The New Keynesian Climate Model

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Introduction

- Climate change will change the macroeconomic landscape in the next decades and the central bank will have to face 2 phenomena [Schnabel 2022]:
  - On the one hand, a warming planet causes damages that will make resources scarcer & prices higher → **climateflation**.
  - On the other hand, the fight against climate change (through increasing carbon taxes) will make fossil fuels & raw materials more expensive → **greenflation**.
- How should the central bank conduct monetary policy in this new landscape?
- Answering this question requires to understand the effects of climate change on the economy.

This paper

- The canonical New Keynesian model is silent on climate developments.
- This paper develops The New Keynesian Climate (NKC) model by:
  - extending the canonical model with a carbon accumulation constraint and a mitigation policy from the Integrated Assessment Model (IAM) literature;
  - estimating this model for the world economy with techniques that take into account nonlinearities resulting from climate change;
  - providing projections up to horizon 2100 under mitigation versus *laissez-faire* policy by changing an exogenous carbon tax rate.
- This allows us to analyze the impact of climate change on inflation and monetary policy.

Methodological breakthrough

- Standard view: stable propagation mechanism with fluctuations naturally decaying over time back to a steady state.
- Climate problem: the way carbon emissions cumulate over time permanently changes the propagation patterns → no steady state.
- We solve our nonlinear model taking into account both long and short term effects using the Fair and Taylor (1983)'s extended path solution method.
- We estimate the model using Bayesian nonlinear techniques based on the inversion filter from Fair and Taylor (1983).

Literature

Our paper is connected to three literatures:

- IAMs analyze the long-term effect of carbon accumulation [Nordhaus 1992; Dietz and Venmans 2019; Barrage and Nordhaus 2023], but take a benign view of fluctuations and price rigidity.
- E-DSGE with nominal rigidities [Annicchiarico and Di Dio 2015; Ferrari and Nispi Landi 2022; Coenen et al. 2023; Del Negro et al. 2023], but no explicit demographic and climate trends.
- Standard New Keynesian models [Woodford 2003; Smets and Wouters 2007], without climate change.

Outline

Plan

Usual ingredients

- Households: ⏩ Households Appendix
  - They choose their consumption, saving and labor supply by maximizing intertemporal utility
  - Demographic trend: population size is exogenous and time-varying.
- Firms: ⏩ Firms Appendix
  - Producers solve a two-stage problem: (i) choose labor to maximize profits and (ii) decide their selling price under a Rotemberg price setting.
- Central bank: ⏩ CB Appendix
  - It chooses interest rate by following a Taylor-type rule.

The Core New Keynesian Model

The core New Keynesian model comprises **three equations**:

IS:

$$(\tilde{y}_t x_t)^{-\sigma_c} = \beta \mathbb{E}_t \frac{\varepsilon_{b,t+1}}{\varepsilon_{b,t}} \frac{r_t}{\pi_{t+1}} (\tilde{y}_{t+1} x_{t+1})^{-\sigma_c}$$

$$x_t = 1 - \kappa \left(\pi_t - \pi_t^*\right)^2$$

PC:

$$\left(\pi_t - \pi_t^*\right) \pi_t = \beta \mathbb{E}_t \; g_{z,t} \tilde{y}_{t+1} / \tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right) \pi_{t+1} + \zeta \kappa^{-1} \varepsilon_{p,t} mc_t + \kappa^{-1} (1 - \zeta)$$

$$mc_t = \psi \left(x_t \tilde{y}_t\right)^{\sigma_c} \tilde{y}_t^{\sigma_n}$$

MP:

$$r_t = r_{t-1}^{\rho} \left[r_r \left(\pi_t^* / \pi\right) \left(\pi_t / \pi_t^*\right)^{\phi_\pi} \left(\tilde{y}_t / \tilde{y}_t^n\right)^{\phi_y} \right]^{1-\rho} \left(\pi_t^* / \pi_{t-1}^*\right)^{\phi_{\pi*}} \varepsilon_{r,t}$$

The Core New Keynesian Model

Detrended GDP

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cons-to-gdp

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Inflation

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$$mc_t = \psi (x_t \tilde{y}_t)^{\sigma_c} \tilde{y}_t^{\sigma_n}$$

marginal cost

$$\textbf{MP:} \quad r_t = r_{t-1}^{\rho} \left[r_r (\pi_t^* / \pi) (\pi_t / \pi_t^*)^{\phi_\pi} (\tilde{y}_t / \tilde{y}_t^n)^{\phi_y} \right]^{1-\rho} (\pi_t^* / \pi_{t-1}^*)^{\phi_{\pi *}} \varepsilon_{r,t}$$

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cons-to-gdp

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Interest rate

The Core New Keynesian Model

The core New Keynesian model comprises three equations **and shocks**:

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x_t &= 1 - \kappa \left(\pi_t - \pi_t^*\right)^2
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\textbf{MP:} \quad r_t = r_{t-1}^{\rho} \left[r_r \left(\pi_t^* / \pi\right) \left(\pi_t / \pi_t^*\right)^{\phi_\pi} \left(\tilde{y}_t / \tilde{y}_t^n\right)^{\phi_y} \right]^{1-\rho} \left(\pi_t^* / \pi_{t-1}^*\right)^{\phi_{\pi *}} \varepsilon_{r,t}
$$

The Core New Keynesian M

Demand shock

The core New Keynesian model comprises three equations **and shocks**:

$$
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\textbf{PC:} \quad \left(\pi_t - \pi_t^*\right) \pi_t &= \beta \mathbb{E}_t \; g_{z,t} \tilde{y}_{t+1} / \tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right) \pi_{t+1} + \zeta \kappa^{-1} \varepsilon_{p,t} mc_t + \kappa^{-1} \left(1 - \zeta\right) \\
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Supply shock

$$
\begin{aligned}
\textbf{PC:} \quad \left(\pi_t - \pi_t^*\right) \pi_t &= \beta \mathbb{E}_t \, g_{z,t} \tilde{y}_{t+1} / \tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right) \pi_{t+1} + \zeta \kappa^{-1} \varepsilon_{p,t} mc_t + \kappa^{-1} \left(1 - \zeta\right) \\
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\textbf{MP:} \quad r_t = r_{t-1}^{\rho} \left[r_r \left(\pi_t^* / \pi\right) \left(\pi_t / \pi_t^*\right)^{\phi_\pi} \left(\tilde{y}_t / \tilde{y}_t^n\right)^{\phi_y} \right]^{1-\rho} \left(\pi_t^* / \pi_{t-1}^*\right)^{\phi_{\pi *}} \varepsilon_{r,t}
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Supply shock

MP shock

$$
\begin{array}{lrcl}
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 & mc_t & = & \psi \left(x_t \tilde{y}_t\right)^{\sigma_c} \tilde{y}_t^{\sigma_n} \\
\textbf{MP:} & r_t & = & r_{t-1}^{\rho} \left[r_r \left(\pi_t^* / \pi\right) \left(\pi_t / \pi_t^*\right)^{\phi_\pi} \left(\tilde{y}_t / \tilde{y}_t^n\right)^{\phi_y} \right]^{1-\rho} \left(\pi_t^* / \pi_{t-1}^*\right)^{\phi_{\pi*}} \varepsilon_{r,t}
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The Core New Keynesian Model

The core New Keynesian model comprises three equations and shocks **and trends**:

IS:

$$\left(\tilde{y}_t x_t\right)^{-\sigma_c} = \beta \mathbb{E}_t \frac{\varepsilon_{b,t+1}}{\varepsilon_{b,t}} \frac{r_t}{\pi_{t+1}} \left(\tilde{y}_{t+1} x_{t+1}\right)^{-\sigma_c}$$

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$x \quad \pi_t - \pi_t^*)^2$

inflation target trend

$$\textbf{PC:} \quad (\pi_t - \pi_t^*)\,\pi_t = \beta\mathbb{E}_t\; g_{z,t}\tilde{y}_{t+1}/\tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right)\pi_{t+1} + \zeta\kappa^{-1}\varepsilon_{p,t} mc_t + \kappa^{-1}(1-\zeta)$$

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inflation target trend

technology growth

PC: $$\left(\pi_t - \pi_t^*\right)\pi_t = \beta \mathbb{E}_t\, g_{z,t} \tilde{y}_{t+1}/\tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right)\pi_{t+1} + \zeta \kappa^{-1} \varepsilon_{p,t} mc_t + \kappa^{-1}(1-\zeta)$$

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Modification: higher discounting

- Discounting is a critical and pervasive issue in macroeconomic analyses.
- In New Keynesian models, long term policies lead to implausibly large effects in present value terms (e.g., the *forward guidance puzzle*).
- This issue is particularly pronounced when considering mitigation of climate change and other long-term environmental policies.
- We introduce: (i) income risk *à la* McKay et al. 2017 $\Rightarrow$ discounted Euler equation and (ii) firms's exit *à la* Bilbiie et al. 2012 $\Rightarrow$ discounted Phillips curve.

