Approximating Fixed-Horizon Forecasts Using Fixed-Event Forecasts

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What we do

- Construct approximations for fixed-horizon forecasts from fixed-event forecasts;
- Approximation uses optimal weighting of fixed-event forecasts derived by minimizing the mean-squared approximation error;
- Explore gains in approximating one-year-ahead mean inflation and GDP growth for the 13 countries from the Consensus Economics (CE) survey...
- and the corresponding cross-sectional forecast disagreement.

Example: Fixed-event forecasts from Consensus Economics

January Survey	Real GDP % increase			Consumer Prices % increase		
	2015	2016	2017	2015	2016	2017
Belgium	1.3	**1.4**	**1.6**	0.6	**1.5**	**1.8**
Canada	1.2	**1.7**	**2.2**	1.1	**1.8**	**2.0**
France	1.1	**1.4**	**1.6**	0.1	**0.8**	**1.4**
Germany	1.7	**1.8**	**1.6**	0.3	**1.1**	**1.8**
Italy	0.7	**1.3**	**1.3**	0.1	**0.6**	**1.3**
Japan	0.6	**1.2**	**0.6**	0.7	**0.6**	**2.0**
Netherlands	2.0	**1.9**	**1.8**	0.6	**1.2**	**1.5**
Norway	1.3	**1.5**	**2.1**	2.1	**2.5**	**2.1**
Spain	3.2	**2.7**	**2.3**	-0.5	**0.7**	**1.4**
Sweden	3.4	**3.2**	**2.7**	0.0	**1.1**	**2.0**
Switzerland	0.8	**1.2**	**1.7**	-1.1	**-0.4**	**0.3**
United Kingdom	2.3	**2.3**	**2.3**	0.0	**1.0**	**1.8**
United States	2.4	**2.4**	**2.5**	0.1	**1.5**	**2.3**

Inflation forecasts from Consensus Economics (CE)

Question: What is the forecast of 11-month-ahead year-on-year inflation rate (**fixed-horizon**), standing at beginning of month t?

$$g_{t+11,t-1} = \frac{p_{t+11} - p_{t-1}}{p_{t-1}} \approx g_{t,t-1} + g_{t+1,t} + \cdots g_{t+11,t+10}.$$

Information at time t: Two CE **fixed-event forecasts**:

- Annual growth rate of price index for **current** calendar year:

$$g_{1,0}^{(12)} = \frac{p_1^{(12)}}{p_0^{(12)}} - 1,$$

  where $p_1^{(12)} = \frac{1}{12}(p_1 + \cdots + p_{12})$, $p_0^{(12)} = \frac{1}{12}(p_{-11} + \cdots + p_0)$.
- Annual growth rate of price index for **next** calendar year:

$$g_{2,1}^{(12)} = \frac{p_2^{(12)}}{p_1^{(12)}} - 1,$$

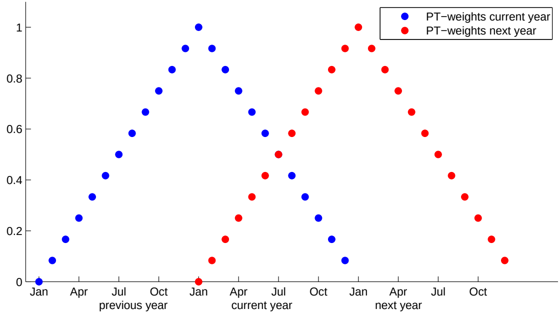
  where $p_2^{(12)} = \frac{1}{12}(p_{13} + \cdots + p_{24})$. Drawbacks of fixed-event forecasts

Annual growth rates and monthly growth rates

CE fixed-event forecasts are well approximated by linear function of monthly growth rates (Patton and Timmermann, 2011):

$$g_{1,0}^{(12)} \approx \omega_1 g_{12,11} + \omega_2 g_{11,10} + \cdots + \omega_{24} g_{-11,-12}$$
$$g_{2,1}^{(12)} \approx \omega_1 g_{24,23} + \omega_2 g_{23,22} + \cdots + \omega_{24} g_{1,0}$$

where $w_k = 1 - \frac{|k-12|}{12}$.



