# The Political Economy of Prudential Regulation 

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#### Abstract

This paper studies the equilibrium level of prudential regulation in a framework with negative borrowing externalities. A debt limit is implemented by a politician appointed through majoritarian elections. If she is committed to enforce it universally, voting allows borrowers to internalize the externality. Without perfect commitment, a captured regulator exempts politically connected borrowers from regulation, distorting voters' policy preferences. Depending on the electoral power of the connected borrowers, the outcome may be an either too lax or too strict policy. Additional results highlight the impact of income inequality on the strictness of prudential regulation.


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[^0]
## 1 Introduction

Prudential policies such as bank capital and liquidity requirements or loan-to-value limits aim to prevent the build-up of excessive risk in financial markets. The academic literature motivates the use of such tools by externalities associated with borrowing (Lorenzoni, 2008), which can arise in economies with incomplete markets (Geanakoplos and Polemarchakis, 1986; Greenwald and Stiglitz, 1986). In the presence of financial constraints borrowing may be excessive as agents do not internalize how their actions exacerbate welfare-reducing firesales (Davila and Korinek, 2018). ${ }^{1}$ Recent contributions identify regulatory tools available to a social planner to improve efficiency in this context. ${ }^{2}$ They abstract, however, from the question of political feasibility of the proposed measures.

Yet, the political economy plays a crucial role in shaping financial policy (Kroszner and Strahan, 1999; Calomiris and Haber, 2015). For example, micro-level studies document that interest groups, such as voter constituencies and lobbies, are effective in influencing politicians' support for financial reforms (Mian et al., 2010; Igan and Mishra, 2014). Moreover, the complexity and opacity of financial regulation make it particularly prone to imperfect enforcement (Gai et al., 2019) benefiting the politically connected (Lambert, 2018).

Motivated by this evidence, this paper aims to deepen the theoretical understanding of how political economy factors affect financial policy. It contributes to the literature on prudential regulation motivated by borrowing externalities by analyzing policy as an equilibrium outcome of an electoral game rather than a choice of a benevolent social planner. This approach offers novel insights into the impact of income inequality and regulatory capture (understood as a favorable treatment of politically connected agents by regulators) on the strictness and efficiency of the equilibrium regulation.

The strictness of equilibrium policy is jointly determined by the balance of electoral powers of different income groups as well as the quality of political institutions, reflected in politicians' susceptibility to capture. High electoral power of low-income borrowers results in strict regulation if politicians are immune to capture, but may lead to lax prudential policy if political institutions are imperfect. The key mechanism is that selective enforcement of regulation due to capture alters voters' policy preference. First, due to reduced effectiveness of the regulation agent's without connections may prefer laxer policy. Second, as capture

[^1]generates heterogeneous exposure to regulation, it also induces the connected borrowers to support excessively strict policy in elections. Through this novel channel, anticipation of capture may result in an inefficiently strict equilibrium policy if connected voters are pivotal.

In my framework prudential regulation is motivated by negative externalities of borrowing modeled as in Davila and Korinek (2018) and Jeanne and Korinek (2013). Borrowers issue debt in order to smooth consumption. A price-dependent collateral constraint limits their ability to borrow at an interim date. Any drop in price tightens the constraint, thereby reducing consumption. A pecuniary externality arises because atomistic borrowers do not internalize the impact of their initial borrowing on prices at the interim date. They overborrow relative to the allocation of a constrained social planner. Regulation in the form of a limit or a tax on initial debt can restore constrained efficiency. My analysis focuses on the debt limit, but the appendix shows that the results are robust to using tax as the regulatory tool.

Prudential policy is implemented by a politician appointed through majoritarian elections. Politicians compete by announcing a limit on debt that they are committed to implement upon victory. In a tradition of probabilistic voting a'la Coughlin and Nitzan (1981) and Lindbeck and Weibull (1987) agents differ in their responsiveness to changes in policy when voting. In equilibrium politicians offer a policy that caters to the more responsive voters. Consequently, the equilibrium policy weighs preferences of the voter groups by their population shares and a measure of their responsiveness to policy. I refer to the latter as a measure of an electoral power per population share.

A key mechanism is that, when voting, borrowers internalize the effect of the policy on the aggregate level of initial debt, and consequently, the equilibrium price. Thus, borrowers have preference for a debt limit that curbs over-borrowing.

In a benchmark setting borrowers are homogenous and the politicians can commit to enforce the policy, so that it applies to all borrowers. In this case, voters fully internalize the borrowing externality. As a consequence, the equilibrium debt limit restores constrained efficiency, defined as the choice of a social planner who maximizes Pareto-weighted social welfare, while facing the same financial constraints as the market.

Having established the benchmark set-up I enrich the model by relaxing the assumptions of homogeneity $(i)$ and universal enforcement (ii). First, borrowers are assumed to receive heterogeneous incomes, while politicians remain committed to enforcing the policy on all borrowers. Next, I assume that politicians may enforce the rule selectively, offering exemptions to connected borrowers. With this reduced-form representation of regulatory capture

I study the equilibrium policy in both a homogenous and a heterogeneous income setting.
In the presence of income heterogeneity $(i)$ fire sales affect borrowers through an additional capital trade channel: high-income borrowers can purchase capital cheaply from the low-income types, and thus partially benefit from falling prices. Both types remain affected by falling prices through tightening of the collateral constraint, however, the capital trade channel generates a policy conflict. Low-income voters suffer a larger negative externality and thus prefer a stricter policy than the high-income types. Politicians offer a policy that weighs borrowers' preferences by their respective electoral powers. As is standard in probabilistic voting models with a committed politician (Coughlin, 1982), the equilibrium policy lies on the Pareto frontier and so it restores constrained efficiency. However, regulation generally differs from the one set by a utilitarian social planner. It is laxer if the electoral power of the high-income types is larger than that of the low-income types and stricter otherwise.

An increase in income inequality exacerbates the policy conflict, shifting the preferred policies of the two types further apart. Since the policy is set in favor of groups with higher electoral power, increasing inequality results in laxer regulation if high-income types have more power and stricter regulation otherwise. This result relates to the hypothesis put forward by Rajan (2011) and Calomiris and Haber (2015) who argue that the lax regulatory environment in the US prior to the recent financial crisis was a means of garnering support of voters from lower income groups. They see this form of tacit redistribution as a response of politicians to growing income inequality. My analysis shows that in an environment with rational agents and the inefficiency deriving from pecuniary externalities of fire sales, the relation may be reversed. In this framework, a preference of low-income voter for lax policy which they postulate could emerge in the presence of government guarantees to borrowers, voter myopia or if enforcement of regulation is selective due to regulatory capture.

Next, I study the equilibrium in a setting in which regulatory capture (ii) leads to selective enforcement of the debt limit. When a fraction of borrowers has political connections allowing them to seek exemption from regulation, two distortions may emerge. First, connected borrowers who are exempt from the limit do not internalize the price impact of their borrowing. They over-borrow and thus impose a pecuniary externality on all borrowers, as in the laissez-faire equilibrium. This reduces the effectiveness of the policy, decreasing the marginal benefits of implementing a strict debt limit. Second, as the cost of regulation is borne only by those without access to politicians, connected borrowers prefer the strictest policy. This allows them to reap the benefits of alleviating the fire sale while shifting the costs of regulation onto borrowers without connections. The anticipation of exemption distorts
their preferences for the ex-ante policy.
The two distortions can affect the equilibrium policy in opposite directions. Consequently, the debt limit implemented after elections may be either too lax or too strict relative to the policy implemented by a constrained social planner with universal enforcement. If the relative electoral power of the connected borrowers is sufficiently high, the outcome is an excessively strict debt limit. Conversely, an inefficiently lax debt limit may emerge in equilibrium if politicians need to cater to borrowers without political connections to win elections.

If income inequality and political access are positively correlated, capital trade channel and selective enforcement distortions work in opposite directions. As fewer of the low-income types are connected, they may prefer an overly lax policy if politicians are highly susceptible to capture. Thus, the policy preferences of the low-income borrowers critically depend on the quality of political institutions.

This analysis relates to the literature studying the role of capture in shaping regulation. Early contributions analyze the emergence of regulatory capture in the context of asymmetric information and the design of optimal contracts that maximize social welfare (Laffont and Tirole, 1991; Laffont et al., 1993). Recent work focuses on how regulatory capture exacerbates the moral hazard problem in financial institutions (Acharya, 2003; Martynova et al., 2019). This paper takes capture as given and explores how anticipation of favorable treatment affects the preference for the strictness of policy ex-ante. It points to a novel source of inefficiency associated with selective enforcement of regulation, working through the political economy channel. Anticipation of exemption distorts the policy preferences of connected borrowers. They support excessively strict ex-ante policy, externalizing the burden of regulation on those without connections. This mechanism is consistent with the empirical findings by Neretina (2018), who documents that firms lobbying on legislature exert a negative externality on their close competitors. Her evidence suggests that lobbying firms can secure their narrow interests at the expense of the non-lobbyists.

This work contributes to a growing theoretical literature on the political economy of financial regulation. Herrera et al. (2014) and Hakenes and Schnabel (2014) show that inefficiently lax regulation can be implemented by a politician concerned with his reputation, if the quality of credit booms or ability to understand complex arguments are a signal of her quality. In a related framework, Almasi et al. (2018) studies the pro-cyclicality of prudential policy. My model abstracts from the problem of reputation and studies the policy set by a politician who implements the policy she announced in the campaign, but may be unwilling to
enforce it universally. It complements the existing studies by deepening the understanding of voters' policy preferences and their impact on strictness of equilibrium regulation in a micro-founded model of inefficient over-borrowing.

The analysis also relates to a large body of literature on the political economy of regulating externalities. Contributions in this field focus on the application to environmental regulation and view its distributive effects as a source of a policy conflict. Fredriksson (1997) shows that in the presence of industrial and environmentalist lobbies, environmental policy may fail to maximize utilitarian social welfare. In the context of voting, Alesina and Passarelli (2014) and Masciandaro and Passarelli (2013) evaluate the equilibrium choice of a regulatory tool and its' level of stringency when agents differ in the cost of producing the externality. Other contributions focus on the potential of a re-distributive use of proceeds from environmental tax and explore how the refund rule affects the equilibrium level of tax (Cremer et al., 2004; Fredriksson and Sterner, 2005; Aidt, 2010). As in Masciandaro and Passarelli (2013), my analysis focuses on the application to financial regulation. In my setting the policy conflict emerges endogenously because of the distributive effects of the capital trade channel of the fire sale. Moreover, my analysis moves beyond the assumption of perfect commitment to policy by politicians, which underlies these models. I explore the problem of selective enforcement, which is of particular relevance in the context of financial regulation, due to its opaque and discretionary nature.

Section 2. provides the model set up and solves the benchmark framework with homogenous borrowers and universal enforcement. In Section 3. I study the implications of income heterogeneity on equilibrium prudential regulation. In Section 4. I allow for selective enforcement of regulation and explore the implications of heterogeneity in political access for the efficiency and strictness of the equilibrium. Section 5. concludes.

## 2 Model

This section introduces the benchmark model in which borrowers are homogenous and the political process is frictionless, ie. politicians enforce the regulation universally. I first outline the elements of the economic environment and then introduced the assumptions on the political process. The section solves for the laissez faire equilibrium, compares it to the allocation of a constrained social planner and then studies the equilibrium with the political game.

### 2.1 The Economic Environment

There are 3 dates, $t=0,1,2$, two goods: capital and a perishable consumption good (the numeraire) and two groups of agents: a unit mass of borrowers and a unit mass of lenders. The role of lenders in this framework is to provide funding to borrowers. Borrower's are the key actors in this setting, as they are directly affected by the inefficiency and engage in externality-generating overborrowing. In the baseline set up borrower's are homogenous, Sections 3 and 4 relax this assumption, and study the equilibrium with heterogeneous borrowers. To nest this case in the exposition of the model, I index borrower types by $B \in \mathbb{B}$, where $\mathbb{B}$ is the set of borrower types and denote by $\theta^{b}$ share of type $b$ in the population of borrowers. In the baseline setting $\mathbb{B}=\{b\}$ and $\theta^{b}=1$.

Agents derive utility from consumption according to:

$$
\begin{align*}
& u^{B}(c)=\log \left(c_{0}^{B}\right)+\log \left(c_{1}^{B}\right)+c_{2}^{B}  \tag{1}\\
& u^{L}(c)=c_{0}^{L}+c_{1}^{L}+c_{2}^{L} \tag{2}
\end{align*}
$$

Thereafter, I refer to these as consumption-utilities. Lenders are risk-neutral at all dates, with linear consumption-utility. Borrower's consumption utility is quasi-linear. The curvature at $t=0$ and $t=1$ generates a consumption smoothing motive, which is key for the welfare impact of a fire sale. Linearity in $t=2$ consumption makes the framework tractable, by ensuring that the demand for consumption at the initial and interim date are independent of income.

Lenders receive large income (endowment of the consumption good) equal to $y^{L} \geq 1$ at $t=0,1,2$. Borrowers are endowed with $y_{1}^{B}=y$ units of the consumption good at $t=1$ and receive no endowment at other dates $y_{0}^{B}=y_{2}^{B}=0$.

Borrowers are endowed with 1 unit of capital at the beginning of $t=1, k_{1}^{B}=1$. Agents can trade capital at $t=1$. The endogenously determined equilibrium price of capital in terms of the consumption good is denoted by $p$. The capital holding of agent $J$ after the trade, so between $t=1$ and $t=2$, is given by $k_{2}^{J}$. Borrower's can use it to produce a consumption good between through a linear production technology, which yields payoff, $f^{b}\left(k_{2}^{B}\right)=k_{2}^{B}$, at $t=2$. Lenders receive no capital endowments, $k_{0}^{L}=0$, and are unproductive, $f^{L}\left(k_{2}^{L}\right)=0$.

At dates $t=0$ and $t=1$, agents $J$ can trade in 1-period risk-free debt $\left(d_{t}^{J}\right)$ with a
promised repayment of $r_{t+1} d_{t}^{J}$ at $t+1$. The resulting budget constraints for agent $J$ are:

$$
\begin{align*}
c_{0}^{J} & \leq d_{0}^{J}+y_{0}^{J}  \tag{BC0-J}\\
r_{1} d_{0}^{J}+c_{1}^{J} & \leq d_{1}^{J}+y^{J}+p\left(k_{1}^{J}-k_{2}^{J}\right)  \tag{BC1-J}\\
r_{2} d_{1}^{J}+c_{2}^{J} & \left.\leq y^{J}+f^{J}\left(k_{2}^{J}\right)\right) \tag{BC2-J}
\end{align*}
$$

Imperfect contract enforcement introduces a financial friction in this model. The debtor can renege on the contract at any time, in which case the creditor can grab a fraction $\phi \leq \frac{1}{2}$ of his assets. This gives rise to collateral constraints that limit borrowers ability to take on debt at any date. The collateral constraint at $t=1$ is given by:

$$
\begin{equation*}
d_{1}^{B} \leq \phi p k_{2}^{B} \tag{CC-B}
\end{equation*}
$$

The constraints of this type are widely used in the macro-finance literature (Kiyotaki and Moore, 1997; Lorenzoni, 2008; Davila and Korinek, 2018). To ensure that it binds in the competitive equilibrium, Assumption 1 (a) below introduces an upper-bound on borrower's income.

