Repo Specialness in the Transmission of Quantitative Easing*

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I show that the repo specialness of sovereign bonds can magnify the transmission of central bank quantitative easing into the real economy. Investors who cannot take advantage of the repo specialness of government bonds substitute for government bonds with riskier assets, such as corporate bonds. The extra demand from this portfolio substitution lowers corporate financing costs. I quantify the magnitude of this repo specialness channel for quantitative easing (QE) transmission in the context of the Public Sector Purchase Program of the Eurosystem.

1. Introduction

This paper shows that the specialness premium of government bonds in repurchase agreement (repo) markets can boost a central bank's effort to lower corporate bond yields by purchasing government bonds. I empirically estimate the magnitude of this channel while the Eurosystem has been buying eurozone government bonds through its Public Sector Purchase Program (PSPP). Yields on 9-year euro-denominated corporate bonds that are rated A by S&P declined by 95.7 basis points from March 3, 2014 to February 29, 2016. I associate 31.9 basis points of this decline with an increase in the repo specialness premium of German federal government bonds (bunds), benchmark bonds in the European sovereign bond market.

Figure 1 provides an illustrative example of a repo transaction. A government bond repo is a contract in which one firm (A) sells a government bond to another firm (B), with a promise to

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purchase it back at a future date. In effect, firm B makes a cash loan to firm A, collateralized with a government bond. The repo rate is the rate of return on the cash investment of firm B, as indicated by the initial sale price and the future re-purchase price of the bond. If the bond is scarce in the repo market, it is costly for firm B to find an alternative counter-party that owns that bond. Consequently, firm B may be willing to lend cash at a repo rate lower than the risk-free rate. The downward deviation of the repo rate (1.5%) from the risk-free rate (2%) is the repo specialness premium (0.5%). By borrowing cash at a low repo rate and subsequently investing it at a higher risk-free rate, firm A earns extra revenue. In repo market terminology, firms A and B are conducting a repo and a reverse repo, respectively. The repo specialness premium tends to be larger if the bonds are harder to locate in the market (Duffie, 1996; Corradin and Maddaloni, 2017; Jank and Mönch, 2018, 2019).

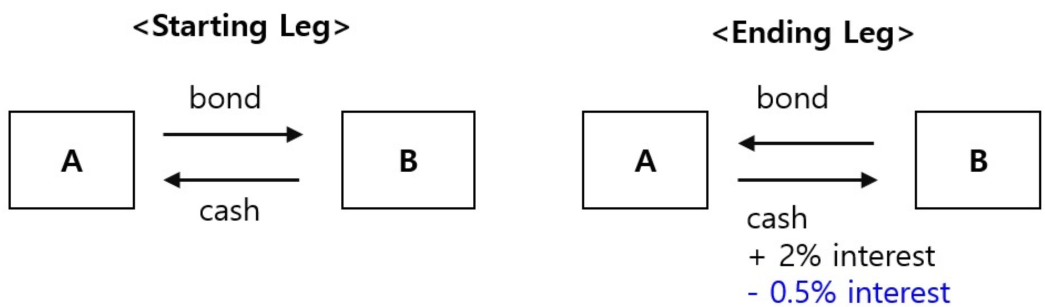

Figure 1: **Illustrative Example of a Repo Transaction.** This diagram illustrates an example of a repo transaction in which firm B reverse repos a bond from firm A. Firm A repos the bond to firm B. The risk-free rate is 2%. The repo rate is 1.5%. The downward deviation of the repo rate from the risk-free rate, which is 0.5% in this example, is the specialness premium.

This paper is motivated by the PSPP through which the Eurosystem has been purchasing the debt instruments of eurozone governments and agencies in the secondary market since March 2015. After their purchases of government bonds, the Federal Reserve or the Bank of Canada supplied back a signification fraction of these bonds to private firms, either through repos or securities lending agreements.[1] In contrast, the Eurosystem has not supplied back to the market most of the bonds that it purchased under the PSPP. For example, in November 2018, the Eurosystem was not willing to repo more than 75 billion euros of securities to private firms. This limit accounts for less than 4% of the total value of bonds that the Eurosystem held for the PSPP at the time.[2] Whether intended or not, this decision of the Eurosystem substantially reduced the availability of eurozone government bonds in the market and, thus, increased the repo specialness of these bonds (Arrata, Nguyen, Rahmouni-Rousseau and Vari, 2018; Jank and Mönch, 2018, 2019). Indeed, Figure 2 shows that the repo specialness premium of a 9-year bund increased steadily during the PSPP period. I show that the heightened repo specialness

[1] See section §D in the appendix for details.

[2] The number is from the European Central Bank website. See section F.3 for more information on the restrictive features of the securities lending facilities that the Eurosystem operated.

premium of securities that were purchased under the PSPP significantly lowered corporate financing costs. Reducing corporate financing costs is one of the most important goals of QE programs (see Praet (2015) and Bernanke (2011) for the objectives of the Eurosystem and the Federal Reserve respectively). Intentionally or not, the Eurosystem magnified the stimulus to the eurozone economy of its PSPP through an increase in repo specialness.

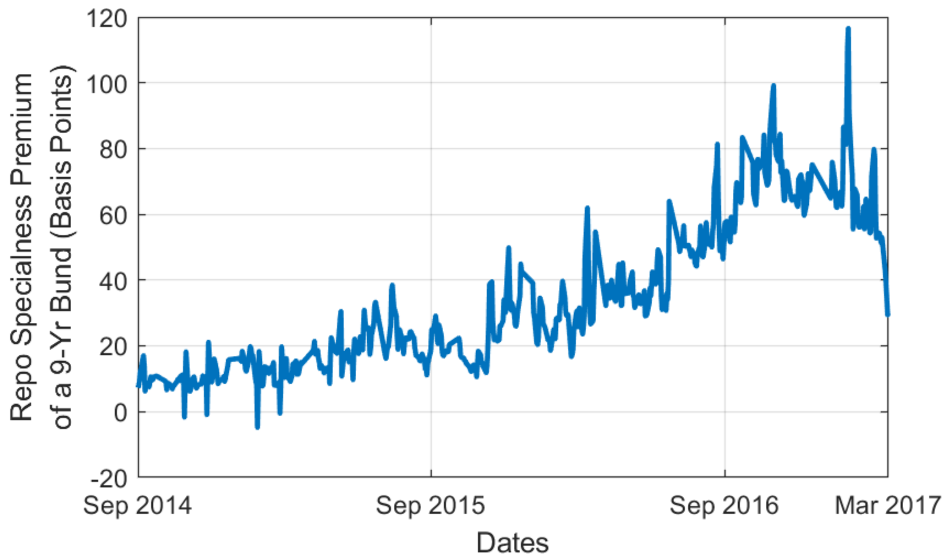

Figure 2: **Bund Repo Specialness Premium.** Data is from BrokerTec. The figure plots the repo specialness premium of a German federal government bond with a remaining maturity of 9 years. If there is no bond with remaining maturity exactly equal to 9 years, Nadaraya-Watson kernel regression is used to estimate the repo rate on a 9-year bond. Repo specialness premium is defined as the 9-year overnight index swap (OIS) rate minus the repo rate. I removed data on quarter ends.

