Mixed frequency models with MA components *

Claudia Foroni † Massimiliano Marcellino ‡ Dalibor Stevanović §

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Abstract

Temporal aggregation in general introduces a moving average (MA) component in the aggregated model. A similar feature emerges when not all but only a few variables are aggregated, which generates a mixed frequency model. The MA component is generally neglected, likely to preserve the possibility of OLS estimation, but the consequences have never been properly studied in the mixed frequency context. In this paper, we show, analytically, in Monte Carlo simulations and in a forecasting application on U.S. macroeconomic variables, the relevance of considering the MA component in mixed-frequency MIDAS and UMIDAS models (MIDAS-ARMA and UMIDAS-ARMA). Specifically, the simulation results indicate that the short-term forecasting performance of MIDAS-ARMA and UMIDAS-ARMA is better than that of, respectively, MIDAS and UMIDAS, and the empirical applications confirm this ranking.

Keywords: Temporal aggregation, MIDAS models, ARMA models.

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†Deutsche Bundesbank

‡Bocconi University, IGIER and CEPR

§Université du Québec à Montréal

1 Introduction

The use of mixed-frequency models has become increasingly popular among academics and practitioners. It is in fact by now well recognised that a good nowcast or short-term forecast for a low frequency variable, such as GDP growth and its components, requires to exploit the timely information contained in higher frequency macroeconomic or financial indicators, such as surveys or spreads. A growing literature has flourished proposing different methods to deal with the mixed-frequency feature. In particular, models cast in state-space form, such as vector autoregressions (VAR) and factor models, can deal with mixed-frequency data, taking advantage of the Kalman filter to interpolate the missing observations of the series only available at low frequency (see, among many others, Mariano and Murasawa (2010) and Giannone et al. (2008) in a classical context, and Chiu et al. (2011) and Schorfheide and Song (2015) in a Bayesian context). A second approach has been proposed by Ghysels (2016). He introduces a different class of mixed-frequency VAR models, in which the vector of endogenous variables includes both high and low frequency variables, with the former stacked according to the timing of the data releases. A third approach is the mixed-data sampling (MIDAS) regression, introduced by Ghysels et al. (2006), and its unrestricted version (UMIDAS) by Foroni et al. (2015). MIDAS models are tightly parameterized, parsimonious models, which allow for the inclusion of many lags of the explanatory variables. Given their non-linear form, MIDAS models need to be estimated by non-linear least squares (NLS). UMIDAS models are the unrestricted counterpart of MIDAS models, which can be estimated by simple ordinary least squares (OLS), but work well only when the frequency mismatch is small.[1]

In this paper, we start from the observation that temporal aggregation generally introduces a moving average (MA) component in the model for the aggregate variable (see, e.g., Marcellino (1999) and the references therein). A similar feature should be present in the mixed frequency models, and indeed we show formally that this is in general the case. The MA component is often neglected, both in same frequency and in mixed frequency

[1]The literature on mixed-frequency approaches is vast. The paper cited in the text are a non-exhaustive list of key contributions to the field. For a review on mixed-frequency literature, see Bai et al. (2013) and Foroni and Marcellino (2013) among many others.

models, likely to preserve the possibility of OLS estimation and on the grounds that it can be approximated by a sufficiently long autoregressive (AR) component.

The effects of neglecting the MA component have been rarely explicitly considered. In a single frequency context, Lutkepohl (2006) shows that VARMA models are especially appropriate in forecasting, since they can capture the dynamic relations between time series with a small number of parameters. Further, Dufour and Stevanovic (2013) showed that a VARMA instead of VAR model for the factors provides better forecasts for several key macroeconomic aggregates relative to standard factor models, as well as producing a more precise representation of the effects and transmission of monetary policy. Leroux et al. (2017) found that ARMA(1,1) models predict well the inflation change and outperform many data-rich models, confirming the evidence on forecasting inflation by Stock and Watson (2007), Faust and Wright (2013) and Marcellino et al. (2006). Finally, VARMA models are often the correct reduced form representation of DSGE models (see, for example, Ravenna (2007)). For mixed frequency models, there are no results available.

We close this gap and analyze the relevance of the inclusion of an MA component in MIDAS and UMIDAS models, with the resulting specifications labeled, respectively, MIDAS-ARMA and UMIDAS-ARMA. We first compare the forecasting performance of the mixed frequency models with and without the MA component in a set of Monte Carlo experiments, using a variety of Data Generating Processes (DGPs). It turns out that the short-term forecasting performance is better when including the MA component, and the gains are higher the more persistent is the series.

Next, we carry out an empirical investigation, where we predict several quarterly macroeconomic variables using timely monthly indicators. In particular, we forecast the quarterly GDP deflator using monthly CPI inflation and the interest rate on 3-month T-Bills. We begin with this example because, as Stock and Watson (2007) show, the MA component for US inflation is important, especially after 1984. In fact, while during the 1970s the inflation process could be very well approximated by a low order AR, after the 1980s this has become less accurate and the inclusion of an MA component more relevant. Evidence on the importance of the MA component for the U.S. inflation is also found by Ng and Perron (2001) and Perron and Ng (1996). Further, we look at two other very relevant quarterly variables: GDP and personal consumption growth. We forecast them with two related monthly variables: industrial production and a consumption index, respectively. Naturally, it would be possible to further improve upon these simple mixed frequency forecasting models. However, they already perform rather well, and the inclusion of an MA component generally improves the forecasting performance substantially. In particular, we obtain good results for GDP deflator inflation and personal consumption growth, with mean squared error (MSE) improvements up to 24% and 19% respectively. Adding the MA part to forecast GDP growth one year ahead ameliorates the MSE up to 10%.

The remainder of the paper proceeds as follows. In Section 2 we show that temporal aggregation generally creates an MA component also in mixed frequency models. In Section 3 we describe parameter estimators for the MIDAS-ARMA and UMIDAS-ARMA models. In Section 4 we present the design and results of the simulation exercises. In Section 5 we develop the empirical applications on forecasting U.S. quarterly variables with monthly indicators. In Section 6 we summarize our main results and conclude.

2 The rationale for an MA component in mixed frequency models

The UMIDAS regression approach can be derived by aggregation of a general dynamic linear model in high frequency, as shown by Foroni et al. (2015), while the MIDAS model imposes specific restrictions on the UMIDAS coefficients in order to reduce their number, which is particularly relevant when the frequency mismatch is large (for example, with daily and quarterly series). In Section 2.1, we briefly review the derivation of the UMIDAS model, highlighting that, in general, there should be an MA component, even though it is generally disregarded. In Section 2.2, we provide two simple analytical examples in which, starting from a high-frequency model without MA term, we end up with a mixed frequency model in which the MA component is present. We discuss estimation of mixed frequency models with an MA component in a separate section.

2.1 UMIDAS regressions and dynamic linear models

Let us assume that the Data Generating Process (DGP) for the variable y and the N variables x is a $VAR(p)$ process, as in Foroni et al. (2015):

$$\begin{pmatrix} \underset{1\times 1}{a(L)} & \underset{1\times N}{-b(L)} \\ \underset{N\times 1}{-d(L)} & \underset{N\times N}{C(L)} \end{pmatrix} \begin{pmatrix} \underset{1\times 1}{y_{t_m}} \\ \underset{N\times 1}{x_{t_m}} \end{pmatrix} = \begin{pmatrix} \underset{1\times 1}{e_{yt_m}} \\ \underset{N\times 1}{e_{xt_m}} \end{pmatrix}. \tag{1}$$

Alternatively, the model can be written as:

$$a(L)y_{t_m} = b_1(L)x_{1t_m} + ... + b_N(L)x_{Nt_m} + e_{yt_m} \tag{2}$$

$$C(L)x_{t_m} = d(L)y_{t_m} + e_{xt_m} \tag{3}$$

where $a(L) = 1 - a_1L - ... - a_pL$, $b(L) = (b_1(L), ..., b_N(L))$, $b_j(L) = b_{j1}L + ... + b_{jp}L^p$, $j = 1, ..., N$, $d(L) = (d_1(L), ..., d_N(L))'$, $d_j(L) = d_{j1}L + ... + d_{jp}L^p$, $C(L) = I - C_1L - ... - C_pL^p$, and the errors are jointly white noise. We assume that the starting values $y_{-p}, ..., y_0$ and $x_{-p}, ..., x_0$ are all fixed and equal to zero (that is, they are fixed at their unconditional expected value). To keep the notation simple, we consider the same lag length of the polynomial in (2) and (3), but different lag lengths can be easily handled.

We then assume that x can be observed for each period t_m, while y can be only observed every m periods. We define $t = 1, ..., T$ as the low frequency (LF) time unit and $t_m = 1, ..., T_m$ as the high frequency (HF) time unit. The HF time unit is observed m times in the LF time unit. As an example, if we are working with quarterly (LF) and monthly (HF) data, it is $m = 3$ (i.e., three months in a quarter). Moreover, L indicates the lag operator at t_m frequency, while L^m is the lag operator at t frequency.