Ad-hoc approximation of fixed-horizon forecasts at time t

Approximate $g_{t+11,t-1}$ by weighting the two CE fixed-horizon forecasts according to:

$$\hat{g}_{t+11,t-1} \approx w_t^{adhoc} g_{1,0}^{(12)} + \left(1 - w_t^{adhoc}\right) g_{2,1}^{(12)},$$

with

$$w_t^{adhoc} = \frac{13 - t}{12}.$$

Example: In beginning of January ($t = 1$) one wants to forecast inflation Dec(current year)-to-Dec(previous year):

$$\hat{g}_{12,0} \approx g_{1,0}^{(12)} \qquad \text{since} \qquad w_t^{adhoc} = 1.$$

Note: Information contained in the second fixed-event forecast $g_{2,1}^{(12)}$ is **ignored** in this case.

Optimal approximation of fixed-horizon forecasts at time t

We propose to determine the weight w_t in:

$$\tilde{g}_{t+11,t-1} = w_t g_{1,0}^{(12)} + (1 - w_t)\, g_{2,1}^{(12)},$$

by minimizing the expected squared approximation error

$$\min_{w} E\left[\left(g_{t+11,t-1} - \tilde{g}_{t+11,t-1}\right)^2\right].$$

Novel approximation accounts correctly for **monthly growth rates embedded** in $g_{t+11,t-1}$ and $\tilde{g}_{t+11,t-1} = w_t g_{1,0}^{(12)} + (1 - w_t)\, g_{2,1}^{(12)}$.

Derive optimal approximation of fixed-horizon forecasts

By introducing the vectors

$$\mathbf{G} = \begin{bmatrix} g_{24,23} \\ g_{23,22} \\ \vdots \\ g_{-11,-12} \end{bmatrix}, \mathbf{A}_{t,12} = \begin{bmatrix} \mathbf{0}_{13-t} \\ \mathbf{1}_{12} \\ \mathbf{0}_{11+t} \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} \mathbf{0}_{12} \\ \omega_1 \\ \omega_2 \\ \vdots \\ \omega_{24} \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_{24} \\ \mathbf{0}_{12} \end{bmatrix}$$

we write the approximation error as linear expression of weight w_t:

$$\begin{aligned} g_{t+11,t-1} - \tilde{g}_{t+11,t-1} &= \mathbf{A}'_{t,12}\mathbf{G} - \left(w_t \mathbf{B}'_1 \mathbf{G} + (1 - w_t)\,\mathbf{B}'_2 \mathbf{G}\right) \\ &= \left(\underbrace{\mathbf{A}'_{t,12} - \mathbf{B}'_2}_{:=\mathbf{M}_t} + w_t \underbrace{\left(\mathbf{B}'_2 - \mathbf{B}'_1\right)}_{:=\mathbf{N}}\right)\mathbf{G}. \end{aligned}$$

$\Rightarrow E\left[\left(g_{t+11,t-1} - \tilde{g}_{t+11,t-1}\right)^2\right]$ is a quadratic expression in w_t.

Optimal weights formula

The optimal weight on the current year fixed-event forecast is:

$$w_t^* = -\mathbf{M}_t \mathbf{\Omega} \mathbf{N}' \; / \; \left(\mathbf{N} \mathbf{\Omega} \mathbf{N}'\right).$$

$\mathbf{\Omega}$ is the covariance matrix of vector $\mathbf{G}$ of monthly growth rates

$$\mathbf{\Omega} = E\left[\left(\mathbf{G} - E\left[\mathbf{G}\right]\right)\left(\mathbf{G}' - E\left[\mathbf{G}'\right]\right)\right].$$

$\mathbf{\Omega}$ needs to account for known and forecasted monthly growth rates in vector $\mathbf{G}$.

Characteristics of the optimal weighting approach

Optimal weighting approach can account for:

- type of growth rates embedded in the fixed-event forecasts (annual growth rates, quarterly growth rates, ...);
- (assumptions about) the data generating process;
- the (assumed) covariance matrix of the data generating process (realized and forecast);
- discretionary forecast horizons (3-month, 6-month, 24-month);
- information from additional fixed-event forecasts,

but does not consider information contained in previous or later forecasts or from other variables.

Properties of the optimal weighting approach

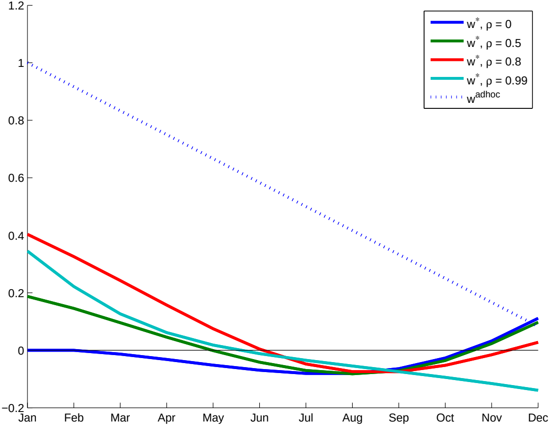
Example:

- Assume data-generating process for the monthly growth rate of the variable:

$$g_{t+1,t} - \mu = \rho \left(g_{t,t-1} - \mu \right) + \varepsilon_{t+1},$$

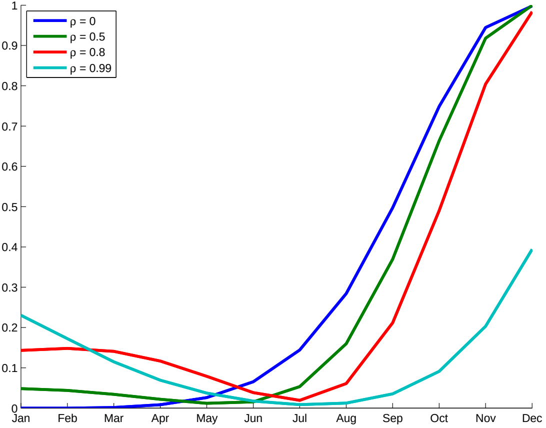
  with ε_{t+1} *iid* $N\left(0, \sigma_\varepsilon^2\right)$.
- Assume that prior's month growth rate is observed, but not the one for the current month.
- The forecaster makes optimal forecasts.

Weight on current-year forecast from two methods



Optimal weights w^* depending on ρ and ad-hoc weights w^{adhoc} for current-year forecast $g_{1,0}^{(12)}$.

Relative approximation errors



Ratio of expected squared approximation error with optimal weights to expected squared approximation error with ad-hoc weights for different values of ρ.

Empirical application based on Consensus Economics data

- Approximate each forecaster's unobservable forecast for CPI and GDP four-quarters-ahead year-on-year quarterly growth rate:

$$g_{t+4,t}^{(3)} = \frac{p_{t+4}^{(3)}}{p_t^{(3)}} - 1,$$

  where $p_t^{(3)} = \frac{1}{3}(p_{3(t-1)+1} + p_{3(t-1)+2} + p_{3(t-1)+3})$, $t = 1, 2, 3, 4$.
- using their individual forecasts for annual growth rates of CPI and GDP for current and next year, i.e. $g_{1,0}^{(12)}$ and $g_{2,1}^{(12)}$;
- in each March, June, September and December.

Assumptions of the optimal approximation method

- inflation data is known up to the previous month;
- GDP growth data is available up to previous quarter;
- both inflation and GDP growth are assumed to be monthly processes;
- growth rates of both variables are assumed to be *iid*, so that

$$\mathbf{\Omega} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sigma_{\varepsilon}^2 \cdot I \end{bmatrix}$$

where I is the identity matrix.

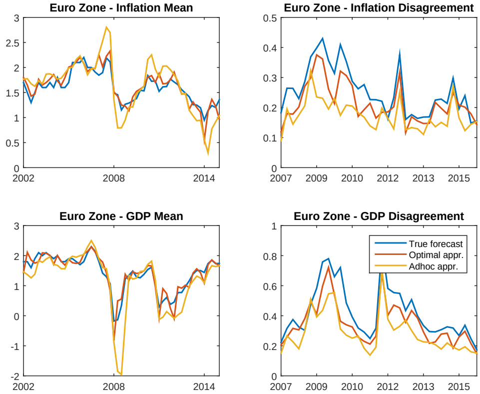
Comparison of optimal and ad-hoc method weights

	w^*		w^{adhoc}
	inflation	GDP growth	
March	0.04	0.00	0.75
June	−0.05	−0.03	0.50
September	−0.07	−0.08	0.25
December	0.08	−0.03	0.00

Weights for current-year forecasts

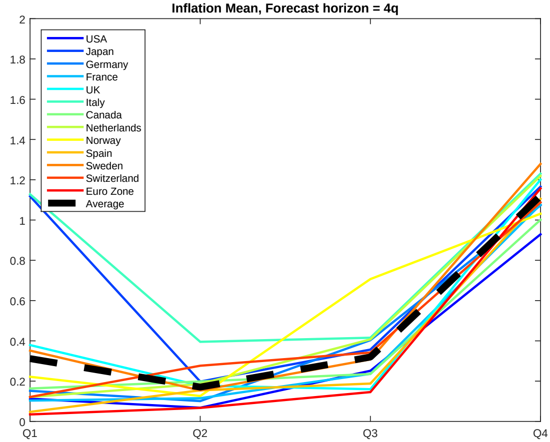
- In the first three quarters, the ad-hoc approach places (much) larger weight on the current-year forecasts

Optimal and ad-hoc approximations for Eurozone (EZ)



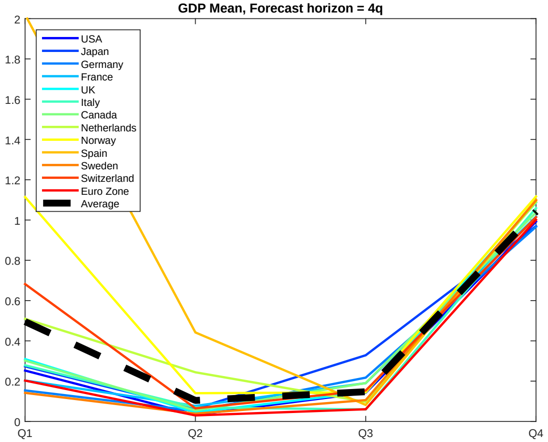
Actual four-quarter-ahead Consensus mean forecasts and the approximations based on optimal and ad-hoc weights for inflation and GDP growth (left panels) and time series of disagreement (measured by the standard deviation) among the actual individual four-quarter-ahead Consensus forecasts and the approximations based on optimal and ad-hoc weights for inflation and GDP growth (right panels).

Relative approximation errors - Mean forecast inflation



Ratios of the average squared approximation errors using the optimal weights to the average squared approximation errors using the ad-hoc weights for inflation mean forecasts four quarters ahead.

Relative approximation errors - Mean forecast GDP growth



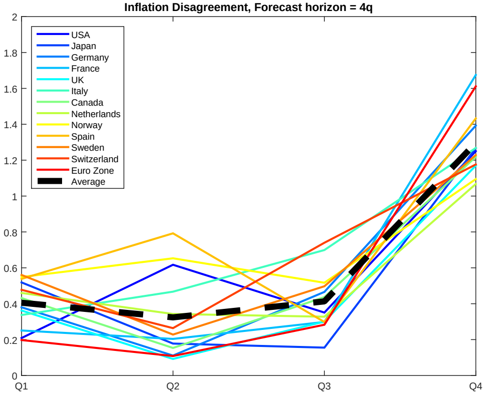
Ratios of the average squared approximation errors using the optimal weights to the average squared approximation errors using the ad-hoc weights for GDP growth mean forecasts four quarters ahead.

Relative approximation errors - mean forecast

	Avg	US	JP	GE	FR	UK	IT	CA	NL	NO	ES	SE	CH	EZ
							Inflation							
MSE ratio	**0.42**	0.3	0.7	0.4	0.3	0.4	0.8	0.3	0.3	0.4	0.3	0.5	0.4	0.2
							GDP growth							
MSE ratio	**0.38**	0.4	0.3	0.3	0.3	0.4	0.3	0.4	0.4	0.5	0.9	0.2	0.5	0.2

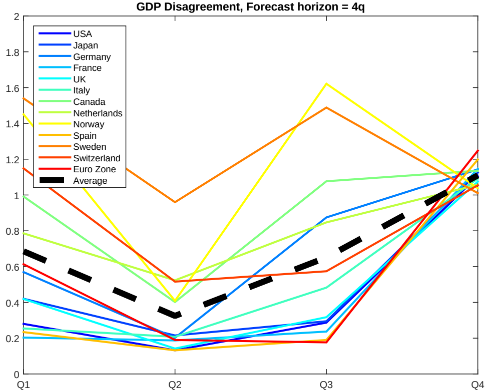
Ratio of the mean-squared approximation error obtained with optimal weights to the mean-squared approximation error obtained with ad-hoc weights in all quarters.

Relative approx. errors - disagreement inflation forecasts



Ratios of the average squared approximation errors using optimal weights to the average squared approximation errors using ad-hoc weights for the standard deviations among individual inflation forecasts four quarters ahead.

Relative approx. errors - disagreement future GDP growth



Ratios of the average squared approximation errors using optimal weights to the average squared approximation errors using ad-hoc weights for the standard deviations among individual GDP growth forecasts four quarters ahead. Nr. Forecasters

Relative approximation errors - forecast disagreement

	Avg	US	JP	GE	FR	UK	IT	CA	NL	NO	ES	SE	CH	EZ
							Inflation							
MSE ratio	0.52	0.6	0.5	0.5	0.3	0.3	0.5	0.6	0.5	0.7	0.6	0.6	0.6	0.3
Bias														
optim.	**-0.07**	**-0.2**	**-0.0**	**-0.1**	**-0.0**	**-0.1**	**-0.0**	**-0.0**	**-0.1**	**-0.1**	**-0.0**	**-0.0**	**-0.1**	**-0.0**
ad-hoc	-0.13	-0.3	-0.1	-0.1	-0.1	-0.2	-0.1	-0.1	-0.2	-0.1	-0.1	-0.1	-0.1	-0.1
Corr. true														
with optim	**0.73**	**0.7**	0.7	**0.7**	**0.9**	**0.7**	**0.8**	**0.8**	**0.6**	**0.6**	**0.7**	**0.6**	**0.7**	**0.9**
with ad-hoc	0.63	0.6	**0.8**	0.6	0.8	0.5	0.7	0.7	0.4	0.5	0.6	0.6	0.6	0.7
							GDP growth							
MSE ratio	0.62	0.4	0.4	0.5	0.4	0.5	0.5	0.8	0.9	1.1	0.3	1.2	0.8	0.4
Bias														
optim.	**-0.08**	**-0.1**	**-0.1**	**-0.1**	**-0.1**	**-0.2**	**-0.1**	**0.0**	**-0.1**	**-0.1**	**-0.1**	**0.0**	**-0.1**	**-0.1**
ad-hoc	-0.15	-0.2	-0.2	-0.1	-0.1	-0.3	-0.1	-0.1	-0.2	-0.1	-0.2	-0.1	-0.2	-0.1
Corr. true														
with optim	0.73	**0.8**	**0.9**	**0.8**	**0.9**	0.8	**0.8**	0.8	0.4	0.4	**0.9**	0.4	0.8	**0.9**
with ad-hoc	**0.74**	0.8	0.7	0.8	0.9	**0.9**	0.8	**0.8**	**0.5**	**0.6**	0.8	**0.6**	**0.8**	0.9

Ratio of the mean-squared approximation error obtained with optimal weights to the mean-squared approximation error obtained with ad-hoc weights in all quarters, together with bias and correlation between true disagreement and disagreement based on the two approximations.

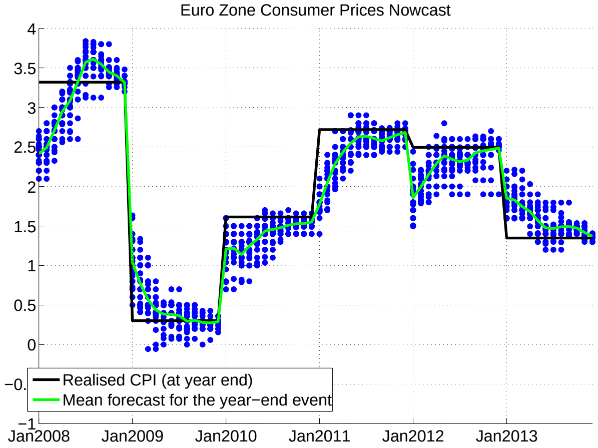
Conclusions

Optimal approximation...

- gives easily-computable weights for constructing fixed-horizon forecasts from fixed-event forecasts;
- is flexible with respect to fixed-horizon forecasts of interest and availability of fixed-event forecasts;
- significantly decreases the mean-squared approximation error for mean forecasts of inflation and GDP growth empirically;
- is a first step towards a better measurement of disagreement among forecasters.

Background slides

Annual growth rates for EZ HICP - forecasts and realizations



Black line represents realized annual growth rate of Eurozone's HICP known (values known just at year end). Blue dots represent monthly individual forecasts of the current year's annual growth rate. Green line represents the mean of these fixed-event forecast. Main

Average number of respondents for fixed-event and fixed-horizon forecasts

	US	JP	GE	FR	UK	IT	CA	NL	NO	ES	SE	CH	EZ
							inflation						
fixed horizon	26	16	17	14	12	6	10	7	5	10	6	10	17
fixed event	28	21	29	21	25	15	16	10	9	16	14	15	27
							GDP growth						
fixed horizon	27	17	19	15	12	7	10	8	5	11	6	10	18
fixed event	28	21	29	21	25	15	16	10	9	16	14	15	27

Average number of forecasters for Consensus Economics fixed-horizon (four-quarter-ahead) and fixed-event forecasts. The number displayed for the fixed-event forecasts is the average of the numbers for current- and next-year forecasts which tend to be virtually identical. main