-interest rate to ensure that the borrower can cope with the loan installments even in an event of an interest rate hike. Despite being slightly different, in modeling terms, it has similar implications for households' borrowing limit as the DSTI constraint (see Appendix A) and thus, in the model part I choose to work with the DSTI constraint for the sake of model universality.

In the Swedish housing market, housing transactions are a result of a bidding process, in which the highest bidder usually acquires the apartment. Before participating in the bidding process, future borrowers obtain from a bank a promise of loan (so-called lånelöfte) stating their borrowing limit. Given household's income, house prices and a minimum downpayment, the KALP constraint determines the borrowing amount and the LTV constraint the value of the house that can be purchased by the borrower (in reality, more choices regarding housing can be made that in the model: size, location, new- vs old-construction etc.). If the discretionary income calculation sets a limit of the loan to B and the LTV requirement is 85%, the maximum value of the house that that customer can acquire (without taking unsecured debt) is $B/LTV = 1.18B$. Given that information and prevailing house prices in different areas, the agent makes the housing decision.[3]

Naturally, the guidelines help the banks in establishing the upper loan limit and if the loan applicant decides to borrow less than the limit, it is possible that neither of the constraints binds. The extent of bindingness of constraints is an empirical question and I address it by looking at the micro-evidence for Sweden.

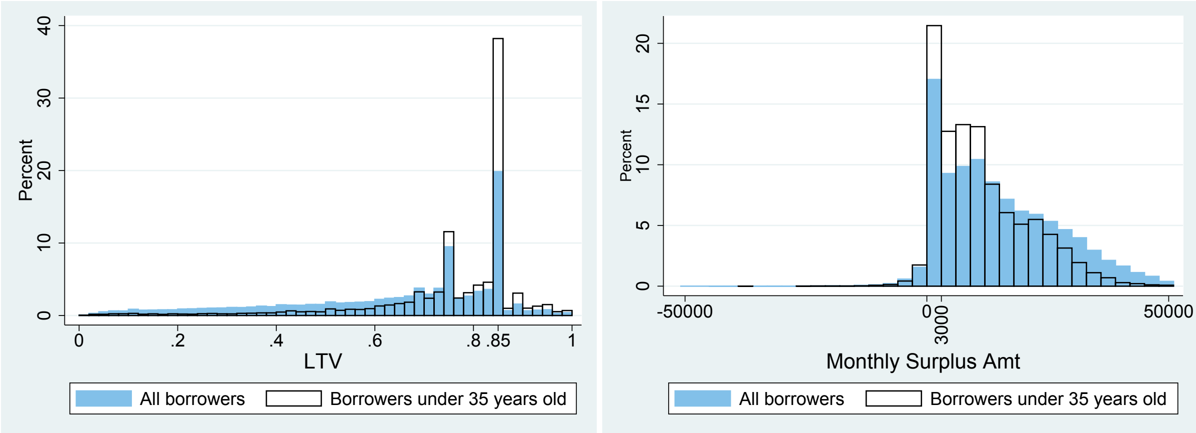
2.2. Descriptive evidence from the Swedish micro-data

To establish the economic importance of the mechanism described above, I refer to the micro-data from the Swedish mortgage survey conducted every year by Finansinspektionen, the Swedish Financial Supervisory Authority. The data used covers all new mortgage loans given out by 8 largest banks in Sweden in years 2011-2015 in a few weeks between August and October.[4] The delivered data contains detailed borrowers and loan characteristics.[5] The

[3] In a static setup with fixed income, the LTV limit determines the housing choice and not the borrowing amount as in a traditional Iacoviello (2005) setup. In the dynamic setup, both housing and labor decisions will be adjusted contemporaneously to account for the existence of both constraints. In this situation, tighter LTV regulation leads to an absolute decrease in household borrowing. However, the debt to GDP ratio does not go down, as wage income, debt and output comove.

[4] Note that the 2016 data is also available, but following Finansinspektionen's request, banks changed in this year their definition of the discretionary income, so that the actual discretionary income used by the banks while granting the loan is not available for 2016. Under certain assumptions, a proxy of banks' discretionary income calculation can be computed and the distribution of borrowers is very similar to years 2011-2015, not changing the fractions used for the calibration of the theoretical model. However, given the coherent variable definitions in years 2011-2015, I choose to restrict my analysis in this paper to these years. Note that before June 2016, no amortization requirement was formally present in Sweden. Since June 2016, a minimum amortization requirement is applied to new mortgages with an LTV above 50%: 1 percent annually for mortgages with LTV between 50 and 70%, and 2 percent annually for mortgages with an LTV above 70% (see Finansinspektionen, 2016). This directly affects the KALP calculations of the banks and could change the distribution of new borrowers across constraints considerably. However, since after the introduction of the amortization requirement, banks lowered other costs included in their discretionary income calculation to offset the strenghtened amortization rules (which can be thought of as a simultaneous reduction in the DSTI limit), in practice, the distributions of borrowers across the constraints did not change much.

[5] Given the survey nature of the dataset, it covers only a subset of new mortgage loans in Sweden every year. However, I compare the aggregate statistics in the survey with the implied characteristics from the UC credit dataset covering all borrowers in Sweden on a monthly basis from 2011 to 2016. Despite the larger size, the UC credit dataset is not suitable for the analysis presented in this paper, as it does not contain house values for borrowers living in apartments (thus, only house owners can be taken into account when considering the LTV ratio and even then, the housing values in the dataset are not corresponding to market values), neither does it contain detailed information on the loan characteristics, such as the type and level of the interest rate, monthly amortization etc. Thus, after making sure that the mortgage survey statistics is representative for the population of new borrowers in Sweden by comparison of borrowers' characteristics in both datasets, I continue the analysis with the mortgage survey data.



(a) The distribution of LTV 2011-2015

(b) The distribution of KALP 2011-2015

Figure 1: Distributions of constraints for new borrowers in Sweden, 2011-2015

mortgage survey data, apart from detailed mortgage characteristics, contains bank's evaluation of the bindingness of the borrower's KALP constraint. If the KALP value in the dataset is 0 (or in exceptional cases, below 0), it indicates that the household maximized its borrowing limit.

Figure 1 presents the distribution of values for LTV and KALP among the borrowers for years 2011-2015, with the whole population indicated with blue bars, and the young population, defined as under 35 years, indicated by transparent bars with black borderline. The bunching of borrowers around the limits suggested by regulatory guidelines indicates that banks follow the guidelines even though they are not established as strict regulation. The left panel presents the histogram for LTV ratios (both collateralized debt and uncollateralized debt to finance the mortgage is taken into account in the LTV calculation, hence values of over 85% are possible), with a visible spike around 85%, the regulatory guideline for new mortgages in Sweden. The spike is more accentuated for borrowers under 35 years old who are more likely to be truly first borrowers (unfortunately, the dataset may contain people who switch banks but are not first-time mortgage borrowers). They can be more affected by constraints due to a shorter work and thus, savings, history. The right panel of Figure 1 presents the distribution of KALP values among the surveyed population of borrowers. Some first time mortgage borrowers have the KALP below 0, which indicates that they borrowed up to the maximum amount and beyond. Moreover, we see that the distribution of KALP values is skewed towards zero, with a spike close to 0, indicating that many more borrowers are approximately constrained by the discretionary income calculation of the bank. Similarly to the LTV distribution, we note that for the borrowers under 35 years old the distribution is skewed towards the maximum specified in Finansinspektionen's guidelines. The shape of the histograms and their spikes around regulatory limits indicate that both constraints are important for the borrowers (and banks).

Figure 1 shows the whole distribution of new borrowers in Sweden across different constraints, which does not mean that they are all actually credit constrained. Given the regulatory guidelines, only borrowers that are reasonably close to the suggested thresholds should be considered credit constrained. For the purpose of this paper, I define a KALP-constrained agent as an individual whose KALP-value in the database is below 3000 SEK (around 300 EUR, it means that a given person has 300 EUR monthly left after the repayment of all debt obligations, housing expenses and minimum consumption expenses in the stressed interest rate scenario). Similarly, given a regulatory LTV limit of 85%, I define as LTV-constrained agents individuals with an LTV larger than 80%. To test for the importance of a (non-existing) debt to income (DTI) constraint, I define DTI-constrained agents as individuals with a DTI larger than 6. The results of this exercise are presented in Figure 2. On average, in years 2011-2015, 33 percent

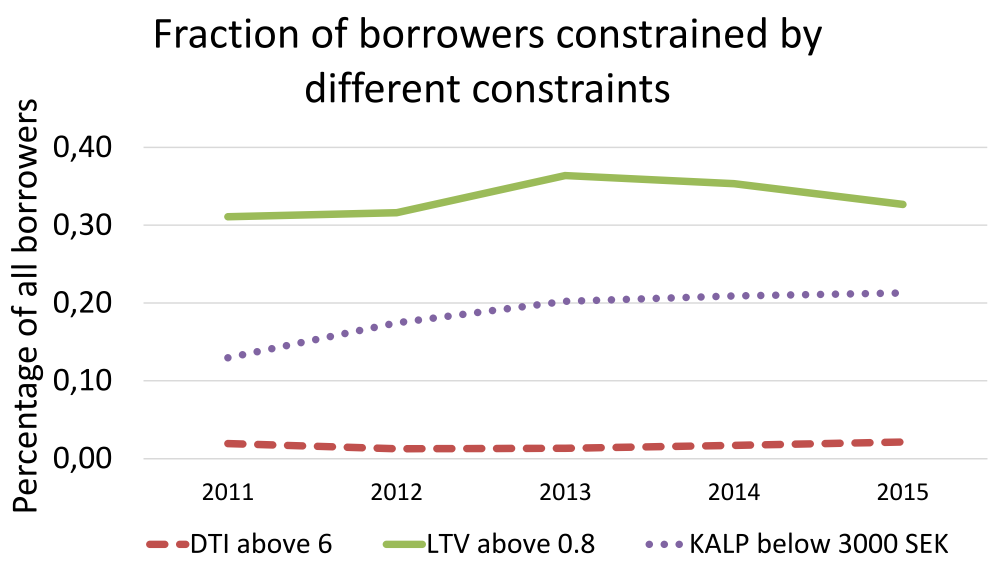


Figure 2: Fraction of borrowers constrained by different constraints in the Swedish mortgage survey data, 2011-2015

of new borrowers were constrained by the LTV constraint, 19 percent of new borrowers were KALP-constrained, and only 2 percent of new borrowers can be defined as DTI-constrained. I consider the remaining borrowers as non-constrained. Given the nonexistence of a DTI constraint in Sweden and a small number of new borrowers that could be potentially constrained by such a regulation, in the following, I focus on the LTV- and KALP-constrained borrowers and I define a subset of new borrowers consisting of people having an LTV higher than 80% and the KALP value below 3000 SEK. Figure 3 presents the distribution of these constrained borrowers among the main types of constraints (aggregate for years 2011-2015). We can see that the share of borrowers constrained by both the LTV- and the KALP-constraint is non-negligible, as it accounts for 14%. Moreover, in the subset of constrained borrowers, ”merely” 60% of households are constrained only by the LTV-constraint - ”merely” with respect to the models used for analysis of the effectiveness of LTV regulation for Sweden, which assume that this share is 100% (see Chen and Columba, 2016 and Finocchiaro et al., 2016). It might seem that the occurrence of agents constrained by both constraints is a relatively unlikely phenomenon, but if we think of a young borrower with little savings and low income, the 14% figure is not surprising. First-time borrowers often maximize their loan amount as given by the DSTI limit, and scrounge money from parents and relatives for the minimum downpayment. In the following, in Section 3, I formulate a simple model with one-period debt to illustrate the idea behind the two borrowing constraints binding at the same time and explain the intuition looking at the long-run and short-run implications of LTV regulation is such a setup, compared to a model with LTV constraint only or DSTI constraint only. In section 4, I extend this setup to consider long-term debt and in a model matching the Swedish data, I study the equilibrium effects of changes in macroprudential policies given the distribution of borrowers across constraints as presented in Figure 3.