The magnification of the stimulus of the PSPP is useful mainly because of the self-imposed rules of the Eurosystem that limit the size of the PSPP. For example, the Eurosystem does not purchase under the PSPP more than 33% of the outstanding debts of any individual government.[3] Some of the rules that the Eurosystem introduced at the beginning of the program became binding constraints and, thus, had to be relaxed for the Eurosystem to keep purchasing bonds for the PSPP (Gros, 2016). Some economists have expressed concerns that the size limit may force the Eurosystem to end the program prematurely, even if the inflation is not high enough (Gros, 2016). Nevertheless, it is politically challenging for the Eurosystem to keep relaxing these rules. A sizable expansion of the PSPP by amending these rules may invite criticisms that the PSPP is violating the European Law on the prohibition of monetary financing (e.g., the direct financing of governments by the central bank) (Mersch, 2016; Boysen-Hogrefe, Fieldler, Jannsen, Kooths and Reitz, 2016).[4] Given these inherent constraints on the

[3] See Article 5 of the Decision 2015/774 of the European Central Bank of 4 March 2015 on a Secondary Market Public Sector Asset Purchase Programme (ECB/2015/10).

[4] The Article 123 of the Treaty on the Functioning of the European Union.

size of the PSPP, it could be valuable to increase the stimulus of the program per unit amount of purchase of government bonds. The repo specialness effect meets this objective, through lowering corporate financing costs.

My theoretical model works roughly as follows. Suppose a central bank purchases government bonds without repoing these bonds back to the market. As fewer bonds circulate in the repo market, the specialness premium of government bonds rise. Consequently, a dealer bank owning these government bonds can earn more revenue in the repo market. Dealers quote higher prices to their non-bank customers, such as insurance companies and pension funds (ICPFs), to reflect opportunities to earn more revenue in the repo market (Duffie, 1996; Jordan and Jordan, 1997; Buraschi and Menini, 2002; Duffie, Garleanu and Pedersen, 2002; Krishnamurthy, 2002; D'Amico, Fan and Kitsul, 2018; DAmico and Pancost, 2017).[5] Nevertheless, these ICPFs cannot earn revenue from the repo specialness nearly as effectively as dealers do.[6] For example, it is costlier for these ICPFs to search for, and then negotiate with, a counter-party to take the other side of the repo (Hill, 2015b). They also cannot effectively manage the risks (e.g., counter-party credit risk) involved in the transaction (Hill, 2017). Finding government bond yields too low, ICPFs substitute for government bonds with corporate bonds. The corporate bonds appreciate as a result.

ICPFs invest heavily in the long-term non-financial corporate bonds market in Europe, so much so that their portfolio allocations can significantly affect these bond prices. For example, in the second quarter of 2016, ICPFs accounted for 43% of private holdings (e.g., excluding holdings by central banks) of eurozone non-financial corporate debt that has remaining maturities of more than one year.[7]

To quantify the repo specialness effect, I run a panel regression of corporate bond yields on bund repo specialness premium. My repo data is from BrokerTec, a major inter-dealer electronic platform for the European repo market. The purchases of government bonds under the PSPP has been profoundly affecting the supply of these government bonds in the repo market and, thus, their specialness premia (Arrata, Nguyen, Rahmouni-Rousseau and Vari, 2018; Jank and Mönch, 2018, 2019). Therefore, I instrument the bund repo specialness using the rules that the Eurosystem used to determine how much to purchase each bund. I associate a third of the decline in yields on 9-year corporate bonds that are rated A by S&P from March 2014 to February 2016 with an increase in the repo specialness premium of bunds.

Section §2 explains the marginal contribution of this paper to the literature. Section §3 describes the setting of my theoretical model. Section §4 solves for equilibrium and illustrates the theoretical intuition of my transmission channel of a QE. In section §5, I numerically calibrate my theoretical model. In section §6, I describe data, and my empirical strategy is detailed in section §7. In section §8, I present the results of my instrumental variable

[5]See Dunne, Hau and Moore (2014); Ejsing and Sihvonen (2009); Schlepper, Hofer, Riordan and Schrimpf (2017) for information on the dealer-to-customer segment of the European government bond market.

[6]See section F.2 for more details.

[7]The data source is the Statistical Data Warehouse of the European Central Bank.

regression. In section §9, I discuss other channels through which the repo specialness premium of government bonds can affect corporate financing costs. I also explore the implication of the repo specialness effect for the fiscal policies as well as the allocative efficiency in the bond and repo market. I conclude in section §10.

2. Literature Review and Marginal Contribution

This paper contributes to a growing strand of theoretical literature that shows how open market operations of the central bank, which is called quantitative easing (QE) programs, can affect asset prices. My paper demonstrates a new channel that involves the repo specialness premium. To my knowledge, the repo-specialness channel for increasing the impact of the central bank's purchase of government bonds on the prices of other assets has not been identified in previous research.

The seminal work of Wallace (1982) shows that open market operations cannot impact asset prices in traditional models with the representative household. Since the representative agent ultimately owns the central bank, a mere exchange of one asset for another on the balance sheet of the central bank does not impact the marginal consumption process of households (Woodford, 2012). Consequently, the representative household price assets with the same pricing kernel. Since it is completely irrelevant whether reserves or other assets are on the central bank balance sheet, open market operations have no impact on asset prices in the world of Wallace (1982).