The Modified New Keynesian Model

Two frictions to attenuate the expectation channel:

$$\textbf{IS:}\quad \left(\frac{\tilde{y}_t x_t - \omega d}{1-\omega}\right)^{-\sigma_c} = \beta \mathbb{E}_t \frac{\varepsilon_{b,t+1}}{\varepsilon_{b,t}} \frac{r_t}{\pi_{t+1}} \left((1-\omega) \left(\frac{\tilde{y}_{t+1} x_{t+1} - \omega d}{1-\omega}\right)^{-\sigma_c} + \omega d^{-\sigma_c} \right)$$

$$x_t = 1 - \kappa \left(\pi_t - \pi_t^*\right)^2 (1-\vartheta) - \vartheta(1 - \varepsilon_{p,t} mc_t)$$

$$\textbf{PC:}\quad \left(\pi_t - \pi_t^*\right) \pi_t = (1-\vartheta)\beta \mathbb{E}_t \, g_{z,t} \tilde{y}_{t+1}/\tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right) \pi_{t+1} + \zeta \kappa^{-1} \varepsilon_{p,t} mc_t + \kappa^{-1} (1-\zeta)$$

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The Modified New Keynesian Model

Euler discounting

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The Modified New Keynesian Model

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PC discounting

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x_t &= 1 - \kappa \left(\pi_t - \pi_t^*\right)^2 (1-\vartheta) - \vartheta(1 - \varepsilon_{p,t} mc_t) \\
\textbf{PC:}\quad \left(\pi_t - \pi_t^*\right) \pi_t &= (1-\vartheta)\beta \mathbb{E}_t \; g_{z,t} \tilde{y}_{t+1}/\tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right) \pi_{t+1} + \zeta \kappa^{-1} \varepsilon_{p,t} mc_t + \kappa^{-1} (1-\zeta) \\
mc_t &= \psi \left(x_t \tilde{y}_t - \omega d\right)^{\sigma_c} \tilde{y}_t^{\sigma_n} \\
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\end{aligned}
$$

The New Keynesian Climate Model

Carbon accumulation and its damages:

$$
\begin{aligned}
\textbf{IS:}\quad \left(\frac{\tilde{y}_t x_t-\omega d}{1-\omega}\right)^{-\sigma_c} &= \beta \mathbb{E}_t \, \frac{\varepsilon_{b,t+1}}{\varepsilon_{b,t}} \frac{r_t}{\pi_{t+1}} \left((1-\omega)\left(\frac{x_{t+1}\tilde{y}_{t+1}-\omega d}{1-\omega}\right)^{-\sigma_c} + \omega d^{-\sigma_c}\right) \\
x_t &= 1-(1-\vartheta)0.5\kappa\left(\pi_t-\pi_t^*\right)^2-\vartheta(1-\varepsilon_{p,t} mc_t) \\
\textbf{PC:}\quad \left(\pi_t-\pi_t^*\right)\pi_t &= (1-\vartheta)\beta\mathbb{E}_t \, g_{z,t}\tilde{y}_{t+1}/\tilde{y}_t\left(\pi_{t+1}-\pi_{t+1}^*\right)\pi_{t+1}+\zeta\kappa^{-1}\varepsilon_{p,t} mc_t+\kappa^{-1}(1-\zeta) \\
mc_t &= \psi\left(x_t\tilde{y}_t-\omega d\right)^{\sigma_c}\tilde{y}_t^{\sigma_n} \; \Phi\left(\tilde{m}_t\right)^{-(1+\sigma_n)} \\
\textbf{MP:}\quad r_t &= r_{t-1}^{\rho}\left[r_r\left(\pi_t^*/\pi\right)\left(\pi_t/\pi_t^*\right)^{\phi_\pi}\left(\tilde{y}_t/\tilde{y}_t^n\right)^{\phi_y}\right]^{1-\rho}\left(\pi_t^*/\pi_{t-1}^*\right)^{\phi_{\pi *}}\varepsilon_{r,t} \\
\textbf{CC:}\quad \tilde{m}_t &= (1-\delta_m)\tilde{m}_{t-1}+\xi_m \; \sigma_t \; z_t \; l_t \; \tilde{y}_t \; \varepsilon_{e,t}
\end{aligned}
$$

The New Keynesian Climate Model

Carbon accumulation and its damages:

$$\textbf{IS:}\quad \left(\frac{\tilde{y}_t x_t - \omega d}{1-\omega}\right)^{-\sigma_c} = \beta \mathbb{E}_t \, \frac{\varepsilon_{b,t+1}}{\varepsilon_{b,t}} \frac{r_t}{\pi_{t+1}} \left((1-\omega) \left(\frac{x_{t+1}\tilde{y}_{t+1} - \omega d}{1-\omega} \right)^{-\sigma_c} + \omega d^{-\sigma_c} \right)$$

$$x_t = 1 - (1-\vartheta) 0.5 \kappa \left(\pi_t - \pi_t^*\right)^2 - \vartheta (1 - \varepsilon_{p,t} mc_t)$$

$$\textbf{PC:}\quad \left(\pi_t - \pi_t^*\right) \pi_t = (1-\vartheta) \beta \mathbb{E}_t \; g_{z,t} \tilde{y}_{t+1} / \tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right) \pi_{t+1} + \zeta \kappa^{-1} \varepsilon_{p,t} mc_t + \kappa^{-1} (1-\zeta)$$

$$mc_t = \psi \left(x_t \tilde{y}_t - \omega d\right)^{\sigma_c} \tilde{y}_t^{\sigma_n} \; \Phi \left(\tilde{m}_t\right)^{-(1+\sigma_n)}$$

$$\textbf{MP:}\quad r_t = r_{t-1}^{\rho} \left[r_r \left(\pi_t^* / \pi\right) \left(\pi_t / \pi_t^*\right)^{\phi_\pi} \left(\tilde{y}_t / \tilde{y}_t^n\right)^{\phi_y} \right]^{1-\rho} \left(\pi_t^* / \pi_{t-1}^*\right)^{\phi_{\pi *}} \varepsilon_{r,t}$$

$$\textbf{CC:}\quad \tilde{m}_t = (1-\delta_m) \tilde{m}_{t-1} + \xi_m \; \sigma_t \; z_t \; l_t \; \tilde{y}_t \; \varepsilon_{e,t}$$

Anthropogenic carbon stock

The New Keynesian Climate Model

Carbon accumulation and its damages:

$$\textbf{IS:}\quad \left(\frac{\tilde{y}_t x_t-\omega d}{1-\omega}\right)^{-\sigma_c} = \beta\mathbb{E}_t\,\frac{\varepsilon_{b,t+1}}{\varepsilon_{b,t}}\frac{r_t}{\pi_{t+1}}\left((1-\omega)\left(\frac{x_{t+1}\tilde{y}_{t+1}-\omega d}{1-\omega}\right)^{-\sigma_c}+\omega d^{-\sigma_c}\right)$$

$$x_t = 1-(1-\vartheta)0.5\kappa\left(\pi_t-\pi_t^*\right)^2-\vartheta(1-\varepsilon_{p,t}mc_t)$$

$$\textbf{PC:}\quad \left(\pi_t-\pi_t^*\right)\pi_t = (1-\vartheta)\beta\mathbb{E}_t\; g_{z,t}\tilde{y}_{t+1}/\tilde{y}_t\left(\pi_{t+1}-\pi_{t+1}^*\right)\pi_{t+1}+\zeta\kappa^{-1}\varepsilon_{p,t}mc_t+\kappa^{-1}\left(1-\zeta\right)$$

$$mc_t = \psi\left(x_t\tilde{y}_t-\omega d\right)^{\sigma_c}\tilde{y}_t^{\sigma_n}\;\Phi\left(\tilde{m}_t\right)^{-(1+\sigma_n)}$$

