Optimal Trend Inflation*

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Abstract

We consider a sticky price model that incorporates firm heterogeneity and plausible firm level productivity trends. Aggregating the model in closed form, we show that the optimal steady state inflation rate generically differs from zero, and that the optimal inflation rate persistently responds to productivity disturbances. Using firm level data from the U.S. Census Bureau to estimate the inflation relevant trends, we find that the optimal trend inflation rate for the U.S. economy was slightly above 2 percent in the year 1986, but continously declined thereafter and reached about 1 percent in the year 2013.

Keywords: optimal inflation rate, sticky prices, firm heterogeneity

JEL Class. No.: E52, E31, E32

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1 Introduction

This paper makes progress by introducing firm level heterogeneity into an otherwise standard sticky price economy. It shows that the resulting generalized sticky price framework yields rather different implications for the optimal inflation rate and can rationalize - amongst other things - positive inflation rates in steady state.

Due to the technical difficulties associated with aggregating heterogeneous firm models, it is standard in the sticky price literature to abstract from all firm level heterogeneity beyond that generated by price adjustment frictions themselves. As is well known, price adjustment frictions then tightly anchor the optimal steady state inflation rate at zero, e.g., Woodford (2003).[1] This rather robust but somewhat puzzling implication of standard sticky price models arises because the homogeneity assumption implies that the productivity of price adjusting firms is equal to that of non-adjusting firms. With economic efficiency requiring relative prices to reflect relative productivities, this feature strongly calls for price adjusting firms to charge the same price as non-adjusting firms, i.e., for zero inflation.[2]

The present paper extends the basic stick price setup by introducing idiosyncratic firm level productivity adjustments that arrive in conjunction with a price adjustment opportunity. This gives rise to a setting with heterogeneous firm level productivities in which the productivity of price adjusting firms is not necessarily equal to that of non-adjusting firms.

In economic terms, one can interpret the combined shock that we introduce in a number of meaningful ways. It may represent, for example, an event in which a firm introduces a new product that is produced with a new technology and that requires the firm to choose a price for this new product. Alternatively, firms may introduce a new model of their existing product that offers new quality features. Again, this would require the firm to choose a price for the new model. Such product or model substitutions take place rather frequently in the data and are typically associated with price changes, but are routinely abstracted from in standard sticky price setups.[3] It is also possible to interpret the shocks we introduce as capturing firm turnover, whereby an existing firm exits the economy and is replaced by a new firm. The productivity adjustment then represents the gap between the technology used by the old and the new firm. Again, it is natural to

[1] Section 2 discusses a range of extensions of the basic framework considered in the literature and their implications for the optimal inflation rate.

[2] Yun (2005) shows, using a setting with homogenous firms, that if initial prices do not reflect initial productivities, the optimal inflation rate can display tranistory deviations from zero.

[3] Section III.C in Nakamura and Steinsson (2008) provides evidence on the rate of product substitution in the U.S. CPI.

assume that the new firm can freely choose its product price upon entry.

To illustrate the implications of such combined firm level shocks for the optimal inflation rate, we introduce them into an otherwise standard sticky price setting with Calvo price adjustment frictions. The choice of a baseline setting with time dependent price adjustment frictions is thereby purely for convenience and we show that our main results equally apply when introducing these shocks instead into menu cost setups.

For simplicity, we shall refer to the arrival of a price adjustment opportunity that occurs in conjunction with an adjustment of firm level productivity as a 'δ-shock' and we let $\delta \geq 0$ denote the idiosyncratic probability that such a shock arrives at a given firm at any point in time. The δ-shocks differ from the idiosyncratic price adjustment shocks present in Calvo price stickiness models, which feature no productivity adjustment. The main point of the paper is that the optimal inflation rate discontinuously jumps when moving from a situation where $\delta = 0$ to one where $\delta > 0$.

To model productivity dynamics, we consider a setting featuring three systematic productivity trends, each of which has different implications for the optimal inflation rate. First, there is a common trend in total factor productivity (TFP), which affects all firms equally, as in a standard homogeneous firm model. This common TFP trend captures general purpose innovations that are adopted by all firms simultaneously. Second, there is an experience trend in firm TFP, which determines how firms accumulate experience since they last received a δ-shock. Depending on the economic interpretation of δ-shocks, the experience trend captures experience accumulation in the production of a specific product or a product quality, or experience accumulation over the lifetime of the firm. One can interpret the experience trend more broadly as capturing learning-by-doing effects. Third, there is a cohort productivity trend, which determines the productivity (or quality) level of the cohort of firms that receives a δ-shock at a given point in time. This trend seeks to capture the fact that new products/qualities/firms bring into the economy new technologies that are not (yet) used by other firms. Firms that receive a δ-shock receive this cohort productivity (in addition to the common TFP component) and then gradually accumulate experience over time.

We show that the optimal steady state inflation rate in this setting depends on the strength of the experience trend relative to the strength of the cohort trend, whenever $\delta > 0$, but is independent of δ, provided $\delta > 0$. The optimal inflation rate is also independent of the common TFP process. This differs notably from a setting without idiosyncratic shocks ($\delta = 0$), where the optimal inflation rate is always equal to zero.

To provide economic intuition for these findings, consider two polar settings. The first setting abstracts from the presence of a cohort trend and considers a setting where the only trend is that firms accumulate experience over time.[4] Firms that receive a δ-shock lose their existing experience stock and become in this setting less productive than the remaining firms. From a welfare standpoint, the optimal price of firms that receive a δ-shock should therefore exceed the average price of the other firms, so as to accurately reflect relative productivities. Achieving this requires either that firms with δ-shocks choose higher prices or that firms without δ-shocks reduce prices, or a combination thereof.

In the presence of sticky prices, price reductions by firms without δ-shocks are costly. In time dependent adjustment models, they lead to inefficient price dispersion due to asynchronous price adjustment; in state-dependent pricing models, they require firms to pay adjustment costs. Therefore, it is optimal to implement the efficient relative price exclusively by having firms with δ-shocks charge higher prices, while all other firms keep their prices. Clearly, this implies that the aggregate inflation rate must be positive in steady state.

As mentioned before, the optimal steady state inflation rate turns out to be independent of the probability $\delta > 0$. A lower value for δ decrease the share of firms receiving δ-shocks, which - ceteris paribus - reduces the optimal rate of inflation. Yet, lower values for δ increase the productivity of firms that do not receive a δ-shock relative to those that do, as the former had - on average - more time to accumulate experience. This increases the optimal relative price for firms with a δ-shock and - ceteris paribus - increases the optimal inflation rate. In net terms, these two effects exactly offset each other.

In the second polar setting, there is no experience effect and the only trend is one where firms that receive a δ-shock become as described by the cohort trend. Firms receiving a δ-shock are then become more productive than the remaining firms, thus should optimally charge lower prices. This makes negative rates of inflation optimal.[5] As before, the strength of this effect is independent of δ, as long as $\delta > 0$. A lower value for δ increases the productivity of firms with δ-shocks relative to the remaining firms, as the latter have received their δ-shocks and associated productivity increase a longer time ago. This effect exactly offsets the fact that there are fewer firms with δ-shocks.

Combining the experience and the cohort effects within a common setting, the optimal steady state inflation rate depends on the strength of the experience effect, which pushes towards positive inflation rates, relative to the strength of the cohort effect, which pushes towards negative inflation rates.

We also determine in closed-form the dynamic response of the optimal inflation rate

[4] As mentioned before, we can abstract from the common TFP trend, as it does not affect the optimal inflation rate.

[5] Due to price setting frictions, it is again not optimal that firms without δ-shocks adjust prices.

following shocks to experience and cohort productivity. We show that such shocks have fairly persistent effects on the optimal inflation rate, especially in settings in which δ-shock intensity is low ($\delta > 0$ but close to zero). A low value for δ causes any persistent level shock to experience or to cohort productivity to disseminate only gradually in the economy. This requires that inflation persistently moves along the transition until the productivity distribution has reached its new steady state. For the limit $\delta \longrightarrow 0$ optimal inflation becomes a random walk.

To obtain a plausible framework for empirical analysis, we extend our results to a multi sector economy. The multi sector setup allows for sector-specific experience and cohort trends, sector specific 'common' TFP trends, as well as sector specific firm turnover rates and degrees of price stickiness. We then show that the inflation rate that maximizes steady state welfare is a weighted average of the inflation rates that would achieve efficient relative prices within each sector individually. The latter depend again on the sector specific cohort and experience trends. Based on this finding, we devise a model-based empirical strategy that allows estimating these sector specific cohort and experience trends and thus the optimal inflation rate from firm level data. As we show, firm level data are required for estimating the inflation relevant trends implied by the model, as these cannot be identified from aggregate data.[6] To estimate the relevant firm level trends, we interpret δ-shocks as entry and exit shocks for productive establishments. We then use the Longitudinal Business Database (LBD) of the U.S. Census Bureau, which covers all private sector establishments in the United States, and estimate the relevant cohort and experience trends and their evolution over time. Our regression results show that the optimal inflation rate implied by our model is positive but approximately halved over the period 1986 to 2013. Depending on the precise value of the elasticity of product demand assumed, the level of the optimal inflation rate varies. For our preferred demand elasticity specification, the optimal inflation rate declined from around 2% in 1986 to approximately 1% in 2013.

The remainder of the paper is structured as follows. Section 2 discusses the related literature. Section 3 presents our heterogeneous firms model with sticky prices. Section 4 analytically aggregates the model and section 5 shows that the flexible price equilibrium is first best when a Pigouvian output subsidy corrects firms' monopoly power. The main result on the optimal rate of inflation is presented in section 6. It derives in closed form the state-contingent inflation rate that replicates the first best allocation in a setting with sticky prices and δ-shocks. The result does not rely on any approximations and apply for a fully stochastic setup. Section 7 discusses the implications of the main result for the optimal steady state inflation rate and steady state welfare. Section 8 shows how the optimal inflation rate jumps discontinuously when moving from a standard sticky price

[6] This holds true even for the baseline setup with a single economic sector.

economy without δ-shocks to one including such shocks. Section 9 determines the utility costs of implementing strict price stability in an economy where the optimal inflation rate according to our model is positive. Section 10 discusses the optimal response of the inflation rate to economic disturbances. Section 11 extends the baseline setup to a multi-sector economy, allowing for sectoral degrees of price stickiness, firm turnover and sector specific productivity trends. Section 12 presents a model-consistent approach for estimating the optimal inflation rate and reports our empirical results regarding the optimal rate of inflation in the U.S. economy. Finally, section 13 discusses the robustness of our findings towards various extensions. A conclusion briefly summarizes. Proofs and technical material is relegated to a series of appendices.

2 Related Literature

A small set of papers discusses the relationship between the optimal inflation rate and productivity trends focusing on aggregate or sectoral productivity trends. All of these papers find that the optimal inflation rate is negative. Amano et al. (2009) consider an economy with aggregate productivity growth in which nominal wages and prices are sticky. They show how monetary policy affects wage and price mark-ups and that this can make it optimal to implement deflation, so as to reduce average wage mark-ups. Wolman (2011) considers a two sector sticky price economy with sectoral productivity trends. He shows that - even in the absence of monetary frictions - the optimal inflation rate is either negative or close to zero, depending on the precise modeling of price adjustment frictions.

Golosov and Lucas (2007) and Nakamura and Steinsson (2010) consider sticky price setups with heterogeneous firms and study monetary non-neutrality within these setups without considering the issue of the optimal inflation rate. Firms in their settings are subject to random idiosyncratic productivity shocks. This differs from the present setup, where the idiosyncratic δ-shocks give rise to systematic productivity adjustments as implied by the cohort and experience trends and also allow for flexible price adjustments. These features together give rise to a situation where the (average) productivity of price adjusting firms differs from the (average) productivity of non-adjusting firms, which ultimately makes non-zero inflation rates optimal. In Golosov and Lucas (2007) and Nakamura and Steinsson (2010), it is mainly the firms with very positive or very negative idiosyncratic productivity shocks that adjust prices, which suggests that the productivity of price adjusting firms is on average similar to that of non-adjusting firms. Although settling this issue needs further study, it suggests that zero inflation is close to optimal in steady state, as in settings with homogeneous firms.

The present paper is also related to a large literature studying the determinants of optimal inflation, most of which find that the optimal inflation rate is either negative or close to zero. None of these papers makes a connection between the optimal inflation rate and firm level productivity dynamics.

In classic work, Kahn, King and Wolman (2003) explore the trade-off between price adjustment frictions, which call for price stability, and monetary frictions that call for a Friedman-type deflation. They document how a slight rate of deflation is optimal in such frameworks. In a comprehensive survey, Schmitt-Grohé and Uribe (2010) document the robustness of these findings to a large number of natural extensions. They show that taxation motives, including the presence of untaxed income, foreign demand for domestic currency (Schmitt-Grohé and Uribe (2012a)), as well as a potential quality bias in measured inflation rates (Schmitt-Grohé and Uribe (2012b)), are all unable to rationalize significantly positive rates of inflation.

Adam and Billi (2006) and Coibion, Gorodnichenko and Wieland (2012) explicitly incorporate a lower bound on nominal interest rates into sticky price economies. Both papers find that optimal monetary policy is consistent with close to zero average rates of inflation. While zero lower bound episodes make it optimal to promise inflation in the future, these promises should only be made conditionally on being at the lower bound, which happens rather infrequently.

A number of papers finds positive inflation rates to be optimal on average when introducing downward nominal wage rigidities into the standard setup. Kim and Ruge-Murcia (2009) argue that such rigidities allow justifying optimal inflation rates of approximately 0.35% on average when using a model with aggregate shocks only. Looking at a setting with idiosyncratic shocks, Benigno and Ricci (2011) also find a positive steady state inflation rate to be optimal.[7] Carlsson and Westermark (2016) consider a setting with nominal wage rigidities and search and matching frictions in the labor market. They show how a standard U.S. calibration of the model implies failure of the Hosios condition and justifies an inflation rate of about 1.16% annually. Schmitt-Grohé and Uribe (2013) analyze the case for temporarily elevated inflation in the Euro Area due to the presence of downward rigidity of nominal wages.

Brunnermeier and Sannikov (2016) show that the optimal inflation rate can also be positive in a model without nominal rigidities. They present a model with undiversifiable idiosyncratic capital income risk in which the optimal inflation rate increases with the amount of idiosyncratic risk.

There is also a literature studying endogenous firm entry decisions in homogenous firm economies, focusing on the effect of inflation on the firm entry margin, e.g., Corsetti and

[7] Since positive inflation has no welfare costs in their setup, they do not quantify the optimal inflation rate.

Bergin (2008), Bilbiie et al. (2008) and Bilbiie, Fujiwara and Ghironi (2014). Bilbiie et al. (2014) document - amongst other things - that the welfare optimal inflation rate is positive, whenever the benefit of additional varieties to consumers falls short of the market incentives for creating these varieties. Inflation then reduces the value of creating varieties and brings firm entry closer to its efficient (lower) level. The present paper abstracts from endogenous firm entry decisions and thus from the implication of monetary policy for the entry margin, instead considers a setting with heterogeneous firms and exogenous entry and exit.

A set of empirical papers decomposes the observed U.S. inflation rate into a trend and cyclical component and show that trend inflation displays substantial low-frequency variation over time, e.g., Cogley and Sargent (2001), Cogley, Primiceri and Sargent (2010). The sticky price literature has reacted to these facts by incorporating trend inflation into their workhorse models, see Ascari and Sbordone (2014) and Cogley and Sbordone (2008). Trend inflation emerges in these setups because the central bank pursues an exogenous inflation target, which is non-zero and potentially time-varying. Primiceri (2006), Sargent (1999), Sargent, Williams and Zha (2006) pesent settings in which policymakers are learning about the Phillips Curve trade-off and show how this can endogenously give rise to the observed low-frequency movements in U.S. inflation. The present paper does not explore to what extent changes in firm level productivity can contribute to explaining the observed U.S. inflation history, as it focuses on the normative implications of these trends. Exploring the positive content of the theory presented in this paper appears to be an interesting avenue for further research.

3 Economic Model

We consider a cashless economy with nominal rigidities and monopolistically competitive firms. The model is entirely standard, except for the more detailed modeling of firm level productivity and price adjustment dynamics. For simplicity, we derive our results within a time-dependent price adjustment model à la Calvo (1983). As we show in section 13.1, our main findings remain unaltered when considering instead a setting where price adjustment frictions take the form of menu costs.

3.1 Technology, Prices and Price Adjustment Opportunities

We consider a setting with a unit mass of monopolistically competitive firms indexed by $j \in [0, 1]$ and discrete time ($t = 0, 1, \ldots$). Each firm j produces output Y_{jt}, which enters as an input into the production of an aggregate consumption/investment good Y_t according to

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}} \mathrm{dj}\right)^{\frac{\theta}{\theta-1}}, \tag{1}$$

where $1 < \theta < \infty$ denotes the price elasticity of product demand. Let P_{jt} denote the price charged by firm j in period t. As in a standard Calvo setup, firms can adjust prices with probability $1-\alpha$ each period ($0 < \alpha < 1$). The arrival of a Calvo price adjustment opportunity is thereby idiosyncratic and independent of all other exogenous random variables in the economy. We augment this standard setting by a second idiosyncratic price adjustment opportunity that arrives with probability $\delta \geq 0$ each period. This second adjustment opportunity is also idiosyncratic but arrives in conjunction with a firm level technology change, as described in detail below. In particular, let $\delta_{jt} \in \{0,1\}$ denote the idiosyncratic i.i.d. random variable governing this second price and technology adjustment and let $\delta_{jt} = 1$ indicate the arrival of such an adjustment event for firm j in period t ($\Pr(\delta_{jt} = 1) = \delta$). We shall informally refer to the event $\delta_{jt} = 1$ as *the arrival of a δ-shock*.

Letting K_{jt} and L_{jt} denote the amount of capital and labor used by firm j, respectively, firm output Y_{jt} is given by

$$Y_{jt} = A_t Z_{jt} \left(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t\right), \tag{2}$$

where A_t captures aggregate productivity, Z_{jt} firm-specific productivity, and $F_t \geq 0$ the potential presence of fixed costs for operating the firm. To be consistent with balanced growth, we assume

$$F_t = f \cdot (\Gamma_t^e)^{1-\frac{1}{\phi}} \tag{3}$$

for some $f \geq 0$, where Γ_t^e captures the growth trend in the balanced growth path, as defined in equation (20) below.[8] Aggregate productivity evolves according to

$$A_t = a_t A_{t-1},$$

firm specific productivity according to

$$Z_{jt} = \begin{cases} g_t Z_{jt-1} & \text{if } \delta_{jt} = 0 \\ Q_t & \text{if } \delta_{jt} = 1 \end{cases},$$

where Q_t is given by

$$Q_t = q_t Q_{t-1}, \tag{4}$$

with $a_t, g_t, q_t > 0$ being stationary productivity growth processes with unconditional mean $a, q, g > 0$, respectively. Productivity dynamics thus feature three trends: (1) the aggregate growth trend a_t; (2) the firm level growth trend g_t, which applies in the absence of

[8] Absent aggregate productivity growth, the formulation of fixed costs in equation (3) corresponds to that used in Melitz (2003).

δ-shocks; and (3) the productivity trend q_t, which determines the effects of δ-shocks on technology. Each growth trend will have a different implications for the optimal inflation rate within our setting.

To understand the productivity dynamics implied by the previous setup, consider first the special case with $\delta = 0$. In the absence of idiosyncratic δ-shocks to firm technology, all firms experience the same productivity growth rate $a_t g_t$. Such a setting with homogeneous productivity growth across all firms is the one routinely considered in the sticky price literature.[9]

Next, consider the case $\delta > 0$ and let s_{jt} denote the number of periods that has elapsed since the firm last experienced a δ-shock (i.e., $\delta_{j,t-s_{jt}} = 1$ and $\delta_{j\widetilde{t}} = 0$ for $\widetilde{t} = t - s_{jt} + 1, ..., t$). Firm-specific technology can then be written as

$$Z_{jt} = G_{jt} Q_{t-s_{jt}},$$

where

$$G_{jt} = \begin{cases} 1 & \text{for } s_{jt} = 0 \\ g_t G_{jt-1} & \text{otherwise,} \end{cases}$$

and where Q_t follows equation (4). This alternative formulation illustrates that all firms that receive a δ-shock in t upgrade idiosyncratic technology to $Z_{jt} = Q_t$, so that Q_t can be interpreted as capturing a 'cohort effect' of productivity dynamics, where cohorts are determined by the arrival time of the last δ-shock. Following any δ-shock, the firm experiences further productivity gains, as described by the process G_{jt}, as long as no further δ-shocks arrive. Since the productivity gains G_{jt} are lost with the arrival of the next δ-shock, one can interpret the process G_{jt} as capturing 'experience' or 'learning-by-doing effects' associated with the cohort production technology $Q_{t-s_{jt}}$. Following a δ-shock in period t, our specification thereby implies that firm technology increases (temporarily decreases), if Q_t has been growing faster (slower) than G_t since the time of arrival of the last δ-shock prior to period t. Note that the long-term growth rate of firms' productivity is determined by the process $a_t q_t$, as the experience growth rates g_t generate - due to the occasional resets - only a level effect for productivity.