Appendix A. 1 provides explicit characterization of $t=0$ collateral constraint. In the analysis I focus on the environments in which the $t=0$ collateral constraint is slack, by assuming that borrower's income is not too low (see Assumption 1 (b) below).

## Assumption 1. Income

Borrowers' income is low: $y<2-\phi$ (a), but not too low $y>\frac{1-\phi}{\phi}$ (b).
The assumptions on borrower's income limit the number of cases that ought to be considered and focus the discussion on the setting in which efficiency can be improved by a policy intervention ${ }^{3}$.

### 2.2 The Political Process

The prudential policy is implemented through a political process governed by standard assumptions of a probabilistic voting model. At the beginning of $t=0$, two politicians, $g=\{A, Z\}$, compete for the governmental office in majoritarian elections. Each agent can cast one vote and the politician who receives the majority of the votes wins. In case of tie,

[^2]the winner is drawn randomly. The politician who takes the governmental office receives exogenous private benefits that give her a motive to run. Politicians compete by announcing the level of prudential policy (limit on debt $\bar{d}_{g}$ ) that they will implement upon winning the elections. In the baseline model I assume that politicians can commit to universally enforce the announced policy, I relax this assumption in Section 4.

Each agent belongs to a voter group $v \in \mathbb{V}$. Lenders and borrowers (potentially of different types) form separate voter groups $\mathbb{V} \in \mathbb{B} \cup\{L\}$. Agents derive consumption-utility from the implemented policy and a political-utility from the identity of the elected politician. The political-utility is an ideological bias towards one of the candidates. It measures the preference for politician A's characteristics that are unrelated to the economic policy, such as individual features, socio-cultural policies, etc. The bias is random and composed of an idiosyncratic ( $b^{i, v}$ ) and an aggregate (b) component. The total utility of agent $i$ in voter group $v$ is given by:

$$
U^{i, v}= \begin{cases}u\left(\bar{d}_{A}\right)+b^{i, v}+b & \text { if } \mathrm{A} \text { wins }  \tag{3}\\ u\left(\bar{d}_{Z}\right) & \text { if } \mathrm{Z} \text { wins }\end{cases}
$$

The idiosyncratic component of the ideological bias, is drawn for each individual $i$ from a voter-group-specific distribution, $b^{i, v} \sim U\left[-\frac{1}{\psi^{v}} ; \frac{1}{\psi^{v}}\right]$. The more ideologically concentrated a given voter group (the higher $\psi^{v}$ ), the more important the economic policy is for it's members when they make their voting choices. The concentration can also be interpreted as a proxy for economic literacy, interest or ease of participation in elections. This is a key parameter which determines the relative importance of different voter groups in equilibrium. The aggregate component is drawn from a uniform distribution $b \sim U\left[-\frac{1}{\Psi} ; \frac{1}{\Psi}\right]$ and measures the average political preference in the population. It helps smoothen the problem of politicians, so that to avoid corner solutions.

The assumption of a probabilistic ideological bias by voters allows for differences in nonpolicy attributes of politicians and in voters preferences for these characteristics. It is widely used in the political economy literature (see for instance Lindbeck and Weibull (1987), Yang (1995), Persson and Tabellini (1999) or Pagano and Volpin (2005)) as it results in a smooth problem of a politician and allows for characteristics other than the population share to affect the relative political influence of different constituencies.

The timing of the political game is as follows: (1) Politicians announce their platforms, (2) Ideological bias is realized, (3) Agents vote in elections (denote by $e^{i, v}=1$ agents' $i$
choice to vote for politician A and by $e^{i, v}=0$ his choice to vote for politician Z) and (4) The policy is implemented ${ }^{4}$.

### 2.3 Competitive Equilibrium

This section derives the equilibrium in the absence of policy. I show that the collateral constraint limits borrowers ability to smooth consumption at the interim date, which results in a low price of capital and generates a positive relationship between price and borrower's income).

## Definition 1. Competitive Equilibrium

A competitive equilibrium is a vector of allocations, $\left\{c_{0}^{J}, c_{1}^{J}, c_{2}^{J}, d_{0}^{J}, d_{1}^{J}, k_{2}^{J}\right\} \forall J \in\{L, B\}$, and prices, $\left\{r_{0}, r_{1}, p\right\}$, such that lenders and borrowers maximize their consumption-utilities (1) and (2), subject to the budget constraints at each date (BC0-J, BC1-J, BC2-J) and the collateral constraint (CC-B). Debt and capital markets clear at all dates:

$$
\begin{array}{r}
d_{t}^{L}+\sum_{B \in \mathbb{B}} \theta^{B} d_{t}^{B}=0, t=0,1 \\
k_{2}^{L}+\sum_{B \in \mathbb{B}} \theta^{B} k_{2}^{B}=1 \tag{5}
\end{array}
$$

The problem of lenders is to maximize (2) subject to budget constraints (BC0-L, BC1-L, BC2-L). Since they are unproductive, lenders do not demand any capital, $k_{2}^{L}=0$. The linearity of their consumption-utility ensures that they are indifferent between consumption at all dates, $c_{1}^{L}+c_{2}^{L}+c_{3}^{L}=3 y^{L}$ and pin down the interest rates on the riskless debt, $r_{1}=r_{2}=1$

### 2.3.1 Borrower's Problem at the Interim Date

Solving the borrower's problem using backwards induction, the consumption and debt repayment at $t=2$ follow directly from the budget constraint. At $t=1$, the problem of a borrower can be expressed as:

$$
V^{B}\left(d_{0}^{B}\right)=\max _{c_{1}^{B}, c_{2}^{B}, d_{1}^{B}, k_{2}^{B}} \log \left(c_{1}^{B}\right)+c_{2}^{B} \text { s.t. CC-B, BC1-B, BC2-B }
$$

Denote by $\lambda_{t}^{b B}$ the Largrange multipliers on budget constraints (Bt-B) and by $\kappa_{1}^{B}$ the multiplier on the collateral constraint (CC-B). The first order conditions with respect to

[^3]consumption require that $\lambda_{1}^{B}=\frac{1}{c_{1}^{B}}$ and $\lambda_{2}^{B}=1$. The first order condition with respect to capital, $k_{1}^{B}$ is:
\[

$$
\begin{equation*}
-p \lambda_{1}^{B}+\kappa_{1}^{B} p \phi+\lambda_{2}^{B}=0 \tag{6}
\end{equation*}
$$

\]

It implies that the borrower is willing to hold any amount of capital as long as the marginal benefits are equal to the marginal costs of holding capital. The former is the value of increased consumption at $t=2$ (captured by $\lambda_{2}^{B}$ ) and the increase in $t=1$ consumption through a more relaxed collateral constraint (captured by $\kappa_{1}^{B} p \phi$ ), while the latter corresponds to the increase in $t=1$ consumption due to gains from selling capital (captured by $p \lambda_{1}^{B}$ ). Borrower's choice of debt at $t=1$ follows from the following Euler equation:

$$
\begin{equation*}
\lambda_{1}^{B}-\lambda_{2}^{B}-\kappa_{1}^{B}=0 \tag{7}
\end{equation*}
$$

There are two cases, the collateral constraint may be either slack or binding. If the collateral constraint is slack, $\kappa_{1}^{B}=0$, the borrower is able to smooth consumption between $t=1$ and $t=2, c_{1}^{B}=1$. The price of capital reflects its' marginal productivity $p=1$. The assumption on the minimum income ensure that this case does not arise in the competitive equilibrium.

If the collateral constraint is binding, $\kappa_{1}^{B}>0$, consumption smoothing is inhibited as borrower's debt is limited by the value of collateral $d_{1}^{B}=\phi p k_{2}^{B}$. Combing the Euler equation (7) with the capital FOC yields (6):

$$
c_{1}^{B}=\frac{(1-\phi) p}{1-\phi p}
$$

The budget constraint ( $\mathrm{BC} 1-\mathrm{B}$ ) pins down the individual demand for capital by borrowers. With the null demand by unproductive lenders, the market clearing requires that equilibrium price solves:

$$
\begin{equation*}
\frac{\sum_{B \in \mathbb{B}} \theta^{B}\left(y^{B}-d_{0}^{B}\right)+p}{p(1-\phi)}-\frac{1}{1-\phi p}=1 \tag{8}
\end{equation*}
$$

Focusing on the homogenous case where $B=b$, and using $D_{0}^{b}$ to denote the aggregate $t=0$ debt of borrowers, the Lemma below that re-establishes the result on the existence of a fire sale from the previous literature.

## Lemma 1. Fire Sale

If borrowers' net income at $t=1$ is high, $1-\phi \leq y-D_{0}^{b}$, the price of capital is $p=1$. If it is low, $1-\phi>y-D_{0}^{b}$, the price of capital solves (8). It is lower than capital's marginal productivity, $p<1$, and increases in borrower's net income, $\frac{\partial p}{\partial y}>0 \& \frac{\partial p}{\partial D_{0}^{b}}<0$.

Proof. Taking a derivative of (8) yields $\frac{\partial p}{\partial y}=\frac{1}{\frac{1-\phi}{(1-\phi p)^{2}}-\phi}$ and $\frac{\partial p}{\partial D_{0}^{b}}=\frac{-1}{\frac{1-\phi}{(1-\phi p)^{2}}-\phi}$. Since $\phi<\frac{1}{2}$, $\frac{\partial p}{\partial y}>0 \& \frac{\partial p}{\partial d_{0}^{b}}<0$.

If borrower's net income at $t=1, y-D_{0}^{b}$, is sufficiently high, consumption smoothing across $t=1$ and $t=2$ is achieved. If it is low, borrower is unable to reach a sufficiently high level of consumption at $t=1$ because the collateral constraint limits his borrowing. The marginal utility of consumption at $t=1$ is higher than at $t=2$, making the output produced by capital at the final date relatively less valuable to the borrower. A decrease in the net income decreases the marginal rate of substitution, which implies a larger discount on the marginal value of $t=2$ output and drives down the equilibrium price. I follow the literature and refer to the above mechanism as a "fire sale".

### 2.3.2 Borrower's Problem at the Initial Date

At $t=0$ the borrower chooses the level of consumption and debt so that to maximize his consumption-utility, taking as given his optimal choices at $t=1$ :

$$
\max _{c_{0}^{B}, d_{0}^{B}} \log \left(c_{0}^{B}\right)+V^{B}\left(d_{0}^{B}\right) \text { s.t. }(\mathrm{BC} 0-\mathrm{B})
$$

With $\lambda_{0}^{B}$ denoting the Lagrange multiplier on the $t=0$ budget constraint, the first order condition of this problem is:

$$
\begin{equation*}
\lambda_{0}^{B}-\lambda_{1}^{B}=0 \tag{9}
\end{equation*}
$$

The borrower's optimal choice is to smooth consumption between $t=0$ and $t=1$, so $c_{0}^{B}=c_{1}^{B}$. He does not internalize the effect that the aggregate debt of has on prices. This yield the equilibrium price of capital:

$$
\begin{equation*}
2 \frac{(1-\phi) p}{1-\phi p}-\phi p=y \tag{10}
\end{equation*}
$$

Under assumption 1 (a) the equilibrium price given by (10) is lower than the productivity of capital and the fire sale characterized in Lemma 1 emerges in equilibrium.

### 2.4 Planner's Policy

In this section I define the welfare benchmark and evaluate the efficiency of the competitive equilibrium. I show that a social planner can restore constrained inefficiency by implementing a debt limit.

Whenever the collateral constraint is binding the competitive equilibrium is inefficient relative to the first best. An unconstrained social planner is not restricted by the constraint arising from the financial friction and can freely choose allocations as long as they respect the resource constraints. Consequently, in the first best the allocation is such that borrowers smooth consumption across all dates. This benchmark is not particularly useful for informing policy, as it assumes that the planner can work around the key friction in the economy, the imperfect contract enforcement.

I follow the literature studying the normative implications and the optimal policy in economies with financial frictions and use constrained efficiency as the welfare benchmark (see for instance: Stiglitz (1982); Greenwald and Stiglitz (1986); Geanakoplos and Polemarchakis (1986); Lorenzoni (2008); He and Kondor (2016); Davila and Korinek (2018)). The constrained efficient allocation is the choice by a constrained social planner who can set the initial allocations (group-specific level of consumption and debt, denoted here with capital symbols) so that to maximize social welfare but has to respect the financial constraints and market clearing.

## Definition 2. Constrained Social Planner

The constrained social planner chooses initial allocations $\left\{C_{0}^{L}, D_{0}^{L}, C_{0}^{B}, D_{0}^{B}\right\}$ so that to maximize the social welfare for given Pareto Weights $\left(\chi^{L}, \chi^{B}\right)$, subject to the same constraints as the market, respecting the debt and capital market clearing conditions and leaving $t=1$ and $t=2$ decisions to private agents.

The full formulation of the planner's problem is provided in Appendix A.3. The first order conditions for consumption and lender's debt coincide with those of private agents, while the condition for borrower's debt reads:

$$
\begin{equation*}
\lambda_{0}^{B}-\lambda_{1}^{B}+\phi K_{2}^{B} \kappa_{1}^{B} \frac{\partial p}{\partial D_{0}^{B}}=0 \tag{11}
\end{equation*}
$$

Comparing the planner's choice of initial debt (11) with the optimal individual choice of borrowers (9), yields the result on constrained inefficiency (as in Jeanne and Korinek (2013) or Davila and Korinek (2018)).

## Proposition 1. Constrained Inefficiency

The competitive equilibrium is constrained inefficient characterized by overborrowing at $t=0$.

Proof. The debt level set by the planner solves (11). Comparing it to (9) and using Lemma 1 yields $D_{0}^{b}<d_{0}^{b}$.