Deterministic Decoupling trend

$$\textbf{MP:}\quad r_t = r_{t-1}^{\rho}\left[\;\cdots\;\tilde{y}_t/\tilde{y}_t^n)^{\phi_y}\right]^{1-\rho}\left(\pi_t^*/\pi_{t-1}^*\right)^{\phi_{\pi*}}\varepsilon_{r,t}$$

$$\textbf{CC:}\quad \tilde{m}_t = (1-\delta_m)\tilde{m}_{t-1}+\xi_m\,\sigma_t\,z_t\,l_t\,\tilde{y}_t\,\varepsilon_{e,t}$$

Anthropogenic carbon stock

The New Keynesian Climate Model

Carbon accumulation and its damages:

$$
\begin{aligned}
\textbf{IS:}\quad \left(\frac{\tilde{y}_t x_t - \omega d}{1-\omega}\right)^{-\sigma_c} &= \beta \mathbb{E}_t \, \frac{\varepsilon_{b,t+1}}{\varepsilon_{b,t}} \frac{r_t}{\pi_{t+1}} \left((1-\omega)\left(\frac{x_{t+1}\tilde{y}_{t+1} - \omega d}{1-\omega}\right)^{-\sigma_c} + \omega d^{-\sigma_c} \right) \\
x_t &= 1 - (1-\vartheta) 0.5 \kappa \left(\pi_t - \pi_t^*\right)^2 - \vartheta (1 - \varepsilon_{p,t} mc_t) \\
\textbf{PC:}\quad \left(\pi_t - \pi_t^*\right) \pi_t &= (1-\vartheta)\beta \mathbb{E}_t \, g_{z,t} \tilde{y}_{t+1}/\tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right) \pi_{t+1} + \zeta \kappa^{-1} \varepsilon_{p,t} mc_t + \kappa^{-1}(1-\zeta) \\
mc_t &= \psi \left(x_t \tilde{y}_t - \omega d\right)^{\sigma_c} \tilde{y}_t^{\sigma_n} \; \Phi\left(\tilde{m}_t\right)^{-(1+\sigma_n)}
\end{aligned}
$$

Deterministic Decoupling trend

$$
\textbf{MP:}\quad r_t = r_{t-1}^{\rho} \left[\ldots \tilde{y}_t/\tilde{y}_t^n)^{\phi_y} \right]^{1-\rho} \left(\pi_t^*/\pi_{t-1}^*\right)^{\phi_{\pi*}} \varepsilon_{r,t}
$$

$$
\textbf{CC:}\quad \tilde{m}_t = (1-\delta_m)\tilde{m}_{t-1} + \xi_m \, \sigma_t \, z_t \, l_t \, \tilde{y}_t \, \varepsilon_{e,t}
$$

Anthropogenic carbon stock

Deterministic TFP trend

Carbon accumulation and its damages:

$$\textbf{IS:} \quad \left(\frac{\tilde{y}_t x_t - \omega d}{1-\omega}\right)^{-\sigma_c} = \beta \mathbb{E}_t \, \frac{\varepsilon_{b,t+1}}{\varepsilon_{b,t}} \frac{r_t}{\pi_{t+1}} \left((1-\omega) \left(\frac{x_{t+1}\tilde{y}_{t+1} - \omega d}{1-\omega}\right)^{-\sigma_c} + \omega d^{-\sigma_c} \right)$$

$$x_t = 1 - (1-\vartheta) 0.5 \kappa \left(\pi_t - \pi_t^*\right)^2 - \vartheta (1 - \varepsilon_{p,t} mc_t)$$

$$\textbf{PC:} \quad \left(\pi_t - \pi_t^*\right) \pi_t = (1-\vartheta)\beta \mathbb{E}_t \, g_{z,t} \tilde{y}_{t+1}/\tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right) \pi_{t+1} + \zeta \kappa^{-1} \varepsilon_{p,t} mc_t + \kappa^{-1} (1-\zeta)$$

$$mc_t = \psi \left(x_t \tilde{y}_t - \omega d\right)^{\sigma_c} \tilde{y}_t^{\sigma_n} \; \Phi \left(\tilde{m}_t\right)^{-(1+\sigma_n)}$$

Deterministic Decoupling trend

Deterministic population trend

$$\textbf{MP:} \quad r_t = r_{t-1}^{\rho} \left[\ldots \tilde{y}_t \ldots \right] \ldots^{\rho_{\pi *}} \varepsilon_{r,t}$$

$$\textbf{CC:} \quad \tilde{m}_t = (1-\delta_m)\tilde{m}_{t-1} + \xi_m \, \sigma_t \, z_t \, l_t \, \tilde{y}_t \, \varepsilon_{e,t}$$

Anthropogenic carbon stock

Deterministic TFP trend

Carbon accumulation and its damages:

$$\textbf{IS:} \quad \left(\frac{\tilde{y}_t x_t - \omega d}{1-\omega}\right)^{-\sigma_c} = \beta \mathbb{E}_t \, \frac{\varepsilon_{b,t+1}}{\varepsilon_{b,t}} \frac{r_t}{\pi_{t+1}} \left((1-\omega)\left(\frac{x_{t+1}\tilde{y}_{t+1} - \omega d}{1-\omega}\right)^{-\sigma_c} + \omega d^{-\sigma_c}\right)$$

$$x_t = 1 - (1-\vartheta)0.5\kappa\left(\pi_t - \pi_t^*\right)^2 - \vartheta(1 - \varepsilon_{p,t} mc_t)$$

$$\textbf{PC:} \quad \left(\pi_t - \pi_t^*\right)\pi_t = (1-\vartheta)\beta \mathbb{E}_t \, g_{z,t}\tilde{y}_{t+1}/\tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right)\pi_{t+1} + \zeta\kappa^{-1}\varepsilon_{p,t} mc_t + \kappa^{-1}(1-\zeta)$$

$$mc_t = \psi\left(x_t\tilde{y}_t - \omega d\right)^{\sigma_c} \tilde{y}_t^{\sigma_n} \, \Phi\left(\tilde{m}_t\right)^{-(1+\sigma_n)}$$

Deterministic Decoupling trend

Deterministic population trend

$$\textbf{MP:} \quad r_t = r_{t-1}^{\rho} \left[\cdots \tilde{y}_t \cdots \right] \cdots \varepsilon_{r,t}$$

Emission AR(1) shock

$$\textbf{CC:} \quad \tilde{m}_t = (1-\delta_m)\tilde{m}_{t-1} + \xi_m \, \sigma_t \, z_t \, l_t \, \tilde{y}_t \, \varepsilon_{e,t}$$