As usual, we define the aggregate price level as

$$P_t \equiv \left(\int_0^1 (P_{jt})^{1-\theta} \, \mathrm{dj} \right)^{\frac{1}{1-\theta}}. \tag{5}$$

[9] For the case $\delta = 0$, our setting still allows for a non-degenerate initial distribution of relative firm productivities. Typically, this initial distribution is also assumed degenerate in the sticky price literature. As we show below, however, the additional assumption of a degenerate initial distribution does not affect the conclusions regarding the optimal inflation rate, as long as initial prices reflect initial productivities, see Yun (2005) for a discussion of this and related issues in a homogeneous firms setting.

Cost-minimization in the production of final output Y_t implies

$$P_t = \int_0^1 \left(\frac{Y_{jt}}{Y_t}\right) P_{jt} \text{ dj},$$

so that the price level is an expenditure weighted average of the prices in the different expenditure categories, in line with the practice at statistical agencies. The inflation rate is defined as

$$\Pi_t \equiv P_t/P_{t-1}.$$

We shall furthermore assume that $a_t = a\epsilon_t^a$, $q_t = q\epsilon_t^q$, and $g_t = g\epsilon_t^g$ with $\epsilon_t^a, \epsilon_t^q, \epsilon_t^g \geq 0$ being stationary shocks with an arbitrary contemporaneous and intertemporal covariance structure, satisfying $E[\epsilon_t^a] = E[\epsilon_t^q] = E[\epsilon_t^g] = 1$. Finally, to obtain a well-defined steady state, we assume

$$(1-\delta)\,(g/q)^{\theta-1} < 1. \tag{6}$$

This condition insures that relative prices in the flexible price economy remain bounded.

3.2 Alternative Interpretations of the Setup

The previous section defined δ-shocks ($\delta_{jt} = 1$) as an idiosyncratic change in firm level productivity that is associated with a price adjustment opportunity. This section presents three alternative interpretations of δ-shocks that clarify why such 'productivity changes' may plausibly be associated with price flexibility at the firm level.

Product substitution. The event $\delta_{jt} = 1$ can be interpreted as an event in which the product previously produced by firm j is no longer demanded by consumers. Firm j reacts to this by introducing a new product, which - for simplicity - is assigned the same product index j. The variable Q_t then captures the productivity level associated with products that are newly introduced in t and G_{jt} captures experience accumulation in producing the new product. Product substitutions, e.g., in the form of new product versions or models, take place rather frequently in the data and are also prevalent in the CPI baskets of statistical agencies. Evidence provided in Moulton and Moses (1997), Bils (2009) and Syed and Myers (2016) shows that the prices of new products are typically higher than those that they replace, even when accounting for quality improvements.[10] It thus appears reasonable to assume price flexibility for new products.

Entry and exit. The event $\delta_{jt} = 1$ can also be interpreted as an event in which firm j becomes unproductive and exits the economy. It is then replaced by a firm to which we

[10] Evidence provided in Bils (2009) shows that inflation for durables ex computers over the period 1988-2006 averaged 2.5% per year, but when only including only matched items, the inflation rate was -3.7% per year.

assign - for simplicity - the same index j. The variable Q_t then captures the productivity level of the cohort of firms that enters in period t and G_{jt} the experience accumulated over the lifetime of the firm. Firm entry and exit rates are high in the United States and range in the order of 8-12% per year, see figure 3 in Decker et al. (2014). It also appears plausible to assume that newly entering firms can choose prices flexibly. We shall use an interpretation of our model along these lines in our empirical exercise in section 12.

Quality improvements. Let Q_{jt} denote the quality of the product produced by firm j in period t. Defining $Q_{jt} = Q_{t-s_{jt}}$, the event $\delta_{jt} = 1$ captures the fact that firm j upgrades the quality of its product from level $Q_{t-1-s_{j,t-1}}$ to level Q_t. Let aggregate output produced with intermediate inputs of different quality be given by

$$Y_t = \left(\int_0^1 \left(Q_{jt} \widetilde{Y}_{jt} \right)^{\frac{\theta-1}{\theta}} \text{dj} \right)^{\frac{\theta}{\theta-1}},$$

let firm output of a given quality level Q_{jt} be given by

$$\widetilde{Y}_{jt} = A_t G_{jt} \left(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t \right),$$

where G_{jt} now captures experience effects associated with producing quality Q_{jt}, and let $\widetilde{P}_{jt}$ denote the price of a unit of good j of quality level Q_{jt}. Assuming that statistical agencies can perfectly adjust the price level for quality, i.e., P_t is given by

$$P_t = \left(\int_0^1 \left(\frac{\widetilde{P}_{jt}}{Q_{jt}} \right)^{1-\theta} \text{dj} \right)^{\frac{1}{1-\theta}},$$

this setup with quality improvements is mathematically identical to the one with productivity improvements spelled out in the previous section.[11] As with new products, it appears natural to assume that firms can flexibly price goods with new quality features.

3.3 Optimal Price Setting

Firms choose prices, capital and hours worked to maximize profits. While price adjustment is subject to adjustment frictions, factor inputs can be chosen flexibly. Letting W_t denote the nominal wage and r_t the real rental rate of capital, firm j chooses the factor input mix so as to minimize production costs $K_{jt} P_t r_t + L_{jt} W_t$ subject to the constraints imposed by the production function (2). Let

$$I_{jt} \equiv F_t + Y_{jt} / (A_t Q_{t-s_{jt}} G_{jt})$$

[11] The quality-adjusted price $\widetilde{P}_{jt}/Q_{jt}$ and the quality adjusted quantities $\widetilde{Y}_{jt} Q_{jt}$ then correspond to the price P_{jt} and quantity Y_{jt}, respectively, in the previous section.

denote the units of factor inputs ($K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}}$) required to produce Y_{jt} units of output. As appendix A.1 shows, cost minimization implies that the marginal costs of I_{jt} are given by

$$MC_t = \left(\frac{W_t}{1/\phi}\right)^{\frac{1}{\phi}} \left(\frac{P_t r_t}{1-1/\phi}\right)^{1-\frac{1}{\phi}}. \tag{7}$$

Now consider a firm in period t that can freely choose its price because it has either received a δ-shock or a Calvo adjustment shock. Letting α denote the probability implied by the Calvo process that the firm has to keep its price ($0 < \alpha < 1$), the firm will not be able to reoptimize its price with probability $\alpha(1-\delta)$ at any future date, i.e., whenever it receives neither a δ- shock nor a Calvo adjustment shock.[12] The price setting problem of a firm that can optimize its price in period t is thus given by

$$\max_{P_{jt}} E_t \sum_{i=0}^{\infty} (\alpha(1-\delta))^i \frac{\Omega_{t,t+i}}{P_{t+i}} \left[(1+\tau) P_{jt+i} Y_{jt+i} - MC_{t+i} I_{jt+i}\right] \tag{8}$$

$$\begin{aligned} s.t. \quad I_{jt+i} &= F_t + Y_{jt+i}/A_{t+i} Q_{t-s_{jt}} G_{jt+i}, \\ Y_{jt+i} &= \left(P_{jt+i}/P_{t+i}\right)^{-\theta} Y_{t+i}, \\ P_{jt+i+1} &= \Xi_{t+i+1,t+i} P_{jt+i}. \end{aligned}$$

where τ denotes a sales tax/subsidy and $\Omega_{t,t+i}$ denotes the representative household's discount factor between periods t and $t+i$. The first constraint captures firm's technology, the second constraint captures the demand function faced by the firm, as implied by equation (1), and the last constraint captures how the firm's price is indexed over time (if at all) in periods in which prices are not reset optimally. We consider general price indexation schemes and allow $\Xi_{t+i+1,t+i}$ to be a function of aggregate variables up to period $t+i$.[13]

Appendix A.2 shows that the optimal price $P_{jt}^{\star}$ can be expressed as

$$\frac{P_{jt}^{\star}}{P_t} \left(\frac{Q_{t-s_{jt}} G_{jt}}{Q_t}\right) = \left(\frac{\theta}{\theta-1} \frac{1}{1+\tau}\right) \frac{N_t}{D_t}. \tag{9}$$

where the variables N_t and D_t are functions of aggregate variables only and evolve recur-

[12] In any period, the firm can adjust its price with probability δ due to the arival of a δ-shock and with probability $(1-\alpha)(1-\delta)$ due to the arrival of a Calvo price adjustment shock.

[13] We only require that price indexation is such that the price setting problem remains well defined, that price indexation does not give rise to multiplicities of the optimal inflation rate and that indexation is such that $\Xi_{t+1,t} = 1$ in a steady state without inflation. For instance, when indexing occurs with respect to lagged inflation according to $\Xi_{t+1,t} = (\Pi_t)^{\kappa}$ with $\kappa \geq 0$, we rule out $\kappa > 1$ to avoid non-existence of optimal plans and rule out $\kappa = 1$ to avoid multiplicities of the steady state inflation rate.

sively according to

$$N_t = \frac{MC_t}{P_t A_t Q_t} + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}\frac{Y_{t+1}}{Y_t}\left(\Xi_{t,t+1}\right)^{-\theta}\left(\frac{P_{t+1}}{P_t}\right)^{\theta}\left(\frac{q_{t+1}}{g_{t+1}}\right)N_{t+1}\right] \tag{10}$$

$$D_t = 1 + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}\frac{Y_{t+1}}{Y_t}\left(\Xi_{t,t+1}\right)^{1-\theta}\left(\frac{P_{t+1}}{P_t}\right)^{\theta-1}D_{t+1}\right]. \tag{11}$$

Equation (9) shows that the optimal reset price of a firm depends only on how its own productivity ($A_t Q_{t-s_{jt}} G_{jt}$) relates to the productivity of a firm hit by a δ-shock in period t ($A_t Q_t$), as well as on aggregate variables. It is precisely this feature, which will permit aggregation of the model in closed form. Equation (9) furthermore shows that more productive firms optimally choose lower prices. For the special case with homogeneous firms, where relative productivity is always equal to one ($Q_{t-s_{jt}} G_{jt}/Q_t = 1$), equations (9)-(11) reduce to those capturing price dynamics in a standard homogeneous firm model.

3.4 Household Problem

There is a representative household with balanced growth consistent preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\frac{\left[C_t V(L_t)\right]^{1-\sigma} - 1}{1-\sigma}\right),$$

where C_t denotes private consumption of the aggregate good, L_t labor supply, ξ_t a preference shock with $E[\xi_t] = 1$ and $\beta \in (0,1)$ the discount factor. We assume $\sigma > 0$ and that $V(\cdot)$ is such that period utility is strictly concave in (C_t, L_t) and that Inada conditions are satisfied. The household faces the flow budget constraint

$$C_t + K_{t+1} + \frac{B_t}{P_t} = (r_t + 1 - d)K_t + \frac{W_t}{P_t}L_t + \int_0^1 \frac{\Theta_{jt}}{P_t}\,\mathrm{dj} + \frac{B_{t-1}}{P_t}(1 + i_{t-1}) - T_t,$$

where K_{t+1} denotes the capital stock, B_t nominal government bond holdings, i_{t-1} the nominal interest rate, W_t the nominal wage rate, r_t the real rental rate of capital, d the depreciation rate of capital, Θ_{jt} nominal profits from ownership of firm j, and T_t lump sum taxes. Household borrowing is subject to a no-Ponzi scheme constraint. The first order conditions characterizing optimal household behavior are entirely standard and are derived in Appendix A.3. To insure existence of a well-defined balanced growth path, we assume throughout the paper that

$$\beta < (aq)^{\phi\sigma}.$$

3.5 Government

To close the model we consider a government which faces the budget constraint

$$\frac{B_t}{P_t} = \frac{B_{t-1}}{P_t}(1 + i_{t-1}) + \tau \int_0^1 \left(\frac{P_{jt}}{P_t}\right) Y_{jt}\,\mathrm{dj} - T_t,$$

where τ denotes a sales subsidy, which will be used to correct for the monopoly power distortions in product markets. The government levies lump sum taxes T_t, so as to implement a bounded state-contingent path for government debt B_t/P_t.[14] Since we consider a cashless limit economy, there are no seigniorage revenues, even though the central bank controls the nominal interest rate. We furthermore assume that monetary policy is not constrained by a lower bound on nominal interest rates.

3.6 Equilibrium

We are now in a position to define the market equilibrium:

Definition 1 *An equilibrium is a state-contingent path for* $\{(P_{jt}, L_{jt}, K_{jt})$ *for* $j \in [0,1]$, $W_t, r_t, i_t, C_t, K_{t+1}, L_t, B_t, T_t\}_{t=0}^{\infty}$ *such that*

1. *the firms' choices* $\{P_{jt}, L_{jt}, K_{jt}\}_{t=0}^{\infty}$ *maximize profits for all* $j \in [0,1]$, *given the price adjustment frictions,*
2. *the household's choices* $\{C_t, K_{t+1}, L_t, B_t\}_{t=0}^{\infty}$ *maximize expected household utility,*
3. *the government flow budget constraint holds each period, and*
4. *the markets for capital, labor, final and intermediate goods and government bonds clear,*

given the initial values $B_{-1}(1+i_{-1}), K_0, P_{j,-1}$, *and* $A_{-1}Q_{-1-s_{j,-1}}G_{j,-1}$, $j \in [0,1]$.

4 Analytical Aggregation with heterogeneous Firms

This section outlines the main steps that allow us to aggregate the model in closed form. In a first step, we derive a recursive representation describing the evolution of the aggregate price level P_t over time. In a second step, we derive a closed form expression for the aggregate production function. In a last step, we show how to appropriately detrend aggregate variables, so as to render them stationary.

Evolution of the aggregate price level. Let $P^{\star}_{t-s,t-k}$ denote the optimal price of a firm that last received a δ-shock in $t-s$ and that has last reset its price in $t-k$ ($s \geq k \geq 0$). In period t, this firm's price is equal to $\Xi_{t-k,t}P^{\star}_{t-s,t-k}$, where $\Xi_{t-k,t} = \prod_{j=1}^{k} \Xi_{t-k+j-1,t-k+j}$ captures the cumulative effect of price indexation (with $\Xi_{t-k,t} = 1$ in the absence of price indexation). Let $\Lambda_t(s)$ denote the weighted average price in period t of the cohort of firms

[14] Household's transversality condition will then automatically be satisfied in equilibrium.

that last received a δ-shock in period $t-s$, where all prices are raised to the power of $1-\theta$, i.e.,

$$\Lambda_t(s) = (1-\alpha)\sum_{k=0}^{s-1}\alpha^k(\Xi_{t-k,t}P^\star_{t-s,t-k})^{1-\theta} + \alpha^s(\Xi_{t-s,t}P^\star_{t-s,t-s})^{1-\theta}. \tag{12}$$

There are α^s firms that have not had a chance to optimally reset prices since receiving the δ-shock and $(1-\alpha)\alpha^k$ firms that have last adjusted $k<s$ periods ago. From equation (5) it follows that one can use the cohort average prices $\Lambda_t(s)$ to express the aggregate price level as

$$P_t^{1-\theta} = \sum_{s=0}^{\infty}(1-\delta)^s\delta\Lambda_t(s), \tag{13}$$

where δ is the mass of firms that receive a δ-shock each period and $(1-\delta)^s$ is the share of those firms that have not received another δ-shock since s periods.

To express the evolution of P_t in a recursive form, consider the optimal price $P^\star_{t-s,t}$ of a firm that received a δ-shock $s>0$ periods ago, but can adjust the price in t due to the the arrival of a Calvo shock. Also, consider the price $P^\star_{t,t}$ of a firm that receives a δ-shock in period t. The optimal price setting equation (9) then implies

$$P^\star_{t,t} = P^\star_{t-s,t}\left(\frac{g_t\times\cdots\times g_{t-s+1}}{q_t\times\cdots\times q_{t-s+1}}\right). \tag{14}$$

The previous equation shows that a stronger cohort productivity trend (higher values for q) causes the firm that receives a δ-shock in period t to choose lower prices relative to firms that received δ-shocks further in the past, as a stronger cohort trend makes this firm relatively more productive. Conversely, the experience effect (higher values for g) increases the optimal relative price of the firm that received a δ-shock in t. The net effect depends on the relative strength of the cohort versus the experience effect.

Appendix A.4 shows how one can combine equations (12), (13), and (14) to obtain a recursive representation for the evolution of the aggregate price level given by

$$P_t^{1-\theta} = \delta(P^\star_{t,t})^{1-\theta} + (1-\alpha)(1-\delta)\frac{(p^e_t)^{\theta-1}-\delta}{1-\delta}(P^\star_{t,t})^{1-\theta} + \alpha(1-\delta)(\Xi_{t-1,t}P_{t-1})^{1-\theta}, \tag{15}$$

where p^e_t depends on the history of shocks to cohort and experience productivity and evolves recursively according to

$$(p^e_t)^{\theta-1} = \delta + (1-\delta)\left(p^e_{t-1}g_t/q_t\right)^{\theta-1}. \tag{16}$$

The last term on the r.h.s. of equation (15) captures the price level effects from the share $\alpha(1-\delta)$ of firms that neither received a Calvo shock nor a δ-shock. These firms keep their old price (P_{t-1} on average), adjusted for possible effects of price indexation, as captured by the indexation term $\Xi_{t-1,t}$. The first term on the r.h.s. of equation (15) captures the price effects of the mass δ of firms that received a δ-shock in period t; these firms optimally charge price $P^{\star}_{t,t}$. The second term captures price resetting by firms that received a Calvo shock in period t; their share is $(1-\alpha)(1-\delta)$ and they set a price that on average differs from the price charged by firms hit by a δ-shock, depending on the value of p^e_t. This latter aspect in equation (15) is the key difference relative to the standard model without firm heterogeneity in productivity. A stronger experience trend (a higher value for g_t), for instance, increases $(p^e_t)^{\theta-1}$, and - ceteris paribus - causes firms that receive a Calvo shock to choose a lower value for the optimal reset price. A stronger cohort trend (a higher value for q_t) has the opposite effect. Overall, the interesting new feature is that price dynamics now depend on the productivity trends.

In a setting where all firms have identical productivity, e.g., where the cohort effect is as strong as the experience effect ($q_t = g_t$ for all t), equation (16) implies that p^e_t converges to one so that the price level eventually evolves according to

$$P_t^{1-\theta} = [\delta + (1-\alpha)(1-\delta)] \left(P^{\star}_{t,t}\right)^{1-\theta} + \alpha(1-\delta)(\Xi_{t-1,t}P_{t-1})^{1-\theta},$$

which is independent of productivity developments. If in addition there are no δ-shocks ($\delta = 0$), the previous equation simplifies further to

$$P_t^{1-\theta} = (1-\alpha)(P^{\star}_{t,t})^{1-\theta} + \alpha(\Xi_{t-1,t}P_{t-1})^{1-\theta},$$

which describes the evolution of the aggregate price level in the standard Calvo model with homogenous firms.