The constrained social planner takes into account that the debt taken up by borrowers at $t=0$ affects the price of collateral at $t=1$. Whenever the borrowers are constrained in the competitive equilibrium, this price effect generates welfare losses as it further tightens the collateral constraint, inhibiting borrowers' ability to smooth consumption between $t=1$ and $t=2$. Thus, the pecuniary effects operate through a collateral channel (captured by $\phi K_{2}^{B} \kappa_{1}^{B} \frac{\partial p}{\partial D_{0}^{B}}$ in planner's FOC) resulting in an inefficiency. An individual borrower is atomistic, so does not take into account the impact of his actions on price. Thus, his choice of initial debt generates a pecuniary externality on all borrowers, as it depresses collateral prices and tightens their collateral constraints.

## Corollary 1. Constrained Efficient Debt Limit

The constrained social planner can implement the efficient allocation using a debt limit $\bar{d}^{S P}=$ $D_{0}^{* B}$.

In the presence of the debt limit, agents optimizing at $t=0$ have to account for an additional constraint:

$$
\begin{equation*}
d_{0}^{B} \leq \bar{d} \tag{DL}
\end{equation*}
$$

The optimal choices of borrowers are affected through the $t=0$ Euler equation, which now reads:

$$
\lambda_{0}^{B}-\lambda_{1}^{B}-\kappa_{0}^{b}=0
$$

where $\kappa_{0}^{B}$ is the Lagrange multiplier of the debt limit. If the debt limit is slack $\kappa_{0}^{B}=0$ and the equilibrium corresponds to the competitive equilibrium. If the debt limit is binding, $\kappa_{0}^{B}>0$, the debt and consumption at $t=0$ are pinned down by the limit $c_{0}^{B}=d_{0}^{B}=\bar{d}$. In what follows I refer to the debt limit set by the social planner as the efficient debt limit and compare the policy implemented by a politician to this benchmark.

### 2.5 Political Equilibrium

With the benchmark of the competitive (laissez faire) economy and the planner's optimal policy, I now turn to studying the equilibrium of the political economy in which the limit on initial debt is implemented by an elected politician. This section derives the solution of the probabilistic voting game and shows that the political equilibrium implements planner's policy in the benchmark model.

## Definition 3. Political Equilibrium

The political equilibrium is a vector of allocations, $\left\{c_{0}^{J}, c_{1}^{J}, c_{2}^{J}, d_{0}^{J}, d_{1}^{J}, k_{2}^{J}\right\} \forall j \in L, B$, policies announced by each politician $\left\{\bar{d}_{A}, \bar{d}_{Z}\right\}$, voting strategies, $\left\{e^{i, v}\right\} \forall i, v$, and prices, $\left\{r_{0}, r_{1}, p\right\}$, such that agents choose debt, consumption and capital holding and decide on voting strategies to maximize their total utilities (3), subject to the budget constraints at each date (BC0-J, BC1-J, BC2-J), the collateral constraint ( $C C-B$ ) and the debt limit (DL). Politicians $A$ and $Z$ maximize the probability of winning, taking as given agent's voting strategies. Debt and capital markets clear (4) and (5).

When choosing his voting strategy, an agent evaluates the policy proposals of the two politicians and votes for the one whose policy offers him a higher total utility. Politicians maximize their probability of winning the elections and thus, receiving private benefits of holding office.

The probability of winning the elections by a politician A depends on the expected share of votes that she receives. To find the vote share, define the voter indifferent between either of the politicians in each of the groups, a group specific swing voter $b^{s, v}=U^{v}\left(\bar{d}_{B}\right)-U^{v}\left(\bar{d}_{A}\right)-b$. The candidate $\mathrm{A}(\mathrm{Z})$ receives the votes from all agents, whose realized bias is larger (smaller) than that of the swing voter of the group. .

Using the uniform distributions of the individual biases and taking into account the distribution of the aggregate bias, the ex-ante probability of winning by the politician A for a given policy of the competitor $\bar{d}_{Z}$ is:

$$
\begin{equation*}
\pi_{A}=\frac{1}{2}+\frac{\Psi}{\bar{\psi}}\left[\sum_{v \in \mathbb{V}} \psi^{v} \theta^{v}\left[U^{v}\left(\bar{d}_{A}\right)-U^{v}\left(\bar{d}_{Z}\right)\right]\right] \tag{12}
\end{equation*}
$$

Where $\bar{\psi}=\psi^{L}+\sum_{B \in \mathbb{B}} \theta^{B} \psi^{B}$ is the average ideological concentration of the two groups of agents. The policy affects the probability of winning directly through its impact on agents' utilities. Each politician chooses the policy platform so that to maximize this probability.

The resulting optimality condition of the problem of the politician $A$ is:

$$
\begin{equation*}
\frac{\mathrm{d} \pi_{A}}{\mathrm{~d} \bar{d}_{A}}=\sum_{v \in \mathbb{V}} \theta^{v} \psi^{v} \frac{\mathrm{~d} U^{v}\left(\bar{d}_{A}\right)}{\mathrm{d} \bar{d}_{A}}=0 \tag{13}
\end{equation*}
$$

Therefore, to maximize her probability of winning the politician chooses a policy that maximizes the sum of weighted utilities of agents within each voter group. The weight assigned to a voter group in the politician's optimization reflects the concentration of ideological biases among its' members. The politician anticipates that voting strategies of agents in groups with a high dispersion of ideological bias are less responsive to changes in the prudential policy. Consequently, voters in these groups are relatively less important in her objective function. Thus, the electoral power of each of the groups per population share is proportional to their ideological concentration (in what follows I use these terms interchangeably).

The problem of the politician Z is symmetric, thus the equilibrium of the political subgame is characterized by the convergence of the policy platforms of the two politicians, $\bar{d}_{A}=\bar{d}_{Z}=\bar{d}^{*}$, where $\bar{d}_{A}$ solves (13). Consequently, the voting strategies of agents are determined by the realization of the ideological biases and each politician has an equal probability of winning in equilibrium.

## Proposition 2. Equilibrium Debt Limit

The equilibrium debt limit is binding and corresponds to the choice of the constrained social planner $\bar{d}^{*}=\bar{d}^{S P}<d_{0}^{* B}$.

Proof. Follows from aggregating the policy preferences according to (13).
Since lenders are not affected by the debt limit, they are indifferent between any policy, so that $\frac{\mathrm{d} U^{l}(\bar{d})}{\mathrm{d} \bar{d}}=0$. Borrower's consider the effect of the debt limit on their ability to smooth consumption between $t=0$ and $t=1$ as well as the indirect effect of the regulation through its impact on capital prices:

$$
\frac{\mathrm{d} U^{b}(\bar{d})}{\mathrm{d} \bar{d}}=\kappa_{0}^{b}+\phi k_{2}^{b} \kappa_{1}^{b} \frac{\partial p}{\partial \bar{d}}
$$

Borrower's preferred level of the debt limit equalizes the marginal benefit of allowing more consumption smoothing between $t=1$ and $t=2$, by limiting the fire sale and thus making the collateral constraint more slack (captured by $\phi k_{2}^{b} \kappa_{1}^{b} \frac{\partial p}{\partial d}$ ) against the marginal cost of limiting the consumption smoothing between $t=0$ and $t=1$ (captured by $\kappa_{0}^{b}$ ). Borrowers
internalize the externality associated with overborrowing when forming their preferences for a universally applicable rule. As a result both politician's offer a debt limit that corresponds to the planner's policy.

In an environment with homogenous borrowers and universal enforcement, the equilibrium prudential policy is constrained efficient. The intuition is that ability to introduce a universally applicable rule allows the borrowers to coordinate so that to limit the selfinflicted externality. Borrowers are the only group of agents affected by the fire sale. They are all harmed by it to an equal extend and agree on the optimal level of regulation. These preferences directly translate to policy because the political process is frictionless.

In the following sections I sequentially relax the assumption of borrower homogeneity (Section 3) and universal enforcement of the policy (Section 4) in order to study their role in yielding this result. The analysis reveals that absence of frictions in the political process is critical for the efficiency of the equilibrium policy.

## 3 Heterogenous Borrowers

In the benchmark case, homogeneity of borrowers ensures that the policy preferences of voters are aligned. This section studies the equilibrium policy in a setting where this no longer holds. It introduces heterogeneity with respect to income among borrowers and studies the resulting conflict over the preferred policies. In this framework, I explore how income inequality affects the strictness of equilibrium regulation.

Assume now that there are two types of borrowers: $\mathbb{B}=\{r, p\}$. A fraction $\theta^{r}$ of borrowers are high-income types and receive $y_{1}^{r}=y^{r}$ at $t=1$. The remaining $1-\theta^{r}=\theta^{p}$ are lowincome types with $y_{1}^{p}=y^{p}<y^{r}$. The type is drawn at the beginning of $t=0$ and is private information. ${ }^{5}$ The two types form separate voter groups, that is their ideological biases are drawn from different distributions: $\psi^{r}, \psi^{p}$.

## Assumption 2. Income and Inequality

(a) The average income of borrowers is low, $\bar{y}=\sum_{B \in \mathbb{B}} \theta^{B} y^{B}<2-\phi$;
(b) The income of each borrower type is not too low, $y^{B}>\frac{1-\phi}{\phi} \forall B$;
(c) The income inequality is not too high, $y^{r}<\rho(\bar{y})$

[^4]Assumptions 2 (a) and 2 (b) ensure that the collateral constraint is binding at $t=1$ but slack at $t=0$. Under Assumption 2 (c) both types of borrowers face a negative externality in the competitive equilibrium (see the Proof of Proposition 3 for the definition of $\rho(\bar{y})$ ).

### 3.1 Competitive Equilibrium

Heterogeneity in income affects borrower's choices at $t=1$. Both types of borrowers are indifferent between any capital holding as long as price satisfies (6). This condition pins down the relation between consumption and price and is the same for both types. Thus, clearing of capital markets requires that high- and low-income borrowers choose the same level of consumption in equilibrium. ${ }^{6}$ Consequently, the collateral constraint is either binding or slack for both.

If the collateral constraint is slack, consumption smoothing between $t=1$ and $t=2$ is uninhibited and both types choose $c_{1}^{B}=1$. The capital markets clear at $p=1$. Borrowing and capital trade are equally costly, so borrowers are indifferent between these two. The budget and collateral constraint pin down the optimal combinations.

If the collateral constraint is binding, neither type is able to choose debt freely. They achieve equal consumption through trade in capital. The budget constraints pin down the choice of capital holding of each type. Aggregating these in the capital market clearing condition yields (8), with $\mathbb{B}=\{r, p\}$. This implicitly pins down the price of capital. It follows that, with heterogeneous borrowers, the price depends on their aggregate net income $\sum_{B \in \mathbb{B}} \theta^{B}\left(y^{B}-d_{0}^{B}\right)$.

## Corollary 2. Effect of Changes in Inequality

A mean preserving increase in inequality in net incomes, $\left(y^{r}-d_{0}^{r}\right)-\left(y^{p}-d_{0}^{p}\right)$, has no effect on capital price. It increases the difference in capital holding of the high- and low-income types at the end of the interim date $\left(k_{2}^{r}-k_{2}^{p}\right)$.

The choice of the initial debt and consumption completes the description of the competitive equilibrium. Quasilinear consumption-utility ensures that both types choose the same level of initial debt: $d_{0}^{B}=c_{1}^{B}$. Since they borrow the same amount, in equilibrium high-income types have higher net income at $t=1$. Consequently, they purchase capital from low-income borrowers $k_{2}^{r}>1>k_{2}^{p}$. Solving for price using the optimal initial debt yields the following Lemma.

[^5]
## Lemma 2. Fire Sale with Income Heterogeneity

In the competitive equilibrium the collateral constraint is binding for both borrower types. A fire sale emerges in equilibrium so that the price solves (8) and is $p<1$.

Proof. Follows from using $d_{0}^{B}=c_{1}^{B}$ in (8). The resulting price is $p=1$ if $\bar{y}=2-\phi$. Using that the price is increasing in income, if income satisfies Assumption 2 (a) the price is $p<1$.

### 3.2 Social Planner

To evaluate the efficiency of the equilibrium allocation, I compare it to the choice of a constrained social planner who assigns Pareto weights $\chi^{L}, \chi^{r}, \chi^{p}$ to lenders, high-income and low-income borrowers respectively. The first order condition of the planner's problem with respect to the initial debt reads:

$$
\begin{equation*}
\lambda_{0}-\lambda_{1}+\underbrace{\frac{\partial p}{\partial D_{0}} \phi \kappa_{1}^{b} \frac{\sum_{h \in \mathbb{B}} \theta^{h} \chi^{h} K_{2}^{h}}{\sum_{h \in \mathbb{B}} \theta^{h} \chi^{h}}}_{X_{c}}+\underbrace{\frac{\partial p}{\partial D_{0}} \lambda_{1}^{b} \frac{1-\sum_{h \in \mathbb{B}} \theta^{h} \chi^{b} K_{2}^{h}}{\sum_{h \in \mathbb{B}} \theta^{h} \chi^{h}}}_{X_{t}}=0 \tag{14}
\end{equation*}
$$

Where $D_{0}=D_{0}^{B}, \lambda_{0}=\lambda_{0}^{B}$ and $\lambda_{1}=\lambda_{1}^{B}$ for all $B \in \mathbb{B}$, as the planner does not observe type.

When choosing the optimal level of debt, the planner takes into account the consumption smoothing motive between $t=0$ and $t=1$ as well as the pecuniary effects of the initial debt. As in the case of homogenous borrowers, the planner accounts for the welfare loss associated with tightening of the collateral constraint at $t=1$. The externality that emerges through this collateral channel (captured by $X_{c}$ ) negatively affects both borrower types. The key difference relative to the benchmark, is that with income heterogeneity, fire sales generate additional, distributive effects through a capital trade channel (captured by $X_{t}$ ).

High-income borrowers experience welfare gains associated with a fall in prices, because it allows them to purchase capital cheaply. The reverse holds for low-income borrowers. They suffer losses through the capital trade channel, as a larger fire sale implies that they need to sell their capital at a lower price. The social planner weighs these gains and losses by the Pareto Weights of the two types and may thus view the capital trade channel as having either a negative (if poor have higher Pareto Weights) or a positive (if rich have higher Pareto Weights) welfare impact.

## Proposition 3. Constrained Inefficiency with Heterogenous Borrowers

If the income inequality is not too high, borrowing at $t=0$ imposes an overall negative externality on both types of borrowers. In this case the competitive equilibrium is constrained inefficient characterized by overborrowing at $t=0$ for any set of Pareto Weights.