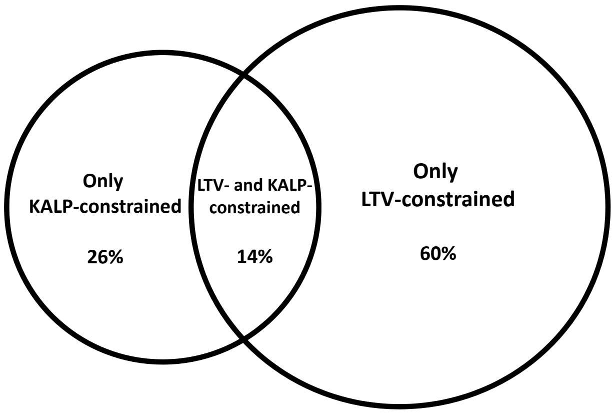


Figure 3: The distribution of constrained borrowers in Sweden among the LTV and the KALP-constraint

3. Simple Model with One-Period Debt

I start the exposition of the main idea from a real business cycle economy, where all debt contracts take the form of one-period debt. Consider an economy populated by firms and households that differ in their degree of impatience. Firms operate in a competitive market and are profit maximizers. They use labor to produce the final good. Households derive utility from leisure, consumption and housing. Patient households (’savers’) provide one-period loans to impatient households (’borrowers’). Only borrowers are subject to credit constraints. They are subject to LTV and DSTI constraints. In the following, I present the borrower's problem (savers' and firms' first-order-conditions (FOCs) are standard and presented in Appendix B).

3.1. Impatient households

Impatient households get utility from goods and housing consumption, as well as leisure. They provide labor to firms and borrow from savers subject to two borrowing constraints.

Impatient households have the following utility function:

$$\max_{b_t^B, h_t^B, L_t^B} E_0 \sum_{t=0}^{\infty} \beta^{B,t} \left(\log c_t^B + j_t \log h_t^B - \frac{l_t^{B\,\eta^B}}{\eta^B} \right), \tag{1}$$

where c_t^B denotes the borrowers' consumption of the final good, j_t is the marginal utility of housing subject to random disturbances (following Iacoviello, 2005, the disturbance is common to patient and impatient households, and is a proxy for a housing demand or housing preference shock), h_t^B is the housing stock held by impatient households, l_t^B denotes labor supply of impatient households. The budget constraint of the impatient household is:

$$c_t^B + q_t(h_t^B - h_{t-1}^B) + R_{t-1}b_{t-1} = b_t + w_t^B l_t^B, \tag{2}$$

where q_t denotes the housing price, $R_t = 1 + i_t$ is the interest rate on mortgage loans, b_t^B is the borrowing, $w_t^B l_t^B$ is labor income .

Households' borrowing is subject to a typical LTV constraint (as in Iacoviello, 2005):

$$R_t b_t \leq m^B q_{t+1} h_t^B, \tag{3}$$

where m^B determines the LTV ratio for borrowers.

In addition, the borrowing is limited by a DSTI constraint:

$$R_t b_t \leq DSTI w_t^B l_t^B. \tag{4}$$

With λ_t being the Lagrangian multiplier on the budget constraint, λ_t^{LTV} being the Lagrangian multiplier on the LTV constraint and λ_t^{DSTI} being the Lagrangian multiplier on the DSTI constraint, the Lagrangian for the described problem is the following:

$$L = \sum_{t=0}^{\infty} \beta^{B,t} \Bigg[logc_t^B + j_t \log h_t^B - \frac{l_t^{B\eta^B}}{\eta^B} + \lambda_t(c_t^B + q_t(h_t^B - h_{t-1}^B) + R_{t-1}b_{t-1} - b_t - w_t^B l_t^B)$$
$$+ \lambda_t^{LTV}(m^B q_{t+1} h_t^B - R_t b_t) + \lambda_t^{DSTI}(DSTI w_t l_t - b_t R_t) \Bigg] \quad (5)$$

The Kuhn-Tucker conditions, necessary for an optimum in a model with inequality constraints, specify, that

$$\lambda_t^{LTV}(m^B q_{t+1} h_t^B - R_t b_t) = 0 \quad (6)$$

and

$$\lambda_t^{DSTI}(DSTI w_t^B l_t^B - b_t R_t) = 0. \quad (7)$$

In order for these conditions to hold, either $\lambda_t^{LTV} = 0$ or $(m^B q_{t+1} h_t^B - R_t b_t) = 0$ and either $\lambda_t^{DSTI} = 0$ or $(DSTI w_t^B l_t^B - b_t R_t) = 0$, which leaves us with four possible regimes. If both Lagrangian multipliers are 0, neither of the two borrowing constraints is binding. If only one of the Lagrangian multipliers is 0 and the other is nonnegative, only one constraint will be binding and if both of Lagrangian constraints are nonnegative, both constraints bind at the same time. The first order conditions of this problem are:

w.r.t. b_t

$$\frac{1}{c_t^B} = \beta^B E_t\left(\frac{R_t}{c_{t+1}^B}\right) + R_t \lambda_t^{LTV} + \lambda^{DSTI} R_t \quad (8)$$

w.r.t. h_t^B

$$\frac{q_t}{c_t^B} = \beta^B E_t\left(\frac{q_{t+1}}{c_{t+1}^B}\right) + \frac{j_t}{h_t^B} + \lambda_t^{LTV} m^B q_{t+1}, \quad (9)$$

w.r.t. l_t^B

$$w_t^B = l_t^{B\eta^B - 1} c_t^B - \lambda_t^{DSTI} DSTI w_t^B c_t^B, \quad (10)$$

If one of the Lagrangian constraints is 0, the terms linked to this constraint will drop out from the stated FOC and we will end up either in a model with the LTV-constraint only, which is a standard case considered in the literature, or in a model with DSTI-constraint only, which is slightly less common. The interesting question is whether both constraints can be binding at the same time, and we can answer this question by looking at the steady state values of the respective Lagrangian multipliers. In order for both constraints to bind, the Lagrangian multipliers have to be nonnegative.

The steady state expression for λ^{LTV}, denoted by the barred variable, can be found from equation 9:

$$\bar{\lambda}^{LTV} = \frac{\bar{q}\bar{h}^B - \beta^B \bar{q}\bar{h}^B - \bar{j}\bar{c}^B}{m^B \bar{q}\bar{h}^B \bar{c}^B}. \tag{11}$$

The steady state expression for λ^{LTV}, denoted by the barred variable, can be found from equation 8:

$$\lambda^{\bar{DSTI}} = \frac{1 - \beta^B \bar{R} - \bar{R}\bar{\lambda}^{LTV}\bar{c}^B}{\bar{R}\bar{c}^B}. \tag{12}$$

The nonnegativity of the two Lagrangian constraints hinges to a large extent on the level of impatience of borrowers - as in Iacoviello (2005), borrowers have to be more impatient than savers and if they are sufficiently impatient, both constraints will bind in equilibrium and close to the steady state. Also, in the model with one-period debt only, the steady state value of housing preference parameter for borrowers, j, determines the bindingness of the LTV constraint. For standard assumptions about parameter values, in this one-period debt setup, the DSTI constraint is always binding, i.e. $\bar{\lambda}^{DSTI}$ is positive. Let us consider fairly standard values for model parameters to examine the sensitivity of $\bar{\lambda}^{LTV}$ to the choice of parameter values. The benchmark calibration of the one-period model is presented in Table 2.[6] Two of the considered parameters mostly determine whether $\bar{\lambda}^{LTV}$ is nonnegative. Figure 4 presents the range of values for the impatience of borrowers, governed by β^B and the preference of households to holding the housing stock, governed by $j = J^B$, for which the Lagrangian multiplier (see the color scale) on the LTV constraint is positive, and thus binding (for β^B, a range 0.8-0.99 was considered, for $j = J^B$, 0.01-0.1). It is clear from the Figure 4 that, ceteris paribus, for low values of housing preference and high levels of impatience of borrowers, the Lagrangian multiplier on the LTV constraint is binding (and so is the Lagrangian multiplier on the DSTI constraint). Values in Table 2 are chosen such that two constraints can bind at the same time. This is needed for steady state comparisons that take into account the existence of possibility that both constraints bind at the same time. For the two other models used for comparison, the calibration is left unchanged and implies a binding respective constraint.

3.2. Long-run and short-run implications of multiple borrowing constraints in the model with one-period debt

Theoretically, the bindingness of both constraints cannot be excluded, and given that the data provides support for considering that case, we can try to understand the consequences for the

[6] It is important to note that the one-period debt model with multiple binding constraints predicts a fairly low indebtedness level in relation to GDP, given that with the DSTI constraint and only one-period constraints present, loans taken out are a fraction $DSTI * (1 - \alpha)$ of output, which can be a low value compared to the observed in the data, like in Sweden (64% debt to GDP). In order to match this data feature better, I later extend the model with one period debt to a model with long-term debt and calibrate it to the Swedish economy.

	Parameter	Value
β^S	savers' discount factor	0.99
β^B	borrowers' discount factor	0.93
m^B	LTV ratio for new loans	0.85
$DSTI$	DSTI ratio for households	0.25
α	savers' wage share	0.8
η^S	savers' labor supply aversion	2
η^B	borrowers' labor supply aversion	2
J^S	savers' weight on housing	0.02
J^B	borrowers' weight on housing	0.02

Table 2: Benchmark calibration of the model with one-period debt

long-run equilibrium implied by the two constraints starting from the one-period debt case. I show that if both constraints bind at the same time, the LTV is not effective as a measure affecting the debt ratios in the economy. However, it is often considered as a macroprudential measure, aimed at lowering aggregate indebtedness, commonly expressed as a ratio in relation to their income or to GDP (Finocchiaro et al., 2016 and Chen and Columba, 2016 use the debt to income ratio for Sweden, while Alpanda and Zubairy, 2017 use the debt to GDP ratio for the U.S). In the present model, the changes in debt to income and debt to GDP ratio are synonymous, given the simple formulation of the production technology.

In order to understand the result of ineffectiveness of the LTV constraint for determining the debt ratios in the one-period debt economy, it is useful to think of what two binding constraints imply in equilibrium. If both DSTI and LTV are binding, we have from equations 3 and 4:

$$DSTI\bar{w^B}\bar{l^B} = \bar{m^B}\bar{q}\bar{h^B}, \tag{13}$$

and so

$$DSTI = \frac{\bar{m^B}\bar{q}\bar{h^B}}{\bar{w^B}\bar{l^B}}. \tag{14}$$

The numerator of the right hand side of equation 14 is the debt and the denominator, impatient households' income $\bar{w^B}\bar{l^B} = (1-\alpha)\bar{y}$, which is a linear function of output. As such, the right hand side of equation 14 is representing the economy's debt to income or debt to GDP ratio and it equals the DSTI limit, which is a constant. Hence, changing the LTV limit in this economy, m^B, despite its effect on house prices, housing stock and labor income of borrowing households, will not move debt ratios at all! It might, however, lower the absolute debt levels. Following the same logic, since DSTI represents the debt to GDP ratio in this economy, changing this limit will lower the debt to GDP or debt to income ratio by the same percentage.