Anthropogenic carbon stock

Deterministic TFP trend

The New Keynesian Climate Model

Carbon accumulation and its damages:

$$\textbf{IS:}\quad \left(\frac{\tilde{y}_t x_t - \omega d}{1-\omega}\right)^{-\sigma_c} = \beta \mathbb{E}_t \, \frac{\varepsilon_{b,t+1}}{\varepsilon_{b,t}} \frac{r_t}{\pi_{t+1}} \left((1-\omega) \left(\frac{x_{t+1}\tilde{y}_{t+1} - \omega d}{1-\omega}\right)^{-\sigma_c} + \omega d^{-\sigma_c} \right)$$

$$x_t = 1 - (1-\vartheta) 0.5 \kappa \left(\pi_t - \pi_t^*\right)^2 - \vartheta (1 - \varepsilon_{p,t} mc_t)$$

$$\textbf{PC:}\quad \left(\pi_t - \pi_t^*\right) \pi_t = (1-\vartheta) \beta \mathbb{E}_t \; g_{z,t} \tilde{y}_{t+1} / \tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right) \pi_{t+1} + \zeta \kappa^{-1} \varepsilon_{p,t} mc_t + \kappa^{-1} (1-\zeta)$$

$$mc_t = \psi \left(x_t \tilde{y}_t - \omega d\right)^{\sigma_c} \tilde{y}_t^{\sigma_n} \; \Phi\left(\tilde{m}_t\right)^{-(1+\sigma_n)} \longleftarrow \boxed{\textbf{Climate damages}}$$

$$\textbf{MP:}\quad r_t = r_{t-1}^{\rho} \left[r_r \left(\pi_t^*/\pi\right) \left(\pi_t / \pi_t^*\right)^{\phi_\pi} \left(\tilde{y}_t / \tilde{y}_t^n\right)^{\phi_y} \right]^{1-\rho} \left(\pi_t^* / \pi_{t-1}^*\right)^{\phi_{\pi*}} \varepsilon_{r,t}$$

$$\textbf{CC:}\quad \tilde{m}_t = (1-\delta_m) \tilde{m}_{t-1} + \xi_m \; \sigma_t \; z_t \; l_t \; \tilde{y}_t \; \varepsilon_{e,t}$$

The New Keynesian Climate Model

Mitigation policies as function of exogenous carbon tax $\tilde{\tau}_t$:

Mitigation expenditures

$$
\begin{aligned}
\textbf{IS:} \quad \left(\frac{\tilde{y}_t x_t - \omega d}{1-\omega}\right)^{-\sigma_c} &= \beta \mathbb{E}_t \frac{\varepsilon_{b,t+1}}{\varepsilon_{b,t}} \frac{r_t}{\pi_{t+1}} \left((1-\omega)\left(\frac{x_{t+1}\tilde{y}_{t+1} - \omega d}{1-\omega}\right)^{-\sigma_c} + \omega d^{-\sigma_c}\right) \\
x_t &= 1 - (1-\vartheta) 0.5 \kappa \left(\pi_t - \pi_t^*\right)^2 - \vartheta (1 - \varepsilon_{p,t} mc_t) - \theta_{1,t} \tilde{\tau}_t^{\theta_2/(\theta_2 - 1)} \\
\textbf{PC:} \quad \left(\pi_t - \pi_t^*\right) \pi_t &= (1-\vartheta) \beta \mathbb{E}_t \, g_{z,t} \tilde{y}_{t+1} / \tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right) \pi_{t+1} + \zeta \kappa^{-1} \varepsilon_{p,t} mc_t + \kappa^{-1} (1 - \zeta) \\
mc_t &= \psi \left(x_t \tilde{y}_t - \omega d\right)^{\sigma_c} \tilde{y}_t^{\sigma_n} \, \Phi \left(\tilde{m}_t\right)^{-(1+\sigma_n)} + \theta_{1,t} \tilde{\tau}_t \left(\theta_2 + (1-\theta_2) \tilde{\tau}_t^{1/(\theta_2 - 1)}\right) \\
\textbf{MP:} \quad r_t &= r_{t-1}^{\rho} \left[r_r \left(\pi_t^* / \pi\right) \left(\pi_t / \pi_t^*\right)^{\phi_\pi} \left(\tilde{y}_t / \tilde{y}_t^n\right)^{\phi_y}\right]^{1-\rho} \left(\pi_t^* / \pi_{t-1}^*\right)^{\phi_{\pi *}} \varepsilon_{r,t} \\
\textbf{CC:} \quad \tilde{m}_t &= (1 - \delta_m) \tilde{m}_{t-1} + \xi_m \, \sigma_t \, z_t \, l_t \, \tilde{y}_t \, \varepsilon_{e,t} \left(1 - \tilde{\tau}_t^{1/(\theta_2 - 1)}\right)
\end{aligned}
$$

The New Keynesian Climate Model

Mitigation policies as function of exogenous carbon tax $\tilde{\tau}_t$:

Mitigation expenditures

$$\textbf{IS:} \quad \left(\frac{\tilde{y}_t x_t - \omega d}{1-\omega}\right)^{-\sigma_c} = \beta \mathbb{E}_t \frac{\varepsilon_{b,t+1}}{\varepsilon_{b,t}} \frac{r_t}{\pi_{t+1}} \left((1-\omega)\left(\frac{x_{t+1}\tilde{y}_{t+1} - \omega d}{1-\omega}\right)^{-\sigma_c} + \omega d^{-\sigma_c}\right)$$

$$x_t = 1 - (1-\vartheta)0.5\kappa\left(\pi_t - \pi_t^*\right)^2 - \vartheta(1 - \varepsilon_{p,t} mc_t) - \theta_{1,t}\tilde{\tau}_t^{\theta_2/(\theta_2-1)}$$

Carbon tax costs

$$\textbf{PC:} \quad \left(\pi_t - \pi_t^*\right)\pi_t = (1-\vartheta)\beta\mathbb{E}_t\, g_{z,t}\tilde{y}_{t+1}/\tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right)\pi_{t+1} + \zeta\kappa^{-1}\varepsilon_{p,t} mc_t + \kappa^{-1}(1-\zeta)$$

$$mc_t = \psi\left(x_t\tilde{y}_t - \omega d\right)^{\sigma_c} \tilde{y}_t^{\sigma_n}\, \Phi\left(\tilde{m}_t\right)^{-(1+\sigma_n)} + \theta_{1,t}\tilde{\tau}_t\left(\theta_2 + (1-\theta_2)\tilde{\tau}_t^{1/(\theta_2-1)}\right)$$

$$\textbf{MP:} \quad r_t = r_{t-1}^{\rho}\left[r_r\left(\pi_t^*/\pi\right)\left(\pi_t/\pi_t^*\right)^{\phi_\pi}\left(\tilde{y}_t/\tilde{y}_t^n\right)^{\phi_y}\right]^{1-\rho}\left(\pi_t^*/\pi_{t-1}^*\right)^{\phi_{\pi*}}\varepsilon_{r,t}$$

$$\textbf{CC:} \quad \tilde{m}_t = (1-\delta_m)\tilde{m}_{t-1} + \xi_m\, \sigma_t\, z_t\, l_t\, \tilde{y}_t\, \varepsilon_{e,t}\left(1 - \tilde{\tau}_t^{1/(\theta_2-1)}\right)$$

The New Keynesian Climate Model

Mitigation policies as function of exogenous carbon tax $\tilde{\tau}_t$:

Mitigation expenditures

$$
\begin{aligned}
\textbf{IS:} \quad \left(\frac{\tilde{y}_t x_t - \omega d}{1-\omega}\right)^{-\sigma_c} &= \beta \mathbb{E}_t \, \frac{\varepsilon_{b,t+1}}{\varepsilon_{b,t}} \frac{r_t}{\pi_{t+1}} \left((1-\omega) \left(\frac{x_{t+1}\tilde{y}_{t+1} - \omega d}{1-\omega}\right)^{-\sigma_c} + \omega d^{-\sigma_c} \right) \\
x_t &= 1 - (1-\vartheta) 0.5 \kappa \left(\pi_t - \pi_t^*\right)^2 - \vartheta (1 - \varepsilon_{p,t} mc_t) - \theta_{1,t} \tilde{\tau}_t^{\theta_2/(\theta_2 - 1)}
\end{aligned}
$$

Carbon tax costs

$$
\begin{aligned}
\textbf{PC:} \quad \left(\pi_t - \pi_t^*\right) \pi_t &= (1-\vartheta) \beta \mathbb{E}_t \, g_{z,t} \tilde{y}_{t+1} / \tilde{y}_t \left(\pi_{t+1} - \pi_{t+1}^*\right) \pi_{t+1} + \zeta \kappa^{-1} \varepsilon_{p,t} mc_t + \kappa^{-1} (1 - \zeta) \\
mc_t &= \psi \left(x_t \tilde{y}_t - \omega d\right)^{\sigma_c} \tilde{y}_t^{\sigma_n} \; \Phi\left(\tilde{m}_t\right)^{-(1+\sigma_n)} + \theta_{1,t} \tilde{\tau}_t \left(\theta_2 + (1-\theta_2) \tilde{\tau}_t^{1/(\theta_2 - 1)}\right)
\end{aligned}
$$

$$
\textbf{MP:} \quad r_t = r_{t-1}^{\rho} \left[r_r \left(\pi_t^* / \pi\right) \left(\pi_t / \pi_t^*\right)^{\phi_\pi} \left(\tilde{y}_t / \tilde{y}_t^n\right)^{\phi_y} \right]^{1-\rho} \left(\pi_t^* / \pi_{t-}^*\right.
$$