Proof. The overall externality on borrower of type $B$ is given by $X^{B}=\frac{1}{c_{1}^{B}} \frac{\partial p}{\partial D_{0}^{B}}\left[1-k_{2}^{B}(1-\right.$ $\left.\phi\left(1-c_{1}^{B}\right)\right)$ ]. It is negative for low-income types, since $k_{2}^{p}<1$. For high-income borrowers it is negative if and only if $k_{2}^{r}<\frac{1}{1-\phi\left(1-c^{r}\right)}$. This is the case if the income inequality satisfies Assumption $2(\mathrm{c}): y^{r}<\bar{y}+\phi p(\bar{y})(1-p(\bar{y})) \equiv \rho(\bar{y})$. Thus, for any set of Pareto Weights, the planner chooses initial debt at a lower level than borrowers.

If the income inequality among borrowers is not too large, the gains of the high-income types through the capital trade are not high enough to compensate for the losses through the collateral channel. In this case, both high- and low-income borrowers suffer from a negative pecuniary externality of the initial debt. There is overborrowing in the competitive equilibrium relative to the constrained planner's allocation.

Corollary 3. The constrained social planner can implement constrained efficient allocation by imposing a debt limit $\bar{d}^{S P}=D_{0}^{S P}$ that solves (14).

Since gains of high-income borrowers through the capital trade channel represent the losses of the low-income borrowers, for a utilitarian social planner $\left(\chi^{r}=\chi^{p}\right)$ the distributive effects of the capital trade channel cancel out. Thus, the utilitarian-constrained efficient level of debt is equivalent to that in the benchmark case with homogenous borrowers whose income satisfies $y^{b}=\bar{y}$. Since low-income borrowers experience larger losses associated with the fire sale, if they have a higher Pareto weight than the high-income types, the inefficiency is larger and planner prefers a lower (stricter) debt limit than in the benchmark. Reverse holds if the utility of high-income borrowers is more important in the social welfare function.

### 3.3 Political Equilibrium

This section evaluates the policy preferences of different types of borrowers and shows that the equilibrium policy lies on the Pareto Frontier. It provides insights on how income inequality and ability to influence election outcomes of the two types affect the strictness of regulation.

The solution of the political sub-game is equivalent to the case of homogenous borrowers, just that now high- and low-income borrowers constitute two separate voter groups. Thus,
the equilibrium policy satisfies the optimality condition of politician's (13) with $\mathbb{V}=\{r, p, L\}$. The politician weighs the preferences of each borrower type by the population share and the measure of sensitivity of their voting behavior to economic policy (the ideological concentration, $\psi^{v}$ ). I denote the policy-sensitivity of the high-income borrowers relative to the sensitivity of low-income borrowers by $\gamma^{r}=\frac{\psi^{r}}{\psi^{p}}$. For equal population shares, this reflects the relative influence of the two groups on the policy choice of the politician. I refer to this measure as the relative electoral power of the high-income types

The policy preferences of borrowers are specific to the type and given by:

$$
\begin{equation*}
\frac{\mathrm{d} U^{B}(\bar{d})}{\mathrm{d} \bar{d}}=\kappa_{0}^{B}+\phi k_{2}^{B} \kappa_{1}^{B} \frac{\partial p}{\partial \bar{d}}+\lambda_{1}^{B}\left(1-k_{2}^{B}\right) \frac{\partial p}{\partial \bar{d}}=0 \tag{15}
\end{equation*}
$$

When determining their policy preferences, high- and low-income borrowers internalize the effects of the debt limit on their utility through both the collateral and the capital trade channels. The distributive effects associated with the trade channel generate a policy conflict. The capital trade channel decreases (increases) the pecuniary externality for highincome (low-income) borrowers, giving them a motive to impose a laxer (stricter) debt limit. Thus, high-income borrowers prefer a laxer debt limit than the low-income types $\bar{d}^{* r}>\bar{d}^{* p}$. The preferred debt limit is lower than the individually optimal choice of debt, $\bar{d}^{r}<d^{* B}$, as long as the total externality on high-income borrowers is negative. Proof of Proposition 3 shows that this is the case if income inequality satisfies Assumption 2 (c).

## Proposition 4. Equilibrium Debt Limit with Heterogenous Borrowers

The equilibrium debt limit is binding and corresponds to the debt limit set by a constrained social planner with the ratio of the Pareto weights of high- and low-income types equal to the relative electoral power of the high-income borrowers $\frac{\chi^{r}}{\chi^{p}}=\gamma^{r}$.

Proof. Follows from using (15) in (13) and comparing with (14).

The outcome of the majoritarian elections is a policy that maximizes weighted social welfare, in which utilities of different voter groups are weighed by their population share and the ideological concentration (proxy for sensitivity to changes in policy). The policy generally differs from the policy of the utilitarian social planner. It is stricter if the low-income borrowers have higher electoral power than high-income borrowers, and laxer otherwise. However, since the problem of the politician corresponds to that of the social planner with appropriate Pareto weights, the equilibrium policy is on the Pareto Frontier.

Critical in arriving at this result is the assumption of a frictionless political process. The heterogeneity in the concentration of ideological biases translate to different sensitivities to policy changes. This affects the relative weighting of preferences in the politician's objective just as Pareto weights do in the planner's problem. As politicians are fully committed to implement and enforce the policy, the majoritarian elections allow direct translation of voters' preferences to policy.

## Proposition 5. Impact of Electoral Power and Inequality

I. The equilibrium debt limit increases in the relative electoral power of high-income types,
II. A mean preserving increase in income inequality increases (decreases) the equilibrium debt limit if the electoral power of high-income types is higher (lower) than that of low-income borrowers.

Proof. See Appendix A. 5
High-income borrowers prefer a lax policy. An increase in their relative electoral power implies that both politicians announce a policy that is more favorable to this group. The outcome is a laxer debt limit which allows high-income types to reap larger benefits of trade.

The policy conflict between high- and low-income types is proportional to the difference in their capital holding. The higher the income inequality, the larger the difference in the amount of capital each type holds at the end of $t=1$. Therefore, an increase in income inequality intensifies the policy conflict, pushing the policy preferences of the two groups further apart. The impact on the equilibrium policy depends on which group is more important in politician's objective. If high-income types have a higher electoral power per population share, $\gamma^{r}>1$, the politician sets a higher (laxer) debt limit when inequality increases. The reverse occurs if low-income borrowers have a higher electoral power than the high-income types, $\gamma^{r}<1$.

These insights contribute to the discussion on the role of inequality in affecting the regulatory environment. Previous contributions point to the possibility that overly lax regulation may be benefiting the poor. Rajan (2011) and Calomiris and Haber (2015) argue that easing access to credit of the poor voters can be seen as a redistributive policy. Politician's implement lax financial regulation in order to gain the support of the lower-income voters. According to this view increasing inequality exacerbated the need for this stealthy redistri-
bution. Critical for this mechanism is that the policy of granting easy access to credit is seen as beneficial by the low-income voters.

In the current framework, in which overborrowing occurs due to the pecuniary externalities associated with the fire sale, this is not the case. Low-income borrowers are particularly harmed by the fire sale and thus view policies that limit ex-ante credit as beneficial. Their policy preferences may be reversed in the presence of government guarantees (such a subsidy for debt repayment at the interim date). In the absence of such guarantees a politician who seeks support of the low-income types would offer a strict prudential policy in order to limit the redistribution towards high-income types through the fire sale. Conversely, a lax regulatory environment would emerge if the high-income types are a key voter constituency for the politicians.

## 4 Selective Enforcement

In this section I relax the assumption of universal enforcement of regulation. As before, the policy platform of the winning politician is introduced after the elections. However, now the politician in office has discretion regarding the extend to which it is enforced. She may choose to exempt some of the borrowers from the regulatory limit.

Borrowers differ in their ability to access politicians. A fraction $\theta^{c}$ of borrowers are politically connected, $B=c$. The politician does not enforce the debt limit on the connected borrowers. ${ }^{7}$ The remaining borrowers $\theta^{n}=1-\theta^{c}$ are non-connected, $B=n$, they face the limit introduced through elections. For tractability, I first analyze the framework under the assumption of homogenous income that satisfies Assumption 1, as in Section 2. I allow for heterogeneity and study the effect of correlation between income and political access in subsection 4.3.

Borrowers privately observe own type prior to elections. An interpretation is that connected borrowers are endowed with technology that enables them to negotiate with politicians or bureaucrats who enforce the policy. It could represent the their own political or persuasion ability or access to information or resources that can be leveraged in a way that grants them access to the enforcer. The connected and the non-connected borrowers may differ in their ideological concentration, and thus constitute separate voter groups: $\mathbb{V}=\{L, c, n\}$. This allows studying how the equilibrium regulation depends on the relative

[^6]electoral power of politically connected borrowers per population share, $\gamma^{c}=\frac{\psi^{c}}{\psi^{n}}$.
As heterogeneity in this setting is directly related to the political process, the competitive equilibrium and the policy implemented by the constrained social planner are the same as in the benchmark model. Therefore, I move directly to solving the political equilibrium. I first study the impact of exempting the connected borrowers from the rule on $t=1$ consumption, debt, capital holding and price. Next, I solve the individual optimization of a connected borrower and the problem of the politician. Finally, I turn to studying the political equilibrium.

### 4.1 Impact of Exemption

This subsection studies how the selective enforcement of the debt limit at $t=0$ affects the allocation and prices at the interim date for a given debt limit. Throughout this analysis I focus on the case in which the debt limit is binding, so that all borrowers without access borrow up to the limit $d_{0}^{n}=\bar{d}$. This will be true in equilibrium as both types prefer to implement a binding debt limit.

### 4.1. 1 Problem at the Interim Date

At $t=1$ the two borrower types may differ in the amount of debt they took on at the previous date. Their respective budget constraints are:

$$
\begin{aligned}
r_{1} d_{0}^{c}+c_{1}^{c} \leq d_{1}^{c}+y+p\left(1-k_{2}^{c}\right) & \text { B1-C } \\
r_{1} \bar{d}+c_{1}^{n} \leq d_{1}^{n}+y+p\left(1-k_{2}^{n}\right) & \text { B1-N }
\end{aligned}
$$

Selective enforcement leads to heterogeneity in net incomes, $y-d_{0}^{b}$, at $t=1$, even with homogenous endowments of the consumption good. The net income of borrowers without political access is at least as high as that of the connected types $\left(y-\bar{d} \leq y-d_{0}^{c}\right)$. Therefore, the optimization at $t=1$ parallels the problem studied in Section 3. In the current setting borrowers differ in the size of their debt repayment and not the endowment. If the collateral constraint binds, the price solves 8 with $\mathbb{B}=\{c, n\}$ and $d_{0}^{n}=\bar{d}$.

The heterogeneity in the net income implies that the two types trade capital in equilibrium. Connected borrowers spend more of their income on repaying debt, leaving them with fewer resources available for consumption. As both types wish to achieve the same consumption levels, connected borrowers sell some of their asset to the borrowers without
political access, $k_{2}^{c}<1<k_{2}^{n}$.

### 4.1.2 Problem at the Initial date

At $t=0$ a politically connected borrower can freely choose the level of debt and consumption. Being atomistic, he does not internalize how his choice of debt affects the prices, so his first order condition with respect to debt is:

$$
\lambda_{0}^{c}-\lambda_{1}^{c}=0
$$

This corresponds to the unconstrained optimal choice of a borrower and is therefore higher than the level of debt required by the limit $d_{0}^{c}=c_{0}^{c}=c_{1}^{c}>\bar{d}$. The following Lemma, describes the equilibrium price for a given level of a debt limit.

## Lemma 3. Fire Sale with Selective Enforcement

If $y-\left(1-\theta^{c}\right) \bar{d}>1-\phi+\theta^{c}$, the collateral constraint is slack and the price of capital is $p=1$. Otherwise, the collateral constraint is binding, a fire sale emerges in equilibrium, so the price of capital is given by (8) with $d_{0}^{c}=c_{1}^{c}$ and $d_{0}^{n}=\bar{d}$. It decreases in:

- the debt limit rule, $\frac{\partial p}{\partial d}<0$,
- the share of politically connected borrowers, $\frac{\partial p}{\partial \theta^{c}}<0$,

Proof. The collateral constraint is binding whenever $p<1$, conditions on $y$ and $\bar{d}$ follow from using $p=1$ and in (8). The comparative statics follow directly from the derivatives.

As politically connected borrowers do not internalize their impact on prices when choosing initial debt, they over-borrow leading to higher aggregate level of initial debt. The repayment of these claims drains the aggregate resources of borrowers at $t=1$, leading to a lower aggregate net income and a larger fire sale. As the share of politically connected increases, more borrowers follow the individual optimization and fewer are subject to the debt limit. This reduces the equilibrium price for any given debt limit.

### 4.2 Political Equilibrium with Selective Enforcement

The optimal decision of the politicians is to propose a debt limit that solves (13) with $\mathbb{V}=\{L, c, n\}$, so that the equilibrium policy weighs the preferences of the two borrower groups by their population share and ideological concentration.

Politically connected borrowers anticipate the exemption from regulation when choosing their preferred policy. They stand to benefit from limiting the fire sale without facing the costs of regulation. The first order derivative of their indirect consumption utility with respect to the policy is given by:

$$
\begin{equation*}
\frac{\mathrm{d} U^{c}(\bar{d})}{\mathrm{d} \bar{d}}=\kappa_{1}^{c} \phi k_{2}^{c} \frac{\partial p}{\partial \bar{d}}+\lambda_{1}^{c}\left(1-k_{2}^{c}\right) \frac{\partial p}{\partial \bar{d}}<0 \tag{16}
\end{equation*}
$$

Since they face a strictly negative externality, $\frac{\mathrm{d} U^{c}(\bar{d})}{\mathrm{d} d}<0$, their preferred policy is a minimum debt limit. That is, either a complete prohibition of borrowing, $\bar{d}=0$, or a debt limit which is strict enough to prevent the fire sale (implicitly defined in $p(\bar{d})=1$ ).

Borrowers without political access bear the costs of regulation. Moreover, as the net buyers of capital they partially benefit from the fire sale through the capital trade channel. Overall, they are subject to full regulatory costs and face a lower negative externality of the initial debt, thus their preferred debt limit is higher than that of the politically connected. Their preferred policy solves:

$$
\begin{equation*}
\frac{\mathrm{d} U^{n}(\bar{d})}{\mathrm{d} \bar{d}}=\kappa_{0}+\kappa_{1}^{n} \phi k_{2}^{n} \frac{\partial p}{\partial \bar{d}}+\lambda_{1}^{n}\left(1-k_{2}^{n}\right) \frac{\partial p}{\partial \bar{d}}=0 \tag{17}
\end{equation*}
$$

## Lemma 4. Policy Preferences with Selective Enforcement

Politically connected borrowers prefer a minimum debt limit $\bar{d}^{* c}=\max \left[0, \hat{d}_{0}\right]$, the debt limit preferred by borrowers who are non-connected solves (17).