Interestingly, in such a setup, lowering LTV, all else equal, will raise the level of equilibrium

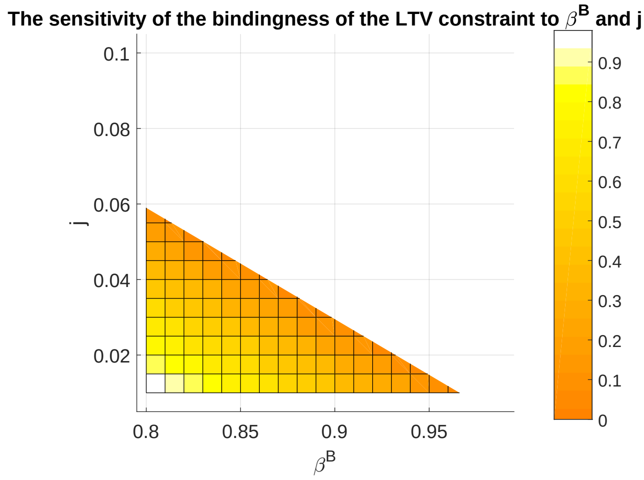


Figure 4: λ^{LTV} as a function of β^B and J^B

house prices. Ceteris paribus, if one unit of housing can pledge less collateral, its value has to increase if it has to collateralize the same amount of debt, as described in an example in Greenwald (2016). This is exactly the mechanism that occurs in this model, as shown in Table 3. The Table presents long-run effects of changes in the LTV ratio, DSTI ratio and the interest rate by 5% in the benchmark model (with multiple constraints), DSTI-only model and LTV-only model. Across the considered experiments, the bindingness of the constraints does not change, i.e. none of the constraints becomes slack. The DSTI-only model is a version of the benchmark model in which the LTV constraint is ignored and the LTV-only model is a model in which the existence of the DSTI constraint is not considered.[7] A decrease in the admissible LTV ratio has a big effect on the debt to GDP/income ratio in a model in which only LTV constraint is considered, but no effect on this ratio in the model with multiple constraints or with a DSTI constraint only. Analogously, in the models with a DSTI constraint present, DSTI limit changes have an effect on the indebtedness ratio. When it comes to changes in the interest rate, a 5 percent increase in the equilibrium mortgage rate leads to a decrease in the debt to GDP/income ratio in all the models, but in the LTV-only model this decline is much more pronounced. The house price decline after an increase in the equilibrium interest rate is very similar across different model specifications. Analyzing the results presented in this table and considering the composition of borrowers observed in the Swedish data, it is clear that models with an LTV constraint only will overstate the effectiveness of the LTV regulation as a measure affecting household indebtedness.

[7] The calibrated parameters are kept fixed across different specifications.

Variable/Model	Benchmark model	DSTI-only model	LTV-only model
		LTV ↓ 5%	
Debt to GDP/income	0%	0%	-16.12%
House prices	+0.18%	0%	-1.38%
Borrowers' housing stock	+5.07%	0%	-10.47%
Output	-0.01%	0%	0%
		DSTI ↓ 5%	
Debt to GDP/income	-5%	-5%	0%
House prices	-0.18%	-0.01%	0%
Borrowers' housing stock	-4.83%	+0.02%	0%
Output	0%-	0.01%	0%
		(R-1) ↑ 5%	
Debt to GDP/income	-0.05%	-0.05%	-2.24%
House prices	-4.55%	-4.55%	-4.42%
Borrowers' housing stock	+4.76%	+4.76%	+2.33%
Output	0%	0%	0%

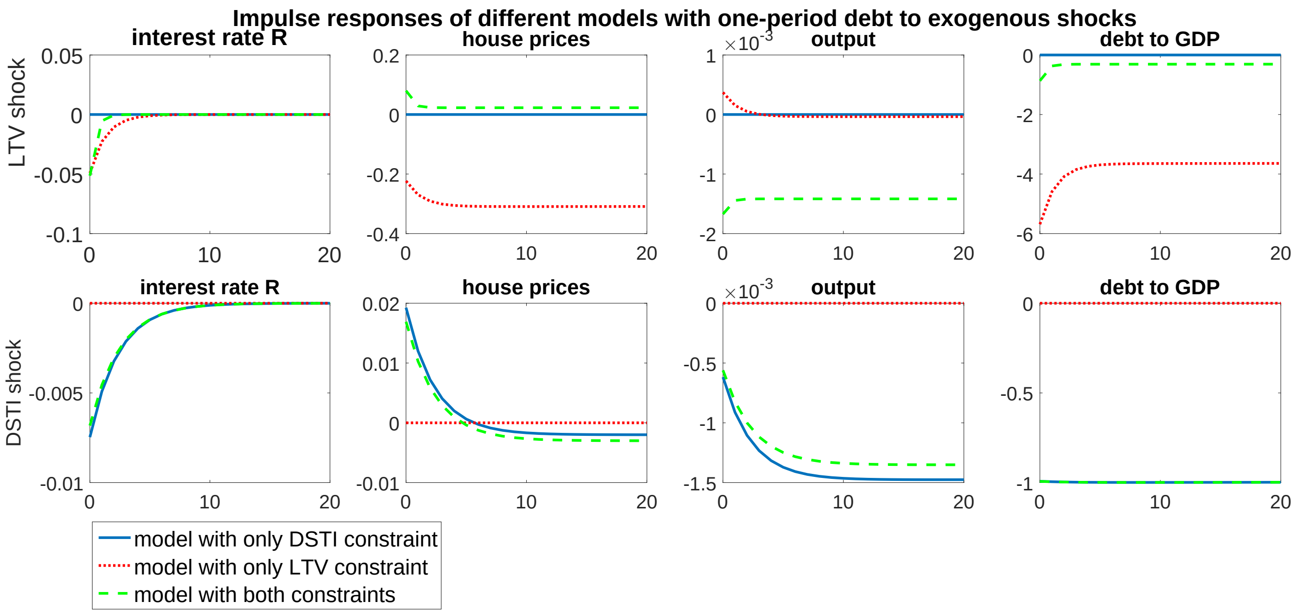
Table 3: Long-term effects of lower LTV, DSTI and higher interest rates in the models with one-period debt

Apart from long-run effects, we can also consider short-run implications of multiple constraints. Using the non-linear solution method presented in Guerrieri and Iacoviello (2015), I consider three models with occasionally binding constraints. Specifically, for the benchmark model with multiple constraints, I consider four possible regimes: in the first regime, both constraints are binding, in the second and third, only one of them is binding, and in the fourth, both constraints are slack. In the DSTI-only and LTV-only model simulations I consider two regimes: either the respective constraint is binding or not.

Figure 5 presents impulse responses to a persistent ($\rho = 0.9999$) 1% decline in the LTV and DSTI ratio in the three considered models. The green dashed line presents impulse responses of the model with both constraints, the red dotted line corresponds to the model with only LTV constraints, and the solid blue line presents impulse responses of the model with only DSTI constraint. We can note that the dynamic simulations of the effects of changes in LTV and DSTI ratios confirm the results presented in Table 3: changing the LTV has only a minor effect on the debt to GDP level in the model with both constraints, while the LTV-only model would suggest a big decline in this ratio for a given change of LTV limit.

Figure 6 presents impulse responses of the three models to a persistent ($\rho = 0.95$) housing preference shock of 5%: a negative shock (upper panel) and a positive shock (lower panel).[8] Given the non-linear solution method, the responses to positive and negative shocks are asymmetric. A negative housing preference shock drives house prices and output down. Debt to

[8] An extended version of this graph including Lagrangian multipliers is presented in Figure 13 in Appendix C.

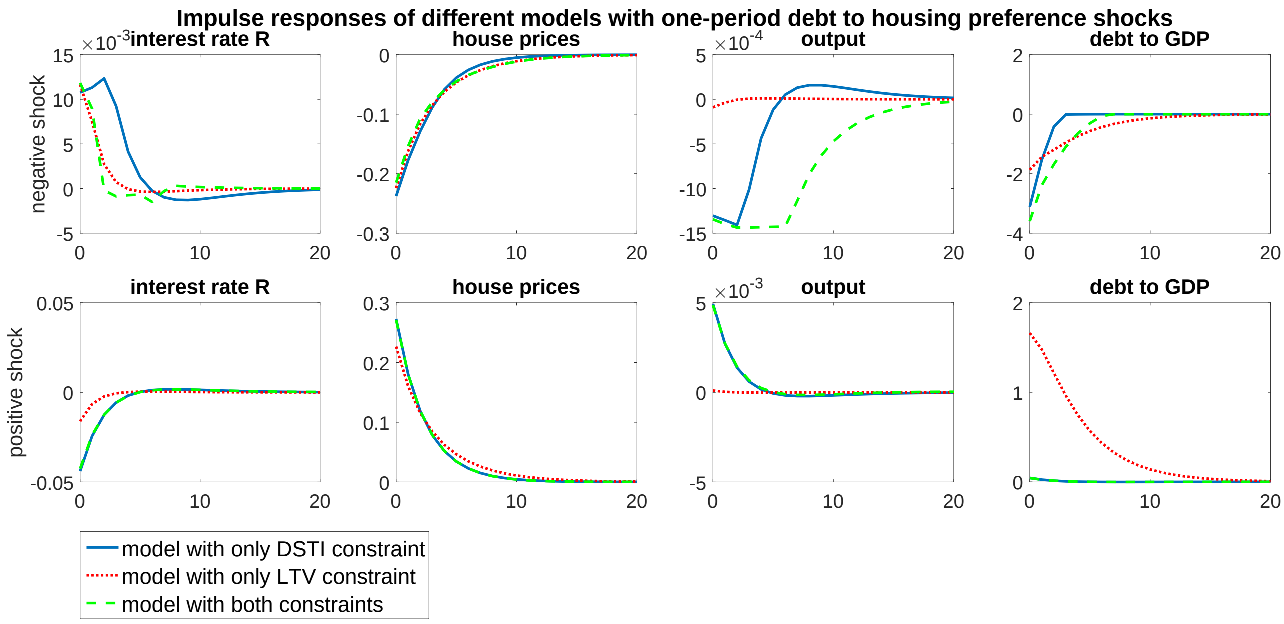


Note: In the presented simulations, the constraints may or may not bind, depending on the shock, its persistence and size. The non-linear solution method in Guerrieri and Iacoviello (2015) allows for endogenous switching between different regimes.