Aggregate production function. In appendix A.5 we show that aggregate output Y_t can be written as

$$Y_t = \frac{A_t Q_t}{\Delta_t}\left(K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t\right), \tag{17}$$

where K_t denotes the aggregate capital stock, L_t aggregate hours worked and

$$\Delta_t = \int_0^1 \left(\frac{Q_t}{G_{jt}Q_{t-s_{jt}}}\right)\left(\frac{P_{jt}}{P_t}\right)^{-\theta} dj \tag{18}$$

evolves recursively according to

$$\Delta_t = \left[\delta + (1-\alpha)(1-\delta)\frac{(p^e_t)^{\theta-1} - \delta}{1-\delta}\right]\left(\frac{P^{\star}_{t,t}}{P_t}\right)^{-\theta} + \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta}\Delta_{t-1}. \tag{19}$$

TFP in the aggregate production function (17) is a function of the TFP of the latest cohort hit by δ-shock, A_tQ_t, and of the adjustment factor Δ_t. The latter is defined in equation (18) and captures a firm's productivity relative to that of the latest cohort, $Q_t/\left(Q_{t-s_{jt}}G_{jt}\right)$, and weighs this relative productivity by the firm's production share

$(P_{jt}/P_t)^{-\theta}$. Equations (17) and (18) thus show how relative price distortions may lead to output losses by negatively affecting aggregate technology, e.g., by allocating more demand to relatively inefficient firms. The evolution of the adjustment factor over time is described by equation (19) and depends on productivity trends - amongst other ways - through the variable p_t^e. In the limit with homogeneous firm trends (i.e., $q_t = g_t$), p_t^e converges to one and the evolution of Δ_t becomes independent of productivity realizations. If - in addition - there are no δ-shocks ($\delta = 0$), then equation (19) simplifies further to

$$\Delta_t = (1-\alpha)\left(\frac{P^{\star}_{t,t}}{P_t}\right)^{-\theta} + \alpha\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta}\Delta_{t-1},$$

which is the equation capturing the potential distortions from price dispersion within the standard homogeneous firm model.

Balanced Growth Path. One can obtain stationary aggregate variables by rescaling them by the aggregate growth trend

$$\Gamma_t^e = (A_t Q_t/\Delta_t^e)^{\phi}, \tag{20}$$

where Δ_t^e denotes the efficient adjustment factor chosen by the planner, defined in equation (24) below. Specifically, the rescaled output $y_t = Y_t/\Gamma_t^e$ and the rescaled capital stock $k_t = K_t/\Gamma_t^e$ are now stationary and the aggregate production function (17) can be written as

$$y_t = \left(\frac{\Delta_t^e}{\Delta_t}\right)\left(k_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - f\right). \tag{21}$$

In the deterministic balanced growth path, Γ_t^e grows at the rate $(aq)^{\phi}$ and hours worked are constant, whenever monetary policy implements a constant inflation rate. Appendices A.6 and A.7 write all model equations using stationary variables only and appendix A.8 determines the resulting deterministic steady state.

5 Efficiency of the Flexible Price Equilibrium

This section derives the efficient allocation and provides conditions under which the flexible price equilibrium is efficient. Appendix B shows that the efficient consumption, hours and capital allocation $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ solves

$$\max_{\{C_t, L_t, K_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\frac{[C_t V(L_t)]^{1-\sigma} - 1}{1-\sigma}\right) \tag{22}$$

$$s.t. \quad C_t + K_{t+1} = (1-d)K_t + \frac{A_t Q_t}{\Delta_t^e}\left((K_t)^{1-\frac{1}{\phi}}(L_t)^{\frac{1}{\phi}} - F_t\right), \tag{23}$$

where

$$\Delta_t^e \equiv \left(\int_0^1 \left(\frac{Q_t}{G_{jt} Q_{t-s_{jt}}} \right)^{1-\theta} \mathrm{dj} \right)^{\frac{1}{1-\theta}}, \tag{24}$$

which evolves according to

$$\left(\Delta_t^e\right)^{1-\theta} = \delta + (1-\delta) \left(\Delta_{t-1}^e q_t / g_t\right)^{1-\theta}. \tag{25}$$

Constraint (23) is the economy's resource constraint, when expressing aggregate output using the aggregate production function (17). The efficient productivity adjustment factor Δ_t^e showing up in the planner's production function is defined in equation (24); its recursive evolution is described by equation (25). The first order conditions of problem (22)-(23) are necessary and sufficient conditions characterizing the efficient allocation.

Decentralizing the efficient allocation requires that firms' prices, which enter Δ_t and thus in the aggregate production function (17), satisfy certain conditions. In particular, equation (18) implies that one achieves $\Delta_t = \Delta_t^e$ if prices satisfy

$$\frac{P_{jt}}{P_t} = \frac{1}{\Delta_t^e} \frac{Q_t}{G_{jt} Q_{t-s_{jt}}}, \tag{26}$$

which requires relative prices to accurately reflect relative productivities. Furthermore, as in models without firm heterogeneity, one has to eliminate firm's monopoly power by a Pigouvian subsidy to obtain efficiency of the market allocation. We thus impose the following condition:

Condition 1 *The sales subsidy corrects firms' market power, i.e.,* $\frac{\theta}{\theta-1}\frac{1}{1+\tau} = 1$.

Appendix C then proves the following result:

Proposition 1 *The flexible price equilibrium ($\alpha = 0$) is efficient, if condition 1 holds.*

The proof of the proposition shows that condition (26) holds under flexible prices, so that one achieves $\Delta_t = \Delta_t^e$ and thereby productive efficiency. In the presence of the assumed sales subsidy, consumer decisions are also undistorted, so that the consumption, hours and capital in the flexible price equilibrium are identical to the values that these variables assume in the efficient allocation.

6 Optimal Inflation with Sticky Prices

This section determines the optimal inflation rate for an economy with sticky prices ($\alpha > 0$). It derives the optimal rate of inflation for the nonlinear stochastic economy with heterogeneous firms in closed form and shows how inflation optimally depends on the productivity growth rates a_t, q_t and g_t. As it turns out, the optimal inflation rate implements the efficient allocation (the flexible price benchmark).

To establish our main result in the most straightforward manner, we impose an assumption on initial conditions, in particular on how firms' initial prices and initial productivities are related. Similar conditions are imposed in sticky price models with homogenous firms, where it is routinely assumed that initial dispersion of prices has reached its stationary outcome. We impose:

Condition 2 *Initial prices in $t=-1$ reflect firms' relative productivities, i.e.,*

$$P_{j,-1} \propto \frac{1}{Q_{-1-s_{j,-1}} G_{j,-1}} \qquad \text{all } j \in [0,1].$$

We discuss the effects of relaxing this condition below. The following proposition states our main result:

Proposition 2 *Suppose conditions 1 and 2 hold. The equilibrium allocation in the sticky price economy is efficient, if monetary policy implements the gross inflation rate*

$$\Pi_t^{\star} = \Xi_{t-1,t}^{\star} \left(\frac{1-\delta \left(\Delta_t^e\right)^{\theta-1}}{1-\delta} \right)^{\frac{1}{\theta-1}} \qquad \text{for all } t \geq 0, \tag{27}$$

where $\Xi_{t-1,t}^{\star}$ captures price indexation between periods $t-1$ and t ($\Xi_{t-1,t}^{\star} \equiv 1$ in the absence of indexation) and Δ_t^e is defined in equation (24) and evolves according to equation (25).

In the absence of price indexation ($\Xi_{t-1,t}^{\star} \equiv 1$), the optimal inflation rate is only a function of the variable Δ_t^e, which captures the distribution of relative productivities between newly entering firms and existing firms, see equation (24). Since these relative productivities are independent of the common TFP growth rate a_t, it follows that the optimal inflation rate does not depend on the realizations of a_t. In contrast, the cohort productivity growth rate q_t and the experience growth rate g_t do affect Δ_t^e, see equation (25). Yet, these trends affect the optimal inflation rate in opposite directions: a stronger cohort productivity growth rate q_t decreases the optimal inflation rate, while a stronger experience growth rate g_t increases the optimal inflation rate.

For the special case in which all firms have identical productivity trends ($\delta = 0$) or even identical productivities ($\Delta_t^e = 1$), the optimal gross inflation rate is equal to one in the absence of price indexation, as in a standard homogenous firm model. Perfect price stability is then optimal at all times.

Price indexation by non-adjusting firms ($\Xi_{t-1,t}^{\star} \neq 1$), say because of indexation to the lagged inflation rate, introduces additional components into the optimal aggregate inflation rate. In particular, it requires that price-adjusting firms, i.e., firms that receive either a δ-shock or a Calvo shock, also adjust their price by the indexation component. This way prices continue to accurately reflect relative productivities at all times. This explains why indexation affects the optimal inflation rate one-for-one.

Although proposition 2 assumes that firms' initial prices accurately reflect the initial relative productivities, the initial productivity distribution itself is unrestricted. We conjecture that for a setting where condition 2 fails to hold, one would obtain additional transitory and deterministic components to the optimal inflation rate, as in the homogeneous firm setting studied by Yun (2005). The inflation rate stated in proposition 2 would then become optimal only asymptotically.

The proof of proposition 2, which is contained in appendix D, establishes that with the optimal inflation rate firms choose relative prices as in the flexible price equilibrium. This result is established by showing that (1) firms that receive a δ-shock choose the same optimal relative price as in the flexible price economy, and that (2) firms that receive a Calvo shock optimally choose not to adjust their price, which avoids the emergence of price dispersion between otherwise identical firms. The optimal inflation rate thus insures that both (1) and (2) are satisfied. This, together with the fact that (3) initial prices reflect initial productivities, ensures that all relative prices are identical to the ones in the flexible price equilibrium. Under the assumed output subsidy, it then follows that household allocations are also identical to the flexible price equilibrium, which has been shown to be efficient, see proposition 1.

Interestingly, it follows from the proof of proposition 2 that the inflation rate (27) continues to insure productive efficiency (but not full efficiency) in settings where condition 1 fails to hold. From the theory of optimal taxation it then follows that it continues to be optimal to implement the inflation rate (27), as it is suboptimal to distort intermediate production as long as (distortionary) taxes on final goods are available.

7 The Optimal Steady State Inflation Rate

This section discusses the optimal steady state inflation rate implied by the model. To simplify the discussion, we abstract from price indexation, unless otherwise stated.

Proposition 2 makes it clear that in the case in which the productivity of all firms grows at the same rate ($\delta = 0$), which includes as a special case a setting with homogeneous firms, we obtain that the optimal inflation rate is $\Pi_t^\star = 1$, independently of all shock processes. For $\delta = 0$, the optimal (gross) steady state inflation rate is thus trivially equal to one.

For the case $\delta > 0$, the optimal inflation rate jumps discontinuously away from $\Pi_t^\star = 1$, but turns out to be independent of the value of δ. The following lemma summarizes this result:

Lemma 1 *Suppose conditions 1 and 2 hold, there are no economic disturbances, there is no price indexation ($\Xi^{\star}_{t-1,t} \equiv 1$) and $\delta > 0$. The optimal inflation rate then satisfies*

$$\lim_{t\to\infty} \Pi^{\star}_t = g/q. \tag{28}$$

Proof. From equations (6) and (25) it follows that $(\Delta^e_t)^{\theta-1} \to [1-(1-\delta)\,(g/q)^{\theta-1}]/\delta$. It then follows from proposition 2 that $\lim_{t\to\infty} \Pi^{\star}_t \to g/q$. ■

Since we allow for arbitrary initial productivity distributions, the absence of shocks does not necessarily imply that the optimal inflation rate is constant from the beginning. This only happens asymptotically, once the productivity distribution converges to its stationary distribution (in detrended terms).[15] The lemma provides the inflation rate that is asymptotically optimal as this stationary distribution is reached.[16]

Interestingly, the optimal long-run inflation rate is completely independent of the intensity of δ-shocks, which may appear surprising. A higher value for δ implies that a larger share of firms receive a δ-shock. In a setting where $g > q$, this implies that many firms will - upon arrival of the δ-shock - loose more in terms of accumulated experience than they gain in terms of cohort productivity, i.e., their productivity initially falls. Since less productive firms should charge higher prices, one may conjecture that higher inflation should be optimal, as higher values for δ imply that there are more of these relatively unproductive firms. Yet, this argument ignores, that a higher intensity of δ-shocks also shortens experience accumulation with old cohort technologies. This makes firms that do not receive a δ-shock relatively less productive than in a setting with lower δ-shock intensity. This second effect suggests lower inflation rates to be optimal. As it turns out, the two effects exactly offset each other and the optimal steady state inflation rate is independent of δ.

It appears empirically plausible to assume that indeed $g > q$, i.e., the experience effects is stronger than the cohort effect. Interpreting the setting as one in which δ-shocks represent product substitutions, $g > q$ implies that new products are more expensive than old products and that their relative price is falling over the life cycle of the product, in line with evidence provided by Syed and Myers (2016). Likewise, interpreting the setting as one in which δ-shocks represent a firm exit shock, with exiting firms being replaced

[15] This is not an issue when $\delta = 0$ as the initial distribution remains then unchanged in detrended terms.

[16] The transitional dynamics can easily be derived from proposition 2 using the initial productivity distribution and equation (25).

by new firms, $g > q$ implies that old firms are larger and more productive than young firms and that firms' output grows over their lifetime, in line with empirical observations. Given this, lemma 1 likely implies positive values for the optimal steady state inflation rate.

Interestingly, aggregate productivity dynamics are not informative about what is the optimal inflation rate in an economy with δ-shocks. The aggregate steady state growth is equal to $(aq)^{\phi}$ and is driven by a factor that does affect the optimal inflation rate (q) and one that does not (a). Moreover, the experience effect (g) has no growth rate implications in the presence of δ-shocks but does affect the optimal inflation rate. Determining the optimal inflation rate thus requires studying firm level productivity trends, as aggregate productivity trends fail to be informative. We shall come back to this issue in section 12.

Finally, we discuss the effects of price indexation. For $\delta > 0$ the optimal long-run inflation rate is then given by $\Xi^{\star}_{t-1,t}\,(g/q)$. For the case where prices are indexed to lagged inflation according to $\Xi^{\star}_{t-1,t} = (\Pi_{t-1})^{\kappa}$ for some $\kappa \in [0, 1)$, one obtains

$$\lim_{t\to\infty} \Pi^{\star}_t = (g/q)^{\frac{1}{1-\kappa}} .$$

The presence of price indexation thus amplifies the divergence of the optimal gross inflation rate from one.

The results in this section show that the optimal inflation rate discontinuously jumps when moving from a setting without δ-shocks to one with $\delta > 0$. Yet, the efficient allocation also discontinuously jumps when moving from $\delta = 0$ to $\delta > 0$, as in the former case efficient aggregate growth is equal to ag and in latter case equal to aq. For this reason, the next section discusses the discontinuity of the optimal steady state inflation rate in greater detail.

8 Discontinuity of the Optimal Inflation Rate

This section compares the optimal inflation rate in an economy with δ-shocks ($\delta > 0$) to the one in the absence of such shocks ($\delta = 0$). We refer to the first economy as the δ-economy and to the latter as the 0-economy. Comparing these two economies is not as straightforward as might initially appear: even if both economies are subject to the same fundamental shocks (a_t, q_t, g_t), the efficient allocation displays a discontinuity when considering the limit $\delta \to 0$.[17]

To properly deal with this issue, we construct a δ-economy whose efficient aggregate allocation (consumption, hours, capital) is identical to the efficient aggregate allocation

[17] This has to do with the fact that aggregate productivity growth in the δ-economy is equal to $a_t q_t$ while it is equal to $a_t g_t$ in the 0-economy.

in the 0-economy.[18] We then compare the optimal inflation rates in these two economies and show that the optimal inflation rate for the δ-economy differs from the optimal rate for the 0-economy, even for the limit $\delta \to 0$.

Let a_t^δ, q_t^δ, g_t^δ denote the productivity disturbances in the δ-economy and let $A_{-1}^\delta G_{j,-1}^\delta Q_{-1-s_{j,-1}}^\delta$ for $j \in [0,1]$ denote the initial distribution of firm productivities. This together with the process $\{\delta_{jt}\}_{t=0}^\infty$ for all $j \in [0,1]$ determines the entire state-contingent values for A_t^δ, Q_t^δ, G_{jt}^δ, and $Q_{t-s_{jt}}^\delta$ for all $j \in [0,1]$ and all $t > 0$.

Next, consider the 0-economy and suppose it starts with the same initial capital stock as the δ-economy. For the 0-economy, we normalize $Q_{t-s_{jt}}^0 \equiv 1$ for all $j \in [0,1]$ and all t and then set the initial firm productivity distribution in the 0-economy equal to that in the δ-economy by choosing the initial conditions

$$A_{-1}^0 = A_{-1}^\delta,$$
$$G_{j,-1}^0 = G_{j,-1}^\delta Q_{-1-s_{j,-1}}^\delta.$$

Finally, let the process for common TFP in the 0-economy be given by

$$A_t^0 = A_t^\delta \left(\int_0^1 \left(Q_{t-s_{jt}}^\delta G_{jt}^\delta \right)^{\theta-1} \mathrm{dj} \right)^{\frac{1}{\theta-1}} \left(\int_0^1 \left(G_{jt}^0 \right)^{\theta-1} \mathrm{dj} \right)^{\frac{-1}{\theta-1}},$$

where G_{jt}^0 is generated by an arbitrary process g_t^0, e.g., $g_t^0 = g_t^\delta$. In this setting, it is easily verified that aggregate productivity associated with the efficient allocation, defined as

$$A_t Q_t / \Delta_t^e = A_t Q_t \left(\int_0^1 \left(G_{jt} Q_{t-s_{jt}} / Q_t \right)^{\theta-1} \mathrm{dj} \right)^{\frac{1}{\theta-1}},$$

is the same in the δ-economy and the 0-economy.[19] We then have the following result:

Proposition 3 *Under the assumptions stated in this section, the efficient allocations in the two economies satisfy*

$$C_t^\delta = C_t^0, L_t^\delta = L_t^0, K_t^\delta = K_t^0$$

for all $t \geq 0$ and all possible realizations of the disturbances.

Proof. Since $A_t^\delta Q_t^\delta / \Delta_t^\delta = A_t^0 Q_t^0 / \Delta_t^0$ for all t, it follows from the planner's problem (22)-(23) and the fact that the initial capital stock is identical that both economies share the same efficient allocation. ■

The following proposition shows that (generically) the optimal inflation rate discontinuously jumps when moving from the 0-economy to the δ-economy, even if both economies are identical in terms of their efficient aggregate dynamics:[20]

[18] The two economies do of course differ in their underlying firm level dynamics.

[19] The fact that $A_t Q_t / \Delta_t^e$ is equal to aggregate productivity in the efficient allocation follows from equations (23) and (24).

[20] Recall that the optimal inflation rates implement the efficient aggregate allocations in these economies.

Lemma 2 *Under the assumptions stated in this section and provided conditions 1 and 2 hold, the optimal inflation rate in the* 0*-economy is* $\Pi_t^{0\star} = 1$ *for all* t*. The optimal inflation rate in the* δ*-economy is given by equation (27); in particular, for* $g_t^N = g$ *and* $q_t^N = q$*, and in the absence of price indexation, the optimal rate of inflation in the* δ*-economy satisfies* $\lim_{t\to\infty} \Pi_t^{\delta\star} = g/q$*.*

Proof. The results directly follow from proposition 2 and lemma 1. ■

The previous result illustrates the fragility of the optimality of strict price stability in standard sticky price models, once firm dynamics and firm heterogeneity are taken into account. Moreover, in combination with proposition 3, it shows that two economies that can be identical in terms of their aggregate efficient allocations may require different inflation rates for implementing these allocations. This highlights the importance of micro-level productivity trends for determining the optimal inflation rate. The next section investigates the welfare consequences of implementing strict price stability when in fact non-zero inflation rates are optimal.

9 The Welfare Costs of Strict Price Stability

This section shows that suboptimally implementing strict price stability, as would be suggested by standard sticky price models, gives rise to strictly positive welfare costs whenever $g \neq q$. We show this fact first for a special case analytically, which allows considering also the limit $\delta \to 0$. In a second step, we use numerical simulations to highlight the source of the welfare losses and their magnitude.

The following proposition shows that - as long as $g \neq q$ - there is a strictly positive welfare loss that is bounded away from zero when implementing strict price stability; this holds true even for the limit $\delta \to 0$.[21]

Proposition 4 *Suppose conditions 1 and 2 hold, there are no economic disturbances, firm turnover is positive (*$\delta > 0$*), fixed costs of production are zero (*$f = 0$*), there is no price indexation (*$\Xi_{t-1,t}^\star \equiv 1$*), and disutility of work is given by*

$$V(L) = 1 - \psi L^\nu,$$

with $\nu > 1$ *and* $\psi > 0$*. Assume* $g/q > \alpha(1-\delta)$*, so that a well-defined steady state with strict price stability exists.*

Consider the limit $\beta(\gamma^e)^{1-\sigma} \to 1$ *and a policy implementing the optimal inflation rate* $\Pi_t^\star$ *from proposition 2, which satisfies* $\lim_{t\to\infty} \Pi_t^\star = \Pi^\star = g/q$*. Let* $c(\Pi^\star)$ *and* $L(\Pi^\star)$

[21] The proof of the proposition is contained in appendix E.