Nevertheless, since the financial crisis, many central banks have initiated QE programs specifically to affect asset prices and make their monetary policies more accommodating. In response, scholars began studying what conditions of Wallace (1982) model need to be relaxed for QE programs to be able to affect asset prices. For example, asset prices can change if the purchased asset gives some non-pecuniary benefit more than central bank deposits do (Woodford, 2012). The convenience yield theory of U.S. Treasuries (Krishnamurthy and Vissing-Jorgensen, 2012a,b) nicely supports this idea. The safety of cash flow from holding Treasuries is not the only reason investors like Treasuries. Treasuries are also useful as collateral for financial transactions. Fed reserves are less valuable for financial transactions because only banks can access them. By purchasing Treasuries with reserves, the Fed reduces the supply of assets that private investors can use to back financial transactions. Consequently, investors are willing to pay higher prices to obtain assets that can back financial transactions. Krishnamurthy and Vissing-Jorgensen (2012b) call this "asset scarcity channel."

My work differs from the asset scarcity channel to the extent that scarcity of the purchased asset in the underlying cash market and that in the repo market do not always go hand in hand. I use "cash market" to refer to the market that involves the outright purchase and sale of government bonds. Throughout the remainder of the paper, the term "cash market" specifically refers to the government bond cash market. QE programs inevitably make it more difficult for private investors to own the asset. Nevertheless, the central bank can make it more or less difficult to borrow the same asset by adjusting its supply of the purchased asset to the repo market.

For the same reason, my work departs from those that focus on QE-induced scarcity of particular class of assets (e.g., assets with long duration) to explain the asset pricing implications of QEs (Altavilla, Carboni and Motto, 2015; Ferrero, Loberto and Miccoli, 2017). Most works are concerned with keeping certain assets out of circulation in the underlying cash market, not the repo market.

In particular, Ferdinandusse, Freier and Ristiniemi (2017) notes that QE may make it more difficult for investors to locate counterparties to trade the asset purchased under the QE. An increase in search frictions moves asset prices a la Duffie, Garleanu and Pedersen (2005). While Ferdinandusse, Freier and Ristiniemi (2017) studies search frictions in the underlying cash market, I focus on search frictions in the repo market.

This paper is also related to empirical literature analyzing asset price movement in response to the PSPP (Altavilla, Carboni and Motto, 2015; Falagiarda, McQuade and Tirpak, 2015; Andrade, Breckenfelder, De Fiore, Karadi and Tristani, 2016; Blattner and Joyce, 2016; De Santis, 2016; Arrata, Nguyen, Rahmouni-Rousseau and Vari, 2018; Arrata and Nguyen, 2017; Corradin and Maddaloni, 2017; De Santis and Holm-Hadulla, 2017; Ferrari, Guagliano and Mazzacurati, 2017; Karadi, 2017; Lemke and Werner, 2017; Schlepper, Hofer, Riordan and Schrimpf, 2017; Albertazzi, Becker and Boucinha, 2018; Koijen, Koulischer, Nguyen and Yogo, 2018; Jank and Mönch, 2018; Pelizzon, Subrahmanyam, Tomio and Uno, 2018; Jank and Mönch, 2019). It is also related to a line of works analyzing how European repo rates are determined (Mancini, Ranaldo and Wrampelmeyer, 2015; Arrata, Nguyen, Rahmouni-Rousseau and Vari, 2018; Boissel, Derrien, Ors and Thesmar, 2017; Corradin and Maddaloni, 2017; Nyborg, 2017; Ferrari, Guagliano and Mazzacurati, 2017; Sangiorgi, 2017; Piquard and Salakhova, 2018; Brand, Ferrante and Hubert, 2019; Nyborg, 2019). Most works tackle the underlying cash market or the repo market separately, with the notable exception of D'Amico, Fan and Kitsul (2018). D'Amico, Fan and Kitsul (2018) estimate the impact of the repo specialness of Treasuries on Treasury yields throughout the QE of the Fed. Growing repo specialness during the implementation of a QE is not only relevant for the prices of the assets purchased but also important in amplifying the transmission of QE programs into the real economy (whether intentionally or not).

Lastly, this paper engages with the literature showing QE programs can lower corporate or household financing costs and, thus, create higher inflation. One of the mechanisms studied in the literature is the portfolio rebalancing channel (Vayanos and Vila, 2009; Krishnamurthy and Vissing-Jorgensen, 2012b; Carpenter, Demiralp, Ihrig and Klee, 2013; Joyce, Liu and Tonks, 2014; Andrade, Breckenfelder, De Fiore, Karadi and Tristani, 2016; Bua and Dunne, 2017; Albertazzi, Becker and Boucinha, 2018; Bergant, Fiodra and Schmitz, 2018; Boermans and Keshkov, 2018; Chadha and Hantzsche, 2018; Goldstein, Witmer and Yang, 2018). For example, in response to QE-induced appreciation of government bonds, banks substitute holdings of government bonds with loans and other riskier assets (Gambetti and Musso, 2017; Albertazzi, Becker and Boucinha, 2018; Paludkiewicz, 2018; Tischer, 2018). This paper differs from earlier works to the extent that repo specialness premium plays the key role in prompting portfolio substitution.

3. Model

I build a continuous-time infinite-period model with risk-neutral agents, in line with Duffie, Garleanu and Pedersen (2002, 2005). Throughout the remainder of the paper, I will color all exogenous variables in blue. Table 4 summarizes the definitions of parameters that appear in the model. Table 5 summarizes random variables. Proofs of all lemma and theorems can be found in the appendix.

Following Duffie, Garleanu and Pedersen (2002), I fix probability space $(\Omega, \mathcal{F}, P)$. I also fix filtration $\{\mathcal{F}_t : t \geq 0\}$ that is assumed to satisfy the usual conditions of Protter (1990). This filtration $\mathcal{F}_t$ models the common information of agents at time t.

There are two assets: government bond and investment-grade corporate bond. The main intuition would go through with any financial asset that non-banks consider substitutable for a government bond. I choose an investment-grade corporate bond because non-banks are likely to consider it the most substitutable asset for a government bond. There is also consumption good that plays the role of numeraire.

There are four markets: over-the-counter government bond repo market, government bond dealer-to-dealer (D2D) cash market, government bond dealer-to-customer (D2C) cash market, and corporate bond cash market. I model the repo market as an over-the-counter market with search frictions. This modeling approach is necessary because specialness premium can arise only if the collateral cannot circulate at infinite speed.