Figure 5: Impulse responses of models with one-period debt to LTV and DSTI shocks

GDP goes down in all of the considered models, but mostly so in the model with both the DSTI and LTV constraint. In the multiconstraint model, after the shock, the LTV constraint becomes more binding and the DSTI constraint slack at the impact. However, as house prices slowly go back to the equilibrium level, the DSTI constraint becomes binding again (along with the binding LTV constraint), driving down households' borrowing and consumption and prolonging the downturn in the economy. When house prices increase, the LTV-only model suggest a big increase in the debt to GDP ratio, a multiple of the increase in house prices, which is inconsistent with the Swedish data. House prices and debt to income or GDP are increasing, but the change in the indebtedness level does not exceed the pace of the house price increases. The model with both constraints and only DSTI constraint suggest a more sluggish increase in the indebtedness ratio as a response to the positive housing preference shock, given that in these models, households' borrowing cannot increase irrespectively of their income. The asymmetry in the responses to the housing preference shock is largest for the model with both constraints, suggesting a limited role of the constraints in the upturn, but amplifying role in the downturn. To obtain this result, I did not have to consider an extremely large shock, which suggests that the existence of multiple occasionally binding constraints amplifies the asymmetry in responses to different shocks compared to models with one constraint only.[9]

[9] A similar analysis is presented in Guerrieri and Iacoviello (2017) for the model with LTV constraint only, but for a shock that drives prices up and down by 20 percent, thus, a very large shock compared to the one considered

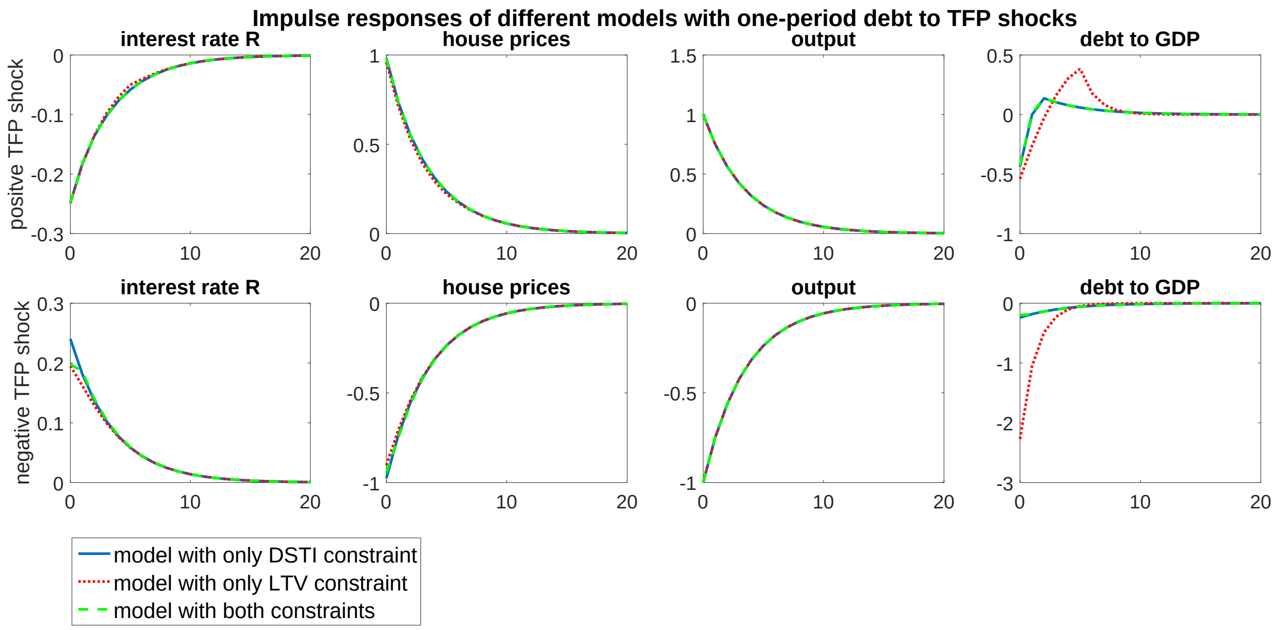


Note: In the presented simulations, the constraints may or may not bind, depending on the shock, its persistence and size. The non-linear solution method in Guerrieri and Iacoviello (2015) allows for endogenous switching between different regimes.

Figure 6: Impulse responses of models with one-period debt to housing preference shock

Figure 7 presents the impulse responses to a positive technology shock of 1% and 0.95 persistence. We see that the model with only LTV constraint binding exhibits much larger fluctuations of mortgage debt and debt to GDP than the remaining two other models. When a positive TFP shock hits the economy, both output and house prices increase on impact, making both constraints slack. New mortgage loans increase, but the increase in TFP is larger so that the debt to GDP ratio goes down in all three economies. When a negative TFP shock hits the economy, both constraints become more binding at the impact. In the LTV only model, mortgage loans go down much more than in the models with DSTI constraints, because in the model with both constraint, after the initial shock is over, the LTV constraint becomes slack, so low house prices do not reduce households' borrowing any more. Interestingly, for the aggregate consumption, output and house price response, in case of a TFP shock, it does not matter much which of the constraints is binding.

in this paper. As seen in Figure 6, also the LTV model exhibits asymmetry in responses to house price increases and decreases, but it is less pronounced than for the model with multiple constraints.



Note: In the presented simulations, the constraints may or may not bind, depending on the shock, its persistence and size. The non-linear solution method in Guerrieri and Iacoviello (2015) allows for endogenous switching between different regimes.

Figure 7: Impulse responses to TFP shock

4. Model with Long-Term Debt

The model with long-term debt is a simple extension of the real business cycle model with short-term debt presented in section 3. It differs from the one-period debt by the inclusion of long-term debt contracts, the introduction of price rigidity due to the presence of retailers, and the consideration of a central bank that sets the interest rates according to the Taylor rule. Debt contracts in this economy are nominal.

4.1. Impatient households

Borrowers get utility from goods (c^B) and housing (h^B) consumption, as well as leisure. They provide labor (l^B) to firms and borrow from savers subject to two constraints.

$$\max_{c_t^B, h_t^B, l_t^B, sb_t} E_0 \sum_{t=0}^{\infty} \beta^{B,t} \left(\log c_t^B + j_t \log h_t^B - \frac{l_t^{B\eta^B}}{\eta^B} \right). \tag{15}$$

The budget constraint of borrowers is:

$$c_t^B + q_t(h_t^B - (1-\delta_h)h_{t-1}^B) + \frac{(R_{t-1} - 1 + \kappa)sb_{t-1}}{\pi_t} = b_t + w_t^B l_t^B, \tag{16}$$

where δ_h is the depreciation of the housing stock, q_t denotes the housing price, $R_t = 1 + i_t$ is the interest rate, κ is the amortization rate, sb_t is the stock of debt, π_t is the inflation rate, b_t is new borrowing, and $w_t^B l_t^B$ is labor income.

Debt evolves according to:

$$sb_t = \frac{(1-\kappa)sb_{t-1}}{\pi_t} + b_t. \tag{17}$$

Substituting for the evolution of the stock of debt, we get

$$c_t^B + q_t(h_t^B - (1-\delta_h)h_{t-1}^B) + \frac{R_{t-1}sb_{t-1}}{\pi_t} = sb_t + w_t^B l_t^B. \tag{18}$$

As in the one-period version of the model, borrowing households face an LTV constraint, restricting their borrowing to a fraction of their collateral:

$$sb_t \leq \frac{(1-\kappa)sb_{t-1}}{\pi_t} + m^B q_t(h_t^B - (1-\delta_h)h_{t-1}^B) \tag{19}$$

This formulation follows Finocchiaro et al. (2016) and Chen and Columba (2016) for Sweden and is similar to one of the formulations in Justiniano, Primiceri, and Tambalotti (2015), implying that not all borrowers refinance their loans every period. The constraint emphasizes that the LTV regulation is applied to the flow of mortgages as defined in 17, which is consistent with the lending practice in Sweden. The right hand side of equation 19 emphasizes that the collateral constraint only applies to adjustments in the housing stock of borrowers (new investment in housing by borrowers), and not their total housing stock. Thus, in the disaggregated setup, we can interpret the constraint as applying only to a fraction of borrowers that actually readjust their mortgages in a given period.

Moreover, each period, a DSTI constraint imposes a limit on borrowing of households. Their debt service, including the amortization on existing loans, cannot exceed a certain fraction of their income, analogously to the one-period debt case.

$$sb_t(R_t + \kappa - 1) \leq DSTI w_t^B l_t^B \tag{20}$$

Notice that the formulation in equation 20 implies that the DSTI constraint is applied to the flow of mortgages that is backed by the labor income of borrowers readjusting their loans in a given period. Constraint 20 can be rewritten as:

$$b_t(R_t + \kappa - 1) \leq DSTI w_t^B l_t^B \mu_t, \tag{21}$$

where $\mu_t = \frac{b_t}{sb_t}$ stands for the fraction of new/refinanced loans in a given period. If we did not multiply equation 21 with the endogenously determined fraction of refinanced loans, the flow of debt would be backed by all borrowers' income and not only the income of borrowers actually taking out a new loan. Given the LTV constraint defined in 19, the formulation of the DSTI constraint as in 20 and 21 ensures that the LTV and DSTI constraints apply only to borrowers that readjust their loans in a given period. Arguably, one could assume that all long-term loans are refinanced each period, but in practice, this does not happen.

The FOCs are (λ_t^{LTV} is the Lagrangian multiplier on the LTV constraint and λ_t^{DSTI} is the Lagrangian multiplier on the DSTI constraint):

w.r.t. sb_t

$$\frac{1}{c_t^B} = \beta^B E_t\left(\frac{R_t}{c_{t+1}^B \pi_{t+1}}\right) + \lambda_t^{LTV} - E_t \frac{\beta^B \lambda_{t+1}^{LTV}(1-\kappa)}{\pi_{t+1}} + \lambda_t^{DSTI}(R_t + \kappa - 1) \tag{22}$$

w.r.t. h_t^B

$$\frac{q_t}{c_t^B} = \frac{j_t}{h_t^B} + \beta^B E_t\left(\frac{(1-\delta_h)q_{t+1}}{c_{t+1}^B} - (1-\delta_h)\lambda_{t+1}^{LTV} m^B q_{t+1}\right) + \lambda_t^{LTV} m^B q_t, \tag{23}$$

w.r.t. l_t^B

$$w_t^B = l_t^{B^{\eta^B - 1}} c_t^B - DSTI c_t^B w_t^B \lambda_t^{DSTI}, \tag{24}$$

As in the one-period model, given the presence of inequality constraints, the model requires the Kuhn-Tucker conditions to hold if we are interested in determining the optimal behavior of households.

The steady state expression for λ^{LTV}, denoted by the barred variable, can be found from equation 23:

$$\bar{\lambda}^{LTV} = \frac{\bar{q}\bar{h}^B - \beta^B \bar{q}\bar{h}^B(1-\delta_h) - \bar{j}\bar{c}^B}{m^B \bar{q}\bar{h}^B \bar{c}^B - \beta^B(1-\delta_h) m^B \bar{q}\bar{h}^B \bar{c}^B}. \tag{25}$$

The steady state expression for λ^{DSTI}, denoted by the barred variable, can be found from equation 22:

$$\bar{\lambda}^{DSTI} = \frac{1 - \beta^B \bar{R} - \bar{\lambda}^{LTV}\bar{c}^B + \beta^B \bar{\lambda}^{LTV}\bar{c}^B(1-\kappa)}{(\bar{R} + \kappa - 1)\bar{c}^B}. \tag{26}$$

As in the model version with one-period debt, if one of the Lagrangian constraints is 0, the terms linked to this constraint will drop out from the stated FOC and we will end up either in a model with the LTV-constraint only, or in a model with DSTI-constraint only. For reasonable parameter values matching the presented economy to the Swedish data, the bindingness of both constraints cannot be excluded, just as in the one-period model case. I present the sensitivity of the values of Lagrangian multipliers to the calibration in section 4.5.1.

4.2. Patient households

Patient households maximize the utility function given by:

$$\max_{sb_t, h_t^S, l_t^S} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t^S + j_t \log h_t^S - \frac{l_t^{S\eta^S}}{\eta^S} \right). \tag{27}$$

The budget constraint of the patient household in real terms is:

$$c_t^S + q_t(h_t^S - (1-\delta_h)h_{t-1}^S) + sb_t = \frac{R_{t-1} sb_{t-1}}{\pi_t} + w_t^S l_t^S, \tag{28}$$

where sb_t denotes the stock of loans, R_t is the interest rate, $w_t^S l_t^S$ is labor income , q_t denotes the housing price.

The First Order Conditions (FOCs) are:

w.r.t. sb_t

$$\frac{1}{c_t^S} = \beta E_t \left(\frac{1}{c_{t+1}^S \pi_{t+1}} \right) R_t, \tag{29}$$

w.r.t. h_t^S

$$\frac{q_t}{c_t^S} = \beta E_t \left(\frac{(1-\delta_h) q_{t+1}}{c_{t+1}^S} \right) + \frac{j_t}{h_t^S}, \tag{30}$$

w.r.t. l_t^S

$$w_t^S = l_t^{S\eta^S - 1} c_t^S. \tag{31}$$

4.3. Firms

Firms are competitive profit maximizers and produce the intermediate good in the economy according to the production function:

$$y_t = a_t l_t^{S\alpha} l_t^{B(1-\alpha)}, \tag{32}$$

where a_t denotes the productivity shock and the parameter α controls for savers' labor share in the production function.