Both types take into account how the debt limit affects their utility through pecuniary externalities. They benefit from higher equilibrium prices through the collateral channel (captured by $\kappa_{1}^{B} \phi k_{s}^{B} \frac{\partial p}{\partial d}$ ). The effect of the capital trade channel is distributive (captured by $\left.\lambda_{1}^{B}\left(1-k_{2}^{B}\right) \frac{\partial p}{\partial d}\right)$. Politically connected borrowers are net sellers of capital, so they reap additional benefits from an increase in prices at the expense of the non-connected borrowers. This policy conflict corresponds to the disagreement between high- and low-income borrowers discussed in Section 3.

A novel source of conflict arises due to the heterogeneous political access. Since the full burden of regulation lies on the non-connected borrowers, only they face the costs associated with the debt limit, namely the inability to smooth consumption between $t=0$ and $t=1$ (captured by $\kappa_{0}^{n}$ ). Connected borrowers do not internalize these costs when forming their policy preferences.

Overall, connected borrowers face larger marginal benefits of strict regulation (due to
the distributive effects) and lower marginal costs relative to the borrowers without political access.

## Lemma 5. Impact of Electoral Power of the Connected

The equilibrium debt limit decreases (becomes more strict) in the relative electoral power of politically connected borrowers.

Proof. Follows directly, as the policy preferred by the politically connected is lower than that of borrowers without political access.

As politically connected borrowers prefer a lower debt limit, the more important they are in politician's optimization (ie. the higher their electoral power per population share), the lower the equilibrium limit. Thus, selective enforcement results in two distortions. First, the exemption from the debt limit increases the overall debt levels at $t=0$, directly lowering the equilibrium price of capital. As a result, politically connected borrowers impose a negative pecuniary externality on all other borrowers by overborrowing (borrowing distortion). Second, as they stand to benefit from the implementation of a low debt limit without incurring the costs, politically connected favor strict policy. If their electoral power is high, they may be imposing a negative externality on non-connected borrowers, by supporting an overly low debt limit (policy preference distortion).

## Proposition 6. Equilibrium Debt Limit with Selective Enforcement

The equilibrium regulation may be too strict or too lax relative to the policy implemented by a constrained social planner with perfect enforcement. The policy is too strict whenever the relative electoral power of the connected is sufficiently high.

Proof. See Figure 1 for existence. Since $\frac{\mathrm{d} U^{c}(\bar{d})}{\mathrm{d} d}<0$ politically connected prefer a minimum debt limit, so $\bar{d}^{* c}<\bar{d}^{S P}$. From Lemma 5, the debt limit increases as the relative electoral power of politically connected decreases. Thus, there exists a threshold $\chi^{c}$ below which the policy is too strict.

The constrained social planner who can implement the policy to all borrowers sets the debt limit so that to satisfy (11) from Section 2. He internalizes the effect of the debt on the price and consequently, borrowers ability to smooth consumption between $t=1$ and $t=2$. Planner weighs these benefits against the costs of impaired consumption smoothing between $t=0$ and $t=1$. The political equilibrium differs from the planner's solution as selective enforcement affects both the marginal benefits and the distribution of marginal costs of regulation.

The borrowing distortion lowers the effectiveness of policy in increasing the equilibrium price as some of the borrowers are exempt from the limit (the marginal effect of debt limit on price is smaller). Thus, for any given debt limit, the price under politician's regulation is lower than the price under the planner. Lower prices, imply a higher marginal value of relaxing the collateral constraint. Therefore, the overall impact of the borrowing distortion is ambiguous. It can generate the pressure pushing the policy either above or below the social planner's choice.

The policy preference distortion implies that connected borrowers do not internalize the costs of the regulation. This puts a downward pressure on the policy implemented by the politician. The higher the electoral power of the connected borrowers, the lower the resulting policy. If their relative electoral power is sufficiently high, the equilibrium policy is stricter than that of a social planner.

A numerical example illustrates, that the equilibrium regulation can be either inefficiently lax or inefficiently strict. The figure below plots the equilibrium debt limit rule implemented by a politician as a function of the share of politically connected borrowers for different values of relative electoral power. The solid line in the figure is the benchmark policy implemented by a constrained social planner with perfect enforcement.

Figure 1: Equilibrium Debt Limit with Selective Enforcement


In this example, the borrowing distortion induces the non-connected borrowers to prefer laxer regulation than the social planner. That is, the negative impact of the exemption of connected borrowers from regulation on the effectiveness of regulation dominates the effect
on its' marginal value. In this case, the preferences of two types are polarized. with one group preferring an inefficiently strict and the other inefficiently lax policy. The impact of selective enforcement on the strictness of ex-ante regulation depends on their relative electoral power.

If the electoral power of connected borrowers is equal to (dash-dotted line) or higher than (dashed line) that of borrowers without political access, the equilibrium policy is stricter than the policy implemented by the planner. It decreases in the share of the politically connected. If the electoral power of borrowers without political access is sufficiently high (dotted line), the equilibrium policy is laxer than planner's policy.

The analysis provides a novel intuition on the impact of selective enforcement on the ex-ante policy. If some agents anticipate the exemption from a rule, they are keen to impose overly strict regulation. The aggregation of preferences through majoritarian elections implies that these views are reflected in the equilibrium policy. Therefore, political access of some may impose a negative externality on others, as it results in the implementation of an overly strict regulation. This result relates to the recent evidence by Neretina (2018) who finds that lobbying on legislation generates negative externalities on the non-lobbying firms. Firms with political access may be able to affect the formulation of policy in a way that favors their narrow interests at the expense of the competitors. The ability to influence the specific shape of legislature may allow lobbyist to codify some form of selective enforcement. In the case of financial regulation, the discretion of the regulators and the complexity of the system may further facilitate unequal enforcement through regulatory capture, making the inefficiency of overall policy particularly relevant in this context.

### 4.3 Selective Enforcement and Heterogenous Income

This subsection relaxes the assumption of income homogeneity and studies the possibility of correlation between income and political access. As in Section 3, assume that fraction $\theta^{r}$ of borrowers receives a high income and remaining $\theta^{p}$ receives low income at $t=1$ and that these endowments satisfy Assumption 2. Let $\rho^{r}$ and $\rho^{p}$ denote the fraction of politically connected borrowers among the high- and low- income types respectively. Thus, the share of the politically connected borrowers in the population is given by:

$$
\theta^{c}=\rho^{r} \theta^{r}+\rho^{p} \theta^{p}
$$

The difference between the share of connected borrowers within each income-type scaled by the share of connected in the population, $\frac{\rho^{r}-\rho^{p}}{\theta^{c}}=\frac{\Delta_{\rho}}{\theta^{c}}$, is a proxy for correlation between political access and income.

There are potentially four types of borrowers in this setting, $\mathbb{B}=\{p n, p c, r n, r c\}$, lowincome non-connected ( $B=p n$ ), low-income connected ( $B=p c$ ), high-income non-connected ( $B=r n$ ) and high-income connected $(B=r c)$. To simplify the exposition, I assume that the ideological concentration of agents is heterogeneous only along the political access dimension. That is unconnected borrowers of high-income and low-income types have the same level of ideological concentration, $\psi^{p n}=\psi^{r n}=\psi^{n}$. The same holds for connected borrowers in both income groups, $\psi^{p c}=\psi^{r c}=\psi^{c}$. This allows, the four agent types to aggregate into two voter groups: $v=\{L, c, b\}$. As before, I denote by $\gamma^{c}=\frac{\psi^{c}}{\psi^{n}}$ the relative electoral power of the connected borrowers per population share.

The problem at $t=1$ now includes four types, each with a different net income $y^{B}-d_{0}^{B}$. As connected borrowers choose their debt freely at $t=0$, opting for higher borrowing than that imposed by regulation, connected low-income types have the lowest net income at $t=1$. Consequently, they are net sellers of capital, $k_{2}^{p c}<1$. Non-connected high-income types have the highest net income at $t=1$ and are thus net buyers of capital, $k_{2}^{r n}>1$. Whether high-income connected and low-income non-connected types buy or sell capital in equilibrium depends on the extend of income heterogeneity and the strictness of the limit. The equilibrium price ensures that the capital markets clear $1=\sum_{B \in \mathbb{B}} \theta^{B} k_{2}^{B}$.

Since all borrower types choose the same level of consumption at $t=1$, the choice of debt by connected borrowers at $t=0$ is independent of income. Both low- and high-income connected borrowers choose it so that to smooth consumption, so that their debt corresponds to the optimal choice of an unconstrained borrower: $d_{0}^{B}=c_{1}^{B}>\bar{d}, B \in\{p c, r c\}$.

The impact of heterogeneous income in the setting with selective enforcement is the same as in the case of universal enforcement. Namely, it leads to heterogeneous capital holding at the end of $t=1$. The distributive effects that result from the capital trade channel benefit high-income types at the expense of the low-income types and non-connected borrowers at the expense of connected borrowers. That is the marginal benefits of regulation are different for each of the four borrower types.

For a given debt limit, non-connected borrowers with high-income face lower marginal benefits of increasing the price of capital than the low-income types. However, as long as the debt limit is not too strict, assumption 2(c) ensures that non-connected borrowers benefit from the increase in price of capital, regardless of their income-type. Both also face the costs
of regulation through impaired consumption smoothing at $t=0$ and $t=1$. Their preferred policy solves:

$$
\begin{equation*}
\frac{\mathrm{d} U^{B}(\bar{d})}{\mathrm{d} \bar{d}}=\kappa_{1}^{B} \phi k_{2}^{B} \frac{\partial p}{\partial \bar{d}}+\lambda_{1}^{B}\left(1-k_{2}^{B}\right) \frac{\partial p}{\partial \bar{d}}+\kappa_{0}=0 \quad \text { if } B \in\{p n, r n\} \tag{18}
\end{equation*}
$$

The connected borrowers do not face the costs of regulation. The marginal benefits of limiting the fire sale are higher for the low-income types than for the high-income types. But, while the magnitude of benefits varies across the income-types, strict regulation is unequivocally beneficial to the connected as both high- and low-income types suffer from the negative externality. They prefer a debt limit to be set at the minimum:

$$
\begin{equation*}
\frac{\mathrm{d} U^{B}(\bar{d})}{\mathrm{d} \bar{d}}=\kappa_{1}^{B} \phi k_{2}^{B} \frac{\partial p}{\partial \bar{d}}+\lambda_{1}^{B}\left(1-k_{2}^{B}\right) \frac{\partial p}{\partial \bar{d}}<0 \quad \text { if } B \in\{p c, r c\} \tag{19}
\end{equation*}
$$

The politician aggregates these preferences weighing them by the population share and the ideological concentration of each group-type according to equation (13). This can be represented as a sum of weighted policy preference schedules of the connected and nonconnected borrowers ( $\frac{\mathrm{d} U^{C}(\bar{d})}{\mathrm{d} \bar{d}}$ and $\frac{\mathrm{d} U^{N}(\bar{d})}{\mathrm{d} \bar{d}}$ respectively) as:

$$
\underbrace{\psi^{c} \sum_{b=p c, r c} \theta^{b} \frac{\mathrm{~d} U^{b}(\bar{d})}{\mathrm{d} \bar{d}}}_{\frac{\mathrm{d} U C(\bar{d})}{\mathrm{d} d}}+\underbrace{\psi^{n} \sum_{b=p n, r n} \theta^{b} \frac{\mathrm{~d} U^{b}(\bar{d})}{\mathrm{d} \bar{d}}}_{\frac{\mathrm{d} U^{N}(\bar{d})}{\mathrm{d} d}}=0
$$

Lemma 6. Consider a change in correlation between political access and income, $\mathrm{d} \Delta_{\rho}$, that leaves the share of politically connected, $\theta^{c}$, unchanged.

- If $\mathrm{d} \Delta_{\rho}>0$, the weighted policy preference schedule of connected borrowers, $\frac{\mathrm{d} U^{C}(\bar{d})}{\mathrm{d} d}$, shifts upwards, while the schedule of non-connected borrowers, $\frac{\mathrm{d} U^{N}(\bar{d})}{\mathrm{d} d}$, shifts downwards;
- If $\mathrm{d} \Delta_{\rho}<0$, the weighted policy preference schedule of connected borrowers, $\frac{\mathrm{d} U^{C}(\bar{d})}{\mathrm{d} d}$, shifts downwards, while the schedule of non-connected borrowers, $\frac{\mathrm{d} U^{N}(\bar{d})}{\mathrm{d} \bar{d}}$, shifts upwards.

Proof. In the Appendix A. 5
Due to the distributive effects of the fire sale, low-income types face higher marginal benefits of regulation than the high-income types in the same group. A higher positive
correlation between income and political access, means that a larger share of connected borrowers is of a high-income type, while more of the unconnected borrower are low-income types. This implies a fall in the weighted marginal benefits of regulation for the whole population of connected borrowers and an increase in that measure for the non-connected borrowers.

## Proposition 7. Impact of Change in Correlation

There exists a threshold relative electoral power of connected borrowers $\hat{\gamma}^{c}$ such that an increase (a decrease) in correlation between income and political access that keeps the share of connected borrowers constant increases (decreases) the equilibrium debt limit if the relative electoral power of connected borrowers is above that threshold, $\gamma>\hat{\gamma}^{c}$.

Proof. In the Appendix A. 5
An increase in correlation between income and political access moves the preferred policies of connected and non-connected borrowers closer to one another. This decreases the policy conflict and so alleviates the impact of selective enforcement. The effect on the equilibrium policy depends on the relative electoral power of the two groups. If connected borrowers dominate, the equilibrium policy becomes more relaxed. If borrowers without political access have a sufficiently high electoral power, the equilibrium policy is stricter as a result of the increase in correlation.

## 5 Conclusions

This paper studies how political factors affect the equilibrium prudential policy in a setting with pecuniary externalities. Borrowers do not internalize the impact of their initial indebtedness on the severity of a welfare reducing fire sale and thus, impose an externality on all borrowers. The possibility to vote on a regulatory rule applicable to all, serves as a coordination device, allowing them to internalize the externality. In a setting with homogenous borrowers and perfect commitment by a politician, this results in a constrained efficient equilibrium policy.