Abatement share

$$
\textbf{CC:} \quad \tilde{m}_t = (1 - \delta_m) \tilde{m}_{t-1} + \xi_m \; \sigma_t \; z_t \; l_t \; \tilde{y}_t \; \varepsilon_{e,t} \left(1 - \tilde{\tau}_t^{1/(\theta_2 - 1)}\right)
$$

Plan

Estimation

- Estimation on world data from 1985Q1 to 2023Q3 (sources: World Bank, OECD and OurWorldInData).
- There are four observable variables:

$$\begin{bmatrix} \text{Real output growth rate} \\ \text{Inflation rate} \\ \text{Short-term interest rate} \\ \text{CO}_2 \text{ emissions growth rate} \end{bmatrix} = 100 \times \begin{bmatrix} \Delta \log(y_t) \\ \pi_t - 1 \\ r_t - 1 \\ \Delta \log(e_t) \end{bmatrix}$$

Estimation

- Our statistical model is an extension of Fair and Taylor (1983) to deal with trends:

$$\tilde{y}_t = g_\Theta\left(y_0, y, 0\right) \tag{1}$$
$$y_t = \mathbb{E}_{t,t+S}\left\{g_\Theta\left(y_{t-1}, \tilde{y}_{t+S+1}, \varepsilon_t\right)\right\} \tag{2}$$
$$\mathcal{Y}_t = h_\Theta\left(y_t\right) \tag{3}$$
$$\varepsilon_t \sim \mathcal{N}\left(0, \Sigma_\varepsilon\right) \tag{4}$$

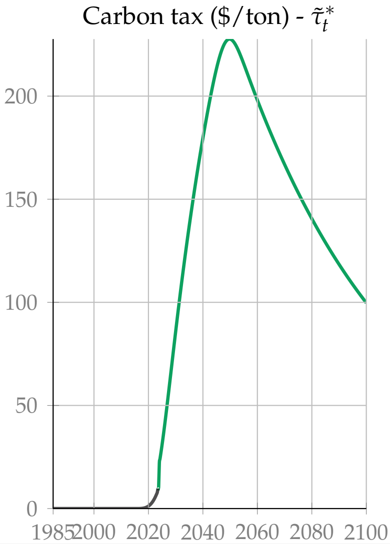
- Compute the deterministic path $\tilde{y}_t$, add stochastic innovations through extended path $\mathbb{E}_{t,t+S}\{\cdot\}$ with the expectation horizon S.
- Maximize sample likelihood $\mathcal{L}(\theta, \mathcal{Y}_{1:T^*})$ & run Metropolis-Hastings to compute uncertainty bands.

Estimation

- Large uncertainty about future carbon tax: implications for estimation in particular at the end of the sample.
- Let $\tilde{\tau}_t^*$ denote the Paris-Agreement tax, with rising carbon tax up to 2050, we let the data inform about the market-based expectations on future carbon mitigation policies:

$$\mathbb{E}_{t,t+S}\{\tilde{\tau}_t\} = \varphi\tilde{\tau}_t^*$$

where $\varphi \in [0,1]$ is the fraction of believers in Paris-Agreement policy.



Calibrated parameters

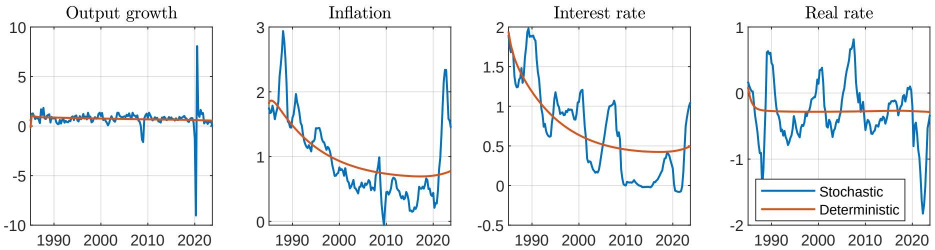
Parameter	Name	Value
Panel A: Climate Parameters		
CO_2 rate of transfer to deep oceans	δ_m	0.00125
Marginal atmospheric retention ratio	ξ_m	0.27273
Pre-industrial stock of carbon (GtC)	m_{1750}	545
Climate damage elasticity	γ	2.379e-05
Initial stock of carbon (GtC)	$m_{1984:4}$	736.98
Panel B: Socio-economic Parameters		
Firm exit shock	ν	0.025
Low productivity worker payoff-to-consumption	d/c	0.85
Initial population (billions)	$l_{1984:4}$	4.85
Terminal population (billions)	l_T	11.42107
Population growth growth	l_g	0.0055
Goods elasticity	ς	6
Decay rate of TFP	$\delta_z \times 400$	0.3
Initial hours worked	$h_{1984:4}$	1
Labor intensity	α	0.7
Initial emissions ($GtCO_2$)	$e_{1984:4}$	5.0825
Initial GDP (trillions USD PPP)	$y_{1984:4}$	11.25
Panel C: Abatement Sector Parameters		
Initial abatement cost	$\theta_{1,1984:4}$	0.30604
Abatement cost	θ_2	2.6
Decay rate of abatement cost	δ_{pb}	0.004277
Initial abatement	$\mu_{1984:4}$	0.0001
Abatement in 2020	$\mu_{2020:1}$	0.05

Estimated parameters

		Prior Distribution			Posterior Distribution		
		Shape	Mean	Std	Mode	Mean	[5%:95%]
Panel A: Shock processes							
Std demand	σ_b	$\mathcal{IG}_2$	0.001	1	0.0245	0.0253	*[0.0231:0.0275]*
Std price	σ_p	$\mathcal{IG}_2$	0.001	1	0.0046	0.0051	*[0.0045:0.0056]*
Std MPR	σ_r	$\mathcal{IG}_2$	0.001	1	0.0009	0.0009	*[0.0008:0.0011]*
Std emissions	σ_e	$\mathcal{IG}_2$	0.001	1	0.0049	0.0047	*[0.0042:0.005]*
AR demand	ρ_b	$\mathcal{B}$	0.5	0.15	0.5974	0.6115	*[0.5965:0.6322]*
AR price	ρ_b	$\mathcal{B}$	0.5	0.15	0.9839	0.9839	*[0.9839:0.9839]*
AR MPR	ρ_r	$\mathcal{B}$	0.5	0.15	0.5407	0.5341	*[0.4711:0.5932]*
AR emissions	ρ_e	$\mathcal{B}$	0.5	0.15	0.9686	0.9707	*[0.9592:0.9823]*
Panel B: Structural parameters							
Initial TFP growth	$g_{z,t_0} \times 400$	$\mathcal{G}$	1.5	0.5	1.735	1.735	*[1.735:1.735]*
Decoupling rate	g_{σ,t_0}	$\mathcal{G}$	1.5	0.5	1.262	1.2254	*[1.1465:1.323]*
Decay TFP	$\delta_z \times 400$	$\mathcal{G}$	0.5	0.35	0.0464	0.0519	*[0.0411:0.0732]*
Risk aversion	σ_c	$\mathcal{G}$	2	0.15	1.1394	1.2608	*[1.1371:1.3715]*
Labor disutility	σ_h	$\mathcal{G}$	2	0.5	0.1708	0.1799	*[0.1649:0.2094]*
Rotemberg Cost	κ	$\mathcal{G}$	25	7.5	117.9202	117.9202	*[117.9194:117.9205]*
Initial inflation trend	$\pi_{*,t_0} \times 400$	$\mathcal{G}$	12	1	12.8434	12.6125	*[11.3687:13.6154]*
Initial inflation trend	$g_\pi \times 400$	$\mathcal{N}$	8	2	9.2703	9.2905	*[9.1232:9.4962]*
Initial interest rate	$r_{t_0} \times 400$	$\mathcal{N}$	12	2	8.7509	8.7746	*[8.3039:9.362]*
Share Low prod.	ω	$\mathcal{B}$	0.05	0.01	0.0512	0.0496	*[0.0422:0.0556]*
Inflation stance	ϕ_π	$\mathcal{G}$	0.75	0.05	0.5883	0.6367	*[0.5702:0.6943]*
MPR GDP stance	ϕ_y	$\mathcal{G}$	0.5	0.1	0.5265	0.5342	*[0.4712:0.6058]*
Discount rate	$(\beta^{-1} - 1) \times 100$	$\mathcal{G}$	1	0.5	0.8055	0.8282	*[0.7976:0.8617]*
Mitigation policy belief	φ	$\mathcal{U}$	0.5	0.2887	0.5264	0.5235	*[0.5047:0.552]*
MPR smoothing	ρ	$\mathcal{B}$	0.5	0.075	0.9127	0.9127	*[0.9127:0.9128]*
Trend stance	ϕ_*	$\mathcal{N}$	0.5	0.5	-0.4482	-0.4434	*[-0.4625:-0.4185]*
Log marginal data density						-3104.54	