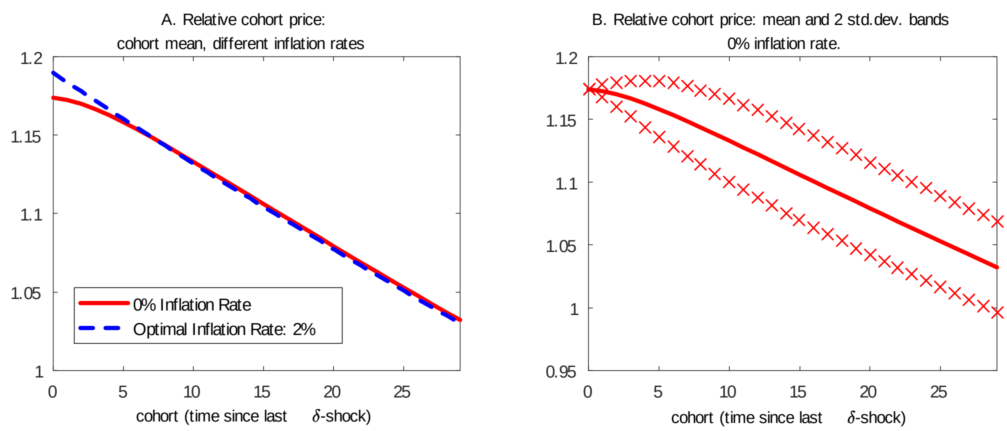

Figure 1: Relative prices and inflation

denote the limit outcomes for $t \to \infty$ for consumption and hours, respectively, under this policy. Similarly, let $c(1)$ and $L(1)$ denote the limit outcomes under the alternative policy of implementing strict price stability. Then,

$$L(1) = L(\Pi^{\star})$$

and

$$\frac{c(1)}{c(\Pi^{\star})} = \left(\frac{1-\alpha(1-\delta)(g/q)^{\theta-1}}{1-\alpha(1-\delta)}\right)^{\frac{\phi\theta}{\theta-1}} \left(\frac{1-\alpha(1-\delta)\,(g/q)^{-1}}{1-\alpha(1-\delta)(g/q)^{\theta-1}}\right)^{\phi} \leq 1. \tag{29}$$

For $g \neq q$ the previous inequality is strict and $\lim_{\delta\to 0} c(1)/c(\Pi^{\star}) < 1$.

We now illustrate the nature of the relative price distortions generated by suboptimal inflation rates. Panel A in figure 1 reports relative prices (y-axis), once for a setting where annualized inflation is 2% (in net terms) and once when inflation is 0%, assuming that the optimal rate is indeed 2%.[22] Panel A depicts the average cohort price (relative to the aggregate price level), where a cohort is defined as the number of quarters that has elapsed since the last δ-shock has hit (x-axis). Panel A shows that young cohorts charge a higher (relative) price and that this price decreases over the lifetime of the cohort. Under the optimal inflation rate (2%) the decline happens at a constant rate.[23] Under strict price stability, firms anticipate that their relative prices will not necessarily fall, due to Calvo price stickiness. This causes them to initially 'front load' prices, i.e., young cohorts charge in an environment with strict price stability initially lower prices than under the optimal inflation rate. Over time, some firms in the cohort get the opportunity to lower

[22] Figure 1 is computed using $g = 1.02^{0.25}, q = 1^{0.25}, \alpha = 0.75, \delta = 0.035, \theta = 3.8$ and assumes that the initial productivity distribution is equal to the stationary distribution (in detrended terms).

[23] The figure assumes that no shocks hit the economy.

their prices in response to Calvo shocks, but the average relative price of the cohort will eventually be higher than under the optimal inflation rate. Beyond these distortions in average cohort prices, the suboptimal inflation rate also generates prices distortions within a cohort of firms. This is illustrates in panel B of figure 1, which shows that strict price stability give rise to substantial amounts of price dispersion within the cohort. Under the optimal inflation rate, there is no price dispersion within the cohort.

Figure 2 reports the steady state value for the ratio Δ_t^e/Δ_t (y-axis) as a function of the implemented steady state inflation rate (x-axis), when the optimal inflation rate is 2% per year.[24] The aggregate production function (17) shows that one can interpret Δ_t^e/Δ_t as a measure of the aggregate productivity distortion that is implied by the relative price distortions associated with suboptimal inflation rates. The figure shows that a 10% shortfall of the inflation rate below its optimal value of 2% is associated with an aggregate productivity loss equal to about 1%. The productivity losses thereby arise rather nonlinearly: a shortfall of inflation of 2% below its optimal value is associated with an aggregate productivity loss of just 0.05%. Furthermore, inflation losses are asymmetric, with above optimal inflation leading to relatively larger losses. For instance, increasing inflation 8% above its optimal value generates a productivity loss of 0.94%, while decreasing inflation by the same amount below its optimal value leads to a productivity losses of only 0.37%.

10 The Variance of the Optimal Inflation Rate

This section discusses the optimal dynamic response of inflation to productivity disturbances. In the absence of δ-shocks, we have

$$\Pi_t^\star = 1 \text{ for all } t,$$

i.e., the optimal inflation rate is then independ of productivity shocks at all times. For $\delta > 0$, it follows from equations (25) and (27) that the optimal nonlinear inflation response to productivity disturbances is given by

$$\frac{1}{1-(1-\delta)\left(\frac{\Pi_t^\star}{\Xi_{t,t-1}}\right)^{\theta-1}} = 1 + \frac{(1-\delta)\left(\frac{g_t}{q_t}\right)^{\theta-1}}{1-(1-\delta)\left(\frac{\Pi_{t-1}^\star}{\Xi_{t-1,t}}\right)^{\theta-1}}. \tag{30}$$

Abstracting from price indexation ($\Xi_{t,t-1} \equiv 1$), a linearization of equation (30) delivers[25]

$$\pi_t^\star = (1-\delta)\pi_{t-1}^\star + \delta\left(\frac{g_t}{q_t} - 1\right). \tag{31}$$

[24] The figure is based on the same parameterization as figure 1.

[25] We log-linearize with respect to the variables $\Pi_t^\star$ and g_t/q_t at the point $(\Pi_t^\star, g_t/q_t) = (1, 1)$.

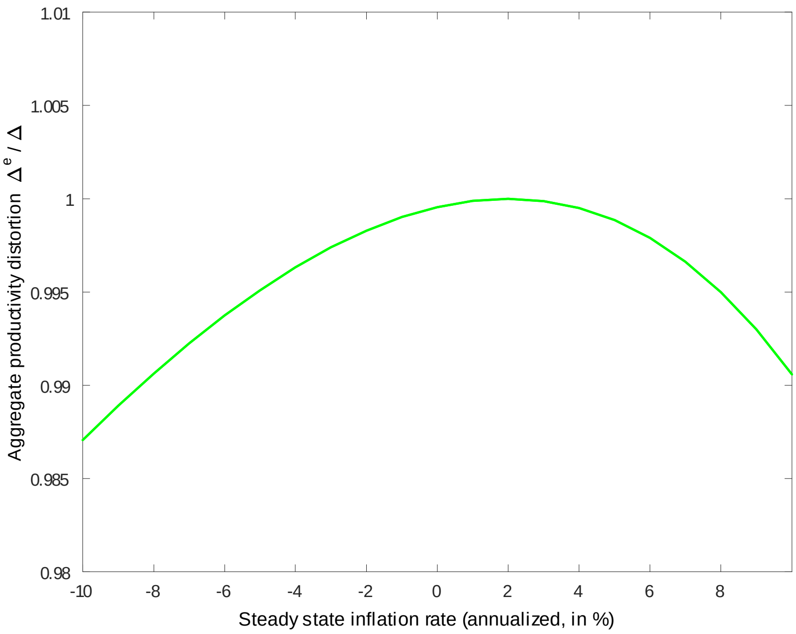

Figure 2: Aggregate productivity as a function of steady state inflation (optimal inflation rate is 2%)

As long as $\delta < 1$, the optimal inflation rate will thus display persistent responses to any deviation of g_t/q_t from its average value. A positive surprise to experience productivity growth g_t, for example, shifts up permanently the experience level of old cohorts. This requires persistently higher inflation rates, as new cohorts (firms that receive a δ-shock) now have to set persistently higher prices, until the productivity distribution reaches again its stationary distribution (in detrended terms). The speed with which the productivity distribution returns to its stationary distribution depends on δ. For $\delta = 1$, the return is immediate and the optimal inflation rate inherits the persistence properties of the exogenous driving process g_t/q_t. For values of δ close to zero, the optimal inflation rate approximately behaves like a random walk whenever g_t/q_t is an iid process, but the unconditional variance of inflation is decrease with δ and approaches zero as $\delta \to 0$.

11 Extension to a Multi-Sector Economy

We now extend the basic sticky price setup to a multi-sector setting. The latter allows for sector-specific productivity trends and sector-specific price stickiness. The extension is of interest when bringing the model to the data, as we do in the following section, because it allows assessing the effects of sectoral heterogeneity for the optimal inflation rate.[26] We show below that the optimal steady state inflation rate in a multi-sector economy is a weighted average of the inflation rates that would achieve efficiency in the respective sectors individually, where inflation rates have to be adjusted by sectoral relative growth trends.

Consider an economy with $z = 1, \ldots, Z$ sectors in which aggregate output Y_t is

$$Y_t = \prod_{z=1}^{Z} (Y_{zt})^{\psi_z},$$

with Y_{zt} denoting output in sector z and $\psi_z \geq 0$ being the sector's expenditure share, with the expenditure shares satisfying $\sum_{z=1}^{Z} \psi_z = 1$. Sectoral output Y_{zt} itself is a Dixit-Stiglitz aggregate of the output of a unit mass of firms j in sector z, in close analogy to the one-sector setup.

Let A_{zt} denote the TFP component, Q_{zt} the cohort specific component and G_{zjt} the experience component to firm productivity in sector $z = 1, \ldots, Z$. Output of the firm producing product $j \in [0, 1]$ in sector z is then given by

$$Y_{jzt} = A_{zt} Q_{t-s_{jzt}} G_{jzt} \left(K_{jzt}^{1-\frac{1}{\phi}} L_{jzt}^{\frac{1}{\phi}} - F_{zt} \right),$$

where s_{jzt} denotes the number of periods since the last δ-shock, K_{jzt} employed capital, L_{jzt} employed labor, and $F_{zt} \geq 0$ is a sector-specific fixed cost of production. The sector-specific growth rates of A_{zt}, Q_{zt} and G_{jzt} are given by $a_{zt} = a_z \varepsilon^a_{zt}$, $q_{zt} = q_z \varepsilon^q_{zt}$ and $g_{zt} = g_z \varepsilon^g_{zt}$, respectively, where a_z, q_z and g_z denote the steady state growth trends. We also allow for sector-specific degrees of price stickiness $\alpha_z \in (0, 1)$ and for sector-specific δ-shock intensities $\delta_z \in (0, 1)$.

The aggregate price level is defined as

$$P_t = \prod_{z=1}^{Z} \left(\frac{P_{zt}}{\psi_z} \right)^{\psi_z},$$

where the sectoral price level P_{zt} depends on the prices charged by firms in sector z in the same way as in the one-sector economy, see equation (5). Further details of the multi-sector economy are provided in a separate technical appendix, jointly with the proof of the following proposition.

Proposition 5 *Suppose condition 1 holds, there are no economic disturbances, there is no price indexation ($\Xi^{\star}_{t-1,t} \equiv 1$), there is positive δ-shock intensity in all sectors ($\delta_z > 0$ for all $z = 1, \ldots Z$), and the discount factor approaches $\beta(\gamma^e)^{1-\sigma} \to 1$. Suppose monetary*

[26] Productivity trends differ, for instance, between the manufacturing and the service sectors.

policy implements the inflation rate $\Pi_t = \Pi$ *for all* t. *The inflation rate* Π^* *that maximizes utility in the steady state of the multi-sector economy is*

$$\Pi^{\star} = \sum_{z=1}^{Z} \omega_z \left(\frac{g_z}{q_z} \frac{\gamma_z^e}{\gamma^e} \right), \tag{32}$$

where

$$\frac{\gamma_z^e}{\gamma^e} = \frac{a_z q_z}{\prod_{z=1}^{Z} (a_z q_z)^{\psi_z}}$$

denotes the growth trend of sector z *relative to the growth trend of the aggregate economy. The sectoral weights* $\omega_z \geq 0$ *are given by*

$$\omega_z = \frac{\tilde{\omega}_z}{\sum_{z=1}^{Z} \tilde{\omega}_z},$$

with

$$\tilde{\omega}_z = \frac{\psi_z \theta \alpha_z (1-\delta_z)(\Pi\gamma^e/\gamma_z^e)^{\theta}(q_z/g_z)}{\left[1-\alpha_z(1-\delta_z)(\Pi\gamma^e/\gamma_z^e)^{\theta}(q_z/g_z)\right]\ \left[1-\alpha_z(1-\delta_z)(\Pi\gamma^e/\gamma_z^e)^{\theta-1}\right]}.$$

The proposition shows that the result from the one sector economy naturally extends to a multi-sector setup. The main new element consists of the fact that the sector specific optimal inflation rates g_z/q_z need to be rescaled by the sectors' relative growth trends γ_z^e/γ_z. This implies that the sector specific optimal inflation rate g_z/q_z is scaled upwards for faster growing sectors ($\gamma_z^e/\gamma_z > 1$) and scaled downwards for sectors that grow slower than the aggregate economy.

Since proposition 5 provides a result specifying the inflation rate that maximizes utility in the limiting steady state, rather than a result about the limit of the optimal inflation rate itself, one does not have to impose condition 2, unlike in proposition 1. Furthermore, unlike in proposition 1, the optimal inflation rate fails to implement the first-best allocation, which would generally require different inflation rates for different sectors. The limiting condition $\beta(\gamma^e)^{1-\sigma} \to 1$ is required in proposition 5 to ensure that the utility losses due to aggregate markup distortions and those due to relative price distortions are minimized by the same inflation rate, namely the one given in the proposition. Absent this condition, minimizing these distortions individually would call for different inflation rates. One could then use sector-specific output subsidies to undo the sectoral markup distortions. The inflation rate $\Pi^{\star}$ stated in proposition 5 is then optimal even if $\beta(\gamma^e)^{1-\sigma}$ is strictly smaller than unity.

For the special case with $a_z q_z = aq$, which implies that $\gamma_z^e = \gamma^e$, and $g_z/q_z = g/q$, we obtain from equation (32) that $\Pi^{\star} = g/q$, which is the result for the one sector economy stated in proposition 1. In this special case, the central bank does not face a trade-off between different sector specific optimal inflation rates and can achieve the first-best allocation in the resulting steady state of the multi-sector economy, despite the presence of sector-specific degrees of price stickiness and sector-specific δ-shock intensities.

Since the closed form expressions for the sector weights ω_z in proposition 5 are difficult to interpret, the subsequent lemma shows that these weights are - to a first order approximation - equal to the sector's expenditure weights ψ_z:

Lemma 3 *The optimal steady state inflation rate in the multi-sector economy is equal to*

$$\Pi^* = \sum_{z=1}^{Z} \psi_z \left(\frac{g_z \gamma_z^e}{q_z \gamma^e} \right) + O(2), \tag{33}$$

where $O(2)$ denotes a second order approximation error and where the approximation to equation (32) has been taken around a point, in which $\frac{g_z}{q_z}\frac{\gamma_z^e}{\gamma^e}$ and $\alpha_z(1-\delta_z)(\gamma^e/\gamma_z^e)^{\theta-1}$ are constant across sectors $z = 1, \ldots Z$.

Interestingly, the optimal steady state inflation rate turns out to be independent of the sector-specific degree of price stickiness (α_z), unlike in Benigno (2004). This happens because the point of approximation is chosen such that sector-specific productivity trends and the effects of sector-specific price stickiness cancel each other. Our result then shows that to a first order approximation, the optimal inflation rate remains independent of α_z in the neighborhood of this point. This contrasts with the effects of sector-specific firm-level productivity trends (q_z/g_z), which do have first order implications for the optimal steady state inflation rate.

12 The Optimal Inflation Rate for the US Economy

This section quantifies the optimal inflation rate for the U.S. economy using the multi-sector setup presented in the previous section. The next section explains our empirical approach and the subsequent section presents the estimation results.

12.1 Empirical Strategy

To quantify the optimal inflation rate implied by microeconomic productivity trends, one would ideally estimate these trends directly at the firm or establishment level. Yet, it is generally not possible to measure physical productivity at the firm or establishment level because output prices are not widely observed at this level of observation.[27] Given this limitation, we proceed instead by using establishment level employment trends to estimate the establishment level productivity trends. Employment and productivity are related to each other via the elasticity of product demand $(\theta-1)$, which maps any firm level productivity (and associated product price) difference into an employment difference.[28] Clearly, to the extent that firms face additional constraints for expanding production beyond being insufficiently productive/competitive (financial constraints, adjustment costs, regulatory constraints), there may be biases in the productivity trends that are estimated from employment trends. To deal with this concern, we shall mainly look at changes in the estimated trends over time, which should remove any fixed effects that arises from other frictions affecting firm employment.

We estimate the employment trends using information provided by the Longitudinal Business Database (LBD) of the US Census Bureau. The LBD reports establishment level employment data and covers all U.S. establishments at an annual frequency. Coverage starts in the year 1976 and we use data up the year 2013. For this period, there is a total of 176 million employment observations at the establishment level. Working with this data source, we shall interpret δ-shocks as an event in which the establishment is closed own, in line with the 'entry and exit' interpretation spelled out in section 3.2. The variable s_{jt} can then be interpreted as the establishment age. Using the multi-sector setup from the previous section, we then can derive a model-implied relationship between establishment level employment, establishment age, both of which are observed in the LBD, and the productivity trends of interest:[29]

[27] As explained in Foster, Haltiwanger and Syverson (2008), the productivity literature usually measures revenue productivity instead of phyiscal productivity at the firm level, which deflates firm level output with some industry level price index. In our setting, firm's revenue productivity is completely unrelated to its physical productivity in the absence of fixed costs of production. For the few industries for which phyiscal and revenue productivities can both be observed, the two productivity measures can be rather different, see Foster, Haltiwanger and Syverson (2008).

[28] An alternative approach would consist of considering product level price data, e.g., the price information entering into the construction of the CPI, as used for example in Nakamura and Steinsson (2008). The results documented Bils (2009) and Moulton and Moses (1997) show that the inflation rate of so-called 'forced substitution' items, i.e., items which become permanently unavailable and are replaced by other 'new' items, is significantly larger than that of so-called matched items, which are products that continue to be available. Our model implies that one can infer the inflation relevant trend g/q from the inflation difference in these two item categories. However, this would requires making accurate quality adjustments in the computation of the the inflation rate for forced substitution items, which is a task that is difficult to achieve.

[29] The proof of the proposition can be found in a separate technical appendix which also spells out the details of the multi-sector setup.

Proposition 6 *Suppose that* $\sum_{i=0}^{t}\ln(\epsilon_{iz}^{q}/\epsilon_{iz}^{g})$ *is a stationary process in* t *and that fixed costs are equal to zero* ($f_z = 0$)*. Employment* L_{jzt} *of the firm producing product* j *in sector* z *at time* t *in the flexible price equilibrium is then equal to*

$$\ln(L_{jzt}) = d_{zt} + \eta_z \cdot s_{jzt} + \epsilon_{jzt}, \tag{34}$$

where d_{zt} *denotes a sector specific time dummy,* s_{jzt} *the age of the firm and* ϵ_{jzt} *a stationary residual term. The regression coefficient* η_z *is given by*

$$\eta_z = (\theta - 1)\ln(g_z/q_z).$$

The requirement that $\sum_{i=0}^{t}\ln(\epsilon_{iz}^{q}/\epsilon_{iz}^{g})$ is stationary is essential for obtaining stationarity of the regression residuals ϵ_{jzt}. It is satisfied, for instance, if $\ln Q_{zt}$ and $\ln G_{jzt}$ are both trend stationary or non-stationary but co-integrated processes. The coefficient of interest, η_z, captures the inflation relevant productivity trends, multiplied by elasticity of product demand $(\theta - 1)$.

Note that proposition 6 derives a property about firm-level employment in the flexible price equilibrium. In the one sector economy, the optimal inflation rate replicates the flexible price equilibrium exactly, so that the flexible price employment would indeed be observed in equilibrium under optimal monetary policy. This fails to be exactly true in the multi-sector economy or when policy is conducted in a suboptimal way. Relative price distortions associated with sticky prices then generally affect the equilibrium distribution of firm-level employment. Since these distortions do not tend to systematically vary with firm age, they will likely get absorbed by the firm level residuals in equation (34). Moreover, since we identify trends by looking at yearly observations, the more short-lived effects of price stickiness at the firm level can plausibly be expected to be averaged out. Note also that equation (34) holds whenever fixed costs of production are zero. From the proof of proposition 6 follows that further nonlinear terms in age can show up on the right-hand side of equation (34), with strictly positive fixed costs. In our empirical analysis, we therefore explore the robustness of the estimates η_z when including also age squared as a regressor on the right-hand side.