We also introduce the aggregation operator

$$\omega(L) = \omega_0 + \omega_1L + ... + \omega_{m-1}L^{m-1}, \tag{4}$$

which characterizes the temporal aggregation scheme. For example, $\omega(L) = 1 + L + ... + L^{m-1}$ indicates the sum of the high-frequency observations over the low-frequency period, typically used in the case of flow variables, while $\omega(L) = 1$ corresponds to point-in-time sampling and is typically used for stock variables. As we will see, different aggregation schemes will play a role in generating MA components.

To derive the generating mechanism for y at mixed frequency (MF), we introduce a polynomial in the lag operator, $\beta(L)$, whose degree in L is at most equal to $pm - p$ and which is such that the product $h(L) = \beta(L)a(L)$ only contains powers of L^m, so that $h(L) = h(L^m)$. It can be shown that such a polynomial always exists, and its coefficients depend on those of $a(L)$, see Marcellino (1999) for details.

In order to determine the AR component of the MF process, we then multiply both sides of (2) by $\omega(L)$ and $\beta(L)$ to get

$$h(L^m)\omega(L)y_{t_m} = \beta(L)b_1(L)\omega(L)x_{1t_m} + ... + \beta(L)b_N(L)\omega(L)x_{Nt_m} + \beta(L)\omega(L)e_{yt_m}. \tag{5}$$

Hence, the autoregressive component only depends on LF values of y. Let us consider now the x variables, which are observable at high frequency t_m. Each HF x_{it_m} influences the LF variable y via a polynomial $\beta(L)b_j(L)\omega(L) = b_j(L)\beta(L)\omega(L)$, $j = 1, ..., N$. We see that it is a particular combination of high-frequency values of x_j, equal to $\beta(L)\omega(L)x_{jt_m}$, that affects the low-frequency values of y.

Only under certain rather strict conditions, it is possible to recover the polynomials $a(L)$ and $b_j(L)$ that appear in the HF model for y from the MF model, and in these cases also $\beta(L)$ can be identified. Therefore, when $\beta(L)$ cannot be identified, we can estimate a model as

$$\begin{aligned} c(L^k)\omega(L)y_{t_m} &= \delta_1(L)x_{1t_m-1} + ... + \delta_N(L)x_{Nt_m-1} + \epsilon_{t_m}, \\ t_m &= m, 2m, 3m, ... \end{aligned} \tag{6}$$

where $c(L^m) = (1 - c_1L^m - ... - c_cL^{mc})$, $\delta_j(L) = (\delta_{j,0} + \delta_{j,1}L + ... + \delta_{j,v}L^v)$, $j = 1, ..., N$.

We can focus now on the error term of equation (5). In general, there is an MA component in the MF model, $q(L^m)u_{yt_m}$, with $q(L^m) = (1 + q_1L^m + ... + q_qL^{mq})$. The order of $q(L^m)$, q, coincides with the highest multiple of m non zero lag in the autocovariance function of $\beta(L)\omega(L)e_{yt_m}$. The coefficients of the MA component have to be such that the implied autocovariances of $q(L^m)u_{yt_m}$ coincide with those of $\beta(L)\omega(L)e_{yt_m}$ evaluated at all multiples of m. Consequently, also the error term ϵ_{t_m} in the approximate mixed frequency model (6), which is the UMIDAS model, in general has an MA structure. It can be shown that the maximum order of the MA structure is p for average sampling and p-1 for point-in-time sampling, where p is the order of the AR component in the high frequency model for y_{t_m} (see, e.g., Marcellino (1999) for a derivation of this results).

2.2 Two analytical examples

In this section, we consider two simple DGPs and show that, even in these basic cases, an MA component appears in the mixed frequency model. In the first example, we consider a bivariate VAR(1) with average sampling. In the second, we consider a bivariate VAR(2) with point-in-time sampling. We consider the case of monthly and quarterly variables, therefore $m = 3$, as in the empirical applications. However, the examples could be easily generalized to consider $N-$variate VARs and different frequency mismatches m.

VAR(1) with average sampling

Let us assume a VAR(1) as HF DGP:

$$\begin{pmatrix} y_{t_m} \\ x_{t_m} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} y_{t_m-1} \\ x_{t_m-1} \end{pmatrix} + \begin{pmatrix} e_{yt_m} \\ e_{xt_m} \end{pmatrix}, \tag{7}$$

where y_{t_m} is the low-frequency variable and x_{t_m} is the high-frequency variable,e_{yt_m} and e_{xt_m} are white noise processes, and t_m is the high-frequency time index.

Let us focus on the dynamic of the LF variable:

$$y_{t_m} = a y_{t_m-1} + b x_{t_m-1} + e_{yt_m}, \tag{8}$$

which can be rewritten as

$$(1 - aL)\, y_{t_m} = bLx_{t_m} + e_{yt_m}. \tag{9}$$

We consider average sampling, and therefore we define $\omega\,(L) = 1 + L + L^2$.

We first introduce a polynomial in the lag operator, $\beta(L)$, which is such that the product $h(L) = \beta(L)\,(1 - aL)$ only contains powers of L^3. This polynomial exists and it is equal to $(1 + aL + a^2L^2)$. We then multiply both sides of equation (8) by $\omega\,(L)$ and $\beta\,(L)$ and we obtain:

$$\begin{aligned} \left(1 + aL + a^2L^2\right)(1 - aL)\left(1 + L + L^2\right) y_{t_m} \quad = \quad & \left(1 + aL + a^2L^2\right) bL \left(1 + L + L^2\right) x_{t_m} + \\ & \left(1 + aL + a^2L^2\right)\left(1 + L + L^2\right) e_{yt_m}, \end{aligned} \tag{10}$$

or equivalently:

$$\begin{aligned} \left(1 - L^3a^3\right) \widetilde{y}_{t_m} \quad = \quad & \left(1 + aL + a^2L^2\right) bL \left(1 + L + L^2\right) x_{t_m} + \\ & \left(1 + (a+1)\,L + \left(a^2 + a + 1\right) L^2 + \left(a^2 + a\right) L^3 + a^2L^4\right) e_{yt_m}, \end{aligned} \tag{11}$$

where $\widetilde{y}_{t_m} = (1 + L + L^2)\, y_{t_m}$ and $t_m = 3, 6, 9, \ldots$.

As we saw it in Section 2.1, the order of the MA component coincides with the highest multiple of 3 non zero lag in the autocovariance function of the error term in equation (11), and it is bounded above by the AR order of the model for y_{t_m}. Actually, in this case the mixed frequency UMIDAS model in equation (11) has an MA component of order 1.

VAR(2) with point-in-time sampling

Let us now assume a VAR(2) as HF DGP:

$$\begin{pmatrix} y_{t_m} \\ x_{t_m} \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \begin{pmatrix} y_{t_m-1} \\ x_{t_m-1} \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} \begin{pmatrix} y_{t_m-2} \\ x_{t_m-2} \end{pmatrix} + \begin{pmatrix} e_{yt_m} \\ e_{xt_m} \end{pmatrix}, \tag{12}$$

where y_{t_m}, x_{t_m}, y_{t_m}, e_{xt_m} and t_m are defined as in the previous example.

The dynamic for the LF variable in this case is:

$$y_{t_m} = a_1 y_{t_m-1} + a_2 y_{t_m-2} + b_1 x_{t_m-1} + b_2 x_{t_m-2} + e_{yt_m}, \tag{13}$$

or, equivalently,

$$\left(1 - a_1 L - a_2 L^2\right) y_{t_m} = \left(b_1 L + b_2 L^2\right) x_{t_m} + e_{yt_m}. \tag{14}$$

We consider point-in-time sampling, and therefore $\omega\left(L\right) = 1$.

As in the previous example, we need to multiply both sides of equation (13) by $\omega(L)$ and find a polynomial $\beta(L)$ such that the product $h(L) = \beta(L)\left(1 - a_1 L - a_2 L^2\right)$ only contains powers of L^3. In can be easily shown that $\beta\left(L\right)$ exists and it is equal to

$$\left(1 + a_1 L + \left(a_1^2 + a_2\right) L^2 - a_1 a_2 L^3 + a_2^2 L^4\right).$$

The resulting process for the low-frequency variable is:

$$\begin{aligned} \left(1 - \left(a_1^3 + 3a_2 a_1\right) L^3 - a_2^3 L^6\right) y_{t_m} &= \left(1 + a_1 L + \left(a_1^2 + a_2\right) L^2 - a_1 a_2 L^3 + a_2^2 L^4\right)\left(b_1 L + b_2 L^2\right) x_{t_m} + \\ &\quad \left(1 + a_1 L + \left(a_1^2 + a_2\right) L^2 - a_1 a_2 L^3 + a_2^2 L^4\right) e_{yt_m}. \end{aligned} \tag{15}$$

Hence, also in this case there is an MA component in the mixed frequency model for y. Its order coincides with the highest multiple of 3 non zero lag in the autocovariance function of $\left(1 + a_1 L + \left(a_1^2 + a_2\right) L^2 - a_1 a_2 L^3 + a_2^2 L^4\right) e_{yt_m}$, and it is bounded above by the AR order of the model for y_{t_m} minus one, which is 1 in this example. Actually, the MA component is of order 1.