Following Iacoviello (2005), I assume that retailers purchase the intermediate good from the firms and produce a final good, imposing a markup X on the goods. This markup appears in the FOC of the firms:

$$w_t^S = \frac{\alpha y_t}{X_t l_t^S}, \tag{33}$$

$$w_t^B = \frac{(1-\alpha) y_t}{X_t l_t^B}. \tag{34}$$

The formulation of the retail sector follows Iacoviello (2005). Retailers are a source of price stickiness in this economy, as they can only adjust their prices with probability $1 - \theta$.

4.4. Monetary policy and market clearing conditions

The central bank follows the Taylor rule in the economy given by:

$$ln\Big(\frac{R_t}{\bar{R}}\Big) = \rho_R ln\Big(\frac{R_{t-1}}{\bar{R}}\Big) + (1-\rho_R)\Big[\rho_\pi ln\Big(\frac{\pi_{t-1}}{\bar{\pi}}\Big) + \rho_y ln\Big(\frac{y_{t-1}}{\bar{y}}\Big)\Big] + \varepsilon_{R,t}, \tag{35}$$

where ρ_R, ρ_π, ρ_y are the parameters determining central banks' reaction to deviations from the steady state interest rate level, inflation and output, and $\varepsilon_{R,t}$ denotes an exogenous disturbance. This formulation is similar to the one used by the Riksbank, with the difference that in my model, the interest rate responds to the change in output, while in the Riksbank model, to the hours worked (see Adolfson, Laséen, Christiano, Trabandt, and Walentin, 2013).

The housing stock is fixed to 1:

$$1 = h_t^S + h_t^B. \tag{36}$$

The aggregate resource constraint is given by:

$$c_t^S + c_t^B + i_h = y_t, \tag{37}$$

where $i_h = \delta_h q_t$ is investment in the housing stock.

4.5. Equilibrium and dynamic implications of multiple borrowing constraints in the model with long-term debt

4.5.1. Calibration

The model is calibrated to the Swedish data (see Table 4). Housing depreciation rate is chosen to match the average LTV in the Swedish data: 65% (UC credit data). The values for DSTI and κ result in a debt to GDP ratio of 62%, between the value of 55% used in Finocchiaro et al. (2016) and the recent data indicator of 64% (for mortgage debt). It is assumed that borrowers earn 20% of wage income in this economy. In Sweden, around 40 % of population holds mortgages (UC credit data), but not all mortgage borrowers are credit-constrained as in the model. As discussed in section 2, a little bit more than half of new borrowers can be defined as credit constrained. In the model, constraints apply both to the stock and flow of mortgages, so setting the share of borrowers' labor income to 20% as at the higher end of calibration, but other studies for Sweden (Finocchiaro et al., 2016 and Chen and Columba, 2016) assume an even higher share of labor income earned by constrained borrowers (40% and 67%). Savers' preference for housing follows Finocchiaro et al. (2016) and Chen and Columba

	Parameter	Value	Source/Target
β^S	savers' discount factor	0.99	4%annual int. rate
β^B	borrowers' discount factor	0.93	high impatience level of borrowers
δ_h	housing depreciation rate	0.0076	average LTV of 65%
m^B	LTV ratio for new loans	0.85	Swedish FSA limit
$DSTI$	DSTI ratio for households	0.25	with κ debt-to-GDP of 62%
κ	quarterly amortization rate	0.01	25 years amortization
α	savers' wage share	0.8	borrowers earn 20% of wage income
η^S	savers' labor supply aversion	2	Frisch labor supply elasticity of 1
η^B	borrowers' labor supply aversion	2	Frisch labor supply elasticity of 1
J^S	savers' weight on housing	0.2	Finocchiaro et al. (2016)
J^B	borrowers' weight on housing	0.8	debt-to-GDP of 62% in the LTV model
θ	degree of price stickiness	0.75	duration of price of 1 year
X	price markup	1.01	4% annual markup
ρ_R	interest rate inertia	0.833	Adolfson et al. (2013)
ρ_π	central bank's response to inflation	1.733	Adolfson et al. (2013)
ρ_y	central bank's response to output	0.051	Adolfson et al. (2013)

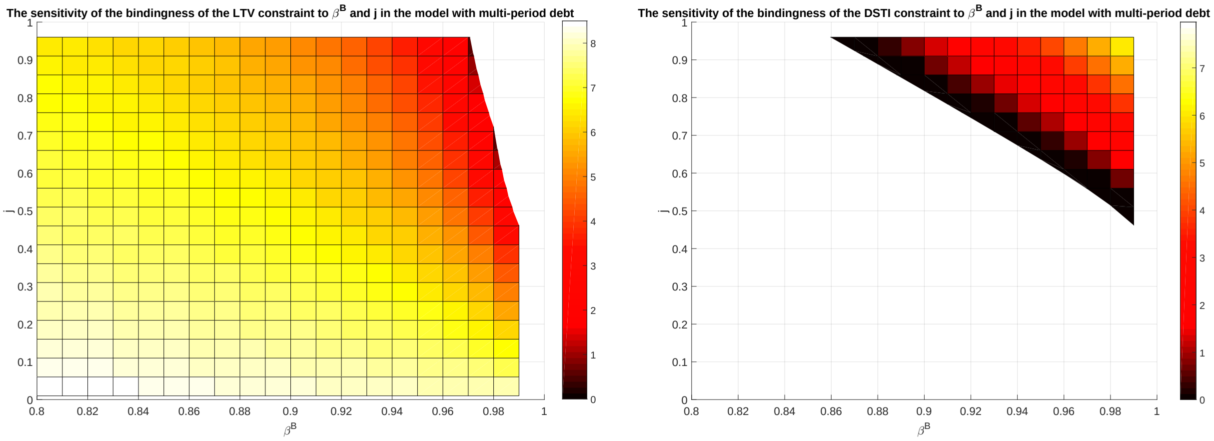
Table 4: Benchmark calibration of the model

(2016). Borrowers preference for housing J'' is chosen to match the debt-to-GDP in the LTV-only economy to 62% and to ensure that both constraints can bind in equilibrium. The value 0.8 is not far from 0.6 used in Finocchiaro et al. (2016). The remaining parameter values are fairly standard. The debt to income of this calibrated economy accounts to 300%, slightly higher than the recent value of 250% observed in the 2015 UC data. The Taylor rule parameters correspond to the estimation results published by the Riksbank in Adolfson et al. (2013).

Figure 8 shows a range of values for borrowers' impatience, governed by β^B and their preference for housing, J^B for which the borrowing constraints are binding (for β^B, the values 0.8-0.99 were considered, for J^B: 0.01 - 0.95). For both constraints to be binding, the level of housing preference of borrowing households has to be substantially high, and borrowers' level of impatience cannot be too low.

4.5.2. Equilibirum comparison

Table 5 presents equilibrium implications of four experiments: lowering LTV and DSTI by 5% and increasing the interest rate and the amortization pace by 5%. In columns 2-4, the table presents the results of these experiments in the benchmark model with two constraints binding, a DSTI-only model and LTV-only model. The calibration is identical for these models and results in a debt to GDP ratio of 62%. Let us first focus on the first three models presented in



(a) λ^{LTV} as a function of β^B and J^B (b) λ^{DSTI} as a function of β^B and J^B

Figure 8: The sensitivity of the bindigness of borrowing constraints in the model with long-term debt

the table. Similarly to the one-period model, a decrease in the admissible LTV ratio has a big effect on the debt to GDP/income ratio in a model in which only LTV constraint is considered, but no effect on this ratio in the model with multiple constraints or with a DSTI constraint only. In the multiconstraint framework, a stricter LTV limit raises house prices in equilibrium, while it lowers them in a LTV-only setup. Conversely, DSTI changes have only an effect on indebtedness in the models in which this constraint is present. When it comes to the effects of equilibrium changes in the interest rate, the model with multiple constraints features the highest house price sensitivity to changes in the interest rate. Across all the models, interest rate increases lower the debt to GDP and house prices, slightly lowering the output. An increase in the amortization pace lowers household indebtedness in relation to the GDP, mostly so in the LTV-only model. It has an ambiguous effect on house prices, and lowers the GDP in the long run. Implementing any of the considered policies seems to be costly in terms of output, however, the output sensitivity to different measures is not too large. House prices react most strongly to changes in the interest rates, across all three models.

Additionally, the fifth column of the table presents the experiment results for the 'Swedish economy', which features a different calibration from the remaining models. 'The Swedish economy' column aims at replicating the existing characteristics of the Swedish mortgage market in 2016 as presented in Figure 3, assuming that in equilibrium, 14% of borrowers are constrained by both constraints at the same time (as in the benchmark economy), 26% of borrowers are constrained by DSTI-constraint only, and 60% of the borrowers are constrained by the LTV constraint only.[10] These shares are achieved by assuming that certain fractions of borrowers'

[10] It is possible that another set of macroprudential policies would change the shares of borrowers in the economy constrained by specific constraints, but the assessment of this effect is beyond the scope of this exercise that

income go to different types of borrowers. If, apart for the income shares, the calibration is left unchanged and is as in the case of other three models, it results in a debt to GDP ratio of over 120%, inconsistent with the data for Sweden. In order to achieve the target debt to GDP of 62% in the changed model with three different types of borrowers, I need to adjust some other parameters accordingly, which may have an impact on the equilibrium results of other variables. In particular, the fraction of savers in economy, α, is assumed to be 0.861 instead of 0.8, and κ 0.0125 instead of 0.01. Without this adjustment, the debt to GDP ratio of the 'Swedish economy' would be unrealistically high, but admittedly, the change may impact the effects of experiments on some variables, particularly output. Among the borrowers, the shares of labor income are assumed as follows in the 'Swedish economy' case: borrowers bound by both constraints earn 0.17 of borrowers' income, borrowers bound by LTV constraint only 0.5 of borrowers' income, and the remaining borrowers' income goes to borrowers constrained by the DSTI constraint. Given the micro-distribution of borrowers in the Swedish economy in 2011-2015, the fifth column of Table 5 gives us the most realistic picture of equilibrium effects of different policies. Note that an equilibrium 5% change in the considered policies leaves the bindingness of Lagrangian multipliers unaffected. That is, given a 5% change in the policies, the long-run distribution of borrowers is unaffected (it might be that in the short run, some of the constraints become slack and I investigate this temporary effect looking at the impulse responses to policy shocks in section 4.5.3.) The impact of LTV changes on the debt-to-GDP ratio in this economy is substantially lowered compared to the LTV-only case (which is intuitive given that only a fraction of borrowers is actually constrained by LTV-constraint only) and lowering the DSTI limit seems to be more effective, with less of a negative impact on equilibrium house prices compared to a stricter LTV policy. In this experiment, higher amortization pace has most effect on debt to GDP without big negative effects for house prices. Changing the interest rate is effective in lowering debt to GDP ratios, but at the cost of substantially lower house prices. While interpreting this results, it is important to bear in mind that in the presented economy, borrowers have no option to circumvent the regulation, which may reduce the effectiveness of macroprudential regulation as recently discussed by Crowe, Dell'Ariccia, Igan, and Rabanal (2013) and Cerutti, Claessens, and Laeven (2017). The equilibrium output effect of all the presented policies is negligible.

4.5.3. Model dynamics

In the following, I present the short-run implications of multiple borrowing constraints in a model with long-term debt where these constraints are occasionally binding. Note that the steady state 'Swedish economy' excercise would be infeasible in the dynamic simulations with

should be thus treated as the assessment of the effectiveness of policies given the distribution of borrowers across the constraints. Presented experiments maintain the bindingness of constraints in equilibrium.