Income inequality between borrowers implies a distributive effect of the fire sale. Highincome borrowers benefit from low prices at the expense of the low-income types, by being able to buy capital at a discount. Both types remain unable to perfectly smooth consumption, making some level of regulation preferred by all. However, trade in capital generates a conflict over the strictness of the debt limit. High-income borrowers prefer a laxer regulation as they
wish to allow for a larger price discount. The equilibrium policy weighs the preferences of each type by their population share and a measure of responsiveness to policy, and thus lies on a Pareto frontier of a constrained social planner.

In this framework an increase in income inequality increases the policy conflict between voters as it increases the volume of capital trade in equilibrium. The impact on equilibrium policy depends on the relative electoral power of the two groups. If high-income borrowers have higher electoral power, the outcome is laxer policy. If low-income borrowers are more influential in the elections, the policy becomes stricter after an increase in inequality.

Regulatory capture, modeled as exemption from regulation of politically connected borrowers results in an inefficient equilibrium regulation, due to two distortions. First, as some borrowers are exempt from the limit, all voters anticipate the regulation to be less effective in curbing the fire sale. This may lower the marginal benefit of regulation relative to the environment in which planner enforces the policy on all borrowers. Second, since the costs of regulation are borne solely by borrowers without political access, the policy preference of connected borrowers are distorted. They prefer an overly strict regulation.

In equilibrium the strictness of policy depends on the relative electoral power of the two groups. The equilibrium regulation may be either too strict or too lax. In environments in which the politically connected have higher electoral power than those without access, the equilibrium debt limit rule is excessively strict. This result is weakened if the politically connected borrowers are more likely to have high income. However, the inefficiently strict limit may result even if the political access and income are perfectly correlated.

This analysis points to the importance of the political context for equilibrium regulation. It provides novel insights on the role of income inequality and political access in shaping the prudential policy. I show that low-income voters may prefer strict regulation, if they stand to lose from fire-sales. Moreover, the analysis underscores the role of political institutions in determine the financial regulation.

In economies with strong political institutions and transparent regulatory environment, which can ensure perfect enforcement of the policies, there is less scope for inefficient regulation. Such political systems may favor redistribution towards specific groups through prudential regulation, but are able to restore constrained efficiency.

In economies with weaker institutions, selective enforcement undermines the effectiveness of regulation. The impact on the equilibrium debt limit depends the ability of the connected groups to influence elections. If they are the pivotal voters, the equilibrium outcome is an inefficiently strict regulation, imposed on the borrowers without political connections. If
the non-connected borrowers are sufficiently important in elections, the regulation may be inefficiently lax.

This study highlights a need for further formal political economy analysis of regulation in the context of other financial frictions. A systematic study of regulation in an economy with investment, a setting with aggregate demand externalities or with government guarantees can help further deepen our understanding of the directions in which various political forces affect financial regulation.

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## A Appendix

## A. 1 Micro-foundations of Collateral Constraint

Consider the environment in which debtors can attempt to renegotiate the debt contract at any time, ie. make a take-it-or-leave-it offer to lower the repayment. If creditors reject the offer they can turn to legal authorities to repossess the assets (current and future consumption and capital goods) of the debtor in order to cover the promised repayment. The authorities are able to repossess only a fraction $\phi$ of the assets. The creditor can then sell the capital goods and consume the proceeds as well as the consumption good.

In this environment, the repayment of debt is incentive compatible for the borrower if:

- $d_{0}^{B}<\phi \frac{u_{1}^{\prime b}\left(c_{1}^{B}\right)}{u_{0}^{\prime B}\left(c_{0}\right)}\left(y^{B}+p k_{1}^{B}\right), \mathrm{CC} 00$, ensures no renegotiation of $t=0$ debt at $t=0$
- $d_{0}^{B}<\phi\left(y^{B}+p k_{1}^{B}\right), \mathrm{CC} 01$, ensures no renegotiation of $t=0$ debt at $t=1$
- $d_{1}^{B}<\phi p k_{2}^{B}$, CC11, ensures no renegotiation of $t=1$ debt at $t=1$
- $d_{1}^{B}<\phi k_{2}^{B}, \mathrm{CC} 12$, ensures no renegotiation of $t=1$ debt at $t=2$

In a competitive equilibrium the individually optimal choice of debt by borrowers is $d_{0}^{B}=\frac{(1-\phi) p}{(1-\phi p)}$ and the MRS between $t=0$ and $t=1$ is $\frac{u_{1}^{\prime} B\left(c_{1}\right)}{u_{0}^{\prime B}\left(c_{0}\right)}=1$. Thus both CC01 and CC11 can be expressed as :

$$
w=\frac{(1-\phi) p}{(1-\phi p)}-\phi p<\phi y^{b}
$$

The left hand side of the inequality increases in p :

$$
\frac{\partial w}{\partial p}=\frac{(1-\phi)(1-\phi p)+\phi(1-\phi) p}{(1-\phi p)^{2}}-\phi=\frac{(1-\phi)}{(1-\phi p)^{2}}-\phi>0
$$

whenever $(1-\phi)>\phi(1-\phi p)^{2}$, which holds for $\left.\phi<\frac{1}{2}\right)$. So if inequality holds for $p=1$ it holds always, which yields: $y^{B}>\frac{1-\phi}{\phi}$. In the case of heterogeneous borrowers (in terms of income or political access), this condition needs to be satisfied for each borrower type.

## A. 2 Individual Optimization

The problem of lenders is:

$$
\begin{array}{r}
\max _{c_{0}^{L}, c_{1}^{L}, c_{2}^{L}, d_{0}^{L}, d_{1}^{L}, k_{2}^{L}} V_{0}^{L}=c_{0}^{L}+c_{1}^{L}+c_{2}^{L}-\lambda_{0}^{L}\left(c_{0}^{L}-d_{0}^{L}-y^{L}\right) \\
-\lambda_{1}^{L}\left(r_{1} d_{0}^{L}+c_{1}^{L}-d_{1}^{L}-y^{L}-p\left(k_{1}^{L}-k_{2}^{L}\right)\right)-\lambda_{2}^{L}\left(r_{2} d_{1}^{L}+c_{2}^{L}-y^{L}\right)
\end{array}
$$

The first order conditions read:

$$
\begin{array}{rr}
1-\lambda_{0}^{L}=0 & \lambda_{0}^{L}-r_{1} \lambda_{1}^{L}=0 \\
1-\lambda_{1}^{L}=0 & \lambda_{1}^{L}-r_{2} \lambda_{2}^{L}=0 \\
1-\lambda_{2}^{L}=0 & -p \lambda_{1}^{L}=0
\end{array}
$$

The problem of the borrower of type $B$ is:

$$
\begin{array}{r}
\max _{c_{0}^{B}, c_{1}^{B}, c_{2}^{B}, d_{0}^{B}, d_{1}^{B}, k_{2}^{B}} V_{0}^{B}=\log \left(c_{0}^{B}\right)+\log \left(c_{1}^{B}\right)+c_{2}^{B}-\lambda_{0}^{B}\left(c_{0}^{B}-d_{0}^{B}\right)- \\
\lambda_{1}^{B}\left(r_{1} d_{0}^{B}+c_{1}^{B}-d_{1}^{B}-y^{B}-p\left(k_{1}^{B}-k_{2}^{B}\right)\right)-\kappa_{1}^{B}\left(d_{0}^{B}-\phi p k_{2}^{B}\right)-\lambda_{2}^{B}\left(r_{2} d_{1}^{B}+c_{2}^{B}-k_{2}^{B}\right)
\end{array}
$$

The first order conditions read:

$$
\begin{aligned}
\frac{1}{c_{0}^{B}}-\lambda_{0}^{B} & =0 & \lambda_{0}^{B}-r_{1} \lambda_{1}^{B} & =0 \\
\frac{1}{c_{1}^{B}}-\lambda_{1}^{B} & =0 & \lambda_{1}^{B}-\kappa_{1}^{B}-r_{2} \lambda_{2}^{B} & =0 \\
1-\lambda_{2}^{B} & =0 & -p \lambda_{1}^{B}+\kappa_{1}^{B} \phi p+\lambda_{2}^{B} & =0
\end{aligned}
$$

## A. 3 Social Planner's Problem

The problem of the constrained social planner with homogenous borrowers can be expressed as:

$$
\begin{array}{r}
\max _{C_{0}^{L}, D_{0}^{L}, C_{0}^{B}, D_{0}^{B}} \chi^{B}\left(\log \left(C_{0}^{B}\right)+V^{B}\left(D_{0}^{B}\right)\right)+\chi^{L}\left(C_{0}^{L}+V^{L}\left(D_{0}^{L}\right)\right) \\
-\mu\left(C_{0}^{B}+C_{1}^{L}-Y^{L}\right)-\omega\left(D_{0}^{B}+D_{0}^{L}\right)
\end{array}
$$

Where $V^{B}\left(D_{0}^{B}\right)$ and $V^{L}\left(D_{0}^{L}\right)$ are the value functions of borrowers and lenders following from their $t=1$ optimization. The first order conditions of the planner problem are:

$$
\begin{aligned}
\chi^{B} \frac{1}{C_{0}^{B}}-\mu & =0 \\
\chi^{L}-\mu & =0 \\
\chi^{B}\left(-\lambda_{1}^{B}+\kappa_{1}^{B} \phi k_{2}^{B} \frac{\partial p}{\partial D_{0}^{B}}\right)-\omega & =0 \\
-\chi^{L} \lambda_{1}^{L}-\omega & =0
\end{aligned}
$$

Using the first two to establish that $\frac{\chi^{B}}{\chi^{L}}=C_{0}^{B}=\frac{\lambda_{0}^{L}}{\lambda_{1}^{L}}$ and the fact that $\lambda_{1}^{L}=1$, the third and fourth FOC yield the following optimality condition for debt:

$$
\lambda_{0}^{B}-\lambda_{1}^{B}+\kappa_{1}^{B} \phi k_{2}^{B} \frac{\partial p}{\partial D_{0}^{B}}=0
$$

In the case of heterogenous borrowers, if the type is private information the social planner can only choose a lender and borrower specific level of consumption and debt $C_{0}^{h}$ and $D_{0}^{h}$ for all $B$. His problem is:

$$
\begin{aligned}
\max _{C_{0}^{L}, D_{0}^{L}, C_{0}^{h}, D_{0}^{h}} & \sum_{B} \chi^{B} \theta^{B}\left(\log \left(C_{0}^{h}\right)+V^{B}\left(D_{0}^{h}\right)\right)+\chi^{L}\left(C_{0}^{L}+V^{L}\left(D_{0}^{L}\right)\right) \\
& -\mu\left(\sum_{B}\left(\theta^{B} C_{0}^{h}\right)+C_{1}^{L}-Y^{L}\right)-\omega\left(\sum_{B}\left(\theta^{B} D_{0}^{h}\right)+D_{0}^{L}\right)
\end{aligned}
$$

Where $V^{B}\left(D_{0}^{h}\right)$ and $V^{L}\left(D_{0}^{L}\right)$ are the value functions of borrowers and lenders following from
their $t=1$ optimization. The first order conditions of the planner problem are:

$$
\begin{aligned}
\sum_{B}\left[\chi^{B} \theta^{B} \frac{1}{C_{0}^{h}}-\theta^{B} \mu\right] & =0 \\
\chi^{L}-\mu & =0 \\
\sum_{B} \chi^{B} \theta^{B}\left[-\lambda_{1}^{B}+\kappa_{1}^{B} \phi K_{2}^{B} \frac{\partial p}{\partial D_{0}^{h}}+\lambda_{1}^{B}\left(1-K_{2}^{B}\right) \frac{\partial p}{\partial D_{0}^{h}}\right]-\sum_{B} \theta^{B} \omega & =0 \\
-\chi^{L} \lambda_{1}^{L}-\omega & =0
\end{aligned}
$$

Using first and second equation to get $\chi^{L}=\sum_{B}\left[\chi^{B} \theta^{B} \frac{1}{C_{0}^{h}}-\theta^{B}\right]$, using this and $\lambda_{1}^{L}=1$ and substituting that into the fourth and then rearranging the third yields:

$$
\sum_{B} \chi^{B} \theta^{B}\left[-\lambda_{1}^{B}+\kappa_{1}^{B} \phi K_{2}^{B} \frac{\partial p}{\partial D_{0}^{h}}+\lambda_{1}^{B}\left(1-K_{2}^{B}\right) \frac{\partial p}{\partial D_{0}^{h}}\right]+\sum_{B}\left[\chi^{B} \theta^{B} \frac{1}{C_{0}^{h}}\right]=0
$$

Since $\lambda_{1}^{B}=\lambda_{1}^{h}$ and $\kappa_{1}^{B}=\kappa_{1}^{h}$ for all $B$, in interior equilibrium:

$$
\lambda_{0}^{h}-\lambda_{1}^{h}+\frac{1}{\sum_{B}\left[\chi^{B} \theta^{B}\right]} \sum_{B} \chi^{B} \theta^{B}\left[\kappa_{1}^{h} \phi K_{2}^{B} \frac{\partial p}{\partial D_{0}^{h}}+\lambda_{1}^{h}\left(1-K_{2}^{B}\right) \frac{\partial p}{\partial D_{0}^{h}}\right]=0
$$

## A. 4 Interior solution as unique equilibrium

When borrowers are ex-post heterogeneous they trade capital at $t=1$. The borrower of type $h$ demand for capital follows from the FOC and is given by:

$$
k_{2}^{B}=\left\{\begin{array}{l}
0 \text { if } p>\frac{c_{1}^{B}}{1-\phi\left(1-c_{1}^{B}\right)} \\
k \in\left(0, \frac{1}{1-\theta^{B}}\right) \text { if } p=\frac{c_{1}^{B}}{1-\phi\left(1-c_{1}^{B}\right)} \\
\frac{1}{1-\theta^{B}} \text { if } p<\frac{c_{1}^{B}}{1-\phi\left(1-c_{1}^{B}\right)}
\end{array}\right.
$$

In the interior solution, both borrowers choose positive level of capital holding, so that $p=\frac{c_{1}^{B}}{1-\phi\left(1-c_{1}^{B}\right)}$ with $c_{0}^{B}=c_{0}^{B} \forall B$. The capital demands follow from their budget constraints:

$$
k_{2}^{B}=\frac{y^{B}-d_{0}^{B}+p}{p(1-\phi)}-\frac{1}{1-\phi p}
$$

The low-income type has positive asset holdings if

$$
\begin{equation*}
y^{p}-d_{0}^{p}>\frac{p(1-\phi)}{1-\phi p}-p \tag{20}
\end{equation*}
$$

Where $p$ solves $1=\frac{\sum_{b} \theta^{B}\left[y^{B}-d_{0}^{B}+p\right]}{p(1-\phi)}-\frac{1}{1-\phi p}$. Since the right-hand side of the inequality is negative (as $\frac{(1-\phi)}{1-\phi p} \leq 1$ ), the condition is satisfied whenever $y^{p}-d_{0}^{p}>0$. In the competitive equilibrium $d_{0}^{p}=c_{1}^{p}<1$, thus if $y^{p}>1$ the interior solution is an equilibrium.