My theoretical model features the SC repo market for government bonds. The specific collateral (SC) market and the general collateral (GC) market are the two main segments of the repo market. In the GC market, the collateral provider can fulfill its contractual obligation by delivering any security from a pre-approved list of securities. In contrast, in the SC market, the collateral provider must deliver the exact asset that its counterparty requires. A need to obtain specific collateral drives SC repo contracts. In contrast, a need to borrow cash drives GC repos. Collateral only serves to secure the transaction.

Figure 3 shows the main players in my model. N dealer banks intermediate both the government bond repo market and government bond cash market. Ex-ante identical dealers are indexed by $i \in \{1, 2, ..., N\}$. Similarly, a continuum of non-banks indexed by j is distributed along the interval $[0, m_n]$ with Lebesgue measure ϕ_n. Each non-bank can be at one of the four states that I further explain in subsection 3.2. k indexes a continuum of cash investors distributed along the interval $[0, m_c]$ with Lebesgue measure ϕ_c. For now, I assume that the probability space $(\Omega, \mathcal{F}, P)$ and investor's measure space has a Fubini extension a la Sun (2006) (Duffie, 2017).

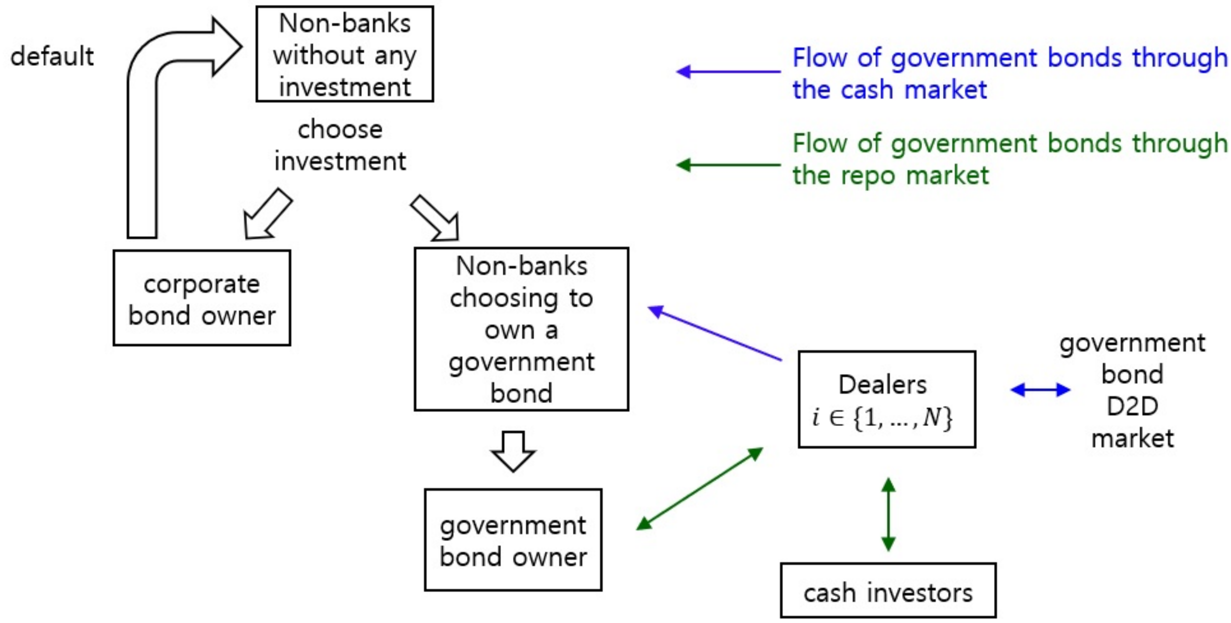

Figure 3: **Overview of the Model.** D2D market stands for the dealer-to-dealer market.

3.1. Assets

A government bond is a perpetual risk-free bond with a flow of coupon c per unit of time. A corporate bond is identical to a government bond except that it can default. The default time is a stopping time τ_η with exogenous intensity η. Default shocks arrive independently across a continuum of all corporate bonds in the economy. All agents in the economy can borrow or lend at the exogenous risk-free rate r. Repos mature stochastically with exogenous rate δ. This assumption is needed to make the investor's problem stationary, as in He and Xiong (2012).

3.2. Non-banks

Non-banks in my model can be most appropriately mapped into insurance companies or pension funds in the real world. Figure 4 gives an overview of the life cycle of a non-bank. Each non-bank in the "N" state decides between owning a government bond or an investment grade (IG) corporate bond. For example, an insurance company regularly re-invest the cash they receive from their insurance clients. To match a long duration on its liability side, the insurance company does not want to hold cash for a long time.

A non-bank can buy a government bond from one of N dealers. After the purchase, the non-bank can generate extra revenue by supplying the purchased bond to the repo market. Alternatively, the non-bank can buy an IG corporate bond from corporate bond dealers. If the corporate bond defaults, the non-bank transitions back to the "N" state.

The non-bank can change its investment decision - whether to purchase a government bond or a corporate bond - in the IG state or the OW state. For example, a non-bank in the IG