Variable/Model	Benchmark	DSTI-only	LTV-only	'Swedish economy'
			LTV ↓ 5%	
Debt to GDP/income	0%	0%	-7.88%	-3.06%
House prices	+1.07%	0%	-2.12%	-3.17%
Borrowers' housing stock	+3.59%	0%	-3.61%	+0.50%
Output	-0.54%	0%	-0.17%	+0.09%
			DSTI ↓ 5%	
Debt to GDP/income	-5%	-6.88%	0%	-3.09%
House prices	-1.50%	+0.09%	0%	-0.21%
Borrowers' housing stock	-3.41%	+1.27%	0%	-2.60%
Output	+0.15%	-0.37%	0%	-0.07%
			(R-1) ↑ 5%	
Debt to GDP/income	-2.45%	-3.72%	-1.82%	-2.69%
House prices	-2.52%	-2.17%	-2.27%	-2.24%
Borrowers' housing stock	-0.01%	+1.97%	+0.53%	+1.50%
Output	-0.08%	-0.30%	-0.11%	-0.15%
			κ ↑ 5%	
Debt to GDP/income	-2.43%	-2.97%	-4.22%	-5.80%
House prices	+0.58%	+0.09%	-0.16%	-0.16%
Borrowers' housing stock	+1.74%	+0.37%	+0.02%	+6.09%
Output	-0.12%	-0.17%	-0.03%	-0.08%

Note: The 'Swedish economy' calibration differs from the remaining three models in order to maintain the same debt-to-GDP ratio in equilbrium. See more discussion in the text.

Table 5: Long-term effects of lower LTV, DSTI and higher interest rates in the models with long-term debt

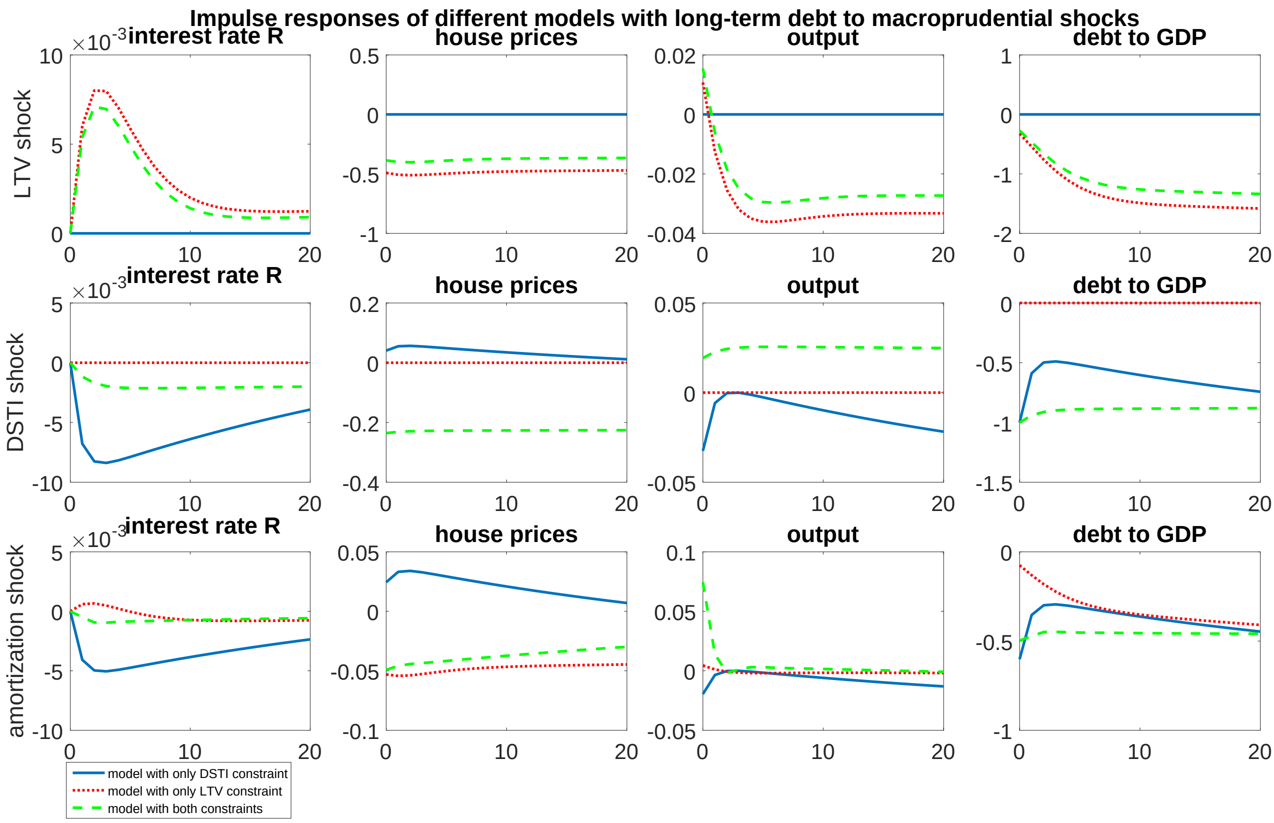
occasionally binding constraints. With three borrowers, each facing four possible regimes, the set of regimes to simulate would have to take into account 24 combinations of slack and/or binding constraints for each of the borrowers. Thus, in the following simulations of benchmark economy, there is only one type of borrower who may face two constraints, hence, the maximum number of considered regimes is four, as in the one-period debt case.

Figure 9 presents impulse responses to highly persistent 1% changes in the LTV and DSTI ratios, as well as the amortization pace ($\rho = 0.9999$). In the model with long-term debt, LTV changes have an impact on debt to GDP ratio even in the benchmark model with both constraints, depicted with a green dashed line (through a different response of interest rates and house prices compared to the model with one-period debt presented in Figure 5), but this impact is lower than in a model with LTV constraint only, presented with the red dotted line. This is because under lower LTV the DSTI constraint becomes slack, but alone the existence of an additional constraint alters the behaviour of agents in economy, hence, the green dashed and red dotted line do not coincide. When it comes to DSTI changes, similarly to the model with one-period debt, they have an impact merely on the models with the DSTI constraint (solid blue line). Lower DSTI limit will make the DSTI constraint more binding and induce borrowers to work more, having a positive impact on output. In the short run, an increase in amortization pace will only moderately lower the debt to GDP level and slightly decrease house prices. It also has a positive effect on house prices through its incentivizing impact on borrowers' labor supply - faster amortization makes the DSTI constraint more binding on impact.

Figure 10 presents impulse responses to a persistent 5% change in the preference for housing ($\rho = 0.95$). After a negative preference shock, house prices go down in all the models, and in the model with both constraints, the DSTI constraint becomes slack for a - hence the benchmark model with two constraints and the model with LTV constraint only result in similar impulse responses. When house prices increase, the LTV-only model suggests the largest increase in the debt ratio, as the household indebtedness in this model is tight to the collateral value of housing, irrespective of their income, which grows much less than housing prices after a housing preference shock, causing only the muted response of the debt to GDP ratio in the economy with two borrowing constraints. This results in an asymmetry in responses to house prices increases and decreases. The asymmetry is much more pronounced in the model with two constraints.[11]

Figure 11 presents impulse responses to a 1% monetary policy shock. When interest rates go down, house prices increase in all the models. The models considering the LTV constraint predict a relatively moderate increase in the debt to GDP. Output increases mostly in the model with two constraints present: in this model, after the initial shock and slack DSTI constraint, rising house prices lead to a slack LTV constraint, but the DSTI constraint becomes more

[11] An extended version of this graph including Lagrangian multipliers is presented in Figure 14 in Appendix D.



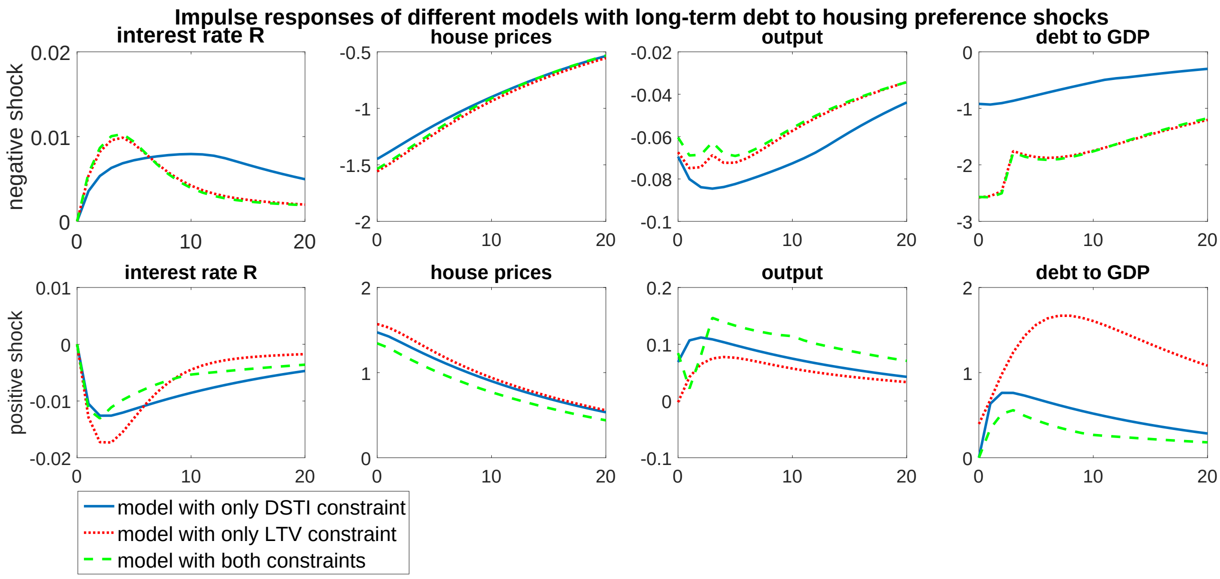
Note: In the presented simulations, the constraints may or may not bind, depending on the shock, its persistence and size. The non-linear solution method in Guerrieri and Iacoviello (2015) allows for endogenous switching between different regimes.

Figure 9: Impulse responses of models with long-term debt to highly persistent LTV, DSTI and amortization rate shocks

binding.[12] Households would like to borrow against the increasing value of their housing, but they cannot do so, unless their wage income increases as well, so in the model with two constraints, the borrowers increase their labor supply relatively more, driving the output up. Increasing interest rates lead to a fall in house prices and economic activity. Output in the model with two constraints goes down only with a lag, as borrowers, trying to keep up their borrowing, increase the labor supply on impact. House prices and debt to GDP seem to react more to an increase in the interest rates rather than decreases for small size shocks. The short-run impact of interest on house prices seems to be limited, but as the analysis in Table 5 shows, in the long run, changes in equilibrium interest rates tend to move house prices considerably.

Comparing the long-run and short-run simulations, we can try to address the question of what could be a potential driver of increasing debt to income in countries like Sweden, where borrowers face multiple borrowing constraints. As shown in Table 5, both macroprudential standards and interest level have a significant impact on the level of debt to GDP and house

[12] More detailed impulse responses are presented in Figure 15 in Appendix D.