In the corner solution, the borrower with high net income buys off all of the capital from the borrower with low net income, so that price reflects his marginal valuation of capital: $\frac{1}{\theta^{r}}=\frac{y^{r}-d_{0}^{r}+p}{p(1-\phi)}-\frac{1}{1-\phi p}$. If low net income types sell off their capital, their consumption follows from the budget constraint $c_{1}^{p}=y^{p}-d_{0}^{p}+p$. The equilibrium is indeed in a corner if the price pinned down by the valuation of high-income types is such that $p>\frac{c_{1}^{p}}{1-\phi\left(1-c_{1}^{p}\right)}$, which requires:

$$
\begin{equation*}
c_{1}^{r}>c_{1}^{p} \Rightarrow y^{p}-d_{0}^{p}<\frac{p(1-\phi)}{1-\phi p}-p \tag{21}
\end{equation*}
$$

Building on the previous argument, this condition is never satisfied if $y^{p}>1$. Thus if $y^{p}>1$ the interior solution is the unique equilibrium.

Assumption 2.II requires that $y^{B}>\frac{1-\phi}{\phi}$, since $\phi>\frac{1}{2}$, this ensures that $y^{B}>1$ for all borrower types.

In the case of heterogenous political access, both borrowers receive the same income $y^{B}$, however politically connected borrowers borrow more than those without access: $d_{0}^{c}=c_{1}^{c}=$ $\frac{p(1-\phi)}{1-\phi p}<1$. Thus, the interior solution constitutes a unique equilibrium for all $y^{B}>1$.

## A. 5 Proof of Proposition 5, Lemma 6 and Proposition 7

## Proof of Proposition 5.

I. High-income borrowers face a lower negative externality, so $\frac{\mathrm{d} U^{r}(\bar{d})}{\mathrm{d} d}>\frac{\mathrm{d} U^{p}(\bar{d})}{\mathrm{d} \bar{d}} \forall \bar{d}$. As the weight of high-income borrowers in politician's objective increases, her FOC shifts upwards resulting in a higher equilibrium debt limit.
II. Let $\psi^{r}=\psi^{p}+\epsilon$, then and aggregating the preferences using politicians' FOC (where $\left.c_{1}^{b}=c_{1}^{r}=c_{1}^{p}\right):$

$$
\left(2 \psi^{p}+\epsilon\right)\left[\frac{1}{\bar{d}}-\frac{1}{c_{1}^{b}}\left(1-\frac{\partial p}{\partial \bar{d}}\right)\right]-\frac{1}{c_{1}^{b}} \frac{\partial p}{\partial \bar{d}}\left(1-\phi\left(1-c_{1}^{b}\right)\right) \underbrace{\left[\psi^{p}\left(\theta^{r} k_{2}^{r}+\theta^{p} k_{2}^{p}\right)+\epsilon \theta^{r} k_{2}^{r}\right]}_{=1+\epsilon \theta^{r} k_{2}^{r}}=0
$$

Using Corrolary 2 a mean preserving change in inequality increases capital trade, so that capital holdings of high income types, $k_{2}^{r}$ raise, but leaves consumption unchanged. Thus, if $\epsilon>0$ increasing inequality shifts politicians' FOC upwards resulting in a higher equilibrium
debt limit. If $\epsilon<0$ increasing inequality shifts politicians' FOC downwards resulting in a higher equilibrium debt limit.

## Proof of Lemma 6.

For a change in correlation $\mathrm{d} \Delta_{\rho}$ to keep the share of connected borrowers fixed, the share of connected poor income borrowers must adjust by $\frac{\mathrm{d} \rho^{p}}{\mathrm{~d} \Delta_{\rho}}=-\theta^{r}$. The impact on the weighted preference of connected and non-connected borrowers respectively:

$$
\begin{aligned}
\frac{\mathrm{d}^{2} U^{C}(\bar{d})}{\mathrm{d} \bar{d} \mathrm{~d} \Delta_{\rho}} & =\psi^{c}\left(1-\theta^{r}\right)\left(\theta^{r}\right)\left(\frac{\mathrm{d} U^{r c}(\bar{d})}{\mathrm{d} \bar{d}}-\frac{\mathrm{d} U^{p c}(\bar{d})}{\mathrm{d} \bar{d}}\right) \\
\frac{\mathrm{d}^{2} U^{N}(\bar{d})}{\mathrm{d} \bar{d} \mathrm{~d} \Delta_{\rho}} & =\psi^{n}\left(1-\theta^{r}\right)\left(\theta^{r}\right)\left(\frac{\mathrm{d} U^{p n}(\bar{d})}{\mathrm{d} \bar{d}}-\frac{\mathrm{d} U^{r n}(\bar{d})}{\mathrm{d} \bar{d}}\right)
\end{aligned}
$$

Since $\frac{\mathrm{d} U^{r c}(\bar{d})}{\mathrm{d} \bar{d}}>\frac{\mathrm{d} U^{p c}(\bar{d})}{\mathrm{d} \bar{d}}$, an increase (decrease) in $\Delta_{\rho}$ increases (decreases) $\frac{\mathrm{d} U^{C}(\bar{d})}{\mathrm{d} \bar{d}}$. The reverse is the case for $\frac{\mathrm{d} U^{N}(\bar{d})}{\mathrm{d} d}$, as $\frac{\mathrm{d} U^{r n}(\bar{d})}{\mathrm{d} \bar{d}}>\frac{\mathrm{d} U^{p n}(\bar{d})}{\mathrm{d} d}$.

## Proof of Proposition 7.

The total derivative of politicians' optimality condition with respect to $\Delta_{\rho}$ is:

$$
\frac{\mathrm{d}^{2} \pi}{\mathrm{~d} \bar{d} \mathrm{~d} \Delta_{\rho}}=\psi^{c} \frac{\mathrm{~d}^{2} U^{C}(\bar{d})}{\mathrm{d} \bar{d} \mathrm{~d} \Delta_{\rho}}+\psi^{n} \frac{\mathrm{~d}^{2} U^{N}(\bar{d})}{\mathrm{d} \bar{d} \mathrm{~d} \Delta_{\rho}}
$$

From Lemma 6, if a change in correlation keeps the share of connected borrowers fixed: $\frac{\mathrm{d}^{2} U^{N}(\bar{d})}{\mathrm{d} \bar{d} \mathrm{~d} \Delta_{\rho}}>0$ and $\frac{\mathrm{d}^{2} U^{C}(\bar{d})}{\mathrm{d} \bar{d} \mathrm{~d} \Delta_{\rho}}<0$. The threshold $\gamma^{c}=\hat{\gamma}^{c}$ is the ratio of the electoral powers at which the two effects exactly cancel out, such that $\frac{\mathrm{d}^{2} \pi}{\mathrm{~d} d \mathrm{~d} \Delta_{\rho}}=0$

## A. 6 Heterogenous Borrowers: public information

With public information about the borrower type, the social planner problem is:

$$
\begin{aligned}
\max _{C_{0}^{L}, D_{0}^{L}, C_{0}^{B}, D_{0}^{B}} & \sum_{B} \\
& \chi^{B} \theta^{B}\left(\log \left(C_{0}^{B}\right)+V^{B}\left(D_{0}^{B}\right)\right)+\chi^{L}\left(C_{0}^{L}+V^{L}\left(D_{0}^{L}\right)\right) \\
& -\mu\left(\sum_{B}\left(\theta^{B} C_{0}^{B}\right)+C_{1}^{L}-Y^{L}\right)-\omega\left(\sum_{B}\left(\theta^{B} D_{0}^{B}\right)+D_{0}^{L}\right)
\end{aligned}
$$

The first order conditions read:

$$
\begin{aligned}
\chi^{B} \theta^{B} \frac{1}{C_{0}^{B}}-\theta^{B} \mu & =0 \forall B \\
\chi^{L}-\mu & =0 \\
-\lambda_{1}^{B}+\sum_{j \in \mathbb{B}} \chi^{j} \theta^{j}\left[+\kappa_{1}^{j} \phi K_{2}^{j} \frac{\partial p}{\partial D_{0}^{B}}+\lambda_{1}^{j \in \mathbb{B}}\left(1-K_{2}^{j}\right) \frac{\partial p}{\partial D_{0}^{B}}\right]-\theta^{B} \omega & =0 \quad \forall B \\
-\chi^{L} \lambda_{1}^{L}-\omega & =0
\end{aligned}
$$

Rearranging yields the following optimality conditions:

$$
\begin{aligned}
& \theta^{r} \chi^{r}\left(\lambda_{0}^{r}-\lambda_{1}^{r}\right)+\sum_{B} \chi^{B} \theta^{B}\left[\kappa_{1}^{B} \phi K_{2}^{B} \frac{\partial p}{\partial D_{0}^{r}}+\lambda_{1}^{B}\left(1-K_{2}^{B}\right)\right] \frac{\partial p}{\partial D_{0}^{r}}=0 \\
& \theta^{p} \chi^{p}\left(\lambda_{0}^{p}-\lambda_{1}^{p}\right)+\sum_{B} \chi^{B} \theta^{B}\left[\kappa_{1}^{B} \phi K_{2}^{B} \frac{\partial p}{\partial D_{0}^{p}}+\lambda_{1}^{B}\left(1-K_{2}^{B}\right)\right] \frac{\partial p}{\partial D_{0}^{p}}=0
\end{aligned}
$$

The price impact of the debt of the two borrower types differs as they constitute different population shares:

$$
\begin{gathered}
\frac{\partial p}{\partial D_{0}^{r}}=\theta^{r} \frac{\partial p}{\partial D_{0}^{b}} \\
\frac{\partial p}{\partial D_{0}^{p}}=\theta^{p} \frac{\partial p}{\partial D_{0}^{b}}
\end{gathered}
$$

So the conditions that define the choice of debt limit for each borrower type are:

$$
\begin{aligned}
& \chi^{r}\left(\lambda_{0}^{r}-\lambda_{1}^{r}\right)+\sum_{B} \chi^{B} \theta^{b}\left[\kappa_{1}^{B} \phi K_{2}^{B} \frac{\partial p}{\partial D_{0}^{r}}+\lambda_{1}^{B}\left(1-K_{2}^{B}\right)\right] \frac{\partial p}{\partial D_{0}^{r}}=0 \\
& \chi^{p}\left(\lambda_{0}^{p}-\lambda_{1}^{p}\right)+\sum_{B} \chi^{B} \theta^{B}\left[\kappa_{1}^{B} \phi K_{2}^{B} \frac{\partial p}{\partial D_{0}^{p}}+\lambda_{1}^{b}\left(1-K_{2}^{B}\right)\right] \frac{\partial p}{\partial D_{0}^{p}}=0
\end{aligned}
$$

The planner weighs the total benefits of limiting the aggregate debt and distributes the costs of regulation between the two groups according to their Pareto Weights. The total benefit of setting a low debt comes from utility gains of limiting the fire sale and correspond to the negative externality of debt. The size of externality is the same as in the case of public information and depends on the Pareto weights of the two types and volume of capital trade between them. Thus, if the Pareto weight on high-income types is low, the negative externality of initial borrowing is larger, than if the Pareto weight is high. This results in the same pressure on the overall tightness of regulation as with private information on types. With public information on types, the social planner distributes the costs of limiting the
externality according to the Pareto weights. For a utilitarian social planner the distributive effects coming through the capital trade channel cancel out, so that all of the externality comes from the collateral channel. He chooses equal level of debt for the two types of borrowers.

- If $\chi^{r}>\chi^{p}$ the negative externality is lower (less need for limiting debt), than under a utilitarian social planner; constrained efficient allocation has higher debt of highincome types relative to low-income types;
- If $\chi^{r}<\chi^{p}$ the negative externality is larger (less need for limiting debt) than under a utilitarian social planner; constrained efficient allocation has lower debt of high-income types relative to low-income types;

If the politician can only implement one policy, she would implement the policy as described in Section 3. In this case the policy would be inefficient as the politician is unable to distribute the costs of regulation between different types.

- If $\chi^{r}>\chi^{p}$ the debt limit is inefficiently strict for the high-income types and inefficiently lax for the low-income types
- If $\chi^{r}<\chi^{p}$ the debt limit is inefficiently lax for the high-income types and inefficiently strict for the low-income types

With public information the equilibrium policy is distributive in itself as higher regulatory burden is placed on the less influential voter groups. Some of the transfers intended by the social planner now take form of different distribution of the policy burden.

If however, the politician can also implement type-specific policies, the first order conditions of his problem are:

$$
\begin{align*}
& \sum_{v=p, r} \theta^{v} \psi^{v} \frac{\partial U^{v}(\bar{d})}{\partial \bar{d}^{r}}=0  \tag{22}\\
& \sum_{v=p, r} \theta^{v} \psi^{v} \frac{\partial U^{v}(\bar{d})}{\partial \bar{d}^{p}}=0 \tag{23}
\end{align*}
$$

Which yield the following optimality conditions:

$$
\begin{aligned}
& \psi^{r}\left(\lambda_{0}^{r}-\lambda_{1}^{r}\right)+\sum_{h \in \mathbb{B}} \psi^{h} \theta^{h}\left[\kappa_{1}^{h} \phi K_{2}^{h} \frac{\partial p}{\partial D_{0}^{r}}+\lambda_{1}^{b}\left(1-K_{2}^{h}\right)\right] \frac{\partial p}{\partial D_{0}^{r}}=0 \\
& \psi^{p}\left(\lambda_{0}^{p}-\lambda_{1}^{b p}\right)+\sum_{h \in \mathbb{B}} \psi^{h} \theta^{h}\left[\kappa_{1}^{h} \phi K_{2}^{h} \frac{\partial p}{\partial D_{0}^{r}}+\lambda_{1}^{h}\left(1-K_{2}^{h}\right)\right] \frac{\partial p}{\partial D_{0}^{p}}=0
\end{aligned}
$$

Thus, the equilibrium policy corresponds to that of the social planner with the Pareto weights that satisfy $\frac{\chi^{r}}{\chi^{p}}=\frac{\psi^{r}}{\psi^{r}}$. With public information the equilibrium policy is distributive in itself as higher regulatory burden is placed on the less influential voter groups.

