Approximating Fixed-Horizon Forecasts Using Fixed-Event Forecasts

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Abstract

In recent years, survey-based measures of expectations and disagreement have received increasing attention in economic research. Many forecast surveys ask their participants for fixed-event forecasts. Since fixed-event forecasts have seasonal properties, researchers often use an ad-hoc approach in order to approximate fixed-horizon forecasts using fixed-event forecasts. In this work, we derive an optimal approximation by minimizing the mean-squared approximation error. Like the approximation based on the ad-hoc approach, our approximation is constructed as a weighted sum of the fixed-event forecasts, with easily computable weights. The optimal weights tend to differ substantially from those of the ad-hoc approach. In an empirical application, it turns out that the gains from using optimal instead of ad-hoc weights are very pronounced. While our work focusses on the approximation of fixed-horizon forecasts by fixed-event forecasts, the proposed approximation method is very flexible. The forecast to be approximated as well as the information employed by the approximation can be any linear function of the underlying high-frequency variable. In contrast to the ad-hoc approach, the proposed approximation method can make use of more than two such information-containing functions.

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1 Introduction

Survey data on expectations have become an important ingredient for many empirical analyses in economics and finance. Yet, for many analyses, the expectation data required does not exactly match the available survey data. An often-encountered problem is given by the need for fixed-horizons forecasts where surveys can only provide fixed-event forecasts. Fixed-event forecasts have seasonal properties – e.g. the dependence of the forecast error's variance on the month of the survey – which hamper many types of empirical analyses. When confronted with this problem, researchers often resort to an ad-hoc approach in order to construct fixed-horizon forecasts from fixed-event forecasts. This ad-hoc approximation has been employed in many studies, including Begg et al. (1998), Alesina et al. (2001), Gerlach (2007), Kortelainen et al. (2011), Dovern et al. (2012), Siklos (2013), D'Agostino and Ehrmann (2014), de Haan et al. (2014), Grimme et al. (2014), Lamla and Lein (2014), and Hubert (2015). However, concerns about the quality of the approximation sometimes give rise to the need of justifying the use of the approach as in Dovern et al. (2012), or to the refusal of its use as in Hubert (2014, p. 1392).

In this work, we derive an optimal approximation for fixed-horizon forecasts using fixed-event forecasts by minimizing the mean-squared approximation error. Like the approximation based on the ad-hoc approach, our approximation is constructed as a weighted sum of the fixed-event forecasts, with easily computable weights. However, these optimal weights tend to differ substantially from those of the ad-hoc approach. Moreover, they depend on several known properties of the fixed-event forecasts (for example, whether these refer to growth rates of annual averages or to growth rates of end-of-current-year on end-of-previous-year values), while these properties are ignored in the ad-hoc approach. The optimal weights also depend on an unknown covariance matrix, but are found to be very robust with respect to misspecifications of that matrix. While our work focusses on the approximation of fixed-horizon forecasts by fixed-event forecasts, the proposed approximation method is very flexible. The forecast to be approximated can be any linear function of the underlying high-frequency variable. The information employed by the approximation can also be any linear function of the underlying high-frequency variable. In contrast to the ad-hoc approach, the proposed approximation method can make use of more than two such functions.

It should be noted that the proposed approach is optimal in the sense that it uses the information from all forecasts for a certain variable made at a certain point in time in order to approximate the forecast of interest for that variable at that point in time. Our approach does not make use of information contained in forecasts for other variables, or in previous or later forecasts. This could be accomplished employing state-space models as in Kozicki and Tinsley (2012). While state-space models can take additional information into account, they also require additional assumptions about the data-generating process, and their implementation tends to be more involved.

In the empirical application, we approximate the one-year-ahead inflation and growth forecasts implied by quarterly forecasts for annual inflation and growth in the current and in the next year published by Consensus Economics for 13 countries. The performance of the approximations can be evaluated based on the quarterly publications of the one-year-ahead inflation and growth forecasts. It turns out that the approach based on optimal weights yields a lower mean-squared approximation error than the ad-hoc approach, and that the optimal approximation is preferable to the ad-hoc approach when trying to capture cross-sectional disagreement prevailing among forecasters.

2 Optimal Approximations

2.1 Growth Rates

Concerning many economic variables, we are interested in their growth rates g. These growth rates usually have a monthly, quarterly or annual frequency, and they can be formulated with respect to the previous period, but also with respect to more distant past periods. In order to convey all this information, define the growth rate from period $t/m-n$ to period t/m of the variable p which has the frequency m by

$$g^{(m)}_{t/m,t/m-n} = \frac{p^{(m)}_{t/m} - p^{(m)}_{t/m-n}}{p^{(m)}_{t/m-n}}$$

with $t/m = 1, 2, ...$, and where $m \geq 1$ refers to the number of high-frequency periods within one low-frequency period. We require t/m to be an integer. $n = 1, 2, ...$ denotes the number of low-frequency periods between the levels of p for which the growth rate is calculated. The growth rate $g^{(1)}_{t,t-n}$ has the highest possible frequency and is indexed by t. For notational convenience, we define

$$g_{t,t-n} = g^{(1)}_{t,t-n}.$$

For example, concerning the price level and assuming that the highest frequency is the monthly frequency, $g_{t,t-12}$ refers to the monthly growth rate with respect to the price level in the same month of the previous year, the so-called monthly year-on-year (y-o-y) inflation rate, whereas $g^{(3)}_{t/3,t/3-4}$ refers to the quarterly growth rate with respect to the price level in the same quarter of the previous year.[1]

The growth rates of low-frequency variables that we observe often refer to the change of the level of these variables between a certain low-frequency period and the following low-frequency period. Patton and Timmermann (2011) show that, usually, these growth rates can be well approximated by a linear function of high-frequency growth rates, namely by

$$g^{(m)}_{t/m,t/m-1} \approx \sum_{k=1}^{2m} \omega_k g_{t-k+1,t-k}. \tag{1}$$

If one is interested in a growth rate which is not concerned with the change from one period to the next, it is useful to employ the approximation

$$g^{(m)}_{t/m,t/m-n} = \sum_{i=1}^{n} g^{(m)}_{t/m-i+1,t/m-i}, \tag{2}$$

where the growth rates on the right-hand side can then be approximated by (1). In what follows, we assume that $g_{t+1,t}$ is covariance stationary.

2.2 The Case of Inflation Forecasts by Consensus Economics

We are first going to focus on a specific example - inflation forecasts by Consensus Economics - in order to illustrate our approach for determining optimal approximations and to prepare the reader for the more complex notation required for the general case. The annual inflation rate forecasted refers to the change in the average price level of one year to the average price level of the next year. The Consensus forecasts are published on a monthly basis, and the fixed-event forecasts reported every month refer to the annual inflation rate for the current and the next year.

Identifying January of the current year by $t = 1$, and denoting the price level in month t by p_t, the fixed-event forecasts reported are thus given by the current-year

[1] In principle, one could drop the requirement that t/m is an integer. For example, if, say, $g^{(3)}_{3/3,3/3-1}$ refers to the growth rate of the first quarter of a year with respect to the last quarter of the previous year, $g^{(3)}_{2/3,2/3-4}$ would refer to the growth rate of the quarter with months December (previous year), January and February (current year) with respect to the quarter with months September, October and November (previous year). However, the possibility to define low-frequency periods in such an uncommon way rather tends to lead to confusion and is irrelevant in our empirical applications.

annual inflation rate

$$g_{1,0}^{(12)} = \frac{p_1^{(12)} - p_0^{(12)}}{p_0^{(12)}}$$

and the next-year annual inflation rate

$$g_{2,1}^{(12)} = \frac{p_2^{(12)} - p_1^{(12)}}{p_1^{(12)}}$$

with

$$p_{t/12}^{(12)} = \frac{1}{12} \sum_{i=1}^{12} p_{t-i+1}.$$

As shown by Patton and Timmermann (2011), if the growth rate of average levels of adjacent low-frequency periods is considered, the weights ω_k equal

$$\omega_k = 1 - |k - m| / m. \tag{3}$$

with $m = 12$ in the case given here. These weights seem to have appeared for the first time in the forecasting literature in the special case of relating monthly and quarterly growth rates, i.e. for $m = 3$, when they were employed in Mariano and Murasawa (2003).