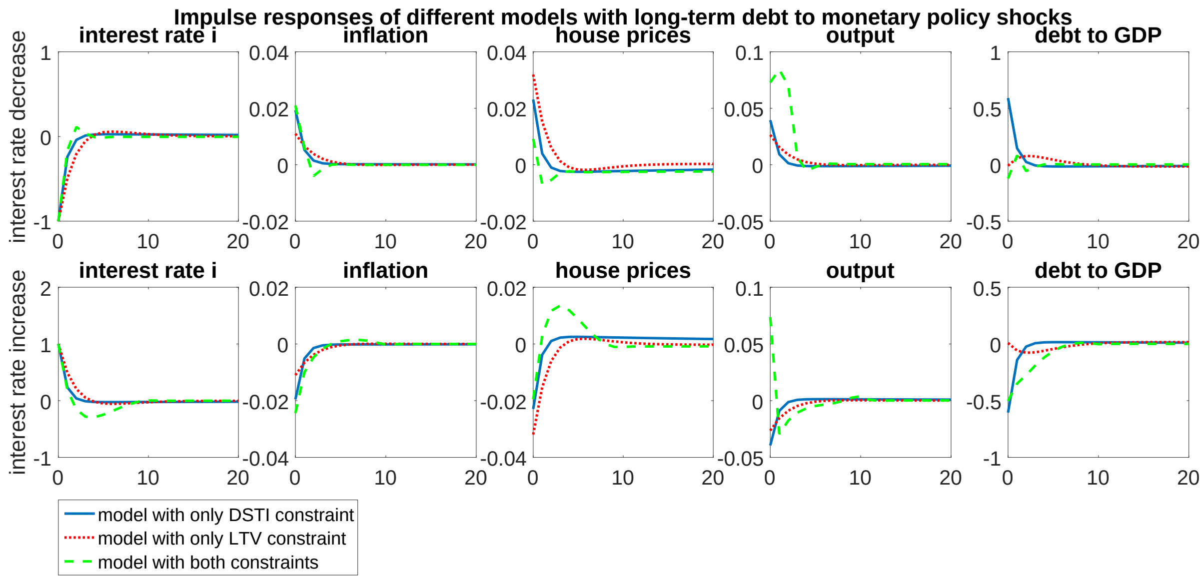


Note: In the presented simulations, the constraints may or may not bind, depending on the shock, its persistence and size. The non-linear solution method in Guerrieri and Iacoviello (2015) allows for endogenous switching between different regimes.

Figure 10: Impulse responses of models with long-term debt to housing preference shocks

prices in the economy. Among all the measures, changing the amortization pace and the DSTI standards has most impact on debt to GDP in relation to house price changes. That indicates that even without house price increases, a liberal amortization policy and DSTI policy could have contributed to rising household indebtedness above the house price increases in the long run. In Sweden, given the lack of the amortization requirement before 2016 and very low interest rates for a prolonged period, it was rather the combined effect of preference for low amortization under very low interest rates automatically relaxing the discretionary income constraint that led to the increase in indebtedness.

Stricter LTV policies and higher interest rates are effective in reducing long-term debt-to-GDP ratio, but they also have a significant negative effect on house prices, unlike DSTI and amortization policies. In the short run, lowering LTV and DSTI is effective in reducing deb-to-GDP ratios, with stricter DSTI having a less negative impact on house prices and output than a stricter LTV policy. Amortization is also effective in reducing household debt without big negative effects for output or house prices. Increasing interest rates in a multiple constraint economy lowers debt-to-GDP without a big negative effect on output. All the policy measures that are aimed at lowering indebtedness and linked to the bindingness of the DSTI constraint: amortization rate, interest rate, DSTI ratio, have a stabilizing effect on output after contractionary shocks, since they make the DSTI constraint more binding and incentivize borrowers to increase their labor supply.



Note: In the presented simulations, the constraints may or may not bind, depending on the shock, its persistence and size. The non-linear solution method in Guerrieri and Iacoviello (2015) allows for endogenous switching between different regimes.

Figure 11: Impulse responses of models with long-term debt to monetary policy shocks

5. Conclusion

In many countries, obtaining a mortgage requires not only a housing collateral, but also sufficient repayment capability. An example of such a country is Sweden, known for high levels of house prices and household indebtedness. In such a case, the question about the right tools affecting the debt ratios becomes an extremely important one. Many studies suggest lowering LTV ratios as an effective macroprudential tool, but they do so mostly using models where the LTV constraint is the only considered constraint. This is not consistent with the mortgage practice in some countries and may overstate the effectiveness of LTV as a policy tool.

Swedish micro-data suggests that while a considerable share of borrowers is indeed constrained by the LTV regulation, a non-negligible share is also constrained by a DSTI constraint, or by both constraints at the same time. If two constraints are potentially limiting the borrowing of households, we can think of four possible situations: either both constraints are binding, or DSTI constraint only, or LTV constraint only, or none of them. First, I construct a model with occasionally binding constraints and establish that both constraints can bind at the same time. This allows me to study long-run implications of different policies in the LTV-only, DSTI-only and multiconstraint world. If borrowers are at both constraints, a tighter LTV regulation will be ineffective in reducing households' debt to GDP ratio in the long run as the ratio is pinned down by a constant equal to the DSTI limit. Stricter LTV ratios will increase house prices in equilibrium and not change the debt ratios at all. I show that this is true in a model with one-period debt and long-term debt, extending the intuition presented in a simple model to a New-Keynesian setup, where I can study the effects of monetary policy in such an environment. Calibrating the model to match the distribution of borrowers across different constraints in the Swedish economy, I show that the effectiveness of LTV in influencing debt to GDP ratios is reduced substantially compared to an LTV-only economy, often used in studies assessing the impact of this ratio. In the short run, house prices do not seem to be very sensitive to monetary policy shocks. Expansionary monetary policy has a much lesser impact on the debt to GDP ratio in economies with the DSTI limit present. DSTI limit precludes leveraging only against rising housing prices, with no support of the real economy, which may reduce rising household indebtedness in cases when it is not accompanied by the wage and productivity growth. In general, the presence of multiple occasionally binding constraints amplifies asymmetries in the business cycles, even for small shocks, which confirms the result obtained in Guerrieri and Iacoviello (2017) for bigger fluctuations.

The model presented in this paper can be a useful tool for thinking about macroprudential policies in countries where multiple constraints are in place. The bindingness of different constraints is an empirical question and has to be studied for each country separately, but the Swedish case suggests that the mechanism presented in this paper can be relevant for policymakers facing high indebtedness levels in the economy. In addition, it can explain why some macroprudential measures may have only short-run effects, as found in the empirical literature.

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A. Appendix A - Discretionary income vs DSTI limit

The KALP-calculation takes into account agent-specific characteristics, such as income, and some lump-sum assumptions not linked to the individual's income, such as the amount of the minimum consumption. It may be represented by a following equation (assuming long-term debt):

$$(1-\tau_l)W_iL_i + TR - C - (I(1-\tau_h) + SR + \kappa)SB_i - HE = 0, \tag{38}$$

where τ_l are labor taxes, TR are transfers, C is the minimum consumption level, I is an average interest rate for a given bank, SR is the shock to the interest rate representing the 'stressed rate' scenario, τ_h is the interest rate tax deduction, κ is the amortization rate, SB the stock of debt, and HE are house expenses.

If we rewrite the KALP constraint, it will take the following form:

$$SB_i = \frac{(1-\tau_l)W_iL_i + TR - C - HE}{I(1-\tau) + SR + \kappa}, \tag{39}$$

The debt service to income ratio imposes a constraint similar to the KALP-constraint in Sweden:

$$\frac{(I(1-\tau_h)+\kappa)SB}{(1-\tau_l)W_iL_i} = DSTI, \tag{40}$$

where $DSTI$ is the ratio set by the regulator.

After rewriting, this constraint reads:

$$SB_i = DSTI\frac{(1-\tau_l)W_iL_i}{I(1-\tau_h)+\kappa}, \tag{41}$$

comparing to

$$SB_i = \frac{(1-\tau_l)W_iL_i}{R(1-\tau)+SR+\kappa} + \frac{TR-C-HE}{I(1-\tau)+SR+\kappa} \tag{42}$$

under the KALP constraint. We see that the DSTI constraint is a linear function of income, while the KALP constraint not, as the second part of equation 42 is applied lump-sum to all borrowers and does not depend on the individual's characteristics. The lump-sum characteristics does not change the slope of the KALP-constraint, but it changes the intercept. Depending on the values of $DSTI$, C, TR, HE and SR, either the DSTI or the KALP constraint can be a steeper function of household's income. However, unless the DSTI ratio is set very high, usually the KALP constraint will result in a borrowing limit being a steeper function of income, as demonstrated in Figure 12. Given that individual characteristics does not play a role in a representative agent world, in terms of modeling, DSTI and KALP constraints differ mostly by the inclusion of additional lump-sum expenses in the constraint that do not affect the dynamics of the model.

B. Appendix B - Simple Model with One-Period Debt

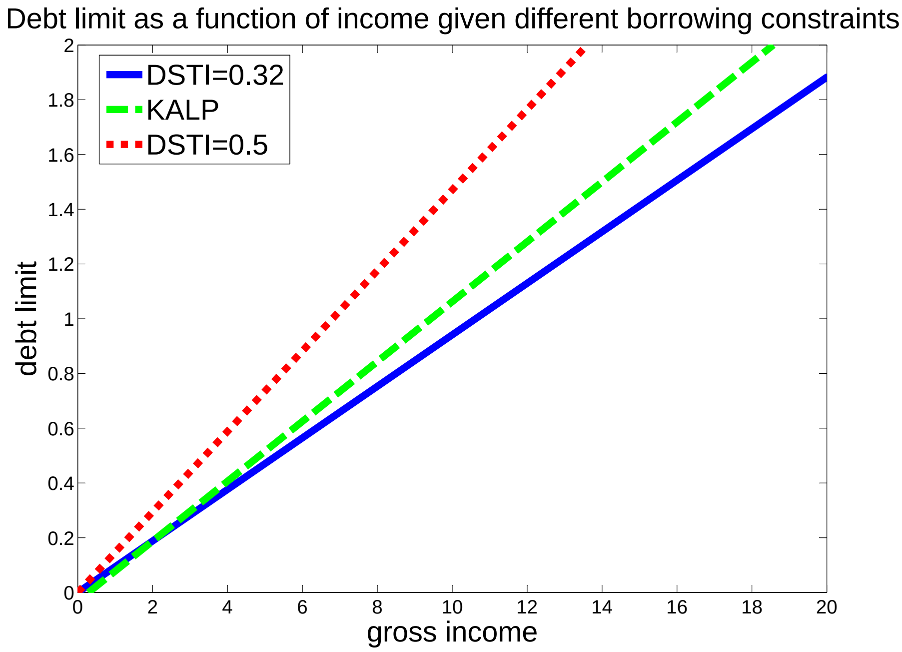
B.1. Patient households

Patient households get utility from goods and housing consumption, as well as leisure. They provide labor to firms and loans to impatient households.

Patient households maximize the utility function given by:

$$\max_{b_t^S, h_t^S, L_t^S} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t^S + j_t \log h_t^S - \frac{{l_t^S}^{\eta^S}}{\eta^S} \right), \tag{43}$$

where c_t^S denotes the consumption of the final good, j_t is the marginal utility of housing subject to random disturbances (following Iacoviello (2005), the disturbance is common to patient and impatient households, and is a proxy for a housing demand or housing preference shock), h_t^S is the housing stock held by savers, l_t^S denotes labor supply of patient households.



Notes: The graph was generated under the following parameter values: $I = 2$,$\tau_h = 0.3$,$C = 0.2$,$TR = 0.1$,$HE = 0.1$,$SR = 3$,$\kappa = 2$,$\tau_l = 0.3$

Figure 12: Debt limit as a function of income given different borrowing constraints

The budget constraint of the patient household in real terms is:

$$c_t^S + q_t(h_t^S - h_{t-1}^S) + b_t = R_{t-1}b_{t-1} + w_t^S l_t^S, \tag{44}$$

where b_t denotes lending, R_t is the interest rate, $w_t^S l_t^S$ is labor income , q_t denotes the housing price.