Equation (34) is of interest because it allows us to identify, when combined with information about the demand elasticity θ, the sector specific relative productivity trends g_z/q_z from establishment level data. The values for g_z/q_z for all sectors together then determine – jointly with the sector specific relative growth trends γ_z/γ and information about sector size ψ_z – the optimal aggregate inflation rate, as implied by lemma 3.

To avoid censoring of the age variable when estimating η_z, we consider a restricted LBD sample that does not include the firms that are present already in the initial year (1976), for which age information is not available. Furthermore, we start estimating

η_z from the year 1986 onwards, so as to minimize any effects from having only young establishments in the sample during the initial years of the database. This leaves us with 147 million establishment-age observations. The mean age for this sample is 8.16 years, with a standard deviation of 7.05 years, so that a wide range of age observations is covered. We then estimate equation (34) for 65 private BEA industries, which is the level of disaggregation at which sectoral GDP information is available. The sectoral GDP information allows us to compute the sector-specific relative growth trends γ_z^e/γ_z showing up in equation (33). To this end, we map the NAICS industries in the LBD database into the 65 private BEA industries (for the early part of the sample we map SIC codes). Finally, the GDP weights ψ_z in equation (34) are computed using sectoral GDP information for the year 2013. As a robustness exercise, we use time-varying weights ψ_{zt} using sectoral GDP information for each of the considered years $t = 1986, \ldots 2013$. Further details are provided in appendix F.

12.2 Empirical Results

For the years 1986 to 2013, we report the time series

$$\Phi_t \equiv \sum_{z=1}^{65} \left(\psi_z \frac{\gamma_z^e}{\gamma^e} \right) \exp(\eta_{zt}), \tag{35}$$

where η_{zt} is the time t estimate for η_z, for sector $z = 1, \ldots 65$. The time series Φ_t is of interest, because it is proportional to the optimal steady state inflation rate[30]

$$\Pi_t^\star - 1 = \frac{1}{\theta - 1}(\Phi_t - 1) + O(2), \tag{36}$$

where $\Pi_t^\star$ is the time t estimate of the steady state inflation rate from lemma 3. Transforming Φ_t into an implied inflation rate thus requires taking a stand on the value of the demand elasticity θ. For statements about the relative evolution of the optimal inflation rate, the value of θ does not matter. For the year 2007, which is the last year before the start of the financial crisis, we report in appendix F.3 detailed information on the cross-sectional estimates η_z, descriptive statistics for the various sectors, and the outcome of a robustness exercise.

Figure 3 presents our baseline estimate for Φ_t. It shows that the optimal inflation rate is positive, in line with the empirical observation that older firms establishments tend to be employ more workers on average ($g_z/q_z > 1$). It also shows that the optimal inflation rate dropped by approximately fifty percent over the period 1986 to 2013. The decline is rather steady over time and there are only weak indications for cyclical fluctuations. The

[30]This follows from $\exp((\theta-1)\ln(g_z/q_z)) = 1 + (\theta-1)(g_z/q_z - 1) + O(2)$ and $\sum_z \left(\psi_z \frac{\gamma_z^e}{\gamma^e}\right) = 1 + O(2)$.

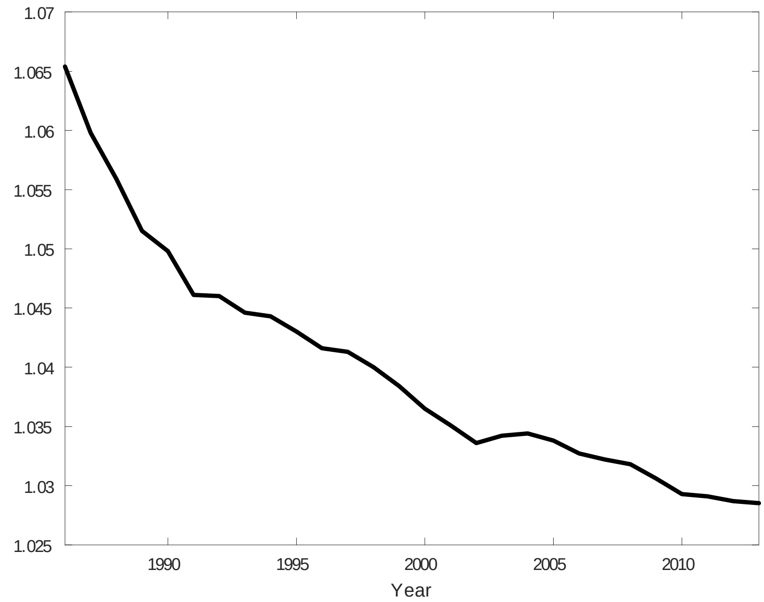

Figure 3: Baseline estimate of Φ_t (fixed 2013 sector weights, linear specification in age)

drop in the estimate implies that either the experience trend in productivity weakened over the considered time period or the cohort productivity trend affecting new entrants accelerated.[31] While we cannot disclose the cross-sectional estimates for the year 2002, comparing 2002 estimates of η_z to the 2007 estimate reported in appendix F, we find that the decline in η_z is widespread in most economic sectors and not driven by a small set of sectors experiencing very large declines.

Figure 4 investigates the robustness of the baseline estimates to alternative estimation approaches. As a first alternative, we consider weights ψ_{zt} that reflect the sectoral GDP share of each economic sector in period t rather than using fixed weights from the year 2013. This is motivated by the observation that the GDP shares of sectors have shifted considerably over the considered period. For instance, the share of manufacturing in private GDP dropped from 21.1 percent in 1986 to 14.0 percent in 2013. As figure 4 shows, using these time-varying weights leads to only negligible changes in the estimates, despite the fact that the GDP weight display significant changes over the consider period. The sectoral rebalancing taking place in the US economy thus does not co-vary significantly with the changes in the η_z (and thus g_z/q_z). This is consistent with the fact that the drop in η_z is present in almost all sectors of the U.S. economy.

Figure 4 also presents estimates for Φ_t when including also a term in age squared

[31] Separating which of the two effects actually drives the decline cannot be identified from the η_z estimates.

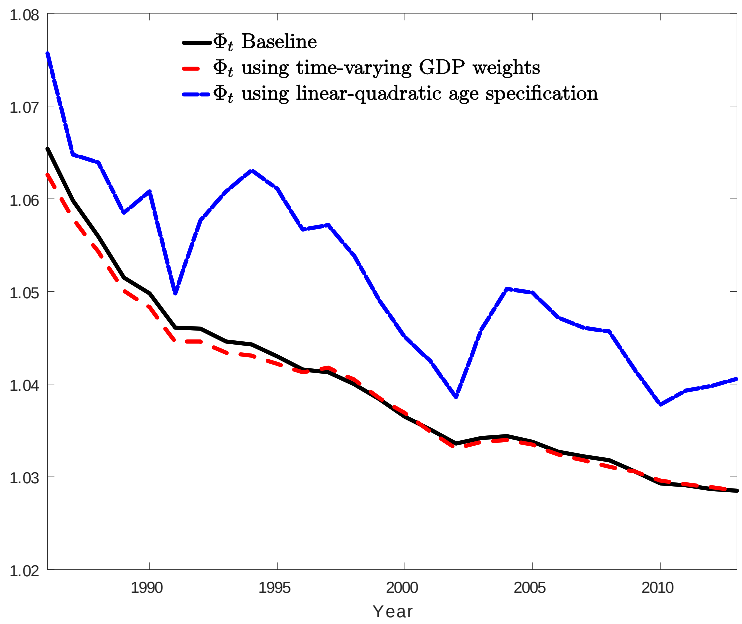

Figure 4: Robustness of Φ_t estimate to alternative estimation approaches

on the right-hand side of the regression equation (34).[32] The Φ_t estimates then become slightly larger in magnitude and also sightly more cyclical, displaying drops around the recession years 1991, 2001, and 2009. The overall message, however, remains unchanged: The optimal inflation rate is positive and approximately halved over the considered time period.

Translating the estimates presented in figures 3 and 4 into optimal inflation rates requires taking a stand on the value of the demand elasticity parameter θ. As our baseline, we follow Bilbiie et al. (2012) and Bernard et al. (2003) who use $\theta = 3.8$ based on a calibration that fits US plant and macro trade data. As a robustness exercise, we also consider $\theta = 5$, based on a calibration in Eusepi et al. (2011) that fits the revenue labor share.[33]

Table 1 reports the outcomes for the optimal inflation rate over the sample period. It shows that for most of the specifications, the optimal inflation rate dropped from a value of close to two percent in 1986 to approximately one percent in 2013.

[32] The definition of Φ_t is still given by equation (35).

[33] Eusepi et al. (2011) also show that the average wholesale markup implies a demand elasticity of 5.1 for the industries in the 1997 Census of Wholesale Trade that Bils and Klenow (2004) could match to consumer goods in the CPI.

Table 1: Optimal Inflation Rate (Net)

	Baseline	TV Weights	LQ Specification	Baseline	TV Weights	LQ Specification
	$\theta = 3.8$			$\theta = 5$		
$\Pi^\star_{1986}$	2.34%	2.24%	2.70%	1.64%	1.57%	1.89%
$\Pi^\star_{2013}$	1.02%	1.02%	1.45%	0.71%	0.71%	1.01%

Notes: "Baseline" refers to the baseline estimate of Φ_t with fixed GDP weights and age as single regressor. "TV Weights" refers to the estimate of Φ_t that is based on time-varying GDP weights. "LQ Specification" refers to the estimate of Φ_t that is based on a specification with both age and age squared as regressors. The parameter θ denotes the product demand elasticity.

13 Extensions and Robustness of Results

This section considers various extensions and alternative model setups. Section 13.1 shows that our main finding about the optimal inflation rate (proposition 2) continues to apply in a setting where price adjustment frictions take the form of menu costs. Section 13.2 discusses the effects of introducing additional firm-specific productivity components.

13.1 Menu Cost Frictions

While our results are illustrated using time-dependent Calvo price setting frictions, our main theoretical finding from proposition 2 extends to a setting in which firms have to pay a fixed cost to adjust their price. Since the optimal inflation rate in proposition 2 replicates the flexible price allocation, firms have – independently of the nature of their price setting frictions – no incentives to adjust their prices, whenever monetary policy implements the optimal inflation rate. Since the flexible price allocation is efficient, see proposition 1, monetary policy also has no incentive to deviate from the flexible price allocation. Both observations together imply that the optimal inflation rate does not depend on whether price setting frictions are state or time dependent.[34]

The previous logic does not fully extend to our results for a multi-sector economy in section 11. The optimal inflation rate there fails to exactly implement the efficient flexible price allocation, because generically each sector has its own sector-specific optimal inflation rate. The precise form of the postulated price setting frictions can then have an influence on the optimal rate of inflation, as it determines the details of how monetary

[34] Obviously, this requires that in a setting with menu cost frictions, δ.-shocks either lead to these menu cost not having to be paid or having to be paid always, independently of whether or not prices are ajdusted. The latter situation appears plausible when interpreting δ-socks as being associated with new products, new product qualities or firm entry and exit, see section 3.2.

policy can get allocations closer to the efficient flexible price benchmark. Determining the optimal inflation rate for menu cost type models, in which monetary policy cannot replicate the efficient allocation, e.g., the setting considered in Golosov and Lucas (2007), is of interest for further research but beyond the scope of the present paper.

13.2 Idiosyncratic Firm Productivity

The model that we present allows for a considerably richer firm specific productivity process than many other sticky price models. At same time, however, it abstracts from a number of potentially interesting additional dimensions of firm level heterogeneity. In particular, one simplifying assumption entertained throughout the paper is that there are no firm-specific productivity components: firms receiving a δ-shock, for example, are homogeneous and - absent further δ-shocks - their productivity grows according to common trends defined by (a_t, g_t).

This said, some of our results continue to hold even in the presence of additional idiosyncratic elements to firm productivity. Adding to the setup, for instance, a multiplicative firm fixed effect to productivity, i.e., letting firm specific productivity be given by

$$Z_{jt} = Q_{t-s_{jt}} G_{jt} \widetilde{Z}_{t-s_{jt}},$$

where $\widetilde{Z}_{t-s_{jt}}$ is chosen in the period in which a δ-shock hits, independently across firms and from a time-invariant distribution with mean one, our main results in proposition 1 and 2 continue to apply. The same holds true if one incorporated instead a time invariant firm-fixed effect $\widetilde{Z}_j$ for productivity, i.e.,

$$Z_{jt} = Q_{t-s_{jt}} G_{jt} \widetilde{Z}_j.$$

For these extended settings, proposition 6, which derives our model implied empirical specification for estimating the relevant sectoral productivity trends, also holds in unchanged form because firm fixed effects get absorbed by the firm specific error term.

The situation is different, if firms specific components are time varying, e.g., take the form

$$Z_{jt} = Q_{t-s_{jt}} G_{jt} \widetilde{Z}_{jt}$$

for some idiosyncratic productivity shock process $\widetilde{Z}_{jt}$ with unconditional mean one. While such idiosyncratic shocks may be present in the data, it is hard to know to what extent they are due to measurement noise. In any case, the presence of such shocks prevents full replication of the flexible price equilibrium, as adjustment frictions then become strictly binding.[35] While this makes it difficult to obtain closed-form solutions for the optimal

[35] This holds true for the case with time dependent pricing frictions and the case with menu cost type frictions.

inflation rate,[36] studying the implications of such time-varying idiosyncratic shocks for the optimal inflation rate appears to be worth exploring further in future work.

14 Conclusions

The conclusions reached in this paper show that firm-level productivity dynamics matter for understanding the inflation rate that is optimal for the aggregate economy. Using Longitudinal Business Database, which covers all establishments in the United States, we show that the existing establishment-level productivity trends can rationalize significantly positive amounts of inflation as being optimal. This differs notably from the inflation rates suggested to be optimal by standard sticky price setups with homogeneous firms, where productivity trends do not affect the optimal rate of inflation. Our estimates show that changes in the micro-level productivity trends taking place over the period 1986 to 2013 in the U.S. economy, caused the optimal inflation rate to fall by approximately 50 percent over this time period. Understanding the economic forces giving rise to these changes in establishment-level productivity trends are certainly worth to be explored further. It also appears interesting to explore to what extend these changes are present also in other advanced economies and imply similarly declining optimal inflation rates.

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Appendix

A Derivation of the Sticky Price Economy

A.1 Cost Minimization Problem of Firms

The cost minimization problem of firm j,

$$\min_{K_{jt},L_{jt}} K_{jt}r_t + L_{jt}W_t/P_t \quad s.t. \quad Y_{jt} = A_t Q_{t-s_{jt}} G_{jt}\left(K_{jt}^{1-\frac{1}{\phi}} L_{jt}^{\frac{1}{\phi}} - F_t\right),$$

yields the first order conditions

$$0 = r_t + \left(1 - \frac{1}{\phi}\right)\lambda_t A_t Q_{t-s_{jt}} G_{jt}\left(\frac{L_{jt}}{K_{jt}}\right)^{\frac{1}{\phi}},$$

$$0 = W_t/P_t + \frac{1}{\phi}\lambda_t A_t Q_{t-s_{jt}} G_{jt}\left(\frac{L_{jt}}{K_{jt}}\right)^{\frac{1}{\phi}-1}.$$

They imply that the optimal capital labor ratio is the same for all $j \in [0,1]$, i.e.,

$$\frac{K_{jt}}{L_{jt}} = \frac{W_t}{P_t r_t}(\phi - 1).$$

Plugging the optimal capital labor ratio into the technology of firm j and solving for the factor inputs yields the factor demand functions

$$L_{jt} = \left(\frac{W_t}{P_t r_t}(\phi - 1)\right)^{\frac{1}{\phi}-1} I_{jt}, \tag{37}$$

$$K_{jt} = \left(\frac{W_t}{P_t r_t}(\phi - 1)\right)^{\frac{1}{\phi}} I_{jt}. \tag{38}$$

Firm j demands these amounts of labor and capital, respectively, to combine them to I_{jt}, which yields Y_{jt} units of output. Accordingly, the firm's cost function to produce I_{jt} is

$$MC_t I_{jt} = W_t\left(\frac{W_t}{P_t r_t}(\phi - 1)\right)^{\frac{1}{\phi}-1} I_{jt} + P_t r_t\left(\frac{W_t}{P_t r_t}(\phi - 1)\right)^{\frac{1}{\phi}} I_{jt}, \tag{39}$$

where MC_t denotes nominal marginal (or average) costs. This equation can be rearrange to obtain equation (7) in the main text.

A.2 Price Setting Problem of Firms

The price setting problem of the firm j, see equation (8), implies that the optimal product price is given by

$$P_{jt}^{\star} = \left(\frac{\theta}{\theta - 1}\frac{1}{1+\tau}\right)\frac{E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}Y_{t+i}\left(\Xi_{t,t+i}/P_{t+i}\right)^{-\theta}\frac{MC_{t+i}/P_{t+i}}{A_{t+i}Q_{t-s_{jt}}G_{jt+i}}}{E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}Y_{t+i}\left(\Xi_{t,t+i}/P_{t+i}\right)^{1-\theta}}. \tag{40}$$

Rewriting this equation yields

$$\frac{P^{\star}_{jt}}{P_t}\left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right)$$
$$= \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}\frac{Y_{t+i}}{Y_t}\left(\frac{\Xi_{t,t+i}P_t}{P_{t+i}}\right)^{-\theta}\frac{MC_{t+i}}{P_{t+i}A_{t+i}Q_{t+i}}\frac{Q_{t+i}/Q_t}{G_{jt+i}/G_{jt}}}{E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}\frac{Y_{t+i}}{Y_t}\left(\frac{\Xi_{t,t+i}P_t}{P_{t+i}}\right)^{1-\theta}}. \quad (41)$$

The multi-period growth rate of the cohort effect relative to the experience effect corresponds to

$$\frac{Q_{t+i}/Q_t}{G_{jt+i}/G_{jt}} = \frac{q_{t+i}\times\cdots\times q_{t+1}}{g_{t+i}\times\cdots\times g_{t+1}},$$

for $i > 0$, and equals unity for $i = 0$. Hence, this growth rate is independent of the index j, because when going forward in time, firms are subject to the same experience effect. Accordingly, we can rewrite the equation (41) according to

$$\frac{P^{\star}_{jt}}{P_t}\left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right) = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_t}{D_t},$$

where the numerator N_t is given by

$$N_t = E_t\sum_{i=0}^{\infty}(\alpha(1-\delta))^i\Omega_{t,t+i}\frac{Y_{t+i}}{Y_t}\left(\frac{\Xi_{t,t+i}P_t}{P_{t+i}}\right)^{-\theta}\frac{MC_{t+i}}{P_{t+i}A_{t+i}Q_{t+i}}\left(\frac{q_{t+i}\times\cdots\times q_{t+1}}{g_{t+i}\times\cdots\times g_{t+1}}\right).$$

It evolves recursively as shown by equation (10). The denominator D_t also evolves recursively, and this yields the recursive pricing equations (9)-(11).

A.3 First Order Conditions to the Household Problem

The first order conditions that belong to the household problem comprise the household's budget constraint, a no-Ponzi scheme condition, the transversality condition, and the following equations:

$$\begin{aligned}
\frac{W_t}{P_t} &= -\frac{U_{Lt}}{U_{Ct}},\\
\Omega_{t,t+1} &= \beta\frac{\xi_{t+1}}{\xi_t}\frac{U_{Ct+1}}{U_{Ct}},\\
1 &= E_t\left[\Omega_{t,t+1}\left(\frac{1+i_t}{\Pi_{t+1}}\right)\right],\\
1 &= E_t\left[\Omega_{t,t+1}(r_{t+1}+1-d)\right].
\end{aligned}$$

Here, we denote by $U(.)$ the period utility function. Our assumptions about $U(.)$ imply

$$\begin{aligned}
U_{Ct} &= C_t^{-\sigma}V(L_t)^{1-\sigma},\\
U_{Lt} &= C_t^{1-\sigma}V(L_t)^{-\sigma}V_{Lt},
\end{aligned}$$

where $U(C_t, L_t) = ([C_tV(L_t)]^{1-\sigma} - 1)/(1-\sigma)$ and $U_{Ct} = \partial U(C_t, L_t)/\partial C_t$ and $V_{Lt} = \partial V(L_t)/\partial L_t$.