Stochastic and deterministic paths

Figure 1: Implied deterministic and stochastic paths



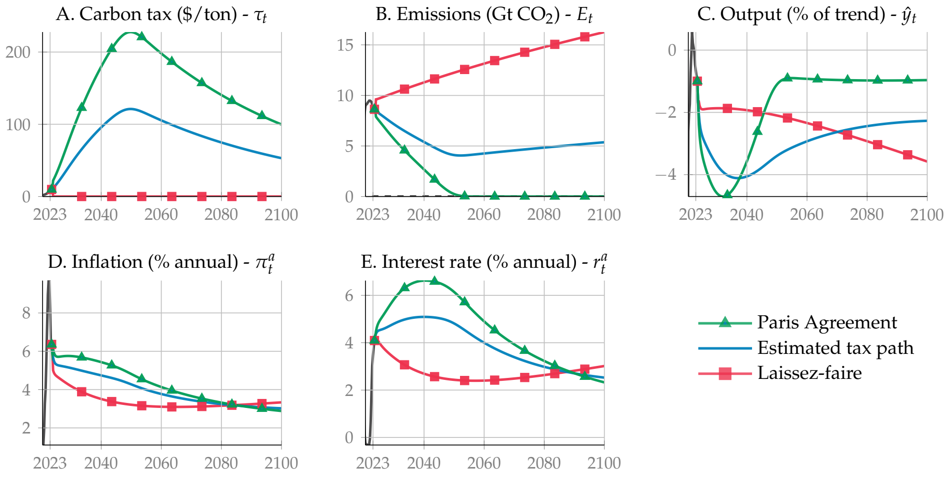
Plan

The Anatomy of Green/Climateflation

- What is the future macroeconomic landscape by the end of the century?
- We consider three alternative scenarios based on the realization of the carbon tax $\varphi\tilde{\tau}_t^*$:
  - Paris-Agreement with $\varphi = 1$.
  - Estimated carbon path with $\varphi = 0.53$.
  - Laissez-fairewith $\varphi = 0$.

Three Transitions

Figure 2: Model-implied projections based on alternative control rates of emissions



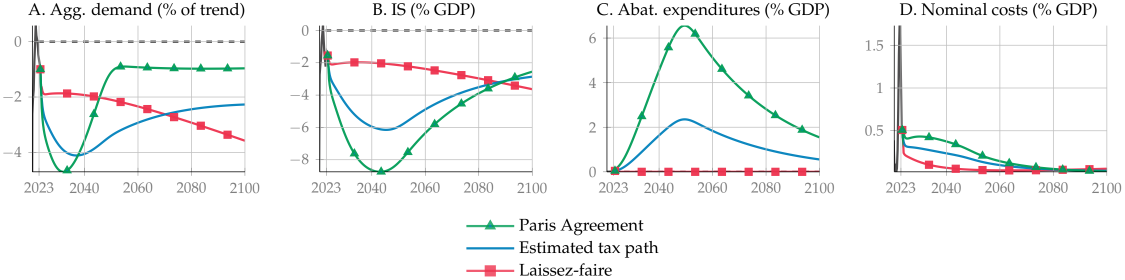
Dissecting the IS curve

One can split the aggregate demand equation into three terms:

$$\hat{y}_t = \underbrace{\widehat{IS}_t}_{\text{forward real interest rate}} + \underbrace{\theta_{1,t}\tilde{\tau}_{e,t}^{\theta_2/(\ \theta_2-1)}}_{\text{green investment}} + \underbrace{(1-\vartheta)\,\frac{\kappa}{2}(\pi_t-\pi)^2+\vartheta\,(1-mc_t)}_{\text{nominal wedge}}\ .$$

- **Forward real interest rate term:** from discounted Euler equation.
- **Green investment term:** from abating more carbon emissions.
- **Nominal wedge term:** from adjusting price and exit shock.

Dissecting the IS curve



- Under Paris-Agreement:
  - The rise in carbon tax triggers a boost in abatement expenditures, and increases aggregate demand.
  - Monetary policy dampens the boom by a real rate increase.
  - Damages are stabilized, but let GDP 1% below the technological trend.

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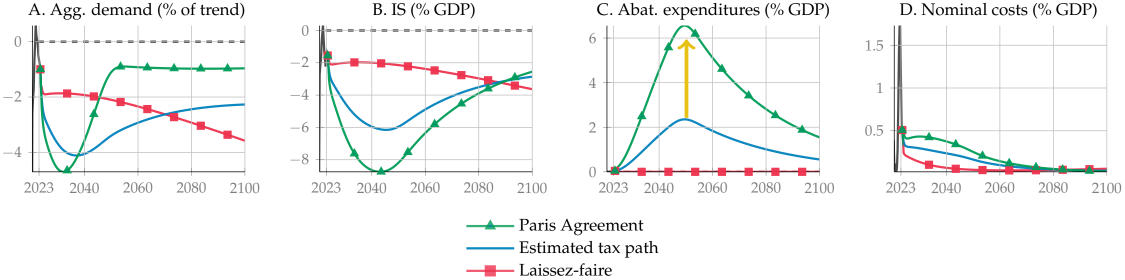

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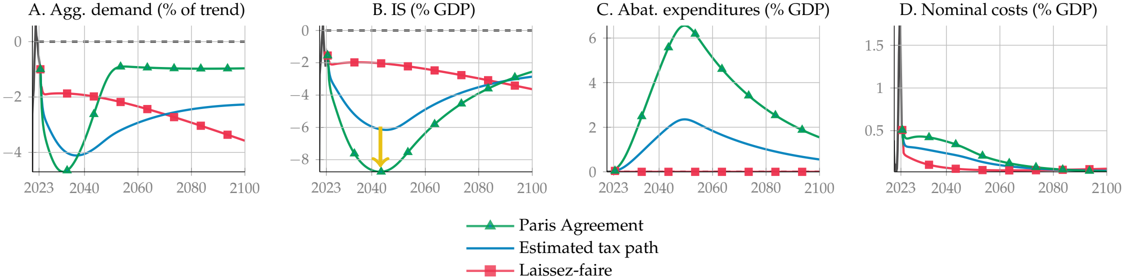

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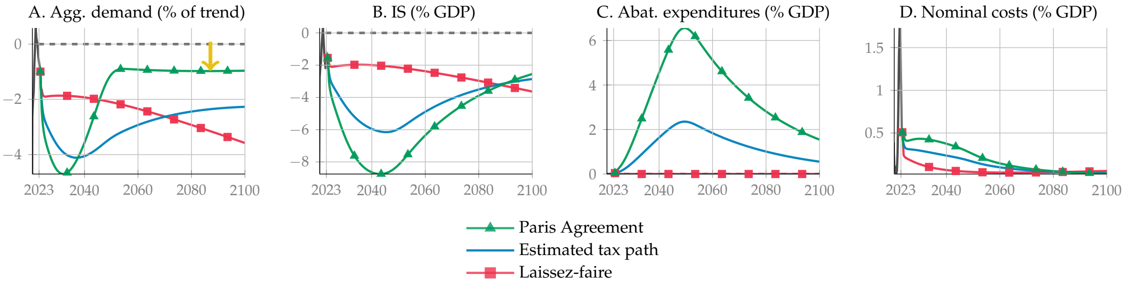