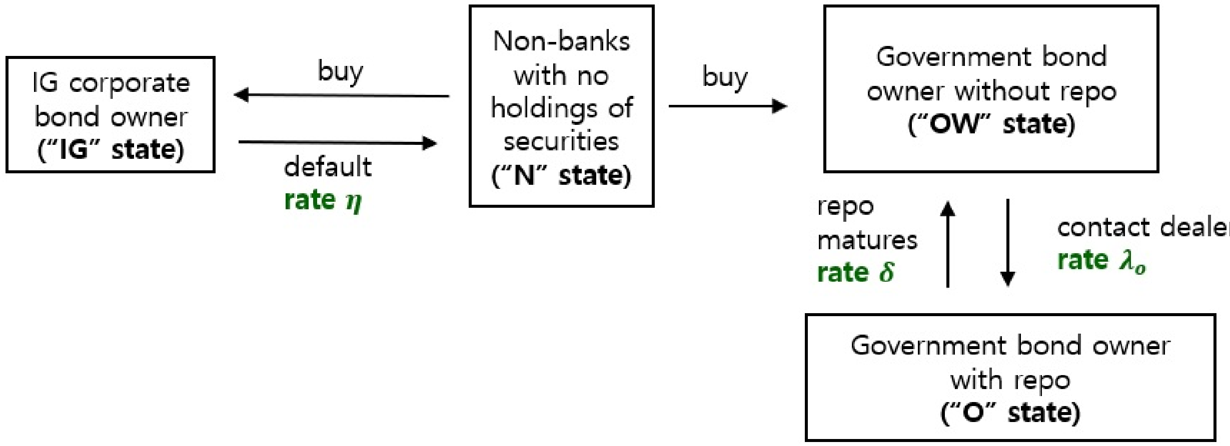

Figure 4: **Type Transition Diagram of Non-Bank**

state can sell its corporate bond and buy a government bond. However, the type transition dynamics in Figure 4 follow a continuous-time Markov chain. In a steady state equilibrium, the non-bank faces the same problem at any point in time. Thus, the non-bank will prefer to own a government bond over a corporate bond in any of the three states - IG state, OW state, and N state.

Let W_t denote the wealth that the non-bank invests in the risk-free bank account. W_t is the only state variable in the non-bank's problem. Let $V_\sigma(W_t)$ denote the value function of the non-bank at the state $\sigma \in \{N, O, OW, IG\}$. Let P and P_{ig} denote the prices at which the non-bank can purchase a government bond or a corporate bond from dealers, respectively. Equation (1) shows the non-bank's problem in the N state.

$$V_N(W_t) = max\{V_{OW}(W_t - P), V_{IG}(W_t - P_{ig})\} \tag{1}$$

3.2.1. Government Bond Owning Non-Banks

The non-bank enters the "OW (owner waiting)" state immediately after purchasing a government bond from a dealer: it has long position in a government bond but has not repoed the bond yet. It immediately starts searching for dealers to repo the purchased bond.

OW non-banks and dealers meet with each other through a pairwise independent random matching process as in Duffie, Garleanu and Pedersen (2005). Let λ_o denote the endogenous search intensity that the OW non-bank chooses. From the perspective of one individual OW non-bank, its encounter with dealers is a Poisson shock process. The contact time is the first arrival time of the Poisson process with intensity λ_o. Let stopping time τ_D denote the contact time. Conditional on the event that a given OW non-bank meets one of the dealers, each dealer is equally likely to be the non-bank's counter-party. The matching process is pairwise independent in the following sense (Duffie, 2017) : for almost every OW non-bank, its matching process with dealers is independent of the matching process of almost every other OW non-bank. As in He and Milbradt (2014), all pairwise random matching processes in this model represent uninsurable uncertainty. Consequently, Poisson shock processes modeling matching between non-banks and dealers are independent of all other random variables in the model.

The OW non-bank wants to make its search intensity λ_o large to monetize repo specialness premium as soon as possible. Nevertheless, as in Duffie, Malamud and Manso (2009), the non-bank incurs search cost $K(\lambda_o)$ per unit of time. $K(\cdot)$ is an exogenous function that is increasing and convex. In addition, $K(0) = 0$ and $K'(0) = 0$. Hence, the non-bank chooses its search intensity that optimally trades-off search cost and delay in monetizing repo specialness premium.

When the OW non-bank and dealer meet, they determine the specialness premium through Nash bargaining (Duffie, Garleanu and Pedersen, 2005). The non-bank gives a unit of government bond to the dealer. In exchange, the non-bank receives cash equivalent to P units of consumption numeraire. I assume zero haircut. The OW non-bank transitions to the O state.

During the lifetime of this repo contract, the non-bank receives cash $c + B$ per unit of time. As the owner of the bond, the non-bank is entitled to the "manufactured payment" c per unit of time. B is the specialness premium determined from Nash bargaining.

At rate δ, the repo matures, and the non-bank transitions back to the OW state. The dealer returns the government bond to the non-bank. Let the random variable $\widetilde{T}$ denote the length of the repo contract. The dealer returns to the non-bank cash equivalent to $P \cdot e^{r\widetilde{T}}$ units of consumption numeraire. The dealer compensates the non-bank for forgone interest that the non-bank could have earned by investing this cash in the risk-free asset.

The lower panel of Figure 5 shows the life cycle of a repo contract between the dealer and the non-bank.

All investors in my model - dealers, cash investors, and non-banks - have a time-discount rate equal to the risk-free rate r. The OW non-bank's problem can be formulated as (2) and (3). W_t is the bank account of the non-bank at time t. dC_{t+s} is a semimartingale consumption process that the non-bank optimizes. Because the problem is stationary, $\lambda_{o,t}$ stays the same over time.

$$V_{OW}(W_t) = \sup_{C,\lambda_o} \mathbb{E}_t \left[\int_{s=0}^{\tau_D} e^{-rs} dC_{t+s} + V_O\left(W_{t+\tau_D} + P\right) \right] \tag{2}$$

$$\text{such that: } dW_t = rW_t dt - dC_t + c dt - K(\lambda_{o,t})\, dt \tag{3}$$

Large repo specialness premium B induces each government bond owner to exert more search effort. Hence, larger quantities of government bonds are supplied to repo dealers per unit of time. This modeling choice reflects the observation that as bonds became more special in the repo market recently, collaterals are circulating in the European repo market faster (see Jank and Mönch (2019) for empirical support). The problem of the non-bank with repo position in