A.4 Recursive Evolution of the Price Level

Plugging the weighted average price of a cohort, equation (12), into the price level, equation (13), yields that

$$P_t^{1-\theta} = \delta(\Xi_{t,t}P_{t,t}^\star)^{1-\theta} + \sum_{s=1}^{\infty}(1-\delta)^s\delta\left[(1-\alpha)\sum_{k=0}^{s-1}\alpha^k(\Xi_{t-k,t}P_{t-s,t-k}^\star)^{1-\theta} + \alpha^s(\Xi_{t-s,t}P_{t-s,t-s}^\star)^{1-\theta}\right].$$

Telescoping the sums yields:

$$\begin{aligned}
P_t^{1-\theta} &= \delta(\Xi_{t,t}P_{t,t}^\star)^{1-\theta} \\
&\quad + \delta(1-\delta)^1\left[(1-\alpha)(\Xi_{t,t}P_{t-1,t}^\star)^{1-\theta} + \alpha(\Xi_{t-1,t}P_{t-1,t-1}^\star)^{1-\theta}\right] \\
&\quad + \delta(1-\delta)^2\left[(1-\alpha)(\Xi_{t,t}P_{t-2,t}^\star)^{1-\theta} + (1-\alpha)\alpha(\Xi_{t-1,t}P_{t-2,t-1}^\star)^{1-\theta} + \alpha^2(\Xi_{t-2,t}P_{t-2,t-2}^\star)^{1-\theta}\right] \\
&\quad + \ldots .
\end{aligned}$$

Collecting optimal prices that were set at the same date yields:

$$\begin{aligned}
&P_t^{1-\theta} = \\
&\delta\Xi_{t,t}^{1-\theta}\left[(P_{t,t}^\star)^{1-\theta} + (1-\alpha)(1-\delta)\left\{(P_{t-1,t}^\star)^{1-\theta} + (1-\delta)(P_{t-2,t}^\star)^{1-\theta} + (1-\delta)^2(P_{t-3,t}^\star)^{1-\theta} + \ldots\right\}\right] \\
&+ [\alpha(1-\delta)]\delta\Xi_{t-1,t}^{1-\theta}\left[(P_{t-1,t-1}^\star)^{1-\theta} + (1-\alpha)(1-\delta)\left\{(P_{t-2,t-1}^\star)^{1-\theta} + (1-\delta)(P_{t-3,t-1}^\star)^{1-\theta} + \ldots\right\}\right] \\
&+ \ldots .
\end{aligned}$$

Using equation (14) and the definition of p_t^e in equation (16), we can replace the terms in square brackets in the previous equation by p_t^e. This yields

$$\begin{aligned}
P_t^{1-\theta} &= \delta(\Xi_{t,t}P_{t,t}^\star)^{1-\theta}\left[1 + (1-\alpha)\left\{\frac{(p_t^e)^{\theta-1}}{\delta} - 1\right\}\right] \\
&\quad + [\alpha(1-\delta)]^1\delta(\Xi_{t-1,t}P_{t-1,t-1}^\star)^{1-\theta}\left[1 + (1-\alpha)\left\{\frac{(p_{t-1}^e)^{\theta-1}}{\delta} - 1\right\}\right] \\
&\quad + [\alpha(1-\delta)]^2\delta(\Xi_{t-2,t}P_{t-2,t-2}^\star)^{1-\theta}\left[1 + (1-\alpha)\left\{\frac{(p_{t-2}^e)^{\theta-1}}{\delta} - 1\right\}\right] + \ldots .
\end{aligned}$$

Rearranging the previous equation yields

$$\begin{aligned}
P_t^{1-\theta} &= (\Xi_{t,t}P_{t,t}^\star)^{1-\theta}\left[\alpha\delta + (1-\alpha)(p_t^e)^{\theta-1}\right] \\
&\quad + \alpha(1-\delta)(\Xi_{t-1,t})^{1-\theta}\left\{(\Xi_{t-1,t-1}P_{t-1,t-1}^\star)^{1-\theta}\left[\alpha\delta + (1-\alpha)(p_{t-1}^e)^{\theta-1}\right]\right. \\
&\quad \left. + \alpha(1-\delta)(\Xi_{t-2,t-1}P_{t-2,t-2}^\star)^{1-\theta}\left[\alpha\delta + (1-\alpha)(p_{t-2}^e)^{\theta-1}\right] + \ldots\right\}.
\end{aligned}$$

The term in curly brackets in this equation corresponds to $P_{t-1}^{1-\theta}$, which yields the price level equation (15) in the main text.

A.5 Aggregate Technology and Aggregate Productivity

To derive the aggregate technology, we combine firms' technology to produce the differentiated product in equation (2) with product demand $Y_{jt}/Y_t = (P_{jt}/P_t)^{-\theta}$ to obtain

$$\frac{Y_t}{A_t Q_t}\left(\frac{Q_t/Q_{t-s_{jt}}}{G_{jt}}\right)\left(\frac{P_{jt}}{P_t}\right)^{-\theta} = \left(\frac{K_{jt}}{L_{jt}}\right)^{1-\frac{1}{\phi}} L_{jt} - F_t.$$

Integrating over all firms with $j \in [0,1]$, using labor market clearing, $L_t = \int_0^1 L_{jt}\, dj$, and the fact that optimizing firms maintain the same (and hence the aggregate) capital labor ratio yields

$$\frac{Y_t}{A_t Q_t}\int_0^1 \left(\frac{Q_t/Q_{t-s_{jt}}}{G_{jt}}\right)\left(\frac{P_{jt}}{P_t}\right)^{-\theta} dj = K_t^{1-\frac{1}{\phi}} L_t^{\frac{1}{\phi}} - F_t.$$

Rearranging this equation and defining the (inverse) endogenous component of aggregate productivity as in equation (18) in the main text yields the aggregate technology (17).

To derive the recursive representation of Δ_t shown in equation (19), we recall the definition of Δ_t, i.e.,

$$\Delta_t = \int_0^1 \left(\frac{Q_t/Q_{t-s_{jt}}}{G_{jt}}\right)\left(\frac{P_{jt}}{P_t}\right)^{-\theta} dj,$$

and rewrite it according to

$$\frac{\Delta_t}{P_t^\theta} = \int_0^1 \left(\frac{q_t \times \cdots \times q_{t-s_{jt}+1}}{g_t \times \cdots \times g_{t-s_{jt}+1}}\right)(P_{jt})^{-\theta}\, dj,$$

using the processes describing the evolution of Q_t and G_{jt}. As for the price level, we aggregate in two steps. First, we aggregate the optimal prices of all firms operating within a particular cohort. Second, we aggregate all cohorts in the economy. To this end, we rewrite Δ_t/P_t^θ in the previous equation according to

$$\frac{\Delta_t}{P_t^\theta} = \sum_{s=0}^{\infty}(1-\delta)^s \delta \widehat{\Lambda}_t(s), \tag{42}$$

using

$$\widehat{\Lambda}_t(s) = \begin{cases} \left(\frac{q_t\times\cdots\times q_{t-s+1}}{g_t\times\cdots\times g_{t-s+1}}\right)\left[(1-\alpha)\sum_{k=0}^{s-1}\alpha^k(\Xi_{t-k,t}P^\star_{t-s,t-k})^{-\theta} + \alpha^s(\Xi_{t-s,t}P^\star_{t-s,t-s})^{-\theta}\right] & \text{if} \quad s \geq 1\ , \\ (\Xi_{t,t}P^\star_{t,t})^{-\theta} & \text{if} \quad s = 0\ . \end{cases}$$

Substituting out for $\widehat{\Lambda}_t(s)$ in equation (42) yields

$$\begin{aligned}\frac{\Delta_t}{P_t^\theta} &= \delta(\Xi_{t,t}P^\star_{t,t})^{-\theta} \\ &+ \delta\sum_{s=1}^{\infty}(1-\delta)^s\left(\frac{q_t\times\cdots\times q_{t-s+1}}{g_t\times\cdots\times g_{t-s+1}}\right)\left[(1-\alpha)\sum_{k=0}^{s-1}\alpha^k(\Xi_{t-k,t}P^\star_{t-s,t-k})^{-\theta} + \alpha^s(\Xi_{t-s,t}P^\star_{t-s,t-s})^{-\theta}\right].\end{aligned}$$

Following corresponding steps to those in appendix A.4 to rearrange the previous equation yields

$$\begin{aligned}\frac{\Delta_t}{P_t^{\theta}} &= (\Xi_{t,t}P^{\star}_{t,t})^{-\theta}\left[\alpha\delta + (1-\alpha)(p^e_t)^{\theta-1}\right]\\ &+ \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)(\Xi_{t-1,t}P^{\star}_{t-1,t-1})^{-\theta}\left[\alpha\delta + (1-\alpha)(p^e_{t-1})^{\theta-1}\right]\\ &+ [\alpha(1-\delta)]^2\left(\frac{q_tq_{t-1}}{g_tg_{t-1}}\right)(\Xi_{t-2,t}P^{\star}_{t-2,t-2})^{-\theta}\left[\alpha\delta + (1-\alpha)(p^e_{t-2})^{\theta-1}\right] + \ldots\end{aligned}$$

We rearrange this equation further to obtain that

$$\begin{aligned}\frac{\Delta_t}{P_t^{\theta}} &= (\Xi_{t,t}P^{\star}_{t,t})^{-\theta}\left[\alpha\delta + (1-\alpha)(p^e_t)^{\theta-1}\right]\\ &+ \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)(\Xi_{t-1,t})^{-\theta}\Big\{(P^{\star}_{t-1,t-1})^{-\theta}\left[\alpha\delta + (1-\alpha)(p^e_{t-1})^{\theta-1}\right]\\ &+ \alpha(1-\delta)\left(\frac{q_{t-1}}{g_{t-1}}\right)(\Xi_{t-2,t-1}P^{\star}_{t-2,t-2})^{-\theta}\left[\alpha\delta + (1-\alpha)(p^e_{t-2})^{\theta-1}\right] + \ldots\Big\}.\end{aligned}$$

The term in curly brackets in the previous equation is equal to $\Delta_{t-1}/P^{\theta}_{t-1}$, which yields

$$\frac{\Delta_t}{P_t^{\theta}} = \left[\alpha\delta + (1-\alpha)(p^e_t)^{\theta-1}\right](\Xi_{t,t}P^{\star}_{t,t})^{-\theta} + \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)(\Xi_{t-1,t})^{-\theta}\left\{\frac{\Delta_{t-1}}{P^{\theta}_{t-1}}\right\}.$$

Multiplying this equation by P_t^{θ} yields equation (19) in the main text.

A.6 Consolidated Budget Constraint

Consolidating household's and government's budget constraints shown in the main text yields

$$C_t + K_{t+1} = (1-d)K_t + r_tK_t + \frac{W_t}{P_t}L_t + \frac{\int_0^1 \Theta_{jt}\ \mathrm{dj}}{P_t} - \tau\left(\frac{\int_0^1 P_{jt}Y_{jt}dj}{P_t}\right). \tag{43}$$

To compute aggregate firm profits $\int_0^1 \Theta_{jt}$ dj, we use marginal costs (39) and combine them with the factor demands for L_{jt} and K_{jt}, equations (37) and (38), which yields that $MC_tI_{jt} = W_tL_{jt} + P_tr_tK_{jt}$. We use this equation and product demand $Y_{jt}/Y_t = (P_{jt}/P_t)^{-\theta}$ to rewrite aggregate firm profits according to

$$\begin{aligned}\int_0^1 \Theta_{jt}\ \mathrm{dj} &= (1+\tau)\int_0^1 P_{jt}Y_{jt}\ \mathrm{dj} - \int_0^1 MC_tI_{jt}\ \mathrm{dj},\\ &= (1+\tau)\int_0^1 P_{jt}Y_{jt}\ \mathrm{dj} - \int_0^1 (W_tL_{jt} + P_tr_tK_{jt})\ \mathrm{dj},\\ &= (1+\tau)P_tY_t - W_tL_t - P_tr_tK_t,\end{aligned}$$

with $L_t = \int_0^1 L_{jt}$ dj and $K_t = \int_0^1 K_{jt}$ dj. Thus, the consolidated budget constraint (43) reduces to

$$K_{t+1} = (1-d)K_t + Y_t - C_t,$$

where $Y_t - C_t$ represents aggregate investment. Dividing the previous equation by trend growth Γ_t^e yields

$$\gamma_{t+1}^e k_{t+1} = (1-d)k_t + y_t - c_t,$$

where $\gamma_t^e = \Gamma_t^e / \Gamma_{t-1}^e$ denotes the (gross) trend growth rate.

A.7 Transformed Sticky Price Economy

We define $p_t^\star = P_{t,t}^\star / P_t$ and $mc_t = MC_t / (P_t (\Gamma_t^e)^{1/\phi})$ and $w_t = W_t / (P_t \Gamma_t^e)$ and $c_t = C_t / \Gamma_t^e$. Also, we use that $p_t^e = 1/\Delta_t^e$, which follows from the equations (16) and (25). This yields the following equations that describe the transformed sticky price economy.

$$1 = \left[\alpha\delta + (1-\alpha)(\Delta_t^e)^{1-\theta}\right](p_t^\star)^{1-\theta} + \alpha(1-\delta)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta-1} \tag{44}$$

$$\Delta_t = \left[\alpha\delta + (1-\alpha)(\Delta_t^e)^{1-\theta}\right](p_t^\star)^{-\theta} + \alpha(1-\delta)\left(\frac{q_t}{g_t}\right)\left(\frac{\Pi_t}{\Xi_{t-1,t}}\right)^{\theta}\Delta_{t-1} \tag{45}$$

$$p_t^\star = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{N_t}{D_t} \tag{46}$$

$$N_t = \frac{mc_t}{\Delta_t^e} + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}\gamma_{t+1}^e\left(\frac{y_{t+1}}{y_t}\right)\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^{\theta}\left(\frac{q_{t+1}}{g_{t+1}}\right)N_{t+1}\right] \tag{47}$$

$$D_t = 1 + \alpha(1-\delta)E_t\left[\Omega_{t,t+1}\gamma_{t+1}^e\left(\frac{y_{t+1}}{y_t}\right)\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}}\right)^{\theta-1}D_{t+1}\right] \tag{48}$$

$$mc_t = \left(\frac{w_t}{1/\phi}\right)^{\frac{1}{\phi}}\left(\frac{r_t}{1-1/\phi}\right)^{1-\frac{1}{\phi}} \tag{49}$$

$$r_t k_t = (\phi-1)w_t L_t \tag{50}$$

$$y_t = \left(\frac{\Delta_t^e}{\Delta_t}\right)\left(k_t^{1-\frac{1}{\phi}}L_t^{\frac{1}{\phi}} - f\right) \tag{51}$$

$$\gamma_{t+1}^e k_{t+1} = (1-d)k_t + y_t - c_t \tag{52}$$

$$\gamma_t^e = (a_t q_t \Delta_{t-1}^e/\Delta_t^e)^{\phi} \tag{53}$$

$$(\Delta_t^e)^{1-\theta} = \delta + (1-\delta)\left(\Delta_{t-1}^e q_t/g_t\right)^{1-\theta} \tag{54}$$

$$w_t = -c_t\left(\frac{V_{Lt}}{V(L_t)}\right) \tag{55}$$

$$1 = E_t\left[\Omega_{t,t+1}\left(\frac{1+i_t}{\Pi_{t+1}}\right)\right] \tag{56}$$

$$1 = E_t\left[\Omega_{t,t+1}(r_{t+1}+1-d)\right] \tag{57}$$

$$\Omega_{t,t+1} = \beta\left(\frac{\xi_{t+1}}{\xi_t}\right)\left(\frac{\gamma_{t+1}^e c_{t+1}}{c_t}\right)^{-\sigma}\left(\frac{V(L_{t+1})}{V(L_t)}\right)^{1-\sigma} \tag{58}$$

After adding a description of monetary policy and a price indexation rule, these seventeen equations determine the paths of the seventeen variables $i_t, \Pi_t, y_t, c_t, k_t, L_t, r_t, w_t, mc_t, \gamma_t^e, \Delta_t, \Delta_t^e, p_t^\star, \Xi_{t-1,t}, N_t, D_t, \Omega_{t-1,t}$. The model contains the four exogenous shocks q_t, g_t, a_t, ξ_t.

A.8 Steady State in the Transformed Sticky Price Economy

We consider a steady state in the transformed sticky price economy, in which g and q are constant and the government maintains a constant inflation rate Π, which also implies a constant rate of price indexation Ξ.

To solve for the model variables in this steady state, we first solve for the ratio Δ/Δ^e as a function of model parameters and the inflation rate Π only. To this end, we derive an expression for $p^\star$ as a function of Δ using the equations (44) and (45). Both equations can be rearranged to obtain, respectively, that

$$(1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}) = \left[\alpha\delta + (1-\alpha)(\Delta^e)^{1-\theta}\right](p^\star)^{1-\theta}, \tag{59}$$

$$\Delta\left(1-\alpha(1-\delta)(\Pi/\Xi)^{\theta}(g/q)^{-1}\right) = \left[\alpha\delta + (1-\alpha)(\Delta^e)^{1-\theta}\right](p^\star)^{-\theta}. \tag{60}$$

Dividing the equation (59) by the equation (60) yields

$$p^\star = \Delta^{-1}\left(\frac{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta}(g/q)^{-1}}\right). \tag{61}$$

We substitute this expression for $p^\star$ into the equation (60), which yields that

$$\left(\frac{\Delta}{\Delta^e}\right)^{1-\theta} = \frac{\alpha\delta(\Delta^e)^{\theta-1}+1-\alpha}{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta}(g/q)^{-1}}\left(\frac{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta}(g/q)^{-1}}\right)^{-\theta}.$$

We use equation (54) to substitute for $(\Delta^e)^{\theta-1}$ on the right hand side of the previous equation and rearrange the result to obtain that

$$\frac{\Delta(\Pi)}{\Delta^e} = \left(\frac{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1-\alpha(1-\delta)(g/q)^{\theta-1}}\right)^{\frac{\theta}{\theta-1}}\left(\frac{1-\alpha(1-\delta)(g/q)^{\theta-1}}{1-\alpha(1-\delta)\left(\Pi/\Xi\right)^{\theta}\left(g/q\right)^{-1}}\right), \tag{62}$$

where we have indicated that $\Delta(\Pi)$ depends on the steady state inflation rate Π. For later use, we define the relative price distortion as

$$\rho(\Pi) = \frac{\Delta^e}{\Delta(\Pi)}. \tag{63}$$

The pricing equations (46) to (48) yield

$$\frac{1}{mc} = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\left(\frac{1}{p^\star\Delta^e}\right)\left(\frac{1-\alpha(1-\delta)[\beta(\gamma^e)^{1-\sigma}]\left(\Pi/\Xi\right)^{\theta-1}}{1-\alpha(1-\delta)[\beta(\gamma^e)^{1-\sigma}]\left(\Pi/\Xi\right)^{\theta}\left(g/q\right)^{-1}}\right).$$

Using the expression for $p^\star$ in equation (61) to substitute for $p^\star$ in the previous equation and the solution for $\Delta(\Pi)/\Delta^e$ in equation (62), we thus obtain a solution for $1/mc$. Again for later use, we denote the average markup by $\mu = 1/mc$ and thus obtain the solution

$$\mu(\Pi) = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\left(\frac{1-\alpha(1-\delta)(\Pi/\Xi)^{\theta-1}}{1-\alpha(1-\delta)(g/q)^{\theta-1}}\right)^{\frac{1}{\theta-1}}\left(\frac{1-\alpha(1-\delta)[\beta(\gamma^e)^{1-\sigma}]\left(\Pi/\Xi\right)^{\theta-1}}{1-\alpha(1-\delta)[\beta(\gamma^e)^{1-\sigma}]\left(\Pi/\Xi\right)^{\theta}\left(g/q\right)^{-1}}\right). \tag{64}$$

Again, we indicate here that $\mu(\Pi)$ depends on the steady state inflation rate.