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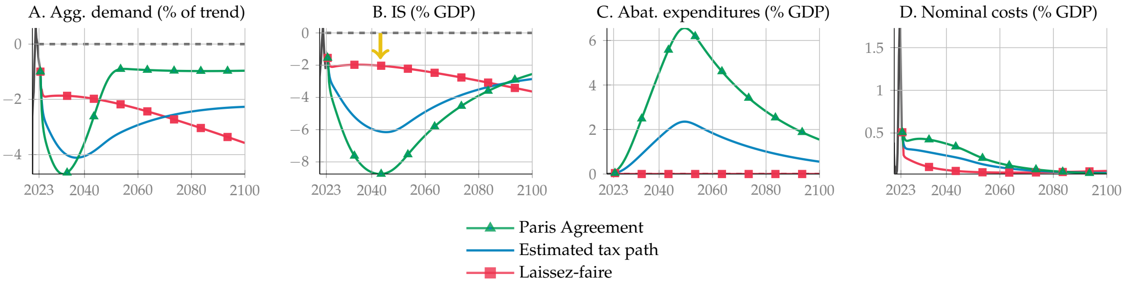
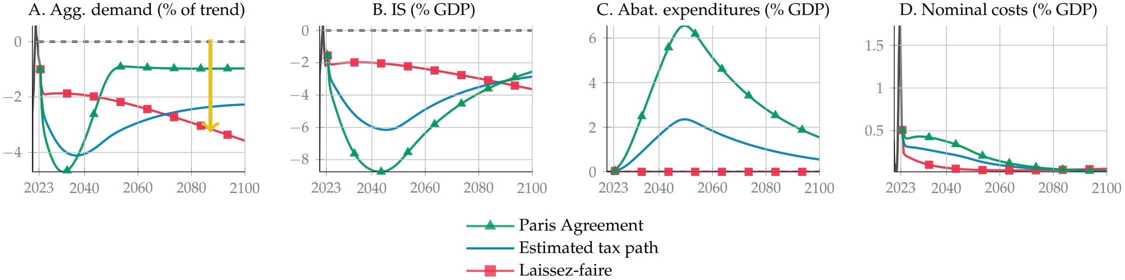


- Under Laissez-faire:
  - Monetary policy contains the surge in inflation by maintaining real rate high.
  - As damages grow, the IS permanently deteriorates aggregate demand.

Dissecting the IS curve



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Dissecting the PC curve

One can split the marginal cost into three term:

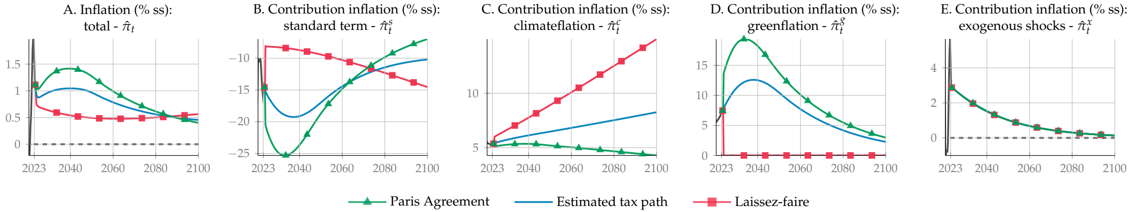
$$mc_t = \underbrace{\tilde{w}_t}_{\text{standard}} / \underbrace{\Phi\left(m_t\right)}_{\text{climateflation}} + \underbrace{\theta_{1,t}\mu_t^{\theta_2} + \tau_{e,t}\sigma_t\left(1-\mu_t\right)\varepsilon_{e,t}}_{\text{greenflation}}, \tag{5}$$

which allows to break down inflation into 4 different forces:

$$\hat{\pi}_t \simeq \underbrace{\hat{\pi}_t^s}_{\text{standard term}} + \underbrace{\hat{\pi}_t^c}_{\text{climateflation}} + \underbrace{\hat{\pi}_t^g}_{\text{greenflation}} + \underbrace{\hat{\pi}_t^x}_{\text{exogenous shocks}} \tag{6}$$

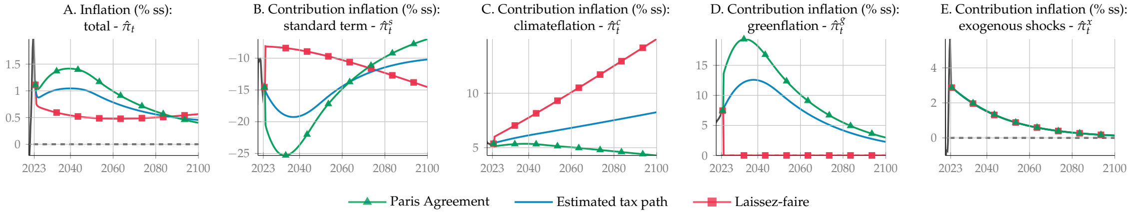
with $\hat{\pi}_t = \pi_t - \pi_t^\star$

Dissecting the PC curve



- Very different inflation dynamics between the 2 regimes.
- What drives this gap?

Dissecting the PC curve



- Under Paris Agreement:
  - The immediate increase in carbon tax fuels inflation.
  - But increasing abatement expenditures reduces both consumption and in turn the wealth effect on the labor supply.
  - Reducing emissions also stabilizes damages and inflation.

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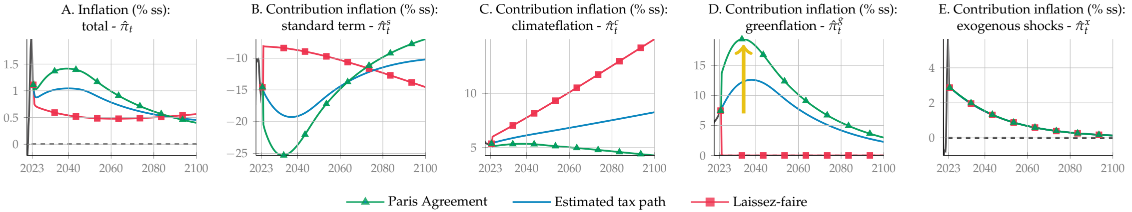

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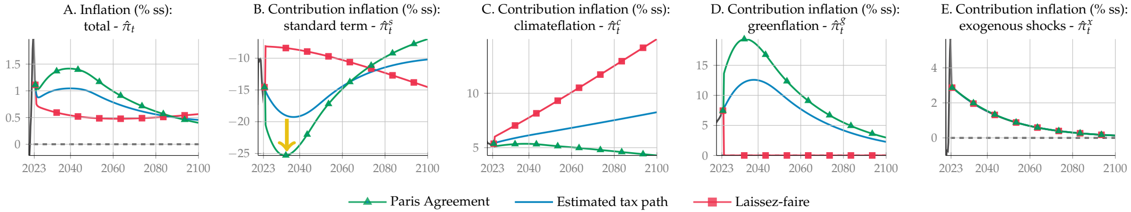

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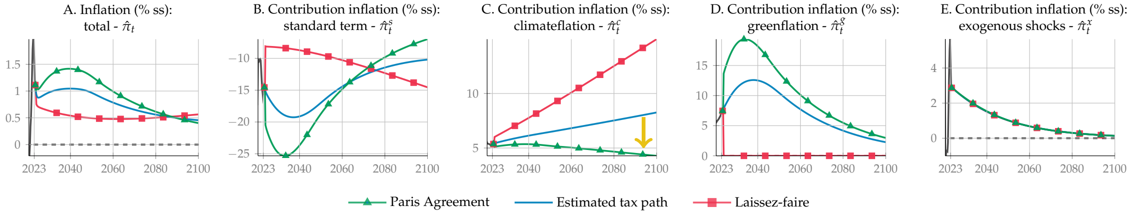

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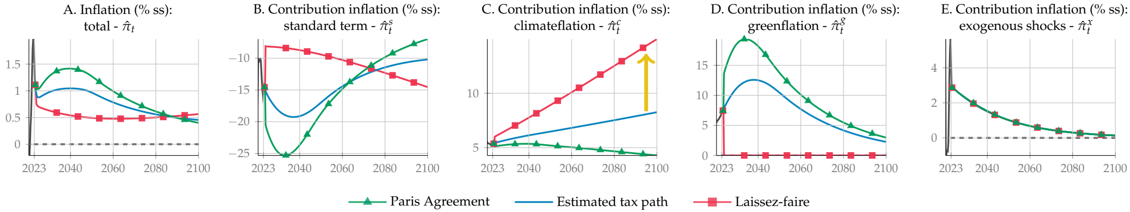

- Under Laissez-faire:
  - The rising damage makes resources scarcer: ever growing inflation as long as planet warms.
  - Disengagement from carbon policy makes carbon price to be zero.
  - Standard term follows the recessionary forces from in-sample inflation, but decreases as climate grows.