3 UMIDAS-ARMA and MIDAS-ARMA: specification and estimation

We describe now in more detail the model specifications we consider and the estimation details. We first recall the main features of the standard MIDAS regression, introduced by Ghysels et al. (2006), and its unrestricted version, as in Foroni et al. (2015). Then, we discuss their extensions to allow for an MA component and we discuss the estimation of the models.

The starting point for our MF models is equation (6). In order to simplify the notation, we assume $\omega(L) = 1$ and only one explanatory variable x_{t_m}[2]. Further, we allow for incorporating leads of the high frequency variable in the projections.

The equation we are going to estimate to generate an h_m-step ahead forecast is the following:

$$y_{t_m} = \tilde{c}(L^m) y_{t_m - h_m} + \delta(L) x_{t_m - h_m + w} + \epsilon_{t_m}, \tag{16}$$

where $\tilde{c}(L^m)$ is a modified lag structure of equation (6) to obtain a direct forecast and w is the number of months with which x is leading y.

Equation (16) represents our UMIDAS-AR model. Given that the model is linear, the UMIDAS-AR regression can be estimated by simple OLS. Empirically, the lag length of the high frequency variable x is often selected by means of an information criterion, such as the BIC.

When extending our UMIDAS to include an MA component, the UMIDAS-ARMA model we estimate is the following:

$$y_{t_m} = \tilde{c}(L^m) y_{t_m - h_m} + \delta(L) x_{t_m - h_m + w} + u_{t_m} + q(L^m) u_{t_m - h_m}, \tag{17}$$

where u_{t_m} is a (weak) white noise with $\mathrm{E}(u_{t_m}) = 0$ and $\mathrm{E}(u_{t_m} u'_{t_m}) = \sigma_u^2 < \infty$, and all the remaining terms stay the same as in equation (16). Given that MIDAS models are

[2]This is an innocuous simplification. It is equivalent to assume more generally $\omega(L) \neq 1$ and redefine $\tilde{y}_{t_m} = \omega(L) y_{t_m}$

direct forecasting tools, we decided to follow a direct approach also when modelling the MA component.

OLS estimation of the UMIDAS-ARMA model is no longer possible, because of the MA component in the residuals. We then have two possible options. First, we can estimate the model as in the standard ARMA literature, by maximum likelihood or, as we will actually do to be coherent with the MIDAS literature, by NLS. Second, we can adapt to the UMIDAS-ARMA model the GLS method proposed in Dufour and Pelletier (2008), which in turn generalizes the regression-based estimation method introduced by Hannan and Rissanen (1983). In this latter case, we proceed as follows: first, we fit a long ARX process to the data, then the lagged innovations in the UMIDAS-AR model are replaced by the residuals of our first step regression. The residuals are, in other words, treated as observables in the second regression[3].

The MIDAS-AR specification is a restricted version of the UMIDAS-AR. The original MIDAS-AR model as in Ghysels et al. (2006) can be written as follows:

$$y_{t_m} = \tilde{c}(L^m)y_{t_m-h_m} + \beta B(L,\theta)x_{t_m-h_m+w} + \epsilon_{t_m}, \tag{18}$$

where

$$\begin{aligned} B(L,\theta) &= \sum_{j=0}^{K} b(j,\theta)L^j, \\ b(j,\theta) &= \frac{exp(\theta_1 j + \theta_2 j^2)}{\sum_{j=0}^{K} exp(\theta_1 j + \theta_2 j^2)}, \end{aligned}$$

and K is the maximum number of lags included of the explanatory variable.

As it is clear by comparing equation (16) and equation (18), the MIDAS model is nested into the UMIDAS model.

The MIDAS-AR model in equation (18) is estimated by non-linear least squares (NLS). Given that it is h_m-dependent, as in the UMIDAS case it has to be re-estimated for each forecast horizon.

Exactly as for the UMIDAS, we extend the MIDAS-AR in equation (18) to incorporate an MA component:

[3]For the asymptotic properties of this estimator, please refer to Dufour and Pelletier (2008).

$$y_{t_m} = \tilde{c}(L^m) y_{t_m - h_m} + \beta B(L, \theta) x_{t_m - h_m + w} + u_{t_m} + q(L^m) u_{t_m - h_m}, \tag{19}$$

where the error term is defined as in (17). Given the nonlinearity of the model, we estimate its parameters by NLS.

4 Monte Carlo evaluation

We now assess the forecasting relevance of including an MA component in MIDAS and UMIDAS models by means of simulation experiments. We use two designs, closely related to the two analytical examples described in Section 2.2. We present first the Monte Carlo designs and then the results.

4.1 Monte Carlo design

In the first design, the DGP is the HF VAR(1):

$$\begin{pmatrix} y_{t_m} \\ x_{t_m} \end{pmatrix} = \begin{pmatrix} \rho & \delta_l \\ \delta_h & \rho \end{pmatrix} \begin{pmatrix} y_{t_m - 1} \\ x_{t_m - 1} \end{pmatrix} + \begin{pmatrix} e_{y,t_m} \\ e_{x,t_m} \end{pmatrix}, \tag{20}$$

where y_{t_m} is only available at LF while x_{t_m} is the HF variable, t_m is the HF time index, the aggregation frequency is $m = 3$ (as in the case of quarterly and monthly frequencies), and t is the LF time index, with $t = 3t_m$. We assume that $\omega(L) = 1 + L + L^2$, corresponding to average sampling.

The shocks e_{y,t_m} and e_{x,t_m} are sampled independently from normal distributions with zero means and variances chosen such that the unconditional variance of y is equal to one. We consider different combinations of ρ and δ_l, representing different degrees of persistence and correlation between the HF and the LF variables. In detail, we evaluate the following parameter sets:

$$(\rho, \delta_l) = \{(0.1, 0.1), (0.5, 0.1), (0.9, 1), (0.94, 1)\}. \tag{21}$$

With no loss of generality, we fix $\delta_h = 0$, so that the HF variable affects the LF variable, but not vice versa. All together, the parameter values are chosen so as to make sure that the VAR satisfies the stationarity condition.

In the second design, the DGP is the HF VAR(2):

$$\begin{pmatrix} y_{t_m} \\ x_{t_m} \end{pmatrix} = \begin{pmatrix} \rho_1 & \delta_{l1} \\ \delta_{h1} & \rho_1 \end{pmatrix} \begin{pmatrix} y_{t_m-1} \\ x_{t_m-1} \end{pmatrix} + \begin{pmatrix} \rho_2 & \delta_{l2} \\ \delta_{h2} & \rho_2 \end{pmatrix} \begin{pmatrix} y_{t_m-2} \\ x_{t_m-2} \end{pmatrix} + \begin{pmatrix} e_{y,t_m} \\ e_{x,t_m} \end{pmatrix}. \tag{22}$$

We still assume $m = 3$ but now $\omega(L) = 1$, so that the LF variable is skip-sampled every $m = 3$ observations.

In this second DGP, we consider the following parameter combinations:

$$(\rho_1, \rho_2, \delta_{l1}, \delta_{l2}) = \{(0.05, 0.1, 0.5, 1), (0.125, 0.5, 0.125, 0.5), (0.25, 0.5, 0.5, 1)\}. \tag{23}$$

As in the first DGP, with no loss of generality, we fix $\delta_{h1} = \delta_{h2} = 0$. All the other design features are as in the first DGP.

We focus on typical sample sizes for the estimation sample, with $T = 50, 100$. The size of the evaluation sample is set to 50, and the estimation sample is recursively expanded as we progress in the recursive forecasting exercise. The number of replications is 500.

The competing forecasting models are the following:

1. A MIDAS-AR model, with 12 lags in the exogenous HF variable and 1 lag in the AR component;

2. A MIDAS-ARMA model, as in the previous point but with the addition of an MA component;

3. A MIDAS-ARMA model, with only 3 lags in the exogenous HF variable and 1 AR lag;

4. A UMIDAS-AR model, with lag length selected according to the BIC criterion, where the maximum lag length is set equal to 12;

5. A UMIDAS-ARMA model, as in the previous point, with the addition of an MA component, estimated by the GLS method presented above (we will refer to this model as UMIDAS-ARMA-GLS)

6. The same model estimated by NLS (we will refer to this model as UMIDAS-ARMA-NLS);

7. The UMIDAS-ARMA-NLS, fixing at 3 the number of lags of the HF exogenous variable.

In all ARMA models there is an MA(1) component, but an higher order can be allowed.