Now, we rewrite marginal costs in equation (49) as

$$mc = \left(\frac{w}{r}(\phi-1)\right)^{\frac{1}{\phi}}\left(\frac{r}{1-1/\phi}\right),$$

and use equation (50) to obtain that $mc = \left(\frac{k}{L}\right)^{\frac{1}{\phi}} \left(\frac{r}{1-1/\phi}\right)$ or

$$r = \mu(\Pi)^{-1} \left(1 - \frac{1}{\phi}\right) \left(\frac{k}{L}\right)^{-\frac{1}{\phi}}, \tag{65}$$

after also using $\mu = 1/mc$. Analogous steps also imply

$$w = \mu(\Pi)^{-1} \left(\frac{1}{\phi}\right) \left(\frac{k}{L}\right)^{1-\frac{1}{\phi}}. \tag{66}$$

The aggregate technology (51), the aggregate resource constraint (52) and household's optimality conditions (55) to (58) imply the following four equations:

$$\begin{aligned} y &= \rho(\Pi) \left(\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} L - f\right), \\ w &= c\left(-\frac{V_L}{V(L)}\right), \\ r &= \frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d, \\ y &= c + (\gamma^e - 1 + d)k, \end{aligned}$$

where we have used that $\rho(\Pi) = \Delta^e/\Delta(\Pi)$. To simplify these four equations further, we use the equations (65) and (66) to substitute out for w and r. Then, we express all the remaining variables in terms of hours worked, which yields the following four equations:

$$\frac{y}{L} = \rho(\Pi) \left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} \left(1 + \rho(\Pi)\frac{f}{y}\right)^{-1} \tag{67}$$

$$\frac{c}{L} = \mu(\Pi)^{-1} \left(\frac{1}{\phi}\right) \left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} \left(-\frac{V(L)}{LV_L}\right) \tag{68}$$

$$\frac{k}{L} = \mu(\Pi)^{-1} \left(1 - \frac{1}{\phi}\right) \left(\frac{k}{L}\right)^{1-\frac{1}{\phi}} \left(\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d\right)^{-1} \tag{69}$$

$$\frac{y}{L} = \frac{c}{L} + (\gamma^e - 1 + d)\frac{k}{L}, \tag{70}$$

We now show that these four equations determine the four variables y, c, L, k, given a steady state inflation rate Π and assuming that the ratio of fixed costs over output, f/y, is a calibrated parameter.

First, we solve for hours worked as a function of Π by substituting the equations (67) to (69) into equation (70). This yields

$$\mu(\Pi)\rho(\Pi) \left(1 + \rho(\Pi)\frac{f}{y}\right)^{-1} = \left(\frac{1}{\phi}\right) \left(-\frac{V(L)}{LV_L}\right) + \left(\frac{\gamma^e - 1 + d}{\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d}\right) \left(1 - \frac{1}{\phi}\right),$$

or

$$\left(-\frac{V(L)}{LV_L}\right) = \phi\mu(\Pi)\rho(\Pi)\left(1+\rho(\Pi)\frac{f}{y}\right)^{-1} - (\phi-1)\left(\frac{\gamma^e - 1 + d}{\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d}\right),$$
$$= \mathcal{L}(\Pi),$$

where $\mathcal{L}(\Pi)$ abbreviates the right-hand-side term, which is a function of the steady state inflation rate. We obtain an explicit solution for L, if we assume a functional form for $V(L)$. Using that $V(L) = 1 - \psi L^\nu$, with $\nu > 1$ and $\psi > 0$ yields that

$$-\frac{V(L)}{LV_L} = \frac{1-\psi L^\nu}{\psi\nu L^\nu}$$

and hence that

$$L(\Pi) = \left(\frac{1}{\psi + \psi\nu\mathcal{L}(\Pi)}\right)^{1/\nu}, \tag{71}$$

where we have indicated that in general, steady state hours worked L depend on the steady state inflation rate Π through $\mathcal{L}(\Pi)$. Recall that in order to compute $\mathcal{L}(\Pi)$, the equations (62), (63) and (64) are required. Then, the solution for k, c, and y can be recursively computed from the equations (67) to (69). These solutions are

$$k(\Pi) = \mu(\Pi)^{-\phi}\left(1-\frac{1}{\phi}\right)^{\phi}\left(\frac{1}{\beta(\gamma^e)^{-\sigma}} - 1 + d\right)^{-\phi} L, \tag{72}$$

$$c(\Pi) = \mu(\Pi)^{-1}\left(\frac{1}{\phi}\right)\left(\frac{k}{L}\right)^{1-\frac{1}{\phi}}\left(-\frac{V(L)}{V_L}\right), \tag{73}$$

$$y(\Pi) = c + (\gamma^e - 1 + d)k. \tag{74}$$

Again, we indicate that these solutions depend on the steady state inflation rate.

B Planner Problem and its Solution

The planner allocates resources across firms and time by maximizing expected discounted household utility subject to firms' technologies and feasibility constraints. The planner problem can be solved in two steps. The first step determines the allocation of given amounts of capital and labor between heterogenous firms at date t. The second step determines the allocation of aggregate capital, consumption and labor over time. Endogenous variables in the planner solution are indicated by superscript e.

B.1 Intratemporal Planner Problem

The intratemporal problem corresponds to

$$\max_{L^e_{jt}, K^e_{jt}} \left(\int_0^1 (Y^e_{jt})^{\frac{\theta-1}{\theta}} \mathrm{dj}\right)^{\frac{\theta}{\theta-1}} \quad s.t. \quad Y^e_{jt} = A_t Q_{t-s_{jt}} G_{jt}\left((K^e_{jt})^{1-\frac{1}{\phi}}(L^e_{jt})^{\frac{1}{\phi}} - F_t\right),$$

and given L_t^e and K_t^e, with $L_t^e = \int_0^1 L_{jt}^e \, \mathrm{dj}$ and $K_t^e = \int_0^1 K_{jt}^e \, \mathrm{dj}$. Optimality conditions yield $K_{jt}^e/L_{jt}^e = K_t^e/L_t^e$ and hence that all firms maintain the same capital labor ratio. Thus, the problem can be recast in terms of the optimal mix of input factors, $I_{jt}^e = (K_{jt}^e)^{1-1/\phi}(L_{jt}^e)^{1/\phi}$:

$$\max_{I_{jt}^e} \left(\int_0^1 \left[A_t Q_{t-s_{jt}} G_{jt} \left(I_{jt}^e - F_t \right) \right]^{\frac{\theta-1}{\theta}} \mathrm{dj} \right)^{\frac{\theta}{\theta-1}} \quad s.t. \quad I_t^e = \int_0^1 I_{jt}^e \, \mathrm{dj},$$

with $I_t^e = (K_t^e)^{1-1/\phi}(L_t^e)^{1/\phi}$ being given. Equating the first order conditions to this problem for two different firms, the firm j and the firm k, to each other yields that

$$Z_{jt} \left[Z_{jt} \left(I_{jt}^e - F_t \right) \right]^{-\frac{1}{\theta}} = Z_{kt} \left[Z_{kt} \left(I_{kt}^e - F_t \right) \right]^{-\frac{1}{\theta}},$$

where $Z_{jt} = A_t Q_{t-s_{jt}} G_{jt}$ denotes productivity of the firm j. Rearranging this condition yields $I_{jt}^e - F_t = (Z_{jt}/Z_{kt})^{\theta-1} \left(I_{kt}^e - F_t \right)$, and aggregating it over all j's yields

$$I_{kt}^e - F_t = \frac{(G_{kt} Q_{t-s_{kt}}/Q_t)^{\theta-1}}{\int_0^1 (G_{jt} Q_{t-s_{jt}}/Q_t)^{\theta-1} \, \mathrm{dj}} (I_t^e - F_t). \tag{75}$$

Thus, the optimal input mix of the firm k net of fixed costs is proportional to the optimal aggregate input mix net of fixed costs, and the factor of proportionality corresponds to the (weighed) productivity of the firm k relative to the (weighed) aggregate productivity in the economy. Thus, equation (75) shows that the productivity distribution determines the efficient allocation of the optimal input mix across firms.

To obtain aggregate technology in the planner economy, we combine equation (75) with equation (2) and the Dixit-Stiglitz aggregator (1). This yields

$$Y_t^e = \left(\int_0^1 \left[A_t Q_{t-s_{jt}} G_{jt} \left(\frac{(Q_{t-s_{jt}} G_{jt})^{\theta-1}}{\int_0^1 (Q_{t-s_{jt}} G_{jt})^{\theta-1} \, \mathrm{dj}} (I_t^e - F_t) \right) \right]^{\frac{\theta-1}{\theta}} \mathrm{dj} \right)^{\frac{\theta}{\theta-1}}.$$

Simplifying this equation yields the aggregate technology in the planner economy,

$$Y_t^e = \frac{A_t Q_t}{\Delta_t^e} \left((K_t^e)^{1-\frac{1}{\phi}} (L_t^e)^{\frac{1}{\phi}} - F_t \right), \tag{76}$$

where the efficient productivity adjustment factor is defined as

$$1/\Delta_t^e = \left(\int_0^1 \left(G_{jt} Q_{t-s_{jt}}/Q_t \right)^{\theta-1} \mathrm{dj} \right)^{\frac{1}{\theta-1}} \tag{77}$$

and evolves recursively. To see this, rewrite equation (77) as

$$\begin{aligned}
(1/\Delta_t^e)^{\theta-1} &= \int_0^1 \left(\frac{q_t \times \cdots \times q_{t-s_{jt}+1}}{g_t \times \cdots \times g_{t-s_{jt}+1}} \right)^{1-\theta} \mathrm{dj} \\
&= \delta \left\{ 1 + \sum_{s=1}^{\infty} (1-\delta)^s \left(\frac{q_t \times \cdots \times q_{t-s+1}}{g_t \times \cdots \times g_{t-s+1}} \right)^{1-\theta} \right\} \\
&= \delta \left\{ 1 + (1-\delta) \left(\frac{q_t}{g_t} \right)^{1-\theta} + (1-\delta)^2 \left(\frac{q_t q_{t-1}}{g_t g_{t-1}} \right)^{1-\theta} + \ldots \right\} \\
&= (p_t^e)^{\theta-1}.
\end{aligned}$$

The last step follows from backward-iterating equation (16) and implies that the efficient productivity adjustment factor equals the relative price of firms that received a δ-shock in period t in the economy with flexible prices,

$$1/\Delta_t^e = p_t^e. \tag{78}$$

It follows also from equation (16) that Δ_t^e evolves recursively as shown in equation (25). The intratemporal planner allocation then consists of equation (75), which determines the efficient allocation of the optimal input mix across firms, and equations (76) and (25), which describe the aggregate consequences of the efficient allocation at the firm level.

B.2 Intertemporal Planner Problem

The intertemporal allocation maximizes expected discounted household utility subject to the intertemporal feasibility condition,

$$\max_{\{C_t^e, L_t^e, K_{t+1}^e\}} E_0 \sum_{t=0}^{\infty} \beta^t \xi_t U(C_t^e, L_t^e) \quad s.t. \tag{79}$$

$$C_t^e + K_{t+1}^e = (1-d)K_t^e + \frac{A_t Q_t}{\Delta_t^e}\left((K_t^e)^{1-\frac{1}{\phi}}(L_t^e)^{\frac{1}{\phi}} - F_t\right), \tag{80}$$

with $U(.)$ denoting the period utility function and Δ_t^e being given by equation (25). The first order conditions to this problem comprise the feasibility constraint and

$$Y_{Lt}^e = -\frac{U_{Lt}^e}{U_{Ct}^e}, \tag{81}$$

$$1 = \beta E_t\left[\frac{\xi_{t+1}}{\xi_t}\frac{U_{Ct+1}^e}{U_{Ct}^e}\left(Y_{Kt+1}^e + 1 - d\right)\right], \tag{82}$$

denoting by Y_{Kt}^e the marginal product of capital and by Y_{Lt}^e the marginal product of labor. Thus, the planner allocation for aggregate variables is characterized by the aggregate technology, equation (76), the efficient adjustment factor, equation (25), the feasibility condition, equation (79), and the two first order conditions (81) and (82).

C Proof of Proposition 1

To show that condition (26) holds under flexible prices, we divide equation (15) by $P_t^{1-\theta}$ and impose $\alpha = 0$ to obtain that the optimal relative price $p_t^\star$ of firms receiving a δ-shock in period t is equal to p_t^e. This and the equations (46) to (48) determining the optimal relative price of firms receiving a δ-shock in t imply that

$$p_t^e = \left(\frac{\theta}{\theta-1}\frac{1}{1+\tau}\right)\frac{mc_t}{\Delta_t^e}.$$

Combining the previous equation with equation (78) yields

$$1 = \left(\frac{\theta}{\theta - 1}\frac{1}{1+\tau}\right) mc_t, \tag{83}$$

which shows that real detrended marginal costs are constant in the economy with flexible prices. From equation (9) follows that the optimal relative price of the firm j (independently of the number of periods s_{jt} elapsed since the last δ-shock) in the flexible price model is

$$\frac{P^{\star}_{jt}}{P_t}(G_{jt}Q_{t-s_{jt}}/Q_t) = \left(\frac{\theta}{\theta - 1}\frac{1}{1+\tau}\right)\frac{mc_t}{\Delta^e_t}.$$

Combining this equation with equation (83), we obtain condition (26):

$$\frac{P_{jt}}{P_t} = \frac{1}{\Delta^e_t}\frac{Q_t}{G_{jt}Q_{t-s_{jt}}}.$$

The flexible price equilibrium thus implements $\Delta_t = \Delta^e_t$, so that the aggregate production function in equation (17) is given by

$$Y_t = \frac{A_t Q_t}{\Delta^e_t}\left((K_t)^{1-\frac{1}{\phi}}(L_t)^{\frac{1}{\phi}} - F_t\right), \tag{84}$$

with $F_t = f \cdot (\Gamma^e_t)^{1-1/\phi}$ and $\Gamma^e_t = (A_t Q_t/\Delta^e_t)^{\phi}$, and the resource constraint (derived in Appendix A.6) is given by

$$K_{t+1} = (1-d)K_t + Y_t - C_t. \tag{85}$$

The two equations (84) and (85) are the same constraints as faced by the planner under the efficient allocation. Combined with the fact that household decisions in the flexible price economy are undistorted in the presence of the corrective sales subsidy, it follows that the allocation of aggregate consumption, capital, labor, and output in the flexible price equilibrium is identical to the efficient allocation.

D Proof of Proposition 2

Establishing (1): First, we show that firms receiving a δ-shock in period t in the sticky price economy chose the same optimal relative price as in the flexible price economy. Let superscript e denote allocations and prices in the flexible price economy, which we have shown reproduces the efficient allocation. Under flexible prices ($\alpha = 0$) and given condition 1, the optimal relative price implied by equation (9) for firms with a δ-shock in period t is given by

$$p^e_t = \frac{(P^{\star}_{t,t})^e}{P^e_t} = \frac{MC^e_t}{P^e_t A_t Q_t}.$$

Under sticky prices ($\alpha > 0$) and the efficient allocation, combining this equation with equation (10) implies

$$\frac{N_t}{p_t^e} = 1 + \alpha(1-\delta)E_t \left[\Omega_{t,t+1}^e \frac{Y_{t+1}^e}{Y_t^e} \left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}} \right)^{\theta} \left(\frac{q_{t+1}}{g_{t+1}} \right) \left(\frac{p_{t+1}^e}{p_t^e} \right) \left(\frac{N_{t+1}}{p_{t+1}^e} \right) \right]. \tag{86}$$

Furthermore, equation (11) implies

$$D_t = 1 + \alpha(1-\delta)E_t \left[\Omega_{t,t+1}^e \frac{Y_{t+1}^e}{Y_t^e} \left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}} \right)^{\theta-1} D_{t+1} \right]. \tag{87}$$

Firms receiving a δ-shock in period t in the sticky price economy chose the same optimal relative price as firms receiving a δ-shock in period t in the flexible price economy, i.e., $P_{t,t}^{\star}/P_t = N_t/D_t = p_t^e$ or equivalently $N_t/p_t^e = D_t$, if it holds that

$$\left(\frac{\Pi_{t+1}}{\Xi_{t,t+1}} \right) \left(\frac{q_{t+1}}{g_{t+1}} \right) \left(\frac{p_{t+1}^e}{p_t^e} \right) = 1, \tag{88}$$

which follows from comparing the equations (86) and (87). To show that equation (88) holds under the optimal inflation rate stated in proposition 2, we lag this equation by one period and rearrange it to obtain

$$\left(\frac{\Pi_t}{\Xi_{t-1,t}} \right) p_t^e = p_{t-1}^e \frac{g_t}{q_t}.$$

Combining this equation with equation (16) implies that the optimal inflation rate as defined in equation (27) satisfies equation (88).

Establishing (2): To show that under the optimal inflation rate, firms that are subject to a Calvo-shock in period t and hence can adjust their price do not find it optimal to change their price, we need to establish that

$$P_{t-k,t}^{\star} = \Xi_{t-k,t}^{\star} P_{t-k,t-k}^{\star}, \tag{89}$$

for all $k > 0$. Dividing this equation by the (optimal) aggregate price level $P_{t-k}^{\star}$ and using the result from step (1), i.e., $p_t^{\star} = p_t^e$, we get that

$$\frac{P_{t-k,t}^{\star}}{P_{t-k}^{\star}} = \Xi_{t-k,t}^{\star} \left(\frac{P_{t-k,t-k}^{\star}}{P_{t-k}^{\star}} \right) = \Xi_{t-k,t}^{\star} p_{t-k}^e.$$

Using equation (14), we can rewrite the previous equation as

$$\frac{P_{t,t}^{\star}}{P_t^{\star}} \left(\frac{q_t \times \cdots \times q_{t-s+1}}{g_t \times \cdots \times g_{t-s+1}} \right) \frac{P_t^{\star}}{P_{t-k}^{\star}} = \Xi_{t-k,t}^{\star} p_{t-k}^e.$$

Again using $P_{t,t}^{\star}/P_t^{\star} = p_t^e$ and that $\Xi_{t-k,t} = \prod_{j=1}^{k} \Xi_{t-k+j-1,t-k+j}$ further delivers

$$\left(\frac{p_t^e}{p_{t-k}^e} \right) \left(\frac{q_t \times \cdots \times q_{t+1-k}}{g_t \times \cdots \times g_{t-s+1}} \right) \left(\frac{\Pi_t^{\star}}{\Xi_{t-1,t}^{\star}} \times \cdots \times \frac{\Pi_{t+1-k}^{\star}}{\Xi_{t-k,t+1-k}^{\star}} \right) = 1.$$

Rewriting the previous equation as

$$\left(\frac{\Pi_t^\star}{\Xi_{t-1,t}^\star}\frac{q_t}{g_t}\frac{p_t^e}{p_{t-1}^e}\right)\times\left(\frac{\Pi_{t-1}^\star}{\Xi_{t-2,t-1}^\star}\frac{q_{t-1}}{g_{t-1}}\frac{p_{t-1}^e}{p_{t-2}^e}\right)\times\cdots\times\left(\frac{\Pi_{t+1-k}^\star}{\Xi_{t-k,t+1-k}^\star}\frac{q_{t+1-k}}{g_{t+1-k}}\frac{p_{t+1-k}^e}{p_{t-k}^e}\right)=1$$

shows that each parenthesis equals unity under the optimal inflation rate, which follows from equation (88). This establishes that firms that can adjust their price maintain the indexed price as given by equation (89).

Establishing (3): We can establish the fact that the condition 2 causes initial prices to reflect initial relative productivities as follows. The pricing equations (9)-(11) imply under flexible prices and no markup distortion that

$$\frac{P_{jt}^\star}{P_t}\left(\frac{Q_{t-s_{jt}}G_{jt}}{Q_t}\right)=\frac{MC_t}{P_tA_tQ_t}.$$

For a firm receiving a δ-shock in period t, this equation yields that

$$p_t^e=\frac{MC_t}{P_tA_tQ_t}.$$

Combining both previous equations yields

$$\frac{P_{jt}^\star}{P_t}=\left(\frac{Q_t}{Q_{t-s_{jt}}G_{jt}}\right)p_t^e.$$

Plugging this equation into the aggregate price level, $P_t^{1-\theta}=\int_0^1 P_{jt}^{1-\theta}\,dj$, yields

$$1=\int_0^1\left(\frac{Q_t}{Q_{t-s_{jt}}G_{jt}}\right)^{1-\theta}(p_t^e)^{1-\theta}dj.$$

Rewriting this equation and using $p_t^e=1/\Delta_t^e$ yields equation (24) for $t=-1$.