Suppose that one is interested in a fixed-horizon forecast for the growth rate $g_{t,t-12}$ denoted by $\hat{g}_{t,t-12}$. This is the case that was considered in the works by Dovern et al. (2012) and others. In order to approximate $\hat{g}_{t,t-12}$, the ad-hoc approach uses the weighted average of both fixed-event forecasts given by

$$\hat{g}_{t,t-12} \approx w^{adhoc} \hat{g}_{1,0}^{(12)} + \left(1 - w^{adhoc}\right) \hat{g}_{2,1}^{(12)} \tag{4}$$

with

$$w^{adhoc} = \frac{24 - t}{12}$$

and $t = 12, 13, ..., 23$. For example, if we are interested in the forecast for the y-o-y inflation rate in December of the current year, i.e. if we are interested in $\hat{g}_{12,0}$, $\hat{g}_{1,0}^{(12)}$ would receive the weight 1 and $\hat{g}_{2,1}^{(12)}$ the weight 0.

To the best of our knowledge, in the setting described, there has been no application of the ad-hoc approach with $n \neq 12$, as it is unclear how the weights should be chosen in this case. In order to harmonize the scales of the variables under study in the case $n \neq 12$, in what follows we assume that the quantity to be approximated is given by $(12/n)\, g_{t,t-n}$ or $(12/n)\, \hat{g}_{t,t-n}$. The approximation is denoted by $\tilde{g}_{t,t-n}$, and, thus, has the property

$$E\left[\tilde{g}_{t,t-n}\right] = E\left[(12/n)\, g_{t,t-n}\right] = E\left[g^{(12)}_{t/12,t/12-1}\right].$$

For our example, this simply means that the approximation always refers to *annualized* inflation rates.

For the moment, we can actually neglect the fact that we are dealing with forecasts, and simply ask the question how $(m/n)\, g_{t,t-n}$ can be approximated using $g^{(12)}_{1,0}$ and $g^{(12)}_{2,1}$. We require the approximation to be unbiased and focus on the linear function

$$\tilde{g}_{t,t-n} = w g^{(12)}_{1,0} + (1-w)\, g^{(12)}_{2,1}$$

where both coefficients have to sum to 1 given that $\tilde{g}_{t,t-n}$ is an annualized inflation rate and assuming that $g^{(12)}_{1,0}$ and $g^{(12)}_{2,1}$ are unbiased. Our aim is the minimization of the expected squared approximation error

$$\min_{w} E\left[\left(g_{t,t-n} - \tilde{g}_{t,t-n}\right)^2\right]. \tag{5}$$

In order to minimize this expression, define the $(36)\times 1$ vectors G, B_1, B_2 and $A_{t,n}$

$$G = \begin{bmatrix} g_{24,23} \\ g_{23,22} \\ \vdots \\ g_{-11,-12} \end{bmatrix}, B_1 = \begin{bmatrix} \mathbf{0}_{12} \\ \omega_1 \\ \omega_2 \\ \vdots \\ \omega_{24} \end{bmatrix}, B_2 = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_{24} \\ \mathbf{0}_{12} \end{bmatrix}, A_{t,n} = \begin{bmatrix} \mathbf{0}_{24-t} \\ \mathbf{1}_n \\ \mathbf{0}_{12+t-n} \end{bmatrix} \tag{6}$$

where $\mathbf{0}_k$ denotes a $k\times 1$ vector of zeros, $\mathbf{1}_k$ denotes a $k\times 1$ vector of ones, and with $24-t\geq 0$ and $12+t-n\geq 0$.

Then, the difference $g_{t,t-n} - \tilde{g}_{t,t-n}$ is given by

$$\begin{aligned} g_{t,t-n} - \tilde{g}_{t,t-n} &= A'_{t,n}G - \left(wB'_1G + (1-w)\, B'_2G\right) \\ &= \Bigg(\underbrace{A'_{t,n} - B'_2}_{:=M} + w\underbrace{(B'_2 - B'_1)}_{:=N}\Bigg) G \\ &= (M + wN)\, G \end{aligned}$$

and the quadratic expression to be minimized thus equals

$$\begin{aligned} E\left[\left(g_{t,t-n}-\tilde{g}_{t,t-n}\right)^2\right] &= \left(M+wN\right)\underbrace{E\left[GG'\right]}_{:=\tilde{\Omega}}\left(M+wN\right)' \\ &= N\tilde{\Omega}N'w^2+2M\tilde{\Omega}N'w+M\tilde{\Omega}M'. \end{aligned} \tag{7}$$

Obviously, the minimum of (5) is attained when w is set to

$$w^* = -M\tilde{\Omega}N' \,/\, \left(N\tilde{\Omega}N'\right). \tag{8}$$

The same result is obtained using

$$w^* = -M\Omega N' \,/\, \left(N\Omega N'\right) \tag{9}$$

with Ω being the covariance matrix of G

$$\Omega = E\left[\left(G-E\left[G\right]\right)\left(G'-E\left[G'\right]\right)\right]$$

whereas $\tilde{\Omega}$ contains the non-central second moments. In Appendix A.1, we briefly explain why both moment matrices yield identical results for w^*.

w^* as defined in (9) delivers optimal weights for the case where the growth rates $g_{t+1,t}$ are known. However, if some of the growth rates contained in the vector G are unknown and have to be forecast, this only affects Ω, the covariance matrix of G, and the formula (9) continues to deliver optimal weights.

In order to illustrate the approach, suppose that $g_{t+1,t}$ is an *i.i.d.* random variable with expectation μ and variance $\sigma^2 = 1$. Moreover, assume that the last known growth rate was observed in December of last year, and that the forecaster produces optimal mean forecasts, i.e. that $\hat{g}_{t+1,t} = \mu$ for $t = 0, 1, ..., 23$. The covariance matrix of G then equals

$$\Omega = \begin{bmatrix} \mathbf{0}_{24,24} & \mathbf{0}_{12,24} \\ \mathbf{0}_{12,24} & I_{12} \end{bmatrix}$$

where $\mathbf{0}_{k,l}$ denotes a $k \times l$ matrix of zeros, and I_k is the $k \times k$ identity matrix.

Additionally assume that we are interested in the y-o-y inflation rate in December of the current year, i.e. in $g_{12,0}$. Using $A_{12,12} = \begin{bmatrix} \mathbf{0}_{12} & \mathbf{1}_{12} & \mathbf{0}_{12} \end{bmatrix}'$ then yields the optimal weight for the current-year forecast which equals $w^* = 0$. This means that the optimal approach puts all the weight on the fixed-event forecast for the *next* year, whereas the ad-hoc approach puts all the weight on the fixed-event forecast for the *current* year. It can easily be seen that here, using the optimal weight actually implies an approximation error of 0, because $\hat{g}_{12,0} = \hat{g}_{2,1}^{(12)} = 12\mu$. In contrast to that, using the weights of the ad-hoc approach and, thus, the fixed-event forecast for the current year, will in general lead to an approximation error, because $\hat{g}_{1,0}^{(12)}$ does not only depend on the forecasts $\hat{g}_{t+1,t} = \mu$, but also on the realized growth rates $g_{t+1,t}$ with $t = -12, -11, ..., -1$.

2.3 The General Case

Here we drop the assumption that there are only two fixed-event forecasts and that $m = 12$. The low-frequency period in which the fixed-event forecasts are made is called the current low-frequency period. We continue to set $t = 1$ for the first high-frequency period in the current low-frequency period.