We evaluate the competing one-step ahead forecasts on the basis of their associated mean square prediction error (MSE), assuming that information on the first two months of the quarter is available (as it is common in nowcasting exercises).

4.2 Results

In Tables 1 to 4 we report the mean relative MSE across simulations, and numbers smaller than one indicate that the model is better than the benchmark (model 1, the standard MIDAS). We also report the $10^{th}, 25^{th}, 50^{th}, 75^{th}$ and 90^{th} percentiles, to provide a measure of the dispersion in the results.

Tables 1 and 2 present the results for the first DGP (the VAR(1) with average sampling), using $T = 100$ in Table 1 and $T = 50$ in Table 2. The corresponding Tables 3 and 4 are based on the second DGP (the VAR(2) with point-in-time sampling).

A few key findings emerge. First, adding an MA component to the MIDAS model generally helps. The gains are not very large but they are visible at all percentiles, with a few exceptions for the second DGP. The gains are larger either with substantial persistence ($\rho = 0.9$ or $\rho = 0.94$ in the first DGP and $\rho_1 = 0.25$, $\rho_2 = 0.5$ in the second DGP) or with low persistence in the first DGP ($\rho = 0.1$), but in the latter case the result is mainly due to a deterioration in the absolute performance of the standard MIDAS model. The more parsimonious specification with 3 lags only of the HF variable is generally better, except when $\rho = 0.5$.

Second, adding an MA component to the UMIDAS model is also generally helpful, though the gains remain small. NLS and GLS estimation yield comparable results, suggesting that the second can be preferable as it is simpler.

Third, in general the MIDAS-ARMA specifications are slightly better than the UMIDAS-ARMA specifications, though the differences are minor. This pattern is in contrast with the findings in Foroni et al. (2015), and suggests that adding the MA component to the MIDAS model helps somewhat in reducing the potential misspecification due to imposing a specific lag polynomial structure.

Finally, results are consistent across sample sizes, and the models do not seem sensitive to short sample sizes.

5 Empirical applications

In this section, we look at the performance of our MA augmented mixed frequency models in a forecasting exercise with actual data. We start with our main empirical application, which consists in forecasting U.S. GDP deflator inflation using monthly CPI inflation and 3-month interest rate on T-Bill. The literature suggests that after the 1980s inflation in the U.S. became harder to approximate with an AR process (see Stock and Watson (2007)). Therefore, adding an MA component can be particularly relevant in this context. Next, we will also discuss forecasting quarterly GDP growth and real consumption growth using, respectively, monthly industrial production and a index for consumption (the real personal consumption expenditures).

The total sample spans over 50 years of data, from the first quarter of 1960 to the end of 2015. The forecasts are computed on progressively expanding samples, with the evaluation period going from 1980Q1 to the end of the sample, covering therefore roughly 35 years. The complete description of data sources and data transformation is available in Table 5. We use the most recent historical data for all series. At each point in time, we compute forecasts up to 4 quarters ahead. The forecasting object is the annualized growth rate. Although the information contained in the monthly variables updates every month, we focus on the case in which the first two months of the quarter are already available.[4]

We evaluate the forecasts both in terms of mean squared errors (MSE) and in terms of mean absolute errors (MAE). We then compare the forecasting performance relative to a standard MIDAS model with an autoregressive component and 12 lags of the explanatory variable (as the model (1) in Section 4).

In Tables 6 to 8 we report the results for, respectively, the GDP deflator inflation rate, the real GDP growth, and the real consumption growth. Each table is organized in the same way: it reports the value of MSE and MAE for each model, the ratio of those criteria for each model relative to the MIDAS-AR, our benchmark model, and the value of the Diebold-Mariano test, to check the statistical significance of the differences in forecast measures with respect to the benchmark (see Diebold and Mariano (1995)). For the sake of conciseness, we report the results for $h = 1, 2, 4$ quarters ahead.

[4]With the MIDAS setup, we could also report the results when no information or only one month of information is available. However, for the sake of conciseness we focus only on the case in which we are two months into the quarter, to have the shortest nowcast horizon. Given that our models are all mixed frequency, we do not advantage any specification with this setup anyways.

The figures and the tables are broadly supportive of the inclusion of the MA component in the mixed frequency models, as the MSE and MAE ratios are often smaller than one for the MIDAS-ARMA and UMIDAS-ARMA models when compared with their versions without MA.[5] More in detail: first, results of MF models with an MA component are good when forecasting GDP deflator growth and real consumption. The best improvement achieves 14% in terms of MSE for GDP deflator inflation when forecasted with the monthly CPI inflation, while it goes up to 24% in case of 3-month TBill. The improvements of the MA models when forecasting the real consumption growth with industrial production or the monthly consumption index growth are more uniform across horizons and top at 19% in terms of both MSE and MAE. Adding the MA part to predict the GDP growth with industrial production does not help at short horizons but improves the MSE up to 10% at four quarters ahead. Second, in many cases the improvement in the forecasting performance is also statistically significant. Third, there is no single model specification that systematically outstands all the others, though models with fewer lags of the explanatory variables seem generally better. The inclusion of the MA component likely compensates for the need of many lags. Fourth, the improvements with the MA component, whenever present, are present at each forecast horizon. Finally, if we decompose the MSE in bias and variance, we find that the MA part helps especially in reducing the bias, suggesting that the MA part is important to well approximate the conditional mean of y (the optimal forecast under the quadratic loss). When the models with the MA component are not performing well, this is due especially to the variance term, instead. Detailed results on the bias/variance decomposition are presented in Table 9.

To have a visual representation of the results, in Figure 1 we report the MSE of the different models relative to the benchmark at 1 to 4 quarters ahead. The figure shows that in most of the cases the relative MSE stays below 1 at the different forecasting horizons. Moreover, the best results (indicated by the grey envelope line in the figure) are in general obtained by models with an MA component.

The MSE and MAE are computed over the entire evaluation sample. We are interested also to check whether the performance of our models is driven by few data points or periods, or it remains good across the entire sample. In Figure 2, we then report the one-quarter ahead forecasts of the benchmark MIDAS-AR model and of one other model, the UMIDAS-ARMA estimated by GLS, together with the realized series. In Figure 3, instead,

[5]The models which include an MA component are indicated in bold in the tables.

we report the 4-quarters ahead forecasts.[6] It turns out that, on average, MIDAS models perform well throughout the sample, both with and without an MA component, except for the striking case of real consumption, where the MA part improves substantially the forecasting performance. However, when looking at the estimated coefficients of the MA components, in all the cases the coefficients are quite different from zero, see Figure 4). Further, except for the period of the early '80s, for most variables and models the estimated MA coefficients remain rather stable across the sample, although their magnitude (and in some cases their sign) change according to the forecast horizon, which is not surprising because of direct estimation.

6 Conclusions

In this paper, we start from the observation that temporal aggregation in general introduces a moving average component in the aggregated model. We show that a similar feature emerges when not all but only a few variables are aggregated, which generates a mixed frequency model. Hence, an MA component should be added to mixed frequency models, while this is generally neglected in the literature.

We illustrate in a set of Monte Carlo simulations that indeed adding an MA component to MIDAS and UMIDAS models further improves their nowcasting and forecasting abilities, though in general the gains are limited and particularly evident in the presence of persistence. Interestingly, the relative performance of MIDAS versus UMIDAS further improves when adding an MA component, with the latter attenuating the effects of imposing a particular polynomial structure in the dynamic response of the low frequency to the high frequency variable.

A similar pattern emerges in an empirical exercise based on actual data. Specifically, we find that the inclusion of an MA component can substantially improve the forecasting performance of GDP deflator growth and real personal consumption growth, while the results for GDP growth are more mixed.

[6]Figures 2 and 3 focus only on a small portion of results that we have available. The same figures for other models, other forecast horizons and other explanatory variables are available upon request.