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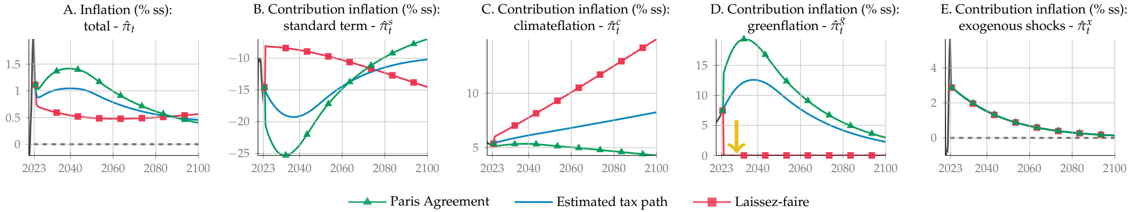

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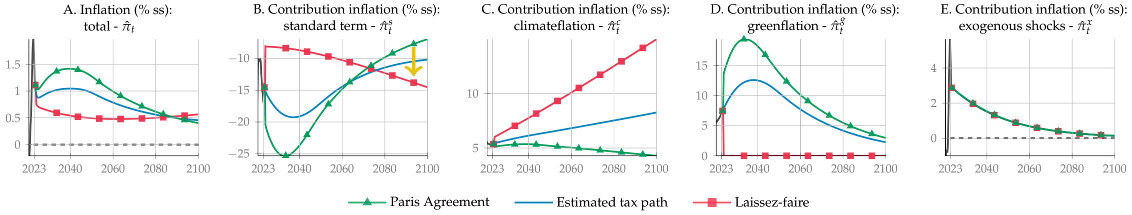

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Plan

Conclusion

- This paper has developed a four-dimensional New Keynesian model with climate externality.
- This framework allows us to identify two phenomena faced by the central bank:
  - The first one is a persistent negative supply shock called *climateflation* that arises from the deleterious effects of climate change itself:
  - The second one is a transitory positive demand shock called *greenflation* that appears following the implementation of a climate mitigation policy;
- Ongoing work: analyzing the conduct of monetary policy in the wake of those two phenomena.

Thank you for your attention

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Households

- Consider a mass of households $l_t = l_{t-1}\left(l_T / l_{t-1}\right)^{\ell_g}$, with convergence rate $\ell_g \in [0,1]$ to population size $l_T \geq 0$;

- The fraction ω of high productive workers receives a wage payments $w_t n_{i,H,t}$;
- The discounted Euler Equation for high and low productive workers $\{H, L\}$:

$$c_{i,H,t}^{-\sigma_c} = \mathbb{E}_t \left\{ \frac{\tilde{\beta}_{t,t+1} \varepsilon_{b,t+1} r_t}{\varepsilon_{b,t} \pi_{t+1}} \left((1-\omega)\, c_{i,H,t+1}^{-\sigma_c} + \omega c_{i,L,t+1}^{-\sigma_c} \right) \right\} \tag{7}$$

$$c_{i,L,t} = D_t \tag{8}$$

$$c_t = (1-\omega)\, c_{H,t} + \omega c_{L,t} \tag{9}$$

- Labor supply equation:

$$w_t = \psi_t n_{i,H,t}^{\sigma_n} c_{i,H,t}^{\sigma_c}. \tag{10}$$

Return

Production

- Monopolistic firm j with production function:

$$y_{j,t} = z_t \Phi\left(m_t\right)\left(n^d_{j,t}\right)^\alpha \tag{11}$$

$z_t = z_{t-1}(1 + g_{z,t})$ exogenous vanishing productivity with $g_{z,t} = g_{z,t-1}\left(1 - \delta_z\right)$

- Following Golosov et al. (2014), exponential economic damages:

$$\Phi(m_t) = \exp(-\gamma(m_t - m_{1750})) \tag{12}$$

$m_t - m_{1750} \geq 0$ the excess carbon in the atmosphere
$\gamma \geq 0$ the damage parameter

Return

Pricing

- Pricing is addressed as a two stage problem.
- 1st stage: determination of the marginal cost

$$\max_{\{y_{j,t},\mu_{j,t}\}} mc_{j,t}y_{j,t} - \underbrace{w_t n_{j,t}^d}_{\text{labor cost}} - \underbrace{\theta_{1,t}\mu_{j,t}^{\theta_2}y_{j,t}}_{\text{Abatement cost}} - \underbrace{\tau_{e,t}e_{j,t}}_{\text{carbon tax}} \tag{13}$$

$\theta_{1,t} = (p_b/\theta_2)(1-\delta_{pb})^{t-t_0}\sigma_t$ exogenous efficiency of abating carbon ($\theta_{1,2020} \simeq 0.1 \rightarrow$10% of output lost if $\mu = 1$, it takes 40 years for $\theta_{1,t}$ to divide by factor 2)

$\tau_{e,t}$ exogenous carbon tax.

⏩ Return

Pricing

- 2nd stage: determination of selling price:

$$\max_{\{p_{j,t}\}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \omega_{j,t+s} \left(y_{j,t+s} \frac{p_{j,t+s}}{p_{t+s}} - \varepsilon_{p,t+s} mc_{t+s} y_{j,t+s} - \frac{\kappa}{2} \left(\frac{p_{j,t+s}}{p_{j,t-1+s}} - \pi^*_{t+s} \right)^2 \frac{y_{t+s}}{l_{t+s}} \right) \right\}$$

$\varepsilon_{p,t}$ an AR(1) cost-push shock
$\omega_{j,t} \in \{0,1\}$ an idiosyncratic exit shock with $\Pr(\omega_{j,t} = 0) = \vartheta$

Return

Monetary policy

Monetary policy rule as in the macro textbook:

$$r_t = r_{t-1}^{\rho} \left[r_r \frac{\pi_t^*}{\pi} \left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{y_t}{y_t^n} \right)^{\phi_y} \right]^{1-\rho} \left(\frac{\pi_t^*}{\pi_{t-1}^*} \right)^{\phi_*} \varepsilon_{r,t} \tag{14}$$

$\varepsilon_{r,t}$ AR(1) shock
ρ coefficient smoothing, inflation ϕ_π and output gap stances ϕ_y
π inflation target, r long run rate and natural output y_t^n

Return

Climate

- The law of motion of atmospheric carbon stock (GtC):

$$m_t - m_{1750} = (1 - \delta_m)(m_{t-1} - m_{1750}) + \xi_m e_t, \tag{15}$$

$\delta_m \in [0,1]$ rate of transfer of atmospheric carbon to the deep ocean;
$\xi_m \geq 0$ physical parameter translating $GtCO_2$ into GtC.

- CO2 emissions are given by:

$$e_t = \sigma_t \left(1 - \mu_t\right) y_t \varepsilon_{e,t} \tag{16}$$

$\sigma_t = \sigma_{t-1}(1 - g_{\sigma,t})$ decoupling rate of carbon emissions
$\varepsilon_{e,t}$ AR(1) emission shock
μ_t abatement share

Return