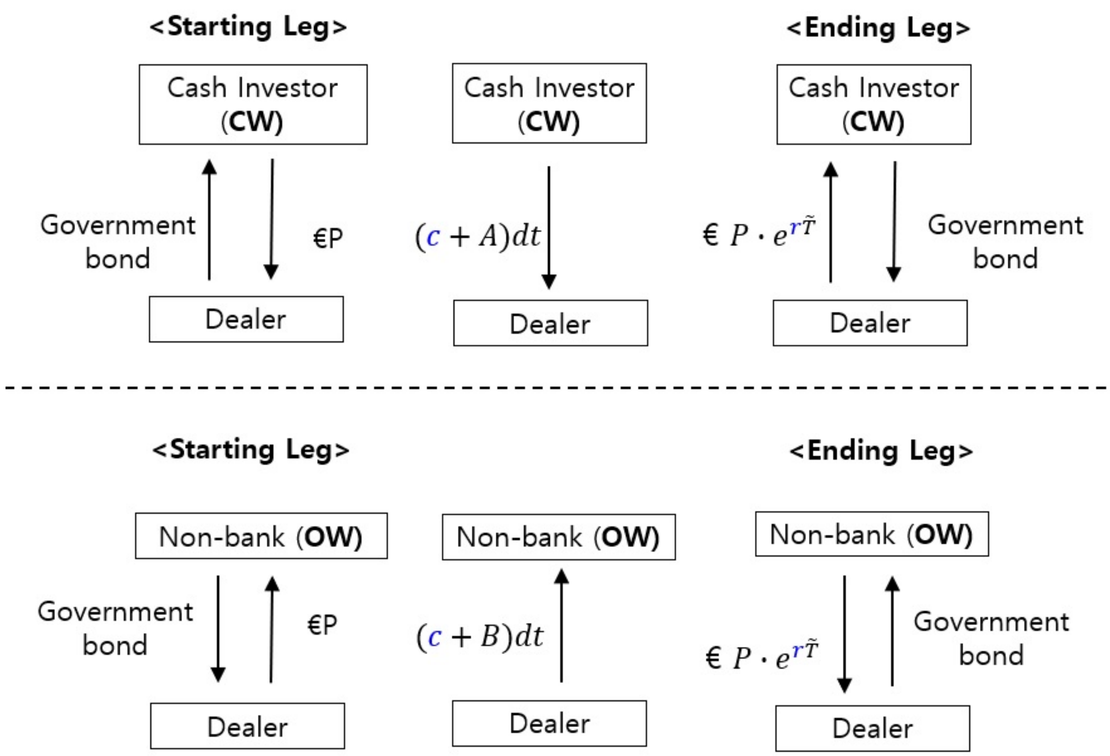

Figure 5: **The Schematics of Repo.** The upper panel describes the life cycle of a repo contract between a dealer and a cash investor. The lower panel describes the life cycle of a repo contract between a dealer and a non-bank.

the O state can be formulated similarly.

$$V_O(W_t) = \sup_{C} \mathbb{E}_t \left[\int_{s=0}^{\tau_\delta} e^{-rs} dC_{t+s} + V_{OW}\left(W_{t+\tau_\delta} - P \cdot e^{r\cdot\tau_\delta}\right) \right] \tag{4}$$

$$\text{such that: } dW_t = rW_t dt - dC_t + (c + B)\, dt \tag{5}$$

3.2.2. Corporate Bond Owning Non-banks

The problem of the non-bank owning a corporate bond can be formulated as follows.

$$V_{IG}(W_t) = \sup_{C} \mathbb{E}_t \left[\int_{s=0}^{\tau_\eta} e^{-rs} dC_{t+s} + V_N\left(W_{t+\tau_\eta} + (1 - \Lambda)\frac{c}{r}\right) \right] \tag{6}$$

$$\text{such that: } dW_t = rW_t dt - dC_t + c dt \tag{7}$$

3.3. Cash Investors

The upper panel of Figure 5 describes the life cycle of a cash investor. Examples of cash investors include hedge funds that want to obtain a government bond for shorting.

The mass of cash investors is exogenously fixed at m_c. Cash investors in the CW ("cash investor waiting") state want to obtain a government bond for an exogenous reason that I do not model here. Cash investors search for repo dealers through the pairwise independent random matching process. Cash investors choose their search intensity λ_c that balances the cost of delaying the possession of a government bond and the search effort. The cash investor and the dealer Nash bargain the specialness premium A, after which the cash investor transitions to the C state. Cash investors receive exogenous flow benefit b per unit of time until the repo matures. When the repo matures, the investor transitions back to the CW state. The problem of a cash investor in the CW state can be formulated as equation (8) and equation (9).

$$V_{CW}(W_t) = \sup_{C,\lambda_c} \mathbb{E}_t \left[\int_{s=0}^{\tau_D} e^{-rs} dC_{t+s} + V_C\left(W_{t+\tau_D} - P\right) \right] \tag{8}$$

$$\text{such that: } dW_t = rW_t dt - dC_t - K(\lambda_{c,t})\, dt \tag{9}$$

The problem of a cash investor in the C state can be stated similarly.

$$V_C(W_t) = \sup_{C} \mathbb{E}_t \left[\int_{s=0}^{\tau_\delta} e^{-rs} dC_{t+s} + V_{CW}\left(W_{t+\tau_\delta} + P \cdot e^{r\cdot\tau_\delta}\right) \right] \tag{10}$$

$$\text{such that: } dW_t = rW_t dt - dC_t + (b - c - A)\, dt \tag{11}$$

3.4. Dealers

Equations (12), (13), (14), (15) and (16) show the dealer's problem. Q_t^{RR} and Q_t^{R} denote the mass of bonds that the dealer reverse repoed and repoed, respectively. The dealer also outright purchases x_t government bonds from the cash market. If x_t is negative, the dealer outright sells bonds. $\theta^{RR}, \theta^R \in \{0,1\}$ are binary variables that indicate whether the dealer wants to intermediate the repo market. For example, the dealer serves incoming customers who want to repo their bonds if and only if $\theta^{RR} = 1$. μ_t^σ is the mass of investors of type $\sigma \in \{CW, C, OW, O, IG, N\}$ at time t. Figure 6 illustrates the dealer's activities in the repo market and in the cash market.

$$V_D(W_t, Q_t^{RR}, Q_t^R) = \sup_{\theta^{RR}, \theta^R, x, C} \mathbb{E}_t \int_{s=0}^{\infty} e^{-rs} dC_{t+s} \tag{12}$$

such that

$$\begin{aligned} dW_t = rW_t dt - dC_t + \theta_t^R \frac{1}{N} \lambda_c \mu_t^{CW} P dt + AQ_t^R dt - \delta Q_t^R \mathbb{E}\left\{e^{r\widetilde{T}}\right\} P dt \\ - \theta_t^{RR} \frac{1}{N} \lambda_o \mu_t^{OW} P dt - BQ_t^{RR} dt + \delta Q_t^{RR} \mathbb{E}\left\{e^{r\widetilde{T}}\right\} P dt + x_t c dt - P dx_t \end{aligned} \tag{13}$$

$$x_t + Q_t^{RR} \geq Q_t^R \tag{14}$$

$$dQ_t^{RR} = -\delta Q_t^{RR} dt + \theta_t^{RR} \frac{1}{N} \lambda_o \mu_t^{OW} dt \tag{15}$$

$$dQ_t^R = -\delta Q_t^R dt + \theta_t^R \frac{1}{N} \lambda_c \mu_t^{CW} dt \tag{16}$$