The First Order Conditions (FOCs) are:

w.r.t. b_t

$$\frac{1}{c_t^S} = \beta E_t \left(\frac{1}{c_{t+1}^S} \right) R_t, \tag{45}$$

w.r.t. h_t^S

$$\frac{q_t}{c_t^S} = \beta E_t \left(\frac{q_{t+1}}{c_{t+1}^S} \right) + \frac{j_t}{h_t^S}, \tag{46}$$

w.r.t. L_t^S

$$w_t^S = l_t^{S\eta^S - 1} c_t^S. \tag{47}$$

B.2. Firms

Firms are competitive profit maximizers.

The production function is:

$$y_t = a_t l_t^{S\alpha} l_t^{B(1-\alpha)}, \tag{48}$$

where a_t denotes the productivity shock and the parameter α controls for savers' labor share in the production function.

Given the competitive environment, wages equal marginal product of labor.

Entrepreneurs' first order conditions

w.r.t. labor:

$$w_t^S = \frac{\alpha y_t}{l_t^S}, \tag{49}$$

$$w_t^B = \frac{(1-\alpha) y_t}{l_t^B}. \tag{50}$$

B.3. Market clearing conditions

The housing stock is fixed to 1:

$$1 = h_t^S + h_t^B, \tag{51}$$

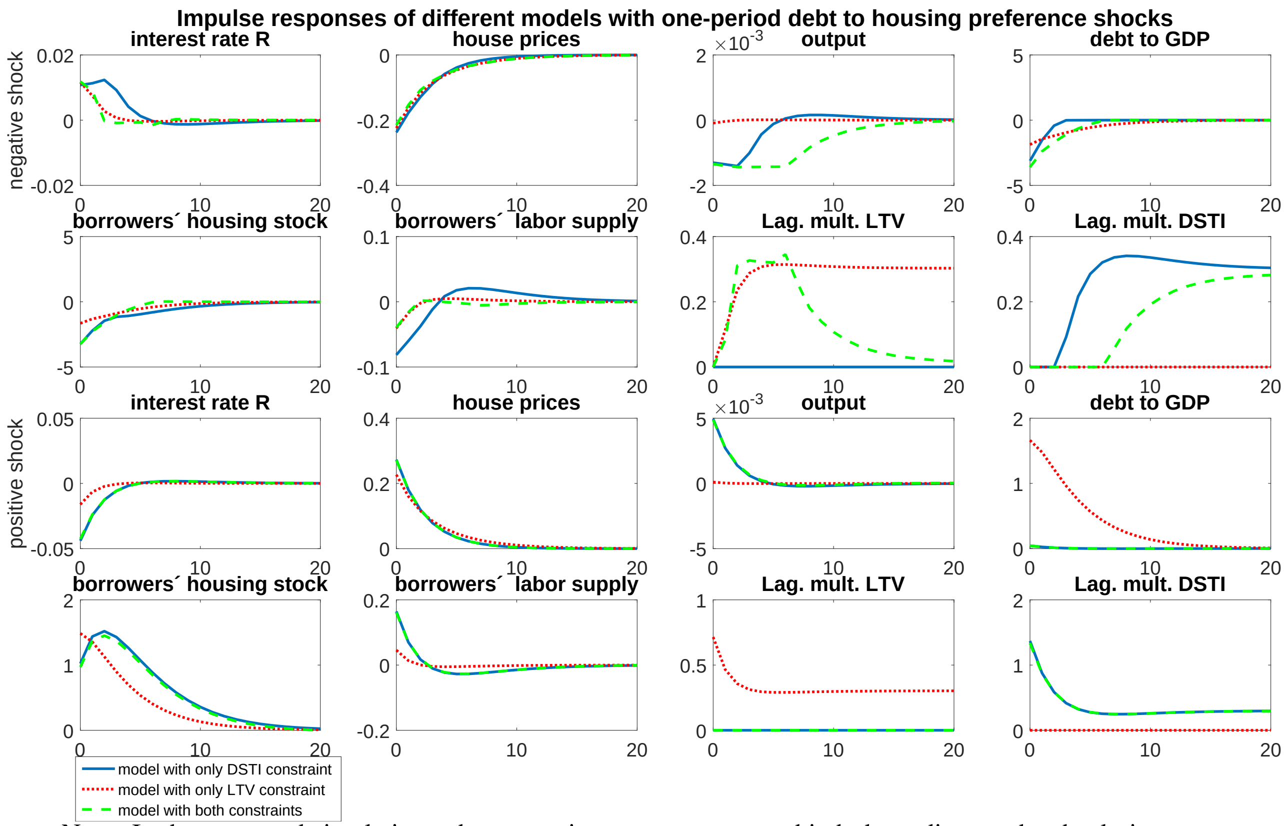
where h^S denotes savers' housing stock and h^B, the borrowers' housing stock.

The aggregate resource constraint is given by:

$$c_t^S + c_t^B = y_t, \tag{52}$$

where c^S denotes savers' consumption and c^B borrowers' consumption.

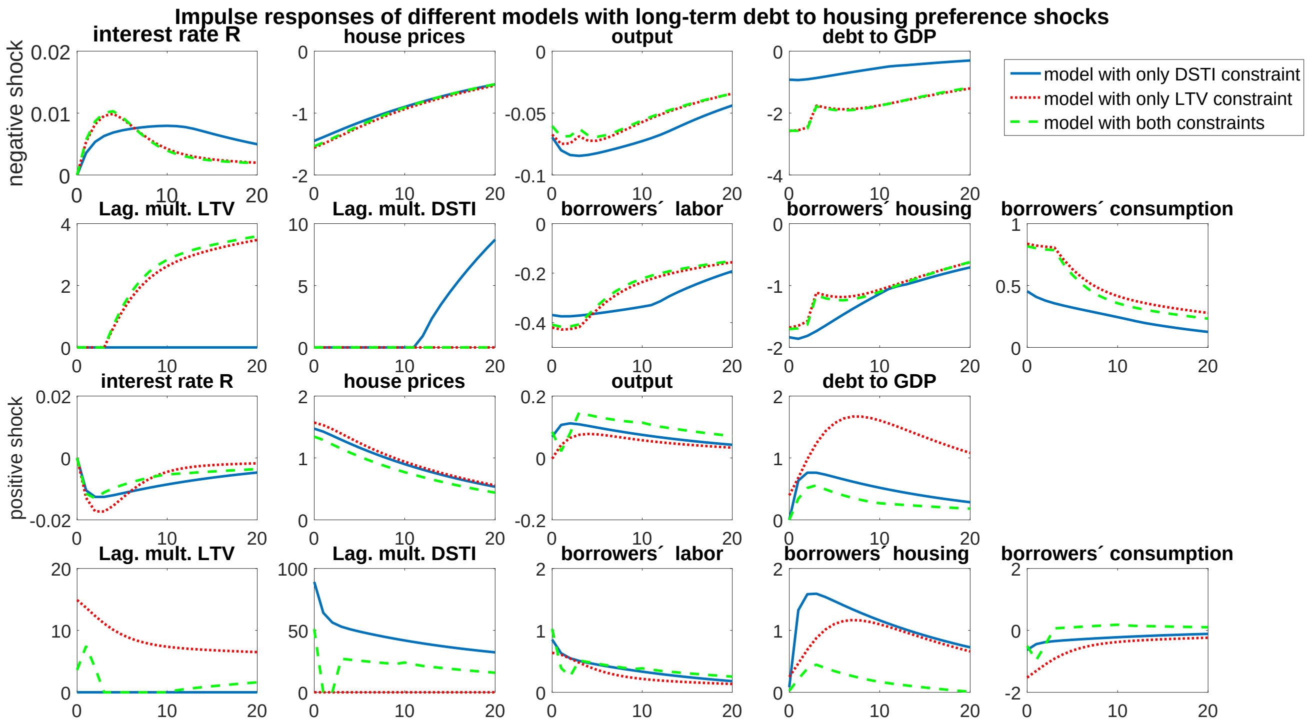
C. Appendix C - Impulse Responses for an Extended Set of Variables in the Model with One-Period Debt



Note: In the presented simulations, the constraints may or may not bind, depending on the shock, its persistence and size. The non-linear solution method in Guerrieri and Iacoviello (2015) allows for endogenous switching between different regimes.

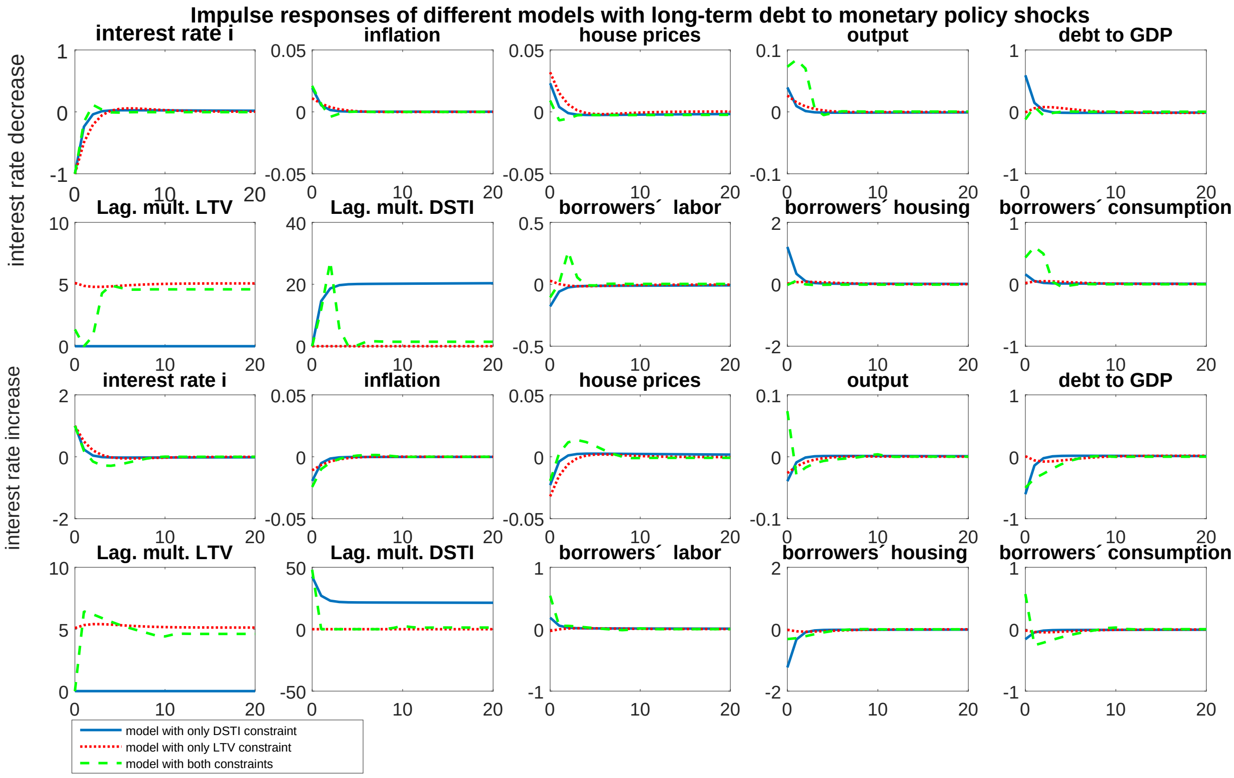
Figure 13: Impulse responses of models with one-period debt to housing preference shock

D. Appendix C - Impulse Responses for an Extended Set of Variables in the Model with Long-Term Debt



Note: In the presented simulations, the constraints may or may not bind, depending on the shock, its persistence and size. The non-linear solution method in Guerrieri and Iacoviello (2015) allows for endogenous switching between different regimes.

Figure 14: Impulse responses of models with long-term debt to housing preference shocks



Note: In the presented simulations, the constraints may or may not bind, depending on the shock, its persistence and size. The non-linear solution method in Guerrieri and Iacoviello (2015) allows for endogenous switching between different regimes.

Figure 15: Impulse responses of models with long-term debt to monetary policy shocks

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