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Table 1: Monte Carlo simulations results - DGP: VAR(1) with average sampling, $T = 100$

	mean	10 prct	25 prct	median	75 prct	90 prct
PANEL (A): $\rho = 0.94, \delta_l = 1, \delta_h = 0$						
MIDAS-ARMA-12 (2)	0.974	0.986	0.981	0.968	0.970	0.967
MIDAS-ARMA-3 (3)	0.966	0.981	0.969	0.962	0.959	0.966
UMIDAS-AR (4)	0.997	0.997	0.986	1.005	0.994	0.990
UMIDAS-ARMA-GLS (5)	0.971	0.973	0.979	0.974	0.968	0.973
UMIDAS-ARMA-NLS (6)	0.969	0.983	0.974	0.970	0.961	0.973
UMIDAS-ARMA-NLS-3 (7)	0.971	0.977	0.974	0.971	0.964	0.975
PANEL (B): $\rho = 0.9, \delta_l = 1, \delta_h = 0$	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.976	0.989	0.984	0.979	0.973	0.976
MIDAS-ARMA-3 (3)	0.975	0.983	0.988	0.969	0.978	0.971
UMIDAS-AR (4)	1.030	1.024	1.023	1.029	1.038	1.040
UMIDAS-ARMA-GLS (5)	1.014	1.019	1.019	1.019	1.019	1.026
UMIDAS-ARMA-NLS (6)	1.019	1.018	1.023	1.012	1.024	1.028
UMIDAS-ARMA-NLS-3 (7)	0.976	0.979	0.984	0.977	0.977	0.981
PANEL (C): $\rho = 0.5, \delta_l = 0.1, \delta_h = 0$	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.986	0.990	0.994	0.984	0.983	0.978
MIDAS-ARMA-3 (3)	1.184	1.197	1.178	1.174	1.202	1.176
UMIDAS-AR (4)	1.005	1.000	1.003	1.006	1.013	0.995
UMIDAS-ARMA-GLS (5)	1.000	1.012	0.994	0.998	0.992	0.991
UMIDAS-ARMA-NLS (6)	1.000	1.005	0.992	0.998	0.993	0.992
UMIDAS-ARMA-NLS-3 (7)	1.182	1.212	1.185	1.175	1.198	1.179
PANEL (D): $\rho = 0.1, \delta_l = 0.1, \delta_h = 0$	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.981	0.985	0.989	0.983	0.980	0.972
MIDAS-ARMA-3 (3)	0.833	0.848	0.846	0.834	0.827	0.828
UMIDAS-AR (4)	0.825	0.837	0.834	0.823	0.824	0.819
UMIDAS-ARMA-GLS (5)	0.832	0.841	0.844	0.830	0.831	0.834
UMIDAS-ARMA-NLS (6)	0.832	0.841	0.844	0.833	0.831	0.836
UMIDAS-ARMA-NLS-3 (7)	0.833	0.846	0.846	0.834	0.829	0.829

Note: The four panels report the results for four different DGPs for 1-quarter ahead horizon (with the information of the first two months of the quarter available). The numbers (2) to (7) refer to the corresponding models described in Section 4. The results reported are the average, median and the $10^{th}, 25^{th}, 75^{th}, 90^{th}$ percentiles of the MSE of the indicated model relative to the average, median and the $10^{th}, 25^{th}, 75^{th}, 90^{th}$ percentiles of the MSE of the benchmark MIDAS (model (1) in Section 4) computed over 500 replications.

Table 2: Monte Carlo simulations results - DGP: VAR(1) with average sampling, $T = 50$

	PANEL (A): $\rho = 0.94, \delta_l = 1, \delta_h = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.985	0.975	0.996	0.983	0.990	0.968
MIDAS-ARMA-3 (3)	0.957	0.949	0.982	0.947	0.957	0.944
UMIDAS-AR (4)	0.982	0.984	0.998	0.968	0.986	0.979
UMIDAS-ARMA-GLS (5)	0.967	0.966	0.983	0.949	0.969	0.971
UMIDAS-ARMA-NLS (6)	0.957	0.950	0.984	0.954	0.965	0.939
UMIDAS-ARMA-NLS-3 (7)	0.968	0.950	1.003	0.962	0.975	0.955
	PANEL (B): $\rho = 0.9, \delta_l = 1, \delta_h = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.997	1.001	1.012	0.994	0.986	0.982
MIDAS-ARMA-3 (3)	0.973	0.978	1.006	0.964	0.961	0.977
UMIDAS-AR (4)	1.033	1.074	1.041	1.025	1.034	1.013
UMIDAS-ARMA-GLS (5)	1.031	1.056	1.050	1.022	1.032	1.014
UMIDAS-ARMA-NLS (6)	1.020	1.041	1.040	1.018	1.019	1.016
UMIDAS-ARMA-NLS-3 (7)	0.981	0.982	1.023	0.968	0.971	0.983
	PANEL (C): $\rho = 0.5, \delta_l = 0.1, \delta_h = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	1.013	1.007	1.010	1.022	0.999	1.014
MIDAS-ARMA-3 (3)	1.188	1.182	1.172	1.179	1.168	1.249
UMIDAS-AR (4)	1.038	1.056	1.054	1.026	1.046	1.064
UMIDAS-ARMA-GLS (5)	1.054	1.060	1.056	1.053	1.044	1.051
UMIDAS-ARMA-NLS (6)	1.061	1.089	1.059	1.049	1.049	1.062
UMIDAS-ARMA-NLS-3 (7)	1.197	1.186	1.181	1.181	1.173	1.241
	PANEL (D): $\rho = 0.1, \delta_l = 0.1, \delta_h = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.984	0.987	0.989	0.983	0.989	0.973
MIDAS-ARMA-3 (3)	0.824	0.809	0.807	0.825	0.830	0.846
UMIDAS-AR (4)	0.810	0.791	0.814	0.819	0.814	0.820
UMIDAS-ARMA-GLS (5)	0.827	0.824	0.833	0.820	0.827	0.857
UMIDAS-ARMA-NLS (6)	0.834	0.824	0.826	0.831	0.834	0.853
UMIDAS-ARMA-NLS-3 (7)	0.830	0.826	0.816	0.827	0.832	0.859

Note: See Table 2.

Table 3: Monte Carlo simulations results - DGP: VAR(2) with point-in-time sampling, $T = 100$

	PANEL (A): $\rho_1 = 0.05, \rho_2 = 0.1, \delta_{l1} = 0.5, \delta_{l2} = 1, \delta_{h1} = 0, \delta_{h2} = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	1.007	1.010	1.003	1.007	1.010	1.025
MIDAS-ARMA-3 (3)	1.006	1.006	0.997	1.003	1.014	1.018
UMIDAS-AR (4)	1.014	1.007	1.016	1.005	1.006	1.030
UMIDAS-ARMA-GLS (5)	1.019	1.009	1.023	1.017	1.018	1.033
UMIDAS-ARMA-NLS (6)	1.015	1.000	1.014	1.007	1.014	1.026
UMIDAS-ARMA-NLS-3 (7)	1.007	0.998	1.004	1.006	1.016	1.023
	PANEL (B): $\rho_1 = 0.125, \rho_2 = 0.5, \delta_{l1} = 0.125, \delta_{l2} = 0.5, \delta_{h1} = 0, \delta_{h2} = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.956	0.955	0.960	0.963	0.949	0.959
MIDAS-ARMA-3 (3)	0.940	0.932	0.950	0.950	0.931	0.943
UMIDAS-AR (4)	0.938	0.921	0.938	0.945	0.929	0.946
UMIDAS-ARMA-GLS (5)	0.920	0.916	0.921	0.920	0.906	0.939
UMIDAS-ARMA-NLS (6)	0.921	0.927	0.922	0.926	0.908	0.939
UMIDAS-ARMA-NLS-3 (7)	0.943	0.921	0.950	0.947	0.932	0.948
	PANEL (C): $\rho_1 = 0.25, \rho_2 = 0.5, \delta_{l1} = 0.5, \delta_{l2} = 1, \delta_{h1} = 0, \delta_{h2} = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	0.984	0.968	0.981	0.985	0.991	0.998
MIDAS-ARMA-3 (3)	0.980	0.981	0.982	0.968	0.981	0.999
UMIDAS-AR (4)	1.021	1.032	1.020	1.006	1.032	1.036
UMIDAS-ARMA-GLS (5)	0.991	0.988	0.997	0.979	1.005	1.014
UMIDAS-ARMA-NLS (6)	0.992	0.987	0.986	0.988	1.001	1.004
UMIDAS-ARMA-NLS-3 (7)	0.983	0.978	0.979	0.980	0.985	0.998

Note: The four panels report the results for three different DGPs. The numbers (2) to (7) refer to the corresponding models described in Section 4. The results reported are the average, median and the $10^{th}, 25^{th}, 75^{th}, 90^{th}$ percentiles of the MSE of the indicated model relative to the average, median and the $10^{th}, 25^{th}, 75^{th}, 90^{th}$ percentiles of the MSE of the benchmark MIDAS (model (1) in Section 4) computed over 500 replications.