In practice, there are several types of fixed-event forecasts not considered here so far. These include low-frequency growth rates of non-adjacent low-frequency periods, averages of several fixed-event forecasts, annual growth rates defined as the level-change from the last high-frequency period of a certain low-frequency period to the last high-frequency period of the following low-frequency or fixed-event forecasts that refer to levels instead of growth rates. However, all common fixed-event forecasts can be expressed as linear functions of high-frequency forecasts and data, in the case of growth rates by making use of (1) and (2). For notational convenience, we again neglect the fact that we are dealing with forecasts. As an example for low-frequency growth rates of non-adjacent low-frequency periods, the growth rate that compares the level in the quarter of a certain year to the level in the same quarter of the previous year, if the high-frequency is the monthly frequency, can be written as

$$g_{t/3,t/3-4}^{(3)} = \frac{1}{3} g_{t,t-1} + \frac{2}{3} g_{t-1,t-2} + \sum_{i=2}^{11} g_{t-i,t-i-1} + \frac{2}{3} g_{t-12,t-13} + \frac{1}{3} g_{t-13,t-14} \tag{10}$$

using the formulas (1) and (2) , which here are given by

$$g_{t/3,t/3-4}^{(3)} = \sum_{i=1}^{4} g_{t/3-i+1,t/3-i}^{(3)}$$

$$g_{t/3,t/3-1}^{(3)} = \sum_{k=1}^{6} (1 - |k-3|/3)\, g_{t-k+1,t-k}.$$

For the general case, define the $q \times 1$ vector

$$G = \begin{bmatrix} g_{\overline{q}m,\overline{q}m-1} \\ g_{\overline{q}m-1,\overline{q}m-2} \\ \vdots \\ g_{\underline{q}m+1,\underline{q}m} \end{bmatrix}$$

with $q = (\overline{q} - \underline{q})\, m$, where $\overline{q} > \underline{q}$. The values $\overline{q}$ and $\underline{q}$ are integers that are chosen such that all fixed-event forecasts and the variable of interest x can be expressed as functions of G.

Let $\breve{x}$, the fixed-horizon forecast of interest, be given by

$$\breve{x} = \breve{A}'G$$

with $\breve{A}$ being a $q \times 1$ vector, and express the r known low-frequency variables as

$$\breve{f}_i = \breve{B}_i'G$$

with $i = 1, 2, ..., r$ and $\breve{B}_i$ being a $q \times 1$ vector. Note that, while we are considering the case that $\breve{x}$ is a fixed-horizon forecast and $\breve{f}_i$ a fixed-event forecast, the setup could just as well be used with x being a fixed-event forecast or $\breve{f}_i$ being a fixed-horizon forecast. The only requirement for the forecast $\breve{x}$ to be approximated and the forecasts $\breve{f}_i$ to be used for this approximation is that they can be represented as linear functions of G.

In order to simplify the interpretation of the weights, it is helpful to rescale the $\breve{B}_i$'s, and thereby the $\breve{f}_i$'s by setting

$$B_i = s_i \breve{B}_i$$
$$f_i = s_i \breve{f}_i$$

where the scalars s_i are chosen such that $B_i \mathbf{1}_q$ equals the same constant for all i and, thus, $E[f_i]$ equals the same constant for all i as well. In what follows, we assume without loss of generality that s_i is given by

$$s_i = m \ / \ \left(\breve{B}_i'\mathbf{1}_q\right)$$

so that

$$B_i'\mathbf{1}_q = m$$

holds for $i = 1, 2, ..., r$. Defining A by

$$A = s\breve{A}$$

with

$$s = m \ / \ \left(\breve{A}'\mathbf{1}_q\right),$$

the difference between $x = AG$ and its approximation

$$\tilde{x} = \sum_{i=1}^{r-1} w_i B_i' G + \left(1 - \sum_{i=1}^{r-1} w_i\right) B_r' G$$

can be written as

$$\begin{aligned} x - \tilde{x} &= \left(\underbrace{A' - B_r'}_{=M} + \sum_{i=1}^{r-1} w_i \underbrace{(B_i' - B_r')}_{=N_i}\right) G \\ &= (M + \mathbf{wN})\, G \end{aligned}$$

with

$$\mathbf{w} = \begin{bmatrix} w_1 & w_2 & \cdots & w_{s-1} \end{bmatrix}$$

$$\mathbf{N} = \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_{s-1} \end{bmatrix}.$$

The minimization of the expected quadratic approximation error gives

$$\begin{aligned} &\min_{\mathbf{w}} E\left[(x - \tilde{x})^2\right] \\ &\Rightarrow \mathbf{w}^{*\prime} = -\left(\mathbf{N}\Omega\mathbf{N}'\right)^{-1} \ M\Omega\mathbf{N}'. \end{aligned}$$

The fact that the covariance matrix Ω can be used instead of the matrix of non-central second moments $\tilde{\Omega}$ again follows from the considerations in Appendix A.1, noting that $M\mathbf{1}_q = 0$ and $N_i\mathbf{1}_q = 0$ hold. The approximation for $\breve{x}$ is given by $\tilde{x}/s$.

3 Properties of the Approximations

The quality of the ad-hoc approximation and the optimal approximation will be studied using the setting of the Consensus inflation forecasts. That is, there are two fixed-event forecasts, and $A_{t,n}, B_1, B_2$ and G are given by (6) and (**??**). Moreover, we assume that the data generating process is an AR(1)-process given by

$$g_{t+1,t} - \mu = \rho \left(g_{t,t-1} - \mu \right) + \varepsilon_{t+1} \tag{11}$$

with ε_{t+1} *iid* $N\left(0, \sigma_\varepsilon^2\right)$. Without loss of generality, we set $\sigma_\varepsilon^2 = 1 - \rho^2$, so that the variance of $g_{t+1,t}$ equals

$$E\left[\left(g_{t+1,t} - \mu \right)^2 \right] = 1.$$

We also assume that, knowing the growth rate $g_{t,t-1}$, the forecaster makes optimal h-step-ahead forecasts

$$\hat{g}^*_{t+h,t+h-1|t} - \mu = \rho^h \left(g_{t,t-1} - \mu \right). \tag{12}$$

While these assumptions might appear simplistic, they allow us to study the importance of persistence, and they allow the straightforward calculation of the covariance matrix Ω. The covariances required are given by

$$\begin{aligned} E\left[\left(g_{t+1,t} - \mu \right) \left(g_{t+1-i,t-i} - \mu \right) \right] &= \rho^i \\ E\left[\left(\hat{g}^*_{t+h,t+h-1|t} - \mu \right) \left(\hat{g}^*_{t+h+i,t+h+i-1|t} - \mu \right) \right] &= \rho^{2h+i} \\ E\left[\left(\hat{g}^*_{t+h,t+h-1|t} - \mu \right) \left(g_{t+1-i,t-i} - \mu \right) \right] &= \rho^i \end{aligned}$$

and the resulting covariance matrix Ω can be found in Appendix A.2.

In line with the Consensus inflation forecasts and the previous example, we now consider the case where the fixed-event forecasts $g_{1,0}^{(12)}$ and $g_{2,1}^{(12)}$ are made from January to December. We want to approximate the fixed-horizon forecasts for the monthly y-o-y inflation rate that cover the first 12 months for which no observations are yet available, so that $n = 12$. We assume that the inflation rate for the month prior to the current month (i.e. the month in which the forecast is made) is observed, but the inflation rate for the current month is not. Thus, in January, $g_{1,0}$ actually has to be forecast, whereas $g_{0,-1}$ is known, and the y-o-y inflation rate in December of the current year is the target of the fixed-horizon forecast. Therefore, for the January forecast, $g_{12,0}$ is the object of interest, and $A_{t,12}$ is given by $A_{0,12}$, for the February forecast these are $g_{13,1}$ and $A_{1,12}$, etc.