E Proof of Proposition 4

Under the assumptions stated in the proposition, it is straightforward to show that the relative price distortion $\rho(\Pi)$ and the markup distortion $\mu(\Pi)$, which are defined in equations (62), (63) and (64), are inversely proportional to each other,

$$\mu(\Pi)=1/\rho(\Pi).$$

As a result, the solution of L determined in equation (71) in appendix A.8 simplifies to

$$L=\left(\frac{1}{\psi(1+\nu)}\right)^{1/\nu},$$

because $\mathcal{L}(\Pi) = 1$ and, therefore, L no longer depends on the steady state inflation rate Π. This result implies that $L(1) = L(\Pi^\star)$, as stated in proposition 4.

In this case, the solutions for capital and consumption, equations (72) and (73), imply

$$k(\Pi) = \rho(\Pi)^\phi \left(1 - \frac{1}{\phi}\right)^\phi (\gamma^e - 1 + d)^{-\phi} L,$$
$$c(\Pi) = \rho(\Pi)^\phi \left(\frac{1}{\phi}\right) \left(1 - \frac{1}{\phi}\right)^{\phi-1} (\gamma^e - 1 + d)^{1-\phi} \left(-\frac{V(L)}{V_L}\right),$$

where we explicitly indicate that steady state capital and consumption depend on Π.

Comparing steady state consumption for the policy implementing the optimal inflation rate $\Pi^\star$ and the alternative policy implementing strict price stability in economies without price indexation yields

$$\frac{c(1)}{c(\Pi^\star)} = \left(\frac{\rho(1)}{\rho(\Pi^\star)}\right)^\phi.$$

Equations (62) and (63) imply that the relative price distortion $\rho(\Pi^\star) = 1$. This yields

$$\begin{aligned} \frac{c(1)}{c(\Pi^\star)} &= \rho(1)^\phi, \\ &= \left(\frac{\Delta^e}{\Delta(1)}\right)^\phi \\ &= \left(\frac{1 - \alpha(1-\delta)(g/q)^{\theta-1}}{1 - \alpha(1-\delta)}\right)^{\frac{\phi\theta}{\theta-1}} \left(\frac{1 - \alpha(1-\delta)\,(g/q)^{-1}}{1 - \alpha(1-\delta)(g/q)^{\theta-1}}\right)^\phi, \end{aligned}$$

which is the expression in proposition 4 .

To show that $c(1)/c(\Pi^\star) \leq 1$, note that $c(1)/c(\Pi^\star) = 1$, if $g = q$ and hence $\Pi^\star = 1$. To show that the inequality holds strictly, $c(1)/c(\Pi^\star) < 1$, for $g \neq q$, we take the derivative of $c(1)/c(\Pi^\star)$ with respect to g/q. This yields

$$\frac{\partial}{\partial(g/q)}\left(\frac{c(1)}{c(\Pi^\star)}\right) = \left[\frac{c(1)}{c(\Pi^\star)}\right]\left[\frac{\alpha(1-\delta)\phi}{(g/q)^2}\right] \frac{1 - (g/q)^\theta}{\left[1 - \alpha(1-\delta)\,(g/q)^{-1}\right]\left[1 - \alpha(1-\delta)(g/q)^{\theta-1}\right]}.$$

Terms in square brackets are positive, because we have assumed that $(1-\delta)(g/q)^{\theta-1} < 1$ (see equation (6)), $\alpha < 1$, and $g/q > \alpha(1-\delta)$. Therefore, the derivative is strictly positive if $1 - (g/q)^\theta > 0$ and thus $g/q < 1$. It is strictly negative if $1 - (g/q)^\theta < 0$ and thus $g/q > 1$. The derivative is zero if $g/q = 1$.

F Data Appendix

F.1 LBD Database

We use data from 1986 to 2013 dropping observations of establishments that were present already in the sample in 1976 for which age information is not available. We only consider establishments with at least one payed employee and truncate employment observations above the 99% percentile in a given industry and year. This leaves us with 147 million establishment-employment observations in our estimation sample.

To improve the consistency of the mapping from SIC codes to NAICS codes, we follow the same establishments over the SIC-NAICS changeover to reverse engineer proper SIC codes for the considered industry z. Using this procedure, only about 0.2 million observations cannot be allocated to a NAICS code, because they have a coarse industry code under SIC. Since their employment share is negligible (0.02%), this should not affect our estimates.

F.2 Sectoral Disaggregation and Sector Weights ψ_z and γ_z^e/γ^e

We use the value added series of the BEA GDP-by-Industry data for 71 industries and focus on the 65 private industries.[37]

To compute the sectoral trend growth rate γ_z^e entering lemma 3, we use the chain-type quantity indexes for value added by industry for the years 1976 to 2013, which is the time span for which the LBD data is available. For a few industries (retail trade, hospitals, nursing and residential care facilities), data is only available from 1997 onwards. For these industries, we use data for the period 1997 to 2013.

To compute the aggregate trend growth rate γ^e entering lemma 3, we use the chain-type quantity index for private industries for the years 1976 to 2013.

To compute expenditure shares ψ_z for $z = 1, \ldots Z$, we use the expenditure shares as implied by the GDP statistics for the year 2013.

To compute time-varying expenditure shares ψ_{zt} for $z = 1, \ldots Z$, we use the expenditure shares as implied by the GDP statistics for the respective year. For a few sectors (retail trade, hospitals, nursing and residential care facilities), expenditure shares are not available for the period 1976 to 1996. We impute these shares using the distribution that we observe in 1997.

Table 2 below reports how we map the LBD NAICS codes into the 65 BEA private industries ($z = 1, 2, \ldots 65$).

Table 2: BEA-NAICS Mapping

z	BEA Code	BEA Title	Related 2007 NAICS Codes
1	111CA	Farms	111, 112
2	113FF	Forestry, fishing, and related activities	113, 114, 115
3	211	Oil and gas extraction	211

Continued on next page

[37]The data is available at http://www.bea.gov/industry/gdpbyind_data.htm and was retrieved on Augst 24, 2016.

Table 2—continued from previous page

z	BEA Code	BEA Title	Related 2007 NAICS Codes
4	212	Mining, except oil and gas	212
5	213	Support activities for mining	213
6	22	Utilities	221
7	23	Construction	230, 233
8	321	Wood products	321
9	327	Nonmetallic mineral products	327
10	331	Primary metals	331
11	332	Fabricated metal products	332
12	333	Machinery	333
13	334	Computer and electronic products	334
14	335	Electrical equipment, appliances, and components	335
15	3361MV	Motor vehicles, bodies and trailers, and parts	3361, 3362, 3363
16	3364OT	Other transportation equipment	3364, 3365, 3366, 3369
17	337	Furniture and related products	337
18	339	Miscellaneous manufacturing	339
19	311FT	Food and beverage and tobacco products	311, 312
20	313TT	Textile mills and textile product mills	313, 314
21	315AL	Apparel and leather and allied products	315, 316
22	322	Paper products	322
23	323	Printing and related support activities	323
24	324	Petroleum and coal products	324
25	325	Chemical products	325
26	326	Plastics and rubber products	326
27	42	Wholesale trade	42
28	441	Motor vehicle and parts dealers	441
29	445	Food and beverage stores	445
30	452	General merchandise stores	452
31	4A0	Other retail	442, 443, 444, 446, 447, 448, 451, 453, 454
32	481	Air transportation	481
33	482	Rail transportation	482
34	483	Water transportation	483
35	484	Truck transportation	484
36	485	Transit and ground passenger transportation	485
37	486	Pipeline transportation	486
38	487OS	Other transportation and support activities	487, 488, 492
39	493	Warehousing and storage	493
40	511	Publishing industries, except internet (includes software)	511
41	512	Motion picture and sound recording industries	512
42	513	Broadcasting and telecommunications	513, 515, 517
43	514	Data processing, internet publishing, and other information services	514, 518, 519
44	521CI	Federal Reserve banks, credit intermediation, and related activities	521, 522
45	523	Securities, commodity contracts, and investments	523
46	524	Insurance carriers and related activities	524
47	525	Funds, trusts, and other financial vehicles	525
48	531	Real estate	531
49	532RL	Rental and leasing services and lessors of intangible assets	532, 533
50	5411	Legal services	5411
51	5415	Computer systems design and related services	5415
52	5412OP	Miscellaneous professional, scientific, and technical services	5412, 5413, 5414, 5416, 5417, 5418, 5419
53	55	Management of companies and enterprises	55
54	561	Administrative and support services	561
55	562	Waste management and remediation services	562
56	61	Educational services	611

Continued on next page

Table 2—continued from previous page

z	BEA Code	BEA Title	Related 2007 NAICS Codes
57	621	Ambulatory health care services	621
58	622	Hospitals	622
59	623	Nursing and residential care facilities	623
60	624	Social assistance	624
61	711AS	Performing arts, spectator sports, museums, and related activities	711, 712
62	713	Amusements, gambling, and recreation industries	713
63	721	Accommodation	721
64	722	Food services and drinking places	722
65	81	Other services, except government	811, 812, 813, 814

F.3 Sectoral Results for the Year 2007

The following table reports for the year 2007 a set of descriptive statistics and the regression outcomes.

Table 3: Descriptive Statistics and Regression Results, LBD, 2007

	Descriptive statistics						Linear specification			Linear-quadratic specification				
z	J	$\sum_j L_j$	$E(\ln L_j)$	$SD(\ln L_j)$	$E(s_j)$	$SD(s_j)$	$100 \cdot \widehat{\eta}$	$100 \cdot SE(\widehat{\eta})$	R^2 (in %)	$100 \cdot \widehat{\widehat{\eta}}$	$100 \cdot SE\left(\widehat{\widehat{\eta}}\right)$	$100 \cdot \widehat{\widehat{\mu}}$	$100 \cdot SE\left(\widehat{\widehat{\mu}}\right)$	R^2 (in %)
1	114200	642400	1.06	1.03	11.33	6.89	1.80	0.04	1.45	1.27	0.12	0.02	0.01	1.47
2	25900	203400	1.27	1.13	11.36	9.13	2.27	0.08	3.35	2.57	0.23	−0.01	0.01	3.36
3	6900	89700	1.48	1.33	12.88	10.73	0.48	0.15	0.15	−0.69	0.57	0.04	0.02	0.22
4	6600	172500	2.31	1.36	14.84	11.88	2.22	0.14	3.75	3.21	0.55	−0.03	0.02	3.80
5	10200	213200	1.89	1.47	9.23	9.24	1.73	0.16	1.18	0.13	0.53	0.06	0.02	1.27
6	21100	561900	2.12	1.48	18.27	12.10	2.67	0.08	4.79	−0.39	0.40	0.09	0.01	5.06
7	687700	5490800	1.34	1.14	10.10	9.22	3.27	0.01	7.04	4.36	0.05	−0.04	0.00	7.11
8	15700	448600	2.42	1.42	14.69	12.01	3.52	0.09	8.92	4.22	0.30	−0.02	0.01	8.95
9	16700	402800	2.34	1.33	16.26	13.53	2.72	0.07	7.63	2.09	0.26	0.02	0.01	7.66
10	4800	366200	3.19	1.67	20.08	14.82	4.48	0.15	15.82	6.77	0.58	−0.05	0.01	16.11
11	57100	1297500	2.24	1.35	17.21	12.92	3.27	0.04	9.84	4.24	0.15	−0.02	0.00	9.91
12	24700	874100	2.51	1.47	18.59	13.11	3.29	0.07	8.63	3.30	0.24	0.00	0.01	8.63
13	13400	719500	2.67	1.64	15.47	11.94	4.11	0.11	8.91	4.51	0.37	−0.01	0.01	8.91
14	5800	334200	2.82	1.65	17.82	13.18	4.68	0.15	14.03	4.84	0.54	0.00	0.01	14.04
15	7900	725700	3.05	1.82	16.02	12.72	3.52	0.16	6.03	1.71	0.54	0.04	0.01	6.18
16	4300	352600	2.75	1.83	14.54	12.75	5.40	0.20	14.12	5.39	0.68	0.00	0.02	14.12
17	20000	373600	1.89	1.36	13.41	10.93	4.00	0.08	10.40	3.63	0.26	0.01	0.01	10.41
18	28200	467900	1.76	1.34	14.38	11.53	3.37	0.07	8.42	3.03	0.21	0.01	0.01	8.43
19	27700	1235800	2.50	1.58	15.59	13.09	4.85	0.07	16.14	3.73	0.23	0.03	0.01	16.22
20	9100	248800	2.04	1.51	15.08	12.17	4.25	0.12	11.66	2.40	0.41	0.05	0.01	11.87
21	9300	175100	1.96	1.35	10.47	10.96	2.77	0.13	5.02	1.20	0.38	0.04	0.01	5.22
22	4900	372900	3.49	1.48	22.14	14.62	4.20	0.13	17.11	4.30	0.53	0.00	0.01	17.11
23	31500	499000	1.84	1.28	16.25	12.01	2.94	0.06	7.58	2.41	0.19	0.01	0.00	7.60
24	2200	80100	2.22	1.52	17.54	14.29	3.70	0.21	12.18	0.38	0.77	0.08	0.02	12.96
25	13100	626800	2.72	1.56	17.60	13.74	3.60	0.09	10.04	4.77	0.34	−0.03	0.01	10.12
26	13800	735600	3.05	1.49	17.63	12.48	3.27	0.10	7.49	4.01	0.34	−0.02	0.01	7.52
27	400800	4596200	1.63	1.22	12.60	10.38	3.29	0.02	7.80	3.19	0.06	0.00	0.00	7.80
28	118400	1695500	1.93	1.20	12.90	10.37	2.56	0.03	4.88	4.05	0.13	−0.05	0.00	5.00
29	130900	2531700	1.88	1.41	11.20	10.09	4.15	0.04	8.78	7.31	0.14	−0.10	0.00	9.17
30	44900	2565300	2.76	1.62	11.13	9.62	5.66	0.07	11.34	11.26	0.27	−0.18	0.01	12.26
31	741800	6797900	1.69	1.03	11.51	9.86	1.58	0.01	2.31	3.81	0.04	−0.07	0.00	2.71

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Table 3—continued from previous page

	Descriptive statistics						Linear specification			Linear-quadratic specification				
z	J	$\sum_j L_j$	$E(\ln L_j)$	$SD(\ln L_j)$	$E(s_j)$	$SD(s_j)$	$100 \cdot \widehat{\eta}$	$100 \cdot SE(\widehat{\eta})$	R^2 (in %)	$100 \cdot \widehat{\widehat{\eta}}$	$100 \cdot SE\left(\widehat{\widehat{\eta}}\right)$	$100 \cdot \widehat{\widehat{\mu}}$	$100 \cdot SE\left(\widehat{\widehat{\mu}}\right)$	R^2 (in %)
32	5400	217700	2.08	1.67	10.93	9.97	5.46	0.22	10.63	7.57	0.77	−0.07	0.02	10.76
33	1200	49000	2.27	1.68	9.92	8.39	4.83	0.56	5.86	13.49	1.84	−0.28	0.06	7.78
34	1600	49800	2.07	1.60	12.08	10.42	4.58	0.37	8.89	6.87	1.38	−0.07	0.04	9.06
35	103300	1060600	1.37	1.28	9.30	8.96	4.68	0.04	10.70	6.19	0.14	−0.05	0.00	10.82
36	16300	398700	2.02	1.55	10.41	9.47	4.26	0.12	6.78	4.86	0.42	−0.02	0.01	6.80
37	2600	30900	1.78	1.18	10.56	10.66	2.01	0.21	3.32	3.95	0.81	−0.06	0.03	3.55
38	52000	810600	1.70	1.35	9.29	8.64	3.35	0.07	4.60	4.83	0.21	−0.05	0.01	4.70
39	14300	574500	2.46	1.54	10.36	9.68	1.68	0.13	1.12	4.60	0.44	−0.09	0.01	1.46
40	29800	711400	2.00	1.46	12.13	11.47	3.66	0.07	8.21	2.27	0.22	0.04	0.01	8.35
41	19000	210400	1.43	1.34	9.61	8.51	2.42	0.11	2.35	5.52	0.35	−0.11	0.01	2.80
42	58000	1074800	1.78	1.43	8.39	9.03	3.60	0.06	5.16	8.97	0.22	−0.18	0.01	6.22
43	23800	427000	1.70	1.45	9.96	10.38	3.74	0.09	7.20	7.93	0.34	−0.13	0.01	7.81
44	219400	2164200	1.70	1.05	10.10	10.32	2.81	0.02	7.64	5.18	0.08	−0.08	0.00	8.07
45	79000	525300	1.05	1.09	7.62	7.73	2.47	0.05	3.06	3.38	0.14	−0.03	0.01	3.12
46	165700	1268900	1.20	1.10	11.45	10.04	2.24	0.03	4.19	1.79	0.10	0.01	0.00	4.20
47	7500	58500	0.94	1.18	9.51	10.65	3.55	0.12	10.32	3.68	0.47	0.00	0.01	10.33
48	266100	1157200	0.93	0.93	9.72	9.40	1.61	0.02	2.65	2.62	0.06	−0.03	0.00	2.75
49	62800	508400	1.61	0.99	9.57	8.37	2.37	0.05	4.00	5.98	0.14	−0.13	0.00	5.08
50	172800	889600	1.05	0.98	12.41	9.75	1.53	0.02	2.33	2.27	0.09	−0.02	0.00	2.37
51	461600	3495500	1.24	1.14	9.56	8.63	3.00	0.02	5.13	4.00	0.06	−0.04	0.00	5.19
52	95100	811900	1.14	1.24	6.12	5.77	3.79	0.07	3.12	6.24	0.18	−0.11	0.01	3.34
53	50000	2132900	2.31	1.69	10.77	10.03	3.84	0.07	5.18	5.30	0.24	−0.05	0.01	5.26
54	309800	5779000	1.68	1.43	8.97	8.29	2.48	0.03	2.06	4.00	0.10	−0.05	0.00	2.15
55	20400	309200	1.90	1.27	10.10	9.01	3.04	0.10	4.68	4.49	0.32	−0.05	0.01	4.78
56	90100	6748600	2.44	1.83	14.11	11.91	8.88	0.04	33.24	−1.12	0.19	0.29	0.01	35.34
57	505500	4373400	1.56	1.06	11.98	9.90	0.93	0.02	0.75	3.64	0.06	−0.09	0.00	1.24
58	6900	5018900	5.75	1.59	19.82	11.53	6.22	0.15	20.38	20.14	0.63	−0.39	0.02	25.95
59	72800	2780600	2.71	1.44	11.08	9.47	5.11	0.05	11.34	3.17	0.18	0.06	0.01	11.48
60	142500	2075000	1.94	1.23	11.56	9.89	2.83	0.03	5.13	4.37	0.12	−0.05	0.00	5.25
61	39500	376700	1.16	1.25	11.23	9.77	3.66	0.06	8.21	1.18	0.22	0.08	0.01	8.53
62	61500	1125900	2.01	1.35	11.87	10.96	2.39	0.05	3.79	4.99	0.20	−0.08	0.01	4.08
63	55700	1318400	2.18	1.38	10.63	9.35	0.23	0.06	0.02	3.10	0.22	−0.09	0.01	0.36

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Table 3—continued from previous page

	Descriptive statistics						Linear specification			Linear-quadratic specification				
z	J	$\sum_j L_j$	$E(\ln L_j)$	$SD(\ln L_j)$	$E(s_j)$	$SD(s_j)$	$100 \cdot \widehat{\eta}$	$100 \cdot SE(\widehat{\eta})$	R^2 (in %)	$100 \cdot \widehat{\widehat{\eta}}$	$100 \cdot SE\left(\widehat{\widehat{\eta}}\right)$	$100 \cdot \widehat{\widehat{\mu}}$	$100 \cdot SE\left(\widehat{\widehat{\mu}}\right)$	R^2 (in %)
64	505500	8893400	2.29	1.16	9.13	8.66	2.40	0.02	3.22	5.36	0.06	−0.11	0.00	3.72
65	691400	4396100	1.30	1.00	14.16	11.06	2.26	0.01	6.24	1.97	0.04	0.01	0.00	6.24

Notes: z denotes the industry number, see table 2; J denotes the number of establishments in industry z; $\sum_j L_j$ denotes total employment; $E(\ln L_j)$ denotes the mean of log employment; $SD(\ln L_j)$ denotes the standard deviation of log employment; $E(s_j)$ denotes the mean of establishment age; $SD(s_j)$ denotes the standard deviation of establishment age; $\widehat{\eta}$ denotes the estimated coefficient obtained from the regression (34); $SE(\widehat{\eta})$ denotes the standard error of this coefficient; R^2 denotes the regression's R-squared statistic; $\widehat{\widehat{\eta}}$ denotes the estimated coefficient on age when augmenting the regression (34) with a regressor age squared; and $\widehat{\widehat{\mu}}$ denotes the estimated coefficient on age squared.