## A. 7 Negotiations between connected borrowers and politician

After the elections are resolved at $t=0$, but before the agents choose their allocations, each politically connected borrower can negotiate with the politician. I assume all bargaining power lies with the borrower. He decides whether to seek exemption $(n=1)$ or to remain subject to the debt limit $(n=0)$ and chooses the level of initial debt $\left(d_{0}^{c}\right)$ and the rents to the politician $(R)$ conditional on seeking exemption, so that to maximize his utility, subject to the budget constraints and the participation constraint of the politician. Let $V_{1}\left(d_{0}^{b}\right)$ be the indirect utility as of $t=1$ of a borrower with the initial debt $d_{0}^{b}$, then the problem is:

$$
\max _{n, d_{0}, R} n\left[\log \left(c_{0}^{c}\right)+V_{1}\left(d_{0}^{c}, p\right)-R\right]+(n-1)\left[\log (\bar{d})+V_{1}(\bar{d}), p\right]
$$

subject to:

$$
\begin{array}{ll}
c_{0}^{c} \leq d_{0}^{c} & \\
R \geq 0 \quad \text { PC-POL }
\end{array}
$$

The connected borrower finds it optimal to offer a minimal rent to the politician so $R=0$ in equilibrium. He chooses the level of debt conditional on exemption, so that to maximize his utility. Being atomistic, he does not internalize how his choice of debt affects the prices, so the first order condition with respect to debt is given by:

$$
\lambda_{0}^{c}-\lambda_{1}^{c}=0
$$

This corresponds to the unconstrained optimal choice of a borrower. So, whenever the debt limit rule binds, all connected borrowers seek exemption and choose $d_{0}^{c}=c_{1}^{c}(p)$.

## A. 8 Lobbying via campaign effort

The political game can be extended to take into account other forms of political activity by voters, such as campaign contributions. Consider the political subgame introduced in Section 2. Assume now that voter groups can form form a lobbies and offer support to politicians
prior to elections. After the politician announced his policy, members of the lobby can put effort in order to increase candidates probability of winning. The campaign effort comes at a cost, it generates a disutility of $h^{J}\left(C^{i}\right)=\alpha^{J}\left(C^{i}\right)^{2} / 2$. The voter groups may differ in the cost of effort (captured by a group specific cost-parameter $\alpha^{J}$. The campaign effort shifts the average bias of borrowers, so that the utility of voter $i$ in group $J$ associated with candidate A winning is given by: $U^{i J}(A$ wins $)=U^{J}\left(\bar{d}_{A}\right)+b^{i J}+b+\sigma\left(C_{A}-C_{B}\right)$

The timing of the political game is as follows: (1) Politicans announce their platforms, (2) Lobbies choose their campaign efforts, (3) Random bias is realized, (4) Elections take place and (5) The policy is implemented.

When voting in the majoritarian elections, agents evaluate the policy proposals of the two politicians and vote for the one whose policy offers them a higher utility accounting for their ideological biases. Lobbies choose their effort to support the politicians in order to maximize the probability of the politician with a favorable platform being elected.

With campaign efforts the ex-ante probability of winning by the politician A for a given policy of the competitor $\bar{d}_{B}$ and lobby efforts $C_{K}=\sum_{J} \theta^{J} C_{A}^{J}$

$$
\begin{equation*}
\pi_{A}=\frac{1}{2}+\frac{\Psi}{\bar{\psi}}\left[\sum_{J} \theta^{J} \psi^{J}\left[U^{J}\left(\bar{d}_{A}\right)-U^{J}\left(\bar{d}_{B}\right)+\sigma\left(C_{A}-C_{B}\right)\right]\right] \tag{24}
\end{equation*}
$$

Where $\bar{\psi}=\sum_{J} \theta^{J} \psi^{J}$ is the average ideological concentration of the two groups. Each of the lobbies observes the policy announcement by the politicians and chooses its' campaign effort so that to maximize the expected utility of its members:

$$
\max _{C_{A}^{J}, C_{B}^{J}} W^{J}=\pi_{A} U^{J}\left(\bar{d}_{A}\right)+\left(1-\pi_{A}\right) U^{J}\left(\bar{d}_{B}\right)-\alpha^{J} \frac{\left(C_{A}^{J}+C_{B}^{J}\right)^{2}}{2}
$$

When deciding on how much effort to put into supporting a candidate with a given policy lobby groups trade off the cost of that effort against it's marginal benefit: the expected increase in utility due to an increase the probability of winning of a candidate with a preferred policy. In equilibirum each lobby supports only one politician. The optimal campaign effort towards politician $A$ for each agent in the lobby is:

$$
C_{A}^{J}=C_{A}^{J}-C_{B}^{J}= \begin{cases}0 & \text { if } U^{J}\left(\bar{d}_{A}\right)-U^{J}\left(\bar{d}_{B}\right) \leq 0 \\ \frac{\sigma}{\alpha^{J}} \frac{\Psi}{\psi}\left[U^{J}\left(\bar{d}_{A}\right)-U^{J}\left(\bar{d}_{B}\right)\right] & \text { otherwise }\end{cases}
$$

The effort exerted by a lobby towards candidate A is proportional to utility gain from electing
the politician with a favorable policy. The intensity of this relation is directly proportional to the impact of tha campaign efforts on voter bias $(\sigma)$ and the aggregate sensitivity to economic policy of the population $\left(\frac{\Psi}{\psi}\right)$. It is inversely proportional to the cost of effort of a given group. The effort towards candidate $B$ can be expressed analogously.

Given these expected campaign contributions, each politician chooses the policy platform so that to maximize his probability of winning. The resulting optimality condition of the problem of politician $A$ is:

$$
\frac{\partial \pi_{A}}{\partial \bar{d}_{A}}=\sum_{J} \theta^{J} \psi^{J}\left(1+\frac{\sigma^{2}}{\alpha^{J}} \frac{\Psi}{\bar{\psi}}\right) \frac{\partial U^{J}\left(\bar{d}_{A}\right)}{\partial \bar{d}_{A}}=0
$$

Therefore, while maximizing his probability of winning the politician practically chooses a policy that maximizes a sum of weighted utilities of the two agents groups. The weight assigned to each of the groups in the politicians optimization reflects the concentration of their ideological biases and the strength of the lobby.

## B Solution with Taxes

Taxes on the initial debt rebated back to borrowers in the form of lump sum transfers are an alternative prudential tool that can restore constrained efficiency. In this section I discuss the planner problem and political equilibrium with taxes as the policy tool.

In the presence of taxes the $t=0$ budget constraint of borrower reads:

$$
c_{0}^{B}-(1-\tau) d_{0}^{B}-T=0 \forall B \in \mathbb{B}
$$

Where through balanced budget constraint, $\tau \sum_{B \in \mathbb{B}} \theta^{B} D_{0}^{B}=T$. The presence of taxes affects borrowers FOC, so that his choice of initial debt is pinned down by:

$$
(1-\tau) \lambda_{0}^{B}-\lambda_{1}^{B}=0
$$

Thus, a higher tax rate drives down the amount of debt that the borrowers take on at $t=0$. When choosing the optimal tax rate the social planner satisfies the following FOC:

$$
\sum_{h \in \mathbb{B}} \chi^{h} \theta^{h}\left[-\lambda_{0}^{h}\left(d_{0}^{h}-\frac{\partial D_{0}^{b}}{\partial \tau} \tau-D_{0}^{b}\right)-\lambda_{1}^{h}\left(1-k_{2}^{h}\right) \frac{\partial p}{\partial \tau}+\kappa_{1}^{h} \phi k_{2}^{h} \frac{\partial p}{\partial \tau}\right]=0
$$

Since both types choose the same level of debt, we have $d_{0}^{h}=D_{0}^{b}, \lambda_{t}^{h}=\lambda_{t}^{b}$, and $\kappa_{1}^{h}=\kappa_{1}^{b}$ for all $h \in \mathbb{B}$. Thus, the optimal tax rate solves:

$$
\tau=\frac{-\sum_{h \in \mathbb{B}} \chi^{h} \theta^{h} \lambda_{1}^{b}\left(1-k_{2}^{h}\right)+\kappa_{1}^{b} \phi k_{2}^{h}}{\lambda_{0}^{b} \sum_{h \in \mathbb{B}} \chi^{h} \theta^{h}} \frac{\partial p}{\partial D_{0}^{b}}
$$

The optimal tax increases as the Pareto Weight of the low-income borrowers increases. The reverse occurs if the Pareto Weight of high-income borrowers is increased. The tax is positive whenever the overall externality is negative, that is if the income inequality is not too large. The same intuition applies in the solutions to politician's problem. The equilibrium tax on debt weighs the preference of different income groups by the electoral power:

$$
\tau=\frac{-\sum_{h \in \mathbb{B}} \psi^{h} \theta^{h} \lambda_{1}^{b}\left(1-k_{2}^{h}\right)+\kappa_{1}^{b} \phi k_{2}^{h}}{\lambda_{0}^{b} \sum_{h \in \mathbb{B}} \psi^{h} \theta^{h}} \frac{\partial p}{\partial D_{0}^{b}}
$$

Like with the debt limit, a mean-preserving increase in inequality increases the disparity between the debt limit preferred by high- and low- income voters. If $\psi^{r}>\psi^{p}$ it results in laxer equilibrium policy. One way to analyze the setting with taxes under imperfect enforcement is to assume that the politically connected borrowers are exempt from the tax and do not receive the benefits associated with the transfers. In this case the total derivative of their utility with respect to tax is:

$$
\lambda_{1}^{c}\left(1-k_{2}^{c}\right) \frac{\partial p}{\partial \tau}+\kappa_{1}^{c} \phi k_{2}^{c} \frac{\partial p}{\partial \tau}>0
$$

Which is strictly positive, so that connected prefer maximum tax rate $\tau=1$ so that to limit the fire sale. The derivative of utility of non-connected with respect to tax pins down their most preferred policy is:

$$
-\lambda_{0}^{c}\left(d_{0}^{N}-D_{0}^{N}-\frac{\partial D_{0}^{N}}{\partial \tau} \tau\right)+\lambda_{1}^{N}\left(1-k_{2}^{N}\right) \frac{\partial p}{\partial \tau}+\kappa_{1}^{N} \phi k_{2}^{N} \frac{\partial p}{\partial \tau}=0
$$

Which translates into the following preferred tax rate, which is lower than the one preferred by the connected.

$$
\tau=-\frac{\lambda_{1}^{N}\left(1-k_{2}^{N}\right) \frac{\partial p}{\partial \tau}+\kappa_{1}^{N} \phi k_{2}^{N} \frac{\partial p}{\partial \tau}-\lambda_{0}^{N}\left(1-\theta^{n}\right) d_{0}^{N}}{\lambda_{0}^{N} \frac{\partial D_{0}^{N}}{\partial \tau}}<1
$$

An alternative way to study the problem of imperfect enforcement in the setting with
taxes is to assume that connected are able to avoid taxes but share in the benefits. In this case, the policy benefits them through a direct transfer, on top of the benefits associated with limit the fire sale. Connected will choose a rate that maximizes these gains (which may be lower than one due to the considerations related to achieving a high transfer). For the non-connected, the transfer to non-connected consitutes an additional cost of tax and so their preferred policy rate is lower than the one in case without transfers to connected.

The total derivative of utility of connected borrowers with respect to tax is:

$$
-\lambda_{0}^{c}\left(-D_{0}^{N}-\frac{\partial D_{0}^{N}}{\partial \tau} \tau\right)+\lambda_{1}^{c}\left(1-k_{2}^{c}\right) \frac{\partial p}{\partial \tau}+\kappa_{1}^{c} \phi k_{2}^{c} \frac{\partial p}{\partial \tau}
$$

Which uses the fact that tax only applies to the non-connected ( $D_{0}^{N}=\theta^{n} d_{0}^{n}$ ). This is strictly positive if $D_{0}^{N}+\frac{\partial D_{0}^{N}}{\partial \tau} \tau>0$. Otherwise setting it equal to zero yields:

$$
\tau=-\frac{\lambda_{1}^{c}\left(1-k_{2}^{c}\right) \frac{\partial p}{\partial \tau}+\kappa_{1}^{c} \phi k_{2}^{c} \frac{\partial p}{\partial \tau}+\lambda_{0}^{c} D_{0}^{N}}{\lambda_{0}^{c} \frac{\partial D_{0}^{N}}{\partial \tau}}
$$

The derivative of utility of non-connected with respect to tax is:

$$
-\lambda_{0}^{c}\left(d_{0}^{N}-D_{0}^{N}-\frac{\partial D_{0}^{N}}{\partial \tau} \tau\right)+\lambda_{1}^{N}\left(1-k_{2}^{N}\right) \frac{\partial p}{\partial \tau}+\kappa_{1}^{N} \phi k_{2}^{N} \frac{\partial p}{\partial \tau}
$$

Which translates into the following preferred tax rate

$$
\tau=-\frac{\lambda_{1}^{N}\left(1-k_{2}^{N}\right) \frac{\partial p}{\partial \tau}+\kappa_{1}^{N} \phi k_{2}^{N} \frac{\partial p}{\partial \tau}-\lambda_{0}^{N}\left(1-\theta^{n}\right) d_{0}^{N}}{\lambda_{0}^{N} \frac{\partial D_{0}^{N}}{\partial \tau}}
$$


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[^1]:    ${ }^{1}$ Fire-sale is a forced sale of an asset at a discount relative to its' fundamental value (Shleifer and Vishny, 1992).
    ${ }^{2}$ See for example: Bianchi (2011); Bianchi and Mendoza (2011); Gersbach and Rochet (2012, 2017); He and Kondor (2016); Jeanne and Korinek (2018, 2013); Perotti and Suarez (2011).

[^2]:    ${ }^{3}$ Inefficient overborrowing at $t=0$, and thus a motive for policy may arise even in the case of a binding $t=0$ collateral constraint. However allowing for that would add more complexity to the exposition, without providing additional insights.

[^3]:    ${ }^{4}$ This framework can be easily extended to allow for ex-ante lobbying by different voter groups. I discuss this setting in Appendix A.8.

[^4]:    ${ }^{5}$ This assumption allows me to focus on a problem with a single debt limit policy. In Appendix A. 6 I show that if type is public information, type-specific debt limits emerge in equilibrium, but the results on the efficiency and strictness derived in this section remain the same.

[^5]:    ${ }^{6}$ This is the interior solution of their problem. In Appendix A. 4 I show that the interior solution constitutes the unique equilibrium under the assumptions on minimum income of each borrower type (Assumption 2 (b)).

[^6]:    ${ }^{7}$ In the Appendix A. 7 I micro-found this by assuming connected borrowers and the politician enter into negotiations after the elections.