Equation (13) shows that cash flow for the dealer comes from three sources: repo, reverse repo, and trade in the cash market. For example, the third, the fourth and the fifth terms show cash flow related to repo positions. Applying the exact law of large numbers (ELLN) (Sun, 2006), the mass of cash investors coming to this dealer to obtain a government bond is $\frac{1}{N} \lambda_c \mu_t^{CW}$ per unit of time almost surely. Each new repo position gives the dealer cash that is equivalent to P units of consumption numeraire. Hence, the third term $\frac{1}{N} \lambda_c \mu_t^{CW} P$ is the cash collateral that the dealer receives from new repo positions per unit of time. The fourth term AQ_t^R is the specialness premium that the dealer collects from outstanding repo positions per unit of time. The dealer has a continuum of outstanding repo positions. Each outstanding repo position matures with rate δ and independently of one another. By the law of large numbers, the mass of repo positions that matures per unit of time is δQ_t^R almost surely. Upon maturity, the dealer returns the cash plus the interest accrued over the lifetime of the repo.

Cash flow from reverse repo positions can be explained similarly. The last two terms ($x_t c dt - P dx_t$) come from trade in the cash market.

Equation (14) implies that in order for the dealer to repo a government bond, the dealer must either purchase the bond or reverse-in the bond. By application of the exact law of large numbers, the dealer's repo position and reverse repo position evolve according to equation (15) and equation (16) almost surely.

I do not allow dealers to trade corporate bonds in the model. As of the second quarter of 2017, European monetary financial institutions held only 9.56% of the eurozone non-financial

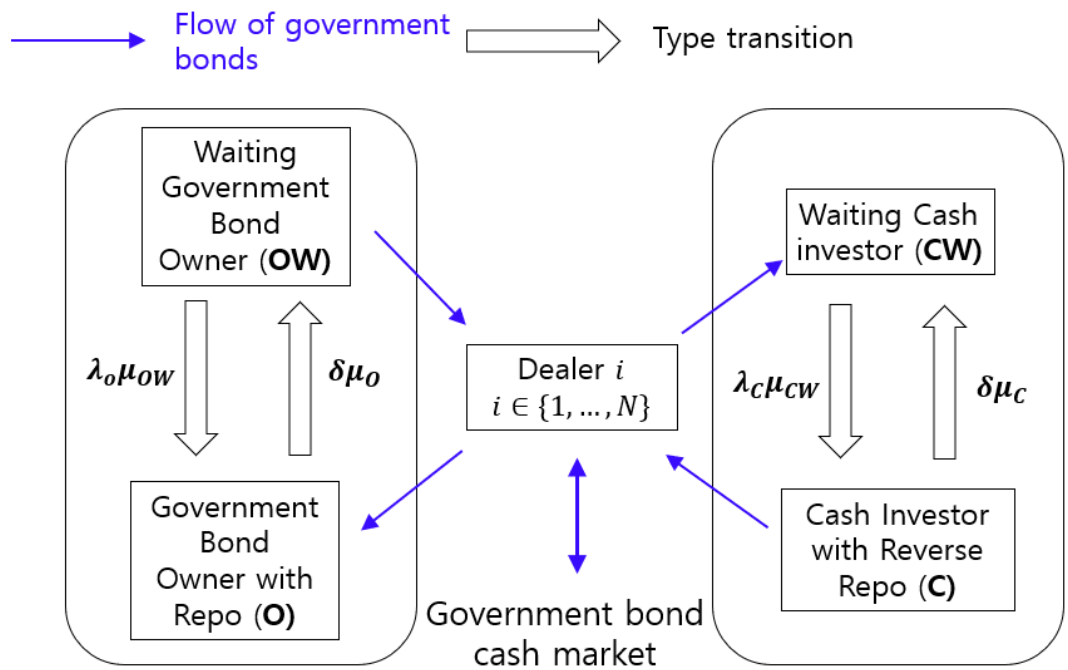

Figure 6: **The Schematics of the Dealer's Problem**

corporate liabilities whose remaining maturity was no shorter than one year.[8] On the other hand, insurance companies and pension funds held 41.8% of long-term euro-area non-financial corporate bonds in the same quarter.

3.5. Agent Mass Dynamics

The value functions of IG non-banks and OW non-banks can be shown to be linearly separable in wealth (Duffie, Garleanu and Pedersen, 2005). Hence, let $V_{IG}(W_t) = J_{IG} + W_t$ and $V_{OW}(W_t) = J_{OW} + W_t$. Non-banks in the N state buy a corporate bond if $J_{IG} > J_{OW}$ and buy a government bond if $J_{IG} < J_{OW}$. Non-banks are indifferent between the two options if $J_{IG} = J_{OW}$. Per unit of time, let f_{IG} denote the mass of N non-banks that buy corporate bonds. Similarly, let f_{OW} denote the mass rate of N non-banks that buy government bonds. Then it must be the case that $f_{IG} = 0$ if $J_{OW} > J_{IG}$ and $f_{OW} = 0$ if $J_{OW} < J_{IG}$.

By the exact law of large numbers, the mass of investors of each type evolve according to (17), (18), (19), (20), (21) and (22) almost surely. Equation (23) is a restriction that the total mass of cash investors summed across all types should be m_c. Equation (24) is a similar restriction for non-banks.

$$\frac{d}{dt}\mu_t^O = \lambda_o \mu_t^{OW} - \delta\mu_t^O \tag{17}$$

[8]The data source is the Securities Holdings Statistics.

$$\frac{d}{dt}\mu_t^{OW} = f_{OW} - \lambda_o \mu_t^{OW} \tag{18}$$

$$\frac{d}{dt}\mu_t^{IG} = -\eta \mu_t^{IG} + f_{IG} \tag{19}$$

$$\frac{d}{dt}\mu_t^{N} = \eta \mu_t^{IG} - f_{IG} - f_{OW} \tag{20}$$

$$\frac{d}{dt}\mu_t^{CW} = \delta \mu_t^{C} - \lambda_c \mu_t^{CW} \tag{21}$$

$$\frac{d}{dt}\mu_t^{C} = -\delta \mu_t^{C} + \lambda_c \mu_t^{CW} \tag{22}$$

$$\mu_t^{C} + \mu_t^{CW} = m_c \tag{23}$$

$$\mu_t^{N} + \mu_t^{IG} + \mu_t^{OW} + \mu_t^{O} = m_n \tag{24}$$

3.6. Nash Bargaining

Nash bargaining between a dealer and a customer (either a cash investor or a non-bank) determines the repo specialness premia A and B. Let z and $1-z$ denote the relative bargaining powers of a dealer and a customer where $0 < z < 1$. See Duffie, Garleanu and Pedersen (2007) for micro-foundations.