Table 4: Monte Carlo simulations results - DGP: VAR(2) with point-in-time sampling, $T = 50$

	PANEL (A): $\rho_1 = 0.05, \rho_2 = 0.1, \delta_{l1} = 0.5, \delta_{l2} = 1, \delta_{h1} = 0, \delta_{h2} = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	1.020	1.003	1.024	1.021	1.009	1.017
MIDAS-ARMA-3 (3)	1.003	0.990	1.015	1.014	0.986	0.994
UMIDAS-AR (4)	1.006	0.982	1.036	1.019	1.011	0.988
UMIDAS-ARMA-GLS (5)	1.023	0.986	1.050	1.040	1.018	1.032
UMIDAS-ARMA-NLS (6)	1.018	0.955	1.033	1.037	1.023	1.030
UMIDAS-ARMA-NLS-3 (7)	1.018	1.000	1.024	1.028	1.021	1.010
	PANEL (B): $\rho_1 = 0.125, \rho_2 = 0.5, \delta_{l1} = 0.125, \delta_{l2} = 0.5, \delta_{h1} = 0, \delta_{h2} = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	1.017	0.980	1.006	1.004	1.019	1.042
MIDAS-ARMA-3 (3)	0.967	0.934	0.970	0.995	0.953	0.991
UMIDAS-AR (4)	0.971	0.973	0.979	0.979	0.961	0.997
UMIDAS-ARMA-GLS (5)	0.965	0.979	0.968	0.969	0.948	0.972
UMIDAS-ARMA-NLS (6)	0.983	0.983	0.977	1.000	0.958	0.980
UMIDAS-ARMA-NLS-3 (7)	1.009	0.970	1.002	1.016	1.000	1.023
	PANEL (C): $\rho_1 = 0.25, \rho_2 = 0.5, \delta_{l1} = 0.5, \delta_{l2} = 1, \delta_{h1} = 0, \delta_{h2} = 0$					
	mean	10 prct	25 prct	median	75 prct	90 prct
MIDAS-ARMA-12 (2)	1.016	1.003	0.991	1.005	1.012	1.010
MIDAS-ARMA-3 (3)	0.990	0.993	0.965	0.984	0.988	0.979
UMIDAS-AR (4)	1.046	1.024	1.016	1.047	1.059	1.039
UMIDAS-ARMA-GLS (5)	1.200	1.013	1.031	1.029	1.049	1.032
UMIDAS-ARMA-NLS (6)	1.041	1.051	1.035	1.018	1.045	1.038
UMIDAS-ARMA-NLS-3 (7)	1.016	1.008	1.014	0.991	1.024	1.018

Note: see Table 3.

Table 5: Data description

Series	Source	Source Code	Transformation	Frequency
GDP Deflator	FRED	GDPDEF	Log-difference	Quarterly
Real GDP	FRED	GDP	Log-difference	Quarterly
Real Consumption	FRED	PCECC96	Log-difference	Quarterly
CPI	FRED	CPIAUCSL	Log-difference	Monthly
Industrial Production	FRED	INDPRO	Log-difference	Monthly
3-month T-bill	FRED	TB3MS	Level	Monthly

Table 6: Forecasting U.S. GDP Deflator

	Explanatory variable: CPI inflation						**Explanatory variable: 3-month Tbill**					
	$h = 1$						$h = 1$					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	0.65	1.00	NaN	0.61	1.00	NaN	1.04	1.00	NaN	0.75	1.00	NaN
MIDAS-ARMA-12lags	0.65	0.99	0.42	0.61	1.00	0.45	0.97	0.93	0.02	0.72	0.97	0.03
MIDAS-ARMA-3lags	0.61	0.94	0.09	0.59	0.97	0.17	0.96	0.92	0.00	0.71	0.95	0.00
UMIDAS-biclags	0.65	0.99	0.42	0.60	0.98	0.22	1.00	0.96	0.06	0.74	0.98	0.06
UMIDAS-ARMA-GLS-biclags	0.61	0.93	0.08	0.60	0.98	0.21	0.96	0.92	0.01	0.71	0.95	0.00
UMIDAS-ARMA-biclags	0.61	0.94	0.09	0.59	0.97	0.17	0.96	0.92	0.01	0.71	0.95	0.00
UMIDAS-ARMA-3lags	0.61	0.94	0.09	0.59	0.97	0.17	1.08	1.03	0.37	0.74	0.98	0.28
	$h = 2$						$h = 2$					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	0.79	1.00	NaN	0.67	1.00	NaN	1.45	1.00	NaN	0.83	1.00	NaN
MIDAS-ARMA-12lags	0.68	0.86	0.08	0.64	0.95	0.09	1.40	0.96	0.10	0.81	0.98	0.11
MIDAS-ARMA-3lags	0.68	0.86	0.13	0.64	0.96	0.16	1.37	0.94	0.04	0.80	0.97	0.09
UMIDAS-biclags	0.74	0.94	0.31	0.66	0.99	0.38	1.34	0.92	0.15	0.80	0.97	0.16
UMIDAS-ARMA-GLS-biclags	0.80	1.02	0.43	0.69	1.03	0.23	1.19	0.82	0.04	0.77	0.93	0.04
UMIDAS-ARMA-biclags	0.70	0.89	0.15	0.67	1.00	0.46	1.37	0.95	0.04	0.80	0.97	0.09
UMIDAS-ARMA-3lags	0.68	0.87	0.13	0.64	0.96	0.16	1.44	0.99	0.47	0.82	0.99	0.42
	$h = 4$						$h = 4$					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	0.86	1.00	NaN	0.74	1.00	NaN	1.83	1.00	NaN	0.97	1.00	NaN
MIDAS-ARMA-12lags	1.08	1.25	0.00	0.84	1.13	0.00	2.28	1.25	0.01	1.10	1.14	0.00
MIDAS-ARMA-3lags	0.85	0.98	0.42	0.73	0.99	0.36	1.72	0.94	0.14	0.92	0.95	0.14
UMIDAS-biclags	1.00	1.16	0.00	0.80	1.07	0.01	1.65	0.91	0.02	0.91	0.94	0.06
UMIDAS-ARMA-GLS-biclags	1.26	1.46	0.00	0.87	1.17	0.00	1.64	0.90	0.06	0.93	0.96	0.16
UMIDAS-ARMA-biclags	1.06	1.23	0.01	0.81	1.09	0.02	2.01	1.10	0.19	0.99	1.03	0.35
UMIDAS-ARMA-3lags	0.98	1.13	0.06	0.77	1.03	0.20	2.14	1.17	0.04	1.07	1.11	0.05

Note: The table reports the results on the forecasting performance of the different models as presented in Section 4. In the columns "value" we report the MSE and the MAE respectively. In the columns "ratio" we report the MSE and MAE of each model relative to the MIDAS-AR benchmark. In the columns "DM" we report the Diebold-Mariano test value. The forecasts are evaluated over the sample 1980Q1-2015Q4.

Table 7: Forecasting U.S. GDP growth

	Explanatory variable: Industrial production growth					
	$h = 1$					
	MSE			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	4.06	1.00	NaN	1.58	1.00	NaN
MIDAS-ARMA-12lags	4.05	1.00	0.41	1.59	1.01	0.13
MIDAS-ARMA-3lags	4.27	1.05	0.07	1.60	1.01	0.22
UMIDAS-biclags	4.21	1.04	0.15	1.58	1.00	0.41
UMIDAS-ARMA-GLS-biclags	4.19	1.03	0.13	1.60	1.01	0.16
UMIDAS-ARMA-biclags	4.18	1.03	0.19	1.59	1.01	0.26
UMIDAS-ARMA-3lags	4.27	1.05	0.07	1.60	1.01	0.22
	$h = 2$					
	MSE			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	8.14	1.00	NaN	2.01	1.00	NaN
MIDAS-ARMA-12lags	8.04	0.99	0.28	1.99	0.99	0.17
MIDAS-ARMA-3lags	7.59	0.93	0.00	1.92	0.95	0.00
UMIDAS-biclags	8.12	1.00	0.45	1.99	0.99	0.19
UMIDAS-ARMA-GLS-biclags	11.26	1.38	0.03	2.44	1.21	0.00
UMIDAS-ARMA-biclags	12.23	1.50	0.01	2.48	1.23	0.01
UMIDAS-ARMA-3lags	8.24	1.01	0.42	1.97	0.98	0.16
	$h = 4$					
	MSE			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	9.14	1.00	NaN	2.11	1.00	NaN
MIDAS-ARMA-12lags	8.63	0.94	0.12	2.08	0.99	0.30
MIDAS-ARMA-3lags	8.26	0.90	0.03	2.02	0.96	0.05
UMIDAS-biclags	8.77	0.96	0.19	2.05	0.97	0.10
UMIDAS-ARMA-GLS-biclags	9.74	1.07	0.33	2.16	1.02	0.35
UMIDAS-ARMA-biclags	8.91	0.97	0.40	2.09	0.99	0.45
UMIDAS-ARMA-3lags	9.07	0.99	0.47	2.06	0.98	0.31

Note: See Table 6.

Table 8: Forecasting U.S. real consumption growth

	Explanatory variable: Industrial production growth						**Explanatory variable: Consumption index growth**					
	$h = 1$						$h = 1$					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	8.84	1.00	NaN	2.26	1.00	NaN	9.07	1.00	NaN	2.25	1.00	NaN
MIDAS-ARMA-12lags	7.45	0.84	0.00	1.88	0.83	0.00	8.07	0.89	0.06	1.95	0.87	0.00
MIDAS-ARMA-3lags	7.42	0.84	0.00	1.89	0.84	0.00	8.50	0.94	0.16	2.09	0.93	0.04
UMIDAS-biclags	8.63	0.98	0.02	2.23	0.99	0.03	8.85	0.98	0.14	2.24	1.00	0.48
UMIDAS-ARMA-GLS-biclags	7.14	0.81	0.00	1.84	0.81	0.00	7.75	0.85	0.00	1.95	0.87	0.00
UMIDAS-ARMA-biclags	7.30	0.83	0.00	1.88	0.83	0.00	8.61	0.95	0.32	1.99	0.89	0.02
UMIDAS-ARMA-3lags	7.47	0.84	0.00	1.91	0.85	0.00	7.63	0.84	0.00	1.92	0.86	0.00
	$h = 2$						$h = 2$					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	8.58	1.00	NaN	2.21	1.00	NaN	8.75	1.00	NaN	2.25	1.00	NaN
MIDAS-ARMA-12lags	7.94	0.93	0.09	2.00	0.90	0.00	8.12	0.93	0.16	2.05	0.91	0.03
MIDAS-ARMA-3lags	7.55	0.88	0.00	1.95	0.88	0.00	7.67	0.88	0.00	1.96	0.87	0.00
UMIDAS-biclags	8.38	0.98	0.07	2.20	0.99	0.04	8.62	0.99	0.32	2.20	0.98	0.08
UMIDAS-ARMA-GLS-biclags	7.30	0.85	0.04	1.84	0.83	0.00	7.76	0.89	0.02	1.94	0.86	0.00
UMIDAS-ARMA-biclags	7.94	0.93	0.16	1.95	0.88	0.00	7.85	0.90	0.03	1.97	0.88	0.00
UMIDAS-ARMA-3lags	7.70	0.90	0.04	1.99	0.90	0.00	7.65	0.87	0.00	1.98	0.88	0.00
	$h = 4$						$h = 4$					
	MSE			**MAE**			**MSE**			**MAE**		
	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM	Value	Ratio	DM
MIDAS-AR-12lags	9.56	1.00	NaN	2.37	1.00	NaN	9.38	1.00	NaN	2.37	1.00	NaN
MIDAS-ARMA-12lags	9.18	0.96	0.27	2.22	0.94	0.08	8.84	0.94	0.17	2.21	0.93	0.05
MIDAS-ARMA-3lags	8.31	0.87	0.00	2.10	0.88	0.00	8.08	0.86	0.00	2.04	0.86	0.00
UMIDAS-biclags	9.23	0.97	0.04	2.34	0.99	0.16	9.26	0.99	0.29	2.34	0.99	0.20
UMIDAS-ARMA-GLS-biclags	7.72	0.81	0.00	1.92	0.81	0.00	8.95	0.95	0.30	2.15	0.91	0.02
UMIDAS-ARMA-biclags	8.95	0.94	0.18	2.16	0.91	0.01	8.44	0.90	0.03	2.11	0.89	0.00
UMIDAS-ARMA-3lags	8.21	0.86	0.01	2.08	0.88	0.00	8.51	0.91	0.05	2.13	0.90	0.00

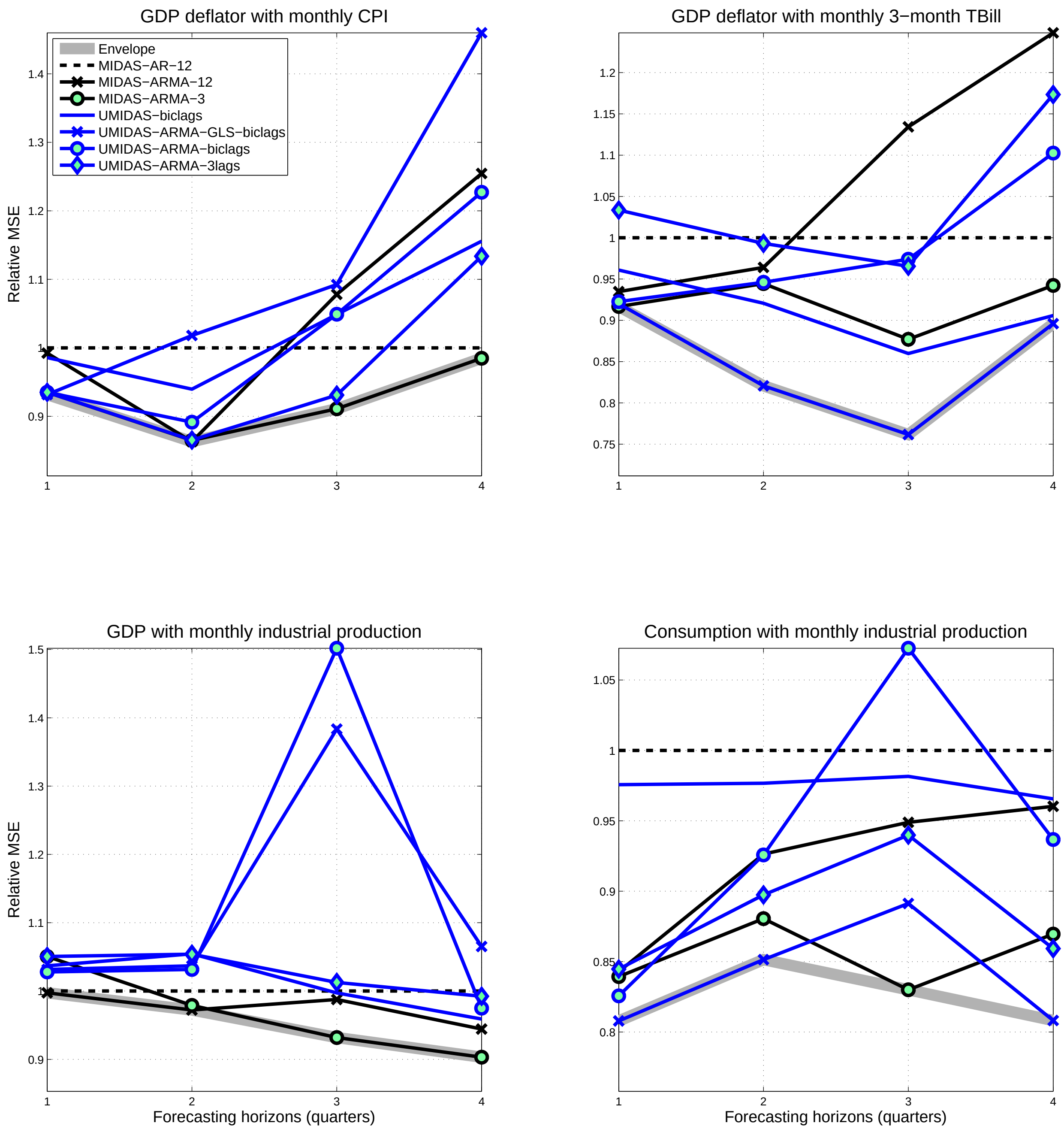
Note: See Table 6

Table 9: Bias/Variance decomposition of MSE

		Bias				**Variance**			
		$h=1$	$h=2$	$h=3$	$h=4$	$h=1$	$h=2$	$h=3$	$h=4$
GDP deflator with CPI inflation	MIDAS-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	1.04	0.89	0.91	0.98	0.98	0.88	1.15	1.35
	MIDAS-ARMA-3lags	0.93	0.78	0.72	0.85	0.94	0.92	1.03	1.07
	UMIDAS-biclags	0.80	0.78	0.80	0.99	1.02	1.01	1.18	1.21
	UMIDAS-ARMA-GLS-biclags	0.94	1.01	0.95	1.07	0.94	1.02	1.15	1.55
	UMIDAS-ARMA-biclags	0.93	0.80	0.77	0.87	0.94	0.95	1.19	1.37
	UMIDAS-ARMA-3lags	0.93	0.78	0.73	0.83	0.94	0.92	1.06	1.27
GDP deflator with TBill	MIDAS-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	1.11	1.02	0.96	1.16	0.91	0.95	1.20	1.21
	MIDAS-ARMA-3lags	0.97	0.99	0.91	0.91	0.91	0.94	0.89	0.99
	UMIDAS-biclags	0.93	0.95	0.89	0.94	0.97	0.93	0.88	0.91
	UMIDAS-ARMA-GLS-biclags	0.97	0.95	0.87	0.94	0.92	0.81	0.76	0.90
	UMIDAS-ARMA-biclags	0.95	1.00	0.96	1.06	0.92	0.94	0.99	1.10
	UMIDAS-ARMA-3lags	0.92	0.90	0.98	1.12	1.05	1.03	0.97	1.14
GDP with Industrial Production	MIDAS-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	0.85	0.92	0.82	0.92	1.00	0.99	1.01	0.95
	MIDAS-ARMA-3lags	0.92	1.05	1.02	0.94	1.06	1.01	0.92	0.90
	UMIDAS-biclags	1.04	1.02	1.04	0.93	1.04	1.06	0.99	0.97
	UMIDAS-ARMA-GLS-biclags	0.96	1.06	1.31	1.07	1.03	1.10	1.36	1.06
	UMIDAS-ARMA-biclags	0.98	1.07	1.12	1.03	1.03	1.04	1.52	0.97
	UMIDAS-ARMA-3lags	0.92	1.11	1.11	0.92	1.06	1.06	1.00	1.00
Consumption with Industrial Production	MIDAS-AR-12lags	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	MIDAS-ARMA-12lags	0.61	0.68	0.79	0.69	1.00	1.08	1.06	1.17
	MIDAS-ARMA-3lags	0.51	0.63	0.61	0.80	1.03	1.04	0.99	0.97
	UMIDAS-biclags	0.99	0.99	1.02	1.00	0.98	0.98	0.96	0.95
	UMIDAS-ARMA-GLS-biclags	0.37	0.15	0.60	0.46	1.03	1.12	1.08	1.07
	UMIDAS-ARMA-biclags	0.49	0.58	0.65	0.80	1.02	1.12	1.30	1.07
	UMIDAS-ARMA-3lags	0.46	0.56	0.62	0.70	1.06	1.09	1.13	1.02

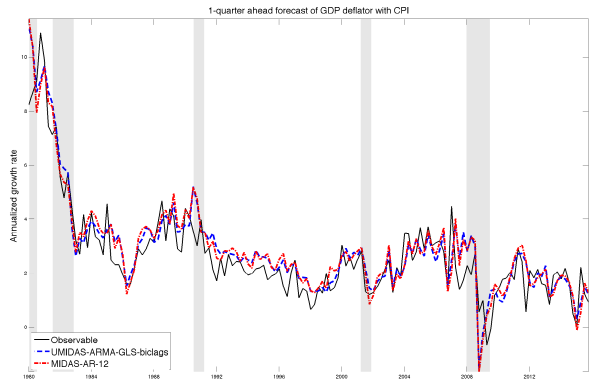
Note: The table the decomposition of the MSE of the different models as presented in Section 4 into bias and variance, for different forecasting horizons. The forecasts are evaluated over the sample 1980Q1-2015Q4.

Figure 1: Relative MSE at different forecasting horizons

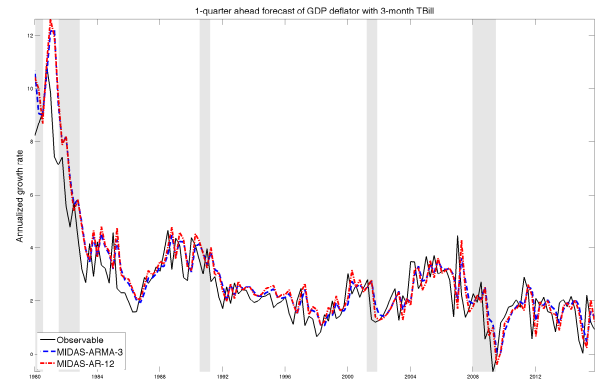


Note: In the figure we report for each model under analysis the ratio of its MSE relative to the benchmark MIDAS-AR. Black lines represent MIDAS models, blue lines UMIDAS models. The grey line connects the lowest MSE at each horizon: it thus represent the best models across horizons.

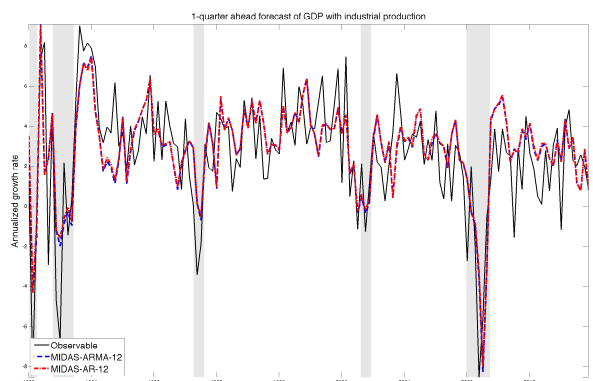
Figure 2: Out-of-sample performance: one-quarter ahead



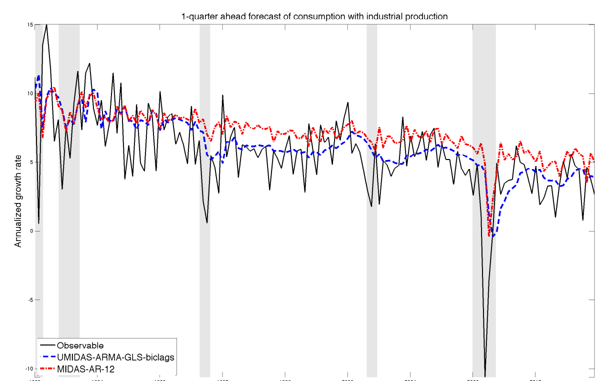
(a) GDP deflator with monthly CPI



(b) GDP deflator with monthly 3-month TBill

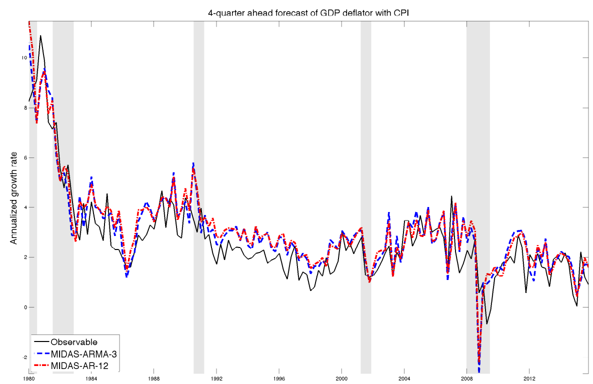


(c) GDP with monthly industrial production

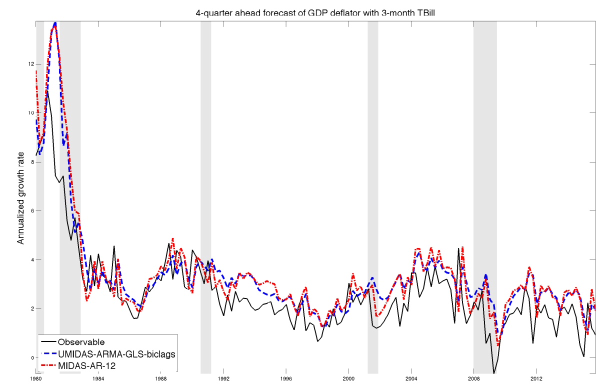


(d) Consumption with monthly industrial production

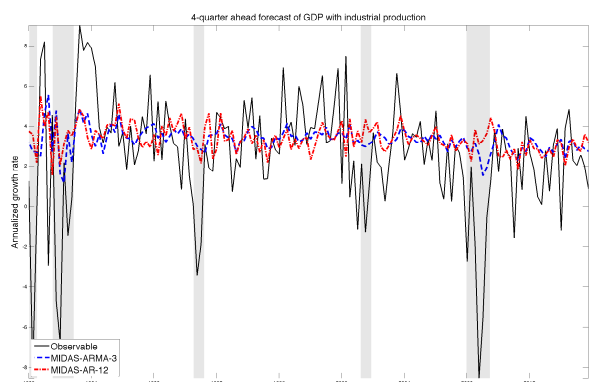
Figure 3: Out-of-sample performance: four-quarters ahead



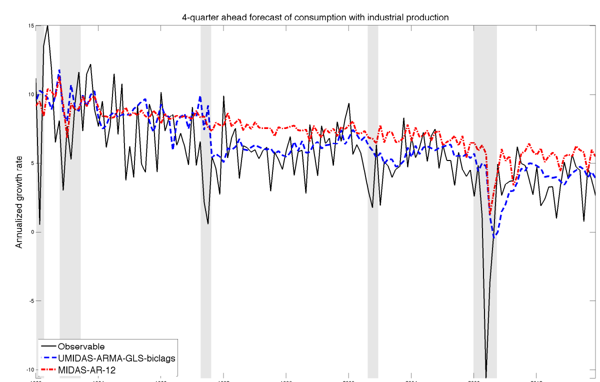
(a) GDP deflator with monthly CPI



(b) GDP deflator with monthly 3-month TBill



(c) GDP with monthly industrial production



(d) Consumption with monthly industrial production

Figure 4: MA coefficients