The optimal weights resulting from this setting are displayed in Figure 1 along with the ad-hoc weights defined in (4). Obviously, the ad-hoc weights w^{adhoc} for the fixed-event forecast $g_{1,0}^{(12)}$ are almost always larger than the optimal weights w^*. Only at the end of the current year both weighting schemes can deliver similar values, and w^* can be smaller than the ad-hoc weight. Interestingly, w^* can become negative,

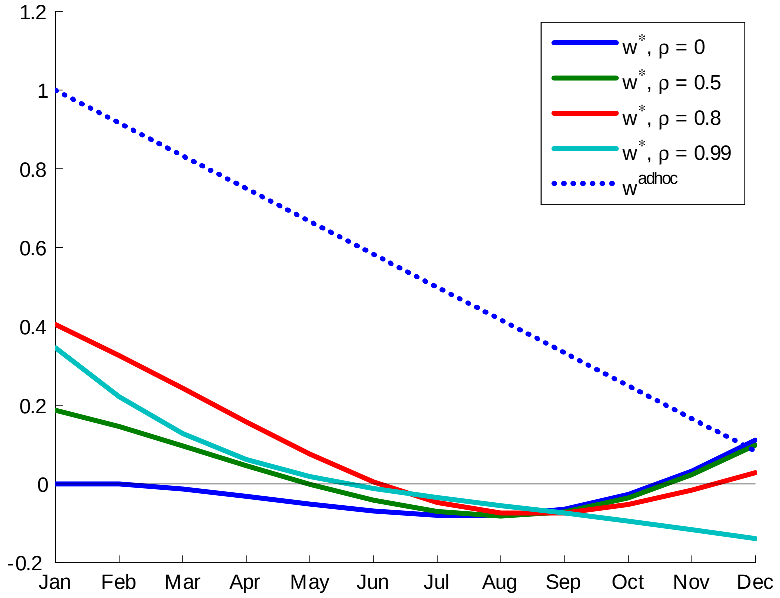

Figure 1: Optimal weights w^* depending on ρ and ad-hoc weights w^{adhoc} for current-year forecasts $g_{1,0}^{(12)}$.

implying that the optimal weight for the fixed-event forecast $g_{2,1}^{(12)}$ can become larger than 1.

The expected squared approximation error of the different weights can be calculated employing (7). We define the ratio of the expected squared approximation error with optimal weights w^* to the expected squared approximation error with ad-hoc weights w^{adhoc} as

$$\phi_t = \frac{E\left[\left(\hat{g}^*_{t+11,t-1|t-1} - \widetilde{\hat{g}}_{t+11,t-1|t-1}\right)^2 \middle| w = w^*\right]}{E\left[\left(\hat{g}^*_{t+11,t-1|t-1} - \widetilde{\hat{g}}_{t+11,t-1|t-1}\right)^2 \middle| w = w^{adhoc}\right]}. \tag{13}$$

The corresponding values of ϕ_t are displayed in Figure 2 for $t = 1, 2, ..., 12$, i.e. for the forecasts from January to December. It turns out that the mean-squared approximation errors can be reduced substantially by using optimal weights instead of ad-hoc weights, especially from the beginning until the middle of the year. At the end of the year, the gains from employing optimal weights are less pronounced, unless the DGP is extremely persistent. The values of ϕ_t obtained with $\rho = 0$ are similar to those with $\rho = 0.5$ and not very different from those with $\rho = 0.8$, suggesting that the impact of the DGP's persistence on ϕ_t is small unless the persistence is very strong.

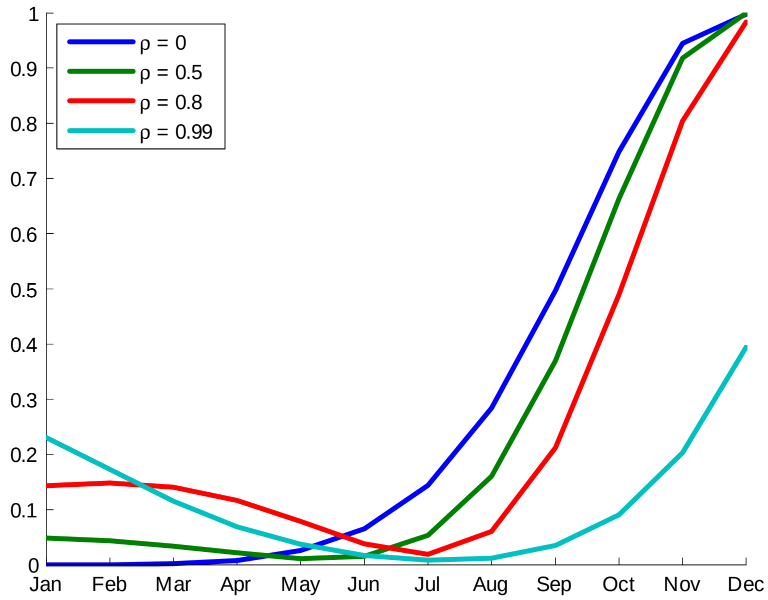

Figure 2: Ratio of expected squared approximation error with optimal weights to expected squared approximation error with ad-hoc weights for different values of ρ

Thus, the question might arise which value of ρ could be adequate for the inflation example, and it should be noted that empirically observed large persistence of the monthly y-o-y inflation rate $g_{t,t-12}$ are not very informative about ρ. For example, given the DGP (11) for $g_{t,t-1}$, in a regression of $g_{t,t-12}$ on a constant and $g_{t-1,t-13}$, the AR-coefficient would converge to 0.917 with $\rho = 0$, to 0.969 with $\rho = 0.5$, and to 0.987 with $\rho = 0.8$. We briefly elaborate on this relation in Appendix A.3.

4 Applications

In the following, we apply the methods developed to the Consensus forecasts for inflation and GDP growth. Consensus forecasts are a collection of individual forecasts. Every month, all forecasters surveyed are asked for their forecasts of annual inflation and GDP growth in the current and in the next year. In addition to these fixed-event forecasts, once per quarter, namely in the months March, June, September, and December, the forecasters are asked for their fixed-horizon forecasts of quarterly inflation and GDP growth for the current and the next six or seven quarters. These growth rates are year-on-year rates, i.e. the growth rates refer to the level of the variable in a certain quarter of a certain year compared to the level of the variable in the same quarter of the previous year.[2]

We consider the situation where the researcher only sees the fixed-event forecasts from March, June, September, or December and then attempts to construct fixed-horizon forecasts from these fixed-event forecasts, either using optimal weights or ad-hoc weights. The results can then be compared to the true fixed-horizon forecasts, and the empirical quality of both approaches can be evaluated.

We assume that inflation data up to the previous month are known when the forecasts are made, whereas for GDP growth only the data up to the previous quarter is available. Moreover, we assume that forecasters know monthly values of GDP growth, which is based on the fact that forecasters observe monthly data like surveys and industrial production which convey information about the development of GDP growth within a quarter. Thus, the A-vector needs to contain the coefficients that can be found in equation 10. Both variables are modelled as *i.i.d.* processes. For inflation, we also assumed AR(1)-processes as DGPs, with ρ estimated based on seasonally adjusted data. The results, however, hardly differ from those obtained with $\rho = 0$, because the estimates of ρ do not tend to be large. Therefore, we only report results for $\rho = 0$. For monthly GDP growth, we also set ρ to 0.[3] Since a forecast horizon of one year is a common choice in the context of macroeconomic forecasts, we focus on this horizon as well. For our quarterly forecasts, this means that, for instance, concerning the March forecasts, we would like to forecast the y-o-y growth rates of the first quarter of the next year, so that the forecast horizon equals $h = 4$ quarters.

The weights for the current-year forecasts are displayed in Table 2. Obviously, for the forecasts from March, June and September, the ad-hoc approach places much larger weights on the current-year forecasts than the optimal approach.[4] It is also interesting to note that the assumption of two more months with observed data can lead to noticeable differences in the optimal weights, as observed for the December

[2]For inflation, this refers to the average level of the price index in the given quarter.

[3]This choice might at least partly be motivated by the estimates for this coefficient based on Consensus forecast data reported in Patton and Timmermann (2011), which vary strongly depending on the estimation method. The values reported are -0.853, 0.586 and 0.996. Note that setting ρ to 0 implies that quarter-on-quarter growth rates are positively correlated. A monthly GDP series could also be constructed based on the method proposed by Chow and Lin (1971), and ρ could be estimated using this series.

[4]Note that the reasoning behind the ad-hoc approach implies that the weights of the ad-hoc approach do not depend on the number of months for which data is already available. For example, the y-o-y growth rate in the first quarter of the next year depends on the quarter-on-quarter growth rates in the second, third, and fourth quarter of the current year and the first quarter of the next year, so that one would always use the weight 0.75 for the current-year forecast if one wants to forecast the y-o-y growth rate in the first quarter of the next year. Obviously, this number does not depend on assumptions about known values of certain months.

	US	JP	GE	FR	UK	IT	CA	NL	NO	ES	SE	CH	EZ
							inflation						
fixed horizon	26	16	17	14	12	6	10	7	5	10	6	10	17
fixed event	28	21	29	21	25	15	16	10	9	16	14	15	27
							GDP growth						
fixed horizon	27	17	19	15	12	7	10	8	5	11	6	10	18
fixed event	28	21	29	21	25	15	16	10	9	16	14	15	27

Note: Average number of forecasters for Consensus fixed-horizon (4-quarter-ahead) and fixed-event forecasts. The number displayed for the fixed-event forecasts is the average of the numbers for current- and next-year forecasts which tend to be virtually identical. 'EZ' denotes the Euro zone. The other 2-letter country codes are ISO codes.

Table 1: Average number of fixed-horizon and fixed-event forecasters

forecasts.

We evaluate the approximations using two quantities, the approximation error with respect to the true fixed-horizon forecasts, calculated for the average forecast of all forecasters surveyed (i.e. the mean forecast), and the approximation error with respect to the true cross-sectional disagreement among forecasters with respect to their individual fixed-horizon forecasts, measured by the standard deviation. In both cases, we use Consensus forecasts for 13 countries. While the fixed-event forecasts of each individual forecaster are published, for the fixed-horizon forecasts only summary statistics like the mean and the standard deviation are available, and also the number of forecasters is given.[5]

Apparently, there are many cases where not all of the fixed-event forecasters issue fixed-horizon forecasts. The distortions arising from this deviation between the samples of fixed-event and fixed-horizon forecasters can be expected to be of minor importance for larger economies like the US, where data from many forecasters are available. However, for smaller economies like Norway these distortions could be relevant, especially for the standard deviation. The average number of forecasters for fixed-horizon and fixed-event forecasts is displayed in Table 1.

In order to illustrate the time series of interest, they are shown for the cases of the US and the Euro zone in Figures 3 and 4. Concerning the mean forecasts, it is obvious that the approach based on optimal weights approximates the true fixed-horizon forecasts better than the ad-hoc approach. Since the ad-hoc approach puts too much weight on the current-year forecasts which depend to a large extent on observed data, the ad-hoc approach gives approximations which are too volatile. Despite of this excess volatility, the disagreement measure obtained with the ad-hoc approach is too small. Again, this is due to the fact that too much weight is put on

[5] Only the mean forecasts of the fixed-horizon forecasts are reported in the booklets published by Consensus Economics. All additional information mentioned for the fixed-horizon forecasts can only be found in the Excel files published by Consensus Economics.

	w^*		w^{adhoc}
	inflation	GDP growth	
March	0.04	0.00	0.75
June	−0.05	−0.03	0.50
September	−0.07	−0.08	0.25
December	0.08	−0.03	0.00

Note: Optimal weights based on assumptions that inflation and GDP growth are monthly i.i.d. processes, that for inflation, data from previous month is known, and that for GDP growth, data from three months before is known. The forecast horizon h equals four quarters.

Table 2: Weights for current-year forecasts

	Avg	US	JP	GE	FR	UK	IT	CA	NL	NO	ES	SE	CH	EZ
							Inflation							
MSE ratio	0.42	0.3	0.7	0.4	0.3	0.4	0.8	0.3	0.3	0.4	0.3	0.5	0.4	0.2
							GDP growth							
MSE ratio	0.38	0.4	0.3	0.3	0.3	0.4	0.3	0.4	0.4	0.5	0.9	0.2	0.5	0.2

Note: The MSE ratio refers to the ratio of the mean-squared approximation error obtained with optimal weights to the mean-squared approximation error obtained with ad-hoc weights. 'EZ' denotes the Euro zone. The other 2-letter country codes are ISO codes. 'Avg' denotes the average over all MSE ratios.

Table 3: Approximation errors of optimal and ad-hoc approach for mean forecasts

observed data which is common knowledge to all forecasters. The approach based on optimal weights also yields a downward-biased disagreement measure, but its bias is less pronounced.

Concerning the mean forecasts, our sample starts in 1989 for about half of the countries, and it ends in 2015.[6] For inflation, we drop those time periods from our sample which are associated with increases in the value-added tax rate by 2 percentage points or more. This occurred in Japan, Germany, France, the UK, the Netherlands, and Spain. We report the ratios of the average squared approximation errors using the optimal weights to the average squared approximation errors using the ad-hoc weights in Figure 5. This is the empirical analogue of the ratio defined in (13) for the monthly case, so that the expectations are estimated by the sample means.

Obviously, using optimal weights instead of ad-hoc weights can lead to extreme reductions in the approximation errors for the four-quarter ahead forecasts from March, June, and September. For example, for the June forecasts, on average, the optimal approach gives squared approximation errors which are about 5 times

[6] The forecasts for the Netherlands, Spain and Sweden start in 1995, the forecasts for Norway and Switzerland in 1998, and the forecasts for the Euro zone in 2002. For Germany, the GDP growth forecasts before December 1995 and the inflation forecasts before December 1996 refer to Western Germany.

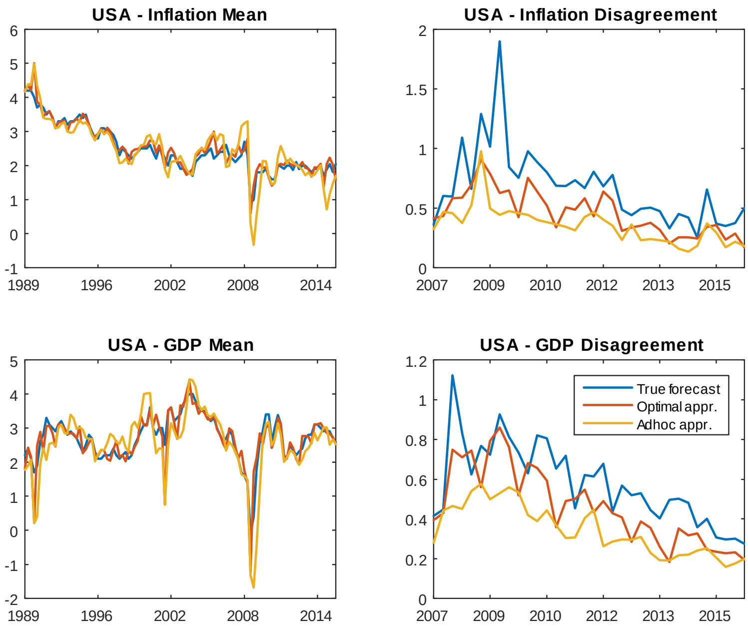

Figure 3: Time series of the actual four-quarter-ahead Consensus mean forecasts and the approximations based on optimal and ad-hoc weights for inflation and GDP growth (left panels) and time series of disagreement (measured by the standard deviation) among the actual individual four-quarter-ahead Consensus forecasts and the approximations based on optimal and ad-hoc weights for inflation and GDP growth (right panels). All data displayed are for the US.

smaller than those of the ad-hoc approach. The average refers to the average over the ratios of all countries for a specific quarter. For the March and September forecasts, the squared approximation errors with optimal weights are about 3 times smaller on average. As to be expected due to the similar weights for the December forecasts, the approximation errors of both approaches attain similar values for these forecasts. Disregarding the December forecasts, the ad-hoc approach delivers better approximations only in two cases, namely for the March forecasts of Japan and Italy.

The results for the mean forecasts of GDP growth shown in Figure 6 resemble those for inflation, with the optimal weights leading to better approximations except for the December forecasts where both approaches use almost identical weights. For the March forecasts, on average, the optimal approach gives squared approximation errors which are about 2 times smaller than those of the ad-hoc approach, with Norway and Spain being the only cases where the ad-hoc approach yields smaller errors than the optimal approach. The squared approximation errors with optimal weights are about 10 times smaller on average for the June forecasts and 6 times smaller for the September forecasts. For the June and the September forecasts, the optimal approach yields better approximations than the ad-hoc approach for each country.

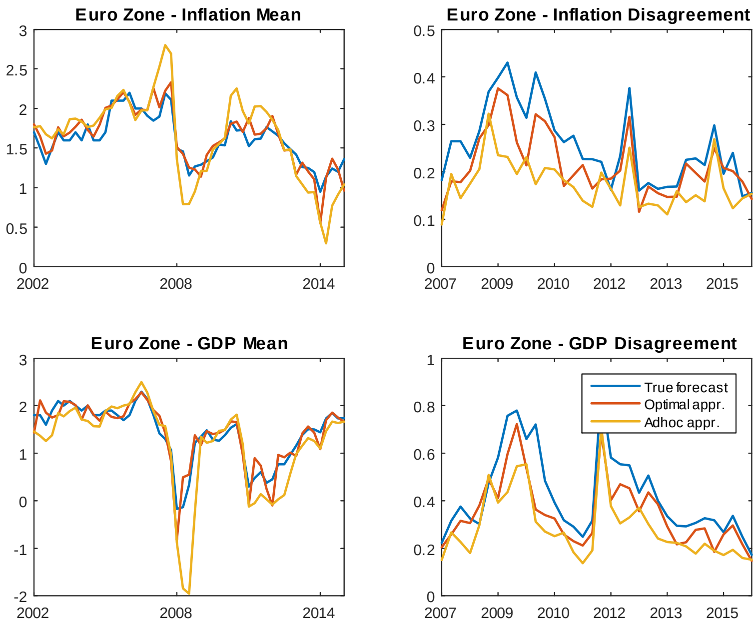

Figure 4: Time series of the actual four-quarter-ahead Consensus mean forecasts and the approximations based on optimal and ad-hoc weights for inflation and GDP growth (left panels) and time series of disagreement (measured by the standard deviation) among the actual individual four-quarter-ahead Consensus forecasts and the approximations based on optimal and ad-hoc weights for inflation and GDP growth (right panels). All data displayed are for the Euro zone.

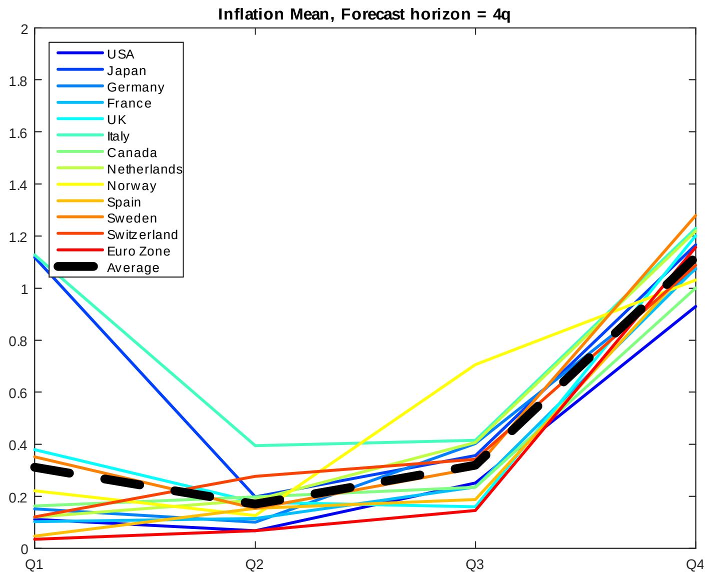

Figure 5: Ratios of the average squared approximation errors using the optimal weights to the average squared approximation errors using the ad-hoc weights for inflation mean forecasts four quarters ahead. The dashed black line denotes the average over all ratios. Q1 (Q2, Q3, Q4) corresponds to the Consensus forecasts from March (June, September, December).

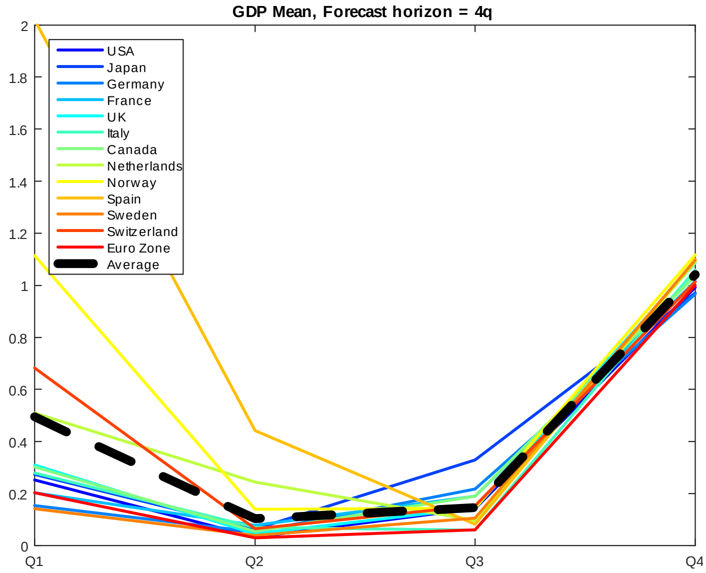

Figure 6: Ratios of the average squared approximation errors using the optimal weights to the average squared approximation errors using the ad-hoc weights for GDP growth mean forecasts four quarters ahead. The dashed black line denotes the average over all ratios. Q1 (Q2, Q3, Q4) corresponds to the Consensus forecasts from March (June, September, December).

When averaging over all forecast dates, the ratios of the mean-squared approximation errors (MSE ratios) displayed in Table 3 are obtained. The optimal weights yield better approximations for all forecasts considered, and on average, the mean-squared approximation errors are more than halved if one switches from ad-hoc weights to optimal weights.

Concerning the cross-sectional disagreement of forecasters, our samples for both variables start in 2007. As mentioned above, we use the standard deviation of the individual forecasts as the measure of cross-sectional disagreement.[7] Due to the small sample size, we do not disregard inflation forecasts related to changes in the VAT rate.[8] Results for the ratios of the means of the squared approximation errors are displayed in Figures 7 and 8.

The results are similar to those obtained for the mean forecasts, with the relative performance of the ad-hoc approach being better in the disagreement case. For example, for the December forecasts, the ad-hoc approach gives moderately better approximations to the true forecast disagreement than the approach with optimal weights. For the inflation forecasts from March, June and September, on average, the squared approximation errors obtained with optimal weights are more than 2

[7] This standard deviation is not available before 2007, which determines the start of our sample.

[8] Moreover, the results for disagreement are relatively robust with respect to such changes, because, by and large, they appear to affect the forecasts of all forecasters by the same amount.

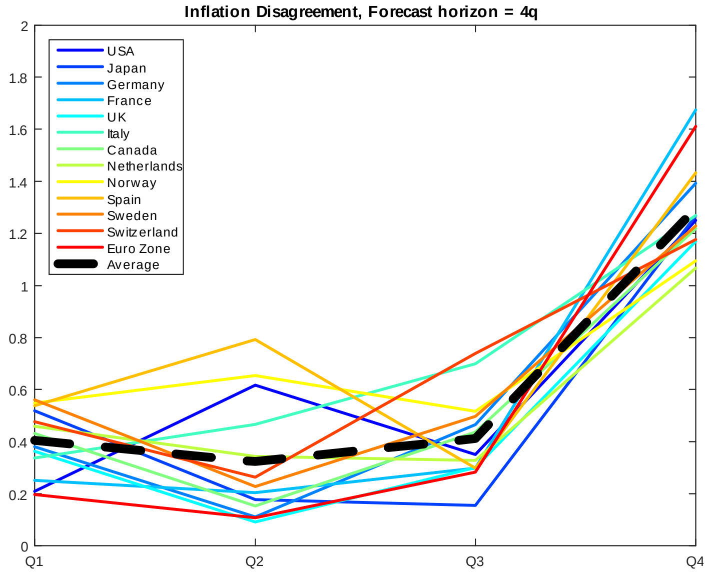

Figure 7: Ratios of the average squared approximation errors using optimal weights to the average squared approximation errors using ad-hoc weights for the standard deviations among individual inflation forecasts four quarters ahead. The dashed black line denotes the average over all ratios. Q1 (Q2, Q3, Q4) corresponds to the Consensus forecasts from March (June, September, December).

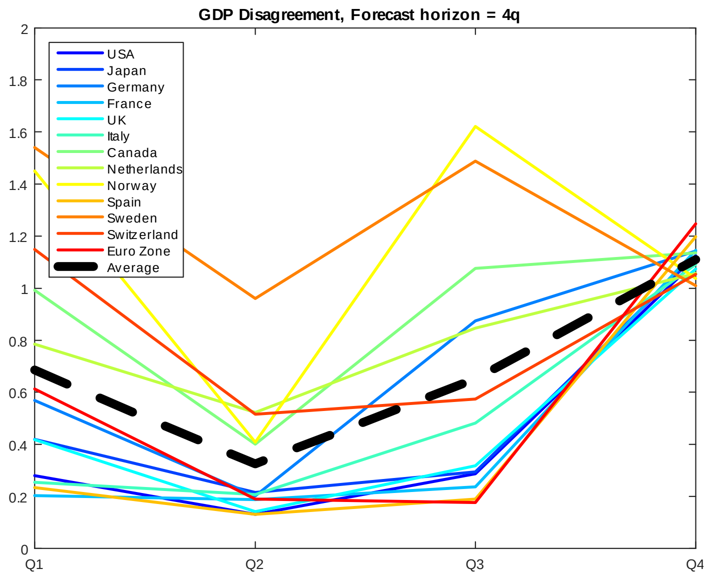

Figure 8: Ratios of the average squared approximation errors using optimal weights to the average squared approximation errors using ad-hoc weights for the standard deviations among individual GDP growth forecasts four quarters ahead. The dashed black line denotes the average over all ratios. Q1 (Q2, Q3, Q4) corresponds to the Consensus forecasts from March (June, September, December).

but less than 3 times larger than their counterparts obtained with ad-hoc weights. A similar result is observed for the GDP growth forecasts from June. For the GDP growth forecasts from March and September, using optimal weights instead of ad-hoc weights reduces the squared approximation errors by about 30 to 40 percent.

When averaging over the results for all forecast dates, the results in Table 4 show that, in general, the cross-sectional disagreement is better approximated by optimal weights, with the disagreement of GDP forecasts for Norway and Sweden being an exception to this rule.[9] Both approaches consistently underestimate the true magnitude of disagreement, but optimal weights do so to a smaller extent, so that the corresponding bias is closer to zero in all cases. The fact that the ad-hoc approach delivers strongly biased results is not too surprising, because it places too much weight on past observations which are common knowledge to all forecasters.

For inflation, except for the case of Japan, the correlations between the true and the approximated disagreement are always larger when using optimal weights. On average, the correlation coefficient is larger by a value of 0.1 if one uses optimal instead of ad-hoc weights. For GDP growth, on average, both approaches yield almost identical correlations, and the ad-hoc approach yields higher correlations for about half of the countries.

One explanation why the ad-hoc approach yields a relatively high correlation between true and approximated disagreement could be given by the correlation between the disagreement of the current-year forecasts and the disagreement of the next-year forecasts. For the forecasts from March and June, this correlation equals about 0.6 on average, while for the forecasts from September and December, it equals about 0.4. This means that for the March and June forecasts, the choice of the weights only matters to a certain extent, because the disagreement of current-year forecasts and the disagreement of the next-year forecasts evolve in a similar way, and, thus, optimal and ad-hoc weights can easily produce similar results. For the September and especially the December forecasts, where the correlation is smaller, the differences between optimal and ad-hoc weights are not too large, so that both methods can again give similar results.

Of course, the sample for the investigation of the behavior of disagreement contains the Great Recession and is relatively short, so that there is a non-negligible uncertainty about the validity of the results for other samples. However, if the ad-hoc weights always yield a correlation between true and approximated disagreement

[9]Note that these are countries where the number of fixed-horizon forecasters is small in absolute and also in relative terms (i.e. when compared to the number of fixed-event forecasters), as displayed in Table 1. Therefore, the measure of 'true' disagreement might actually be distorted to a non-negligible extent.

	Avg	US	JP	GE	FR	UK	IT	CA	NL	NO	ES	SE	CH	EZ
							Inflation							
MSE ratio	0.52	0.6	0.5	0.5	0.3	0.3	0.5	0.6	0.5	0.7	0.6	0.6	0.6	0.3
Bias														
optim.	**-0.07**	**-0.2**	**-0.0**	**-0.1**	**-0.0**	**-0.1**	**-0.0**	**-0.0**	**-0.1**	**-0.1**	**-0.0**	**-0.0**	**-0.1**	**-0.0**
ad-hoc	-0.13	-0.3	-0.1	-0.1	-0.1	-0.2	-0.1	-0.1	-0.2	-0.1	-0.1	-0.1	-0.1	-0.1
Corr. true														
with optim	**0.73**	**0.7**	0.7	**0.7**	**0.9**	**0.7**	**0.8**	**0.8**	**0.6**	**0.6**	**0.7**	**0.6**	**0.7**	**0.9**
with ad-hoc	0.63	0.6	**0.8**	0.6	0.8	0.5	0.7	0.7	0.4	0.5	0.6	0.6	0.6	0.7
							GDP growth							
MSE ratio	0.62	0.4	0.4	0.5	0.4	0.5	0.5	0.8	0.9	1.1	0.3	1.2	0.8	0.4
Bias														
optim.	**-0.08**	**-0.1**	**-0.1**	**-0.1**	**-0.1**	**-0.2**	**-0.1**	**0.0**	**-0.1**	**-0.1**	**-0.1**	**0.0**	**-0.1**	**-0.1**
ad-hoc	-0.15	-0.2	-0.2	-0.1	-0.1	-0.3	-0.1	-0.1	-0.2	-0.1	-0.2	-0.1	-0.2	-0.1
Corr. true														
with optim	0.73	**0.8**	**0.9**	**0.8**	**0.9**	0.8	**0.8**	0.8	0.4	0.4	**0.9**	0.4	0.8	**0.9**
with ad-hoc	**0.74**	0.8	0.7	0.8	0.9	**0.9**	0.8	**0.8**	**0.5**	**0.6**	0.8	**0.6**	**0.8**	0.9

Note: The MSE ratio refers to the ratio of the mean-squared approximation error obtained with optimal weights to the mean-squared approximation error obtained with ad-hoc weights. Concerning bias, the value of the method that yields a bias closer to zero is shown in bold. Concerning correlations, the value of the method that yields a higher correlation is shown in bold. 'EZ' denotes the Euro zone. The other 2-letter country codes are ISO codes. 'Avg' denotes the average over all MSE ratios, biases and correlations, respectively.

Table 4: Approximation errors of optimal and ad-hoc approach for cross-sectional standard deviation among forecasters

that is similar to that obtained with optimal weights, the results of studies which rely on this correlation like, for example, Dovern et al. (2012), are obviously robust with respect to the choice of the weights.

5 Conclusion

In the empirical literature, one frequently encounters the need to construct of fixed-horizon forecasts from fixed-event forecasts. No well-founded approach has been proposed to perform this construction, and researchers had to resort to ad-hoc approaches. This work attempts to close this gap. We derive easily-computable optimal weights for the fixed-event forecasts which allow the construction of fixed-horizon forecasts that minimize the squared approximation error with respect to the actual fixed-horizon forecasts.

In the empirical applications, we find that the gains from using optimal instead of ad-hoc weights tend to be very large. The mean-squared approximation error, on average, is more than halved. Concerning the disagreement among forecasters, optimal weights should also be preferred for the construction of the individual forecasts, because they substantially reduce the bias of the disagreement measure. The dynamics of disagreement, however, appear to be captured reasonably well by both types of weights.

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A Appendix

A.1 Using the Covariance Matrix for the Calculation of w^*

If $E\left[g_{t+1,t}\right] = \mu$ for $t = -11, -10..., 23$, the non-central matrix of second moments Ω can be decomposed as

$$\tilde{\Omega} = \mu^2 \mathbf{1}_{3m} \mathbf{1}_{3m}' + \Omega$$

with $\mathbf{1}_{3m}$ being a $(3m) \times 1$ vector of ones. Plugging this into (8) yields

$$\begin{aligned} w^* &= -\frac{M\left(\mu^2 \mathbf{1}_{3m} \mathbf{1}_{3m}' + \Omega\right) N'}{N\left(\mu^2 \mathbf{1}_{3m} \mathbf{1}_{3m}' + \Omega\right) N'} \\ &= -\frac{\mu^2 \left(M \mathbf{1}_{3m}\right) \left(N \mathbf{1}_{3m}\right)' + M \Omega N'}{\mu^2 \left(N \mathbf{1}_{3m}\right) \left(N \mathbf{1}_{3m}\right)' + N \Omega N'} \\ &= -\frac{M \Omega N'}{N \Omega N'} \end{aligned}$$

because $M\mathbf{1}_{3m} = 0$ and $N\mathbf{1}_{3m} = 0$, since $A_{t,n}\mathbf{1}_{3m} = n$, $B_1\mathbf{1}_{3m} = m$, and $B_2\mathbf{1}_{3m} = m$.

A.2 The Covariance Matrix for an AR(1)-Process

Suppose that the DGP is given by (11) and that forecasts are made according to (12). If the vector G contains n_k observed growth rates and n_u forecasts, i.e. if the largest forecast horizon equals $h = n_u$, the covariance matrix of G is given by

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{12}' & \Omega_{22} \end{bmatrix}$$

where Ω_{11} is the covariance matrix of the forecasts,

$$\Omega_{11} = \begin{bmatrix} \rho^{2n_u} & \rho^{2n_u-1} & \rho^{2n_u-2} & \cdots & \rho^{n_u+1} \\ \rho^{2n_u-1} & \rho^{2n_u-2} & \rho^{2n_u-3} & \cdots & \rho^{n_u} \\ \rho^{2n_u-2} & \rho^{2n_u-3} & \rho^{2n_u-4} & \cdots & \rho^{n_u-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{n_u+1} & \rho^{n_u} & \rho^{n_u-1} & \cdots & \rho^2 \end{bmatrix}$$

Ω_{22} is the covariance matrix of the growth rates,

$$\Omega_{22} = \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{n_k-1} \\ \rho & 1 & \rho & \cdots & \rho^{n_k-2} \\ \rho^2 & \rho & 1 & \cdots & \rho^{n_k-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{n_k-1} & \rho^{n_k-2} & \rho^{n_k-3} & \cdots & 1 \end{bmatrix}$$

and Ω_{12} contains the covariances of growth rates and forecasts

$$\Omega_{12} = \begin{bmatrix} \rho^{n_u} & \rho^{n_u+1} & \rho^{n_u+2} & \rho^{n_u+3} & \cdots & \rho^{n_u+n_k-1} \\ \rho^{n_u-1} & \rho^{n_u} & \rho^{n_u+1} & \rho^{n_u+2} & \cdots & \rho^{n_u+n_k-2} \\ \rho^{n_u-2} & \rho^{n_u-1} & \rho^{n_u} & \rho^{n_u+1} & \cdots & \rho^{n_u+n_k-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho & \rho^2 & \rho^3 & \rho^4 & \cdots & \rho^{n_k} \end{bmatrix}$$

If we are in the case of the Consensus forecasts, $n_k + n_u = 36$. Assuming that the value $g_{t+1,t}$ of the previous month is known, but the value of the current month is unknown, and given the fact that current- and next-year forecasts are produced from January to December, n_k increases from 12 in January to 23 in December, so that n_u decreases from 24 in January to 13 in December.

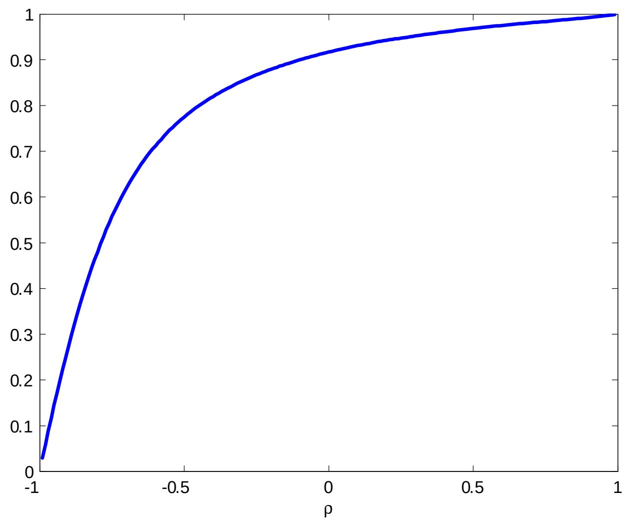

Figure 9: $plim\left(\hat{\lambda}\right)$ for different values of ρ

A.3 The Persistence of $g_{t,t-n}$ with $n > 1$

Assuming that the variable $g_{t,t-1}$ follows an AR(1)-process

$$g_{t,t-1} = \rho g_{t-1,t-2} + \varepsilon_t,$$

the ARMA$(1, m)$-process of the variable $g_{t,t-m}$ is given by

$$g_{t,t-m} = \sum_{k=1}^{m} g_{t-k+1,t-k} = \rho g_{t-1,t-m-1} + \sum_{k=1}^{m} \varepsilon_{t-k+1}.$$

When the misspecified equation

$$g_{t,t-m} = \lambda g_{t-1,t-m-1} + u_t$$

with $m = 12$ is estimated consistently, the estimator $\hat{\lambda}$ converges to the values displayed in Figure 9 for $-0.99 < \rho < 0.99$. Obviously, the persistence as measured by $plim\left(\hat{\lambda}\right)$ attains large values even if the process for $g_{t,t-1}$ exhibits only weak persistence.

The values displayed in Figure 9 can be obtained by determining the autocovariances of $g_{t,t-m}$ and applying the Yule-Walker estimator. The required formulas can be found in Lütkepohl (1993).