3.7. Cash Markets

I model the government bond market as a two-tiered system consisting of the dealer-to-dealer (D2D) segment and the dealer-to-customer (D2C) segment. The D2D market is assumed to be a competitive market. In the D2C segment, a non-bank buys a government bond from dealers at a price equal to the D2D market price. This set-up reflects the actual structure of the European sovereign bond market (Ejsing and Sihvonen, 2009; Schlepper, Hofer, Riordan and Schrimpf, 2017), especially the bund market (Huszar and Simon, 2017). I assume the corporate bond market is a perfectly competitive market in which only non-banks participate.

The outstanding quantity of government bonds is exogenously fixed at S_g. n_c units of new corporate bonds are issued per unit of time. Because corporate bonds default in my model, issuance of new corporate bonds is necessary for analysis of a steady state equilibrium.

4. Equilibrium

In this paper, I only focus on a stationary equilibrium in which the agent masses and market prices do not change over time. Let superscript i imply that the variable is associated with dealer $i \in \{1, 2, ..., N\}$. When defining an equilibrium, I do not consider the equilibrium consumption decisions of agents. This approach is without loss of generality because all agents are risk-neutral with the discount rate equal to the risk-free rate.

A continuum of ex-ante identical bond owners faces the same stationary problem. Let $\lambda_{o,j,t}$ denote the search intensity of non-bank j at time t. Similarly, let $\lambda_{c,k,t}$ denote the search intensity of cash investor k at time t. There are constants λ_o and λ_c such that $\lambda_{o,j,t} = \lambda_o$ for $\forall j, t$ and $\lambda_{c,k,t} = \lambda_c$ for $\forall k, t$.

Definition 1. *Let $\boldsymbol{\mu} = (\mu_{OW}, \mu_O, \mu_{CW}, \mu_C, \mu_{IG})$ and $\boldsymbol{V} = (V_{OW}, V_O, V_{CW}, V_C, V_{IG}, V_N)$. An equilibrium consists of investor masses $\boldsymbol{\mu}$, investor value functions $\boldsymbol{V}$, market prices (A, B, P, P_{ig}), search intensities (λ_o, λ_c), dealer's willingness to intermediate the repo market $\left(\theta_i^{RR}, \theta_i^R\right)_{i=1...N}$ and non-bank's bond purchase rate (f_{IG}, f_{OW}) such that:*

(1) Each OW non-bank chooses search intensity λ_o that solves the problem (2) and (3). Each CW cash investor chooses the search intensity λ_c that solves the problem (8) and (9).

(2) Investor masses evolve according to (17), (18), (19), (20), (21) and (22) almost surely.

(3) Non-bank's bond purchase rate in the N state is incentive compatible. $f_{IG} = 0$ if $V_{OW} > V_{IG}$ and $f_{OW} = 0$ if $V_{OW} < V_{IG}$.

(4) Investor's value functions $\boldsymbol{V}$ are defined by (1), (2), (3), (5), (6), (7), (8), (9), (10), (11), (12), (13), (14), (15) and (16).

(5) *Market clearing for the corporate bond market: $\mu_t^{IG} = \frac{n_c}{\eta}$*

(6) *Market clearing for the government bond market: $\mu_t^O + \mu_t^{OW} + \sum_{i=1}^{N} x_t^i = S_g$*

(7) *Dealer's intermediation in the repo market is incentive compatible.* θ_t^{RR} and θ_t^R solves the dealer's problem (12), (13), (14), (15) and (16).

I assume that the total mass of non-banks m_n equals the combined sum of outstanding quantities of corporate bonds and government bonds. That is, $m_n = \frac{n_c}{\eta} + S_g$. Hence in equilibrium, the risk-neutral non-bank must be indifferent between owning a government bond and owning a corporate bond. If the non-bank strictly prefers owning a corporate bond over owning a government bond, then there is excess demand. If the non-bank's preference is the other way around, then there is an excess supply in the corporate bond market. This modeling assumption is mainly for simplicity. The same intuition holds even if I relax this assumption, but with much more complex algebra.

Theorem 1. ***Unique stationary equilibrium.*** *A unique stationary equilibrium exists. The repo specialness premia (A, B), the search intensities (λ_o, λ_c) and the government bond price P are determined by (25), (26), (27), (28) and (29). The corporate bond price P_c is determined by (30).*

$$b - c - A + K(\lambda_c) = K'(\lambda_c)(r + \lambda_c + \delta) \tag{25}$$

$$B + K(\lambda_o) = K'(\lambda_o)(r + \lambda_o + \delta) \tag{26}$$

$$A = z\left\{K(\lambda_c) + b - c\right\} + (1 - z)(rP - c) \tag{27}$$

$$B = -z \cdot K(\lambda_o) + (1 - z)(rP - c) \tag{28}$$

$$m_c \frac{\lambda_c}{\delta + \lambda_c} = \left(m_n - \frac{n_c}{\eta}\right) \frac{\lambda_o}{\delta + \lambda_o} \tag{29}$$

$$P_c = \frac{c}{r} - (1-\Lambda)\frac{c \cdot \eta}{r(r+\eta)} + \frac{B + K(\lambda_o)}{1-z}\frac{r+\delta+z\cdot\lambda_o}{r+\delta+\lambda_o}\frac{1}{r+\eta} \tag{30}$$

Proof. See section B.1. □

Equations (27) and (28) are consistent with Duffie, Garleanu and Pedersen (2002). Let $\psi = rP - c$. It can be shown that ψ is positive in equilibrium. ψ can be interpreted as the premium that investors are willing to pay for purchasing a government bond. ψ is increasing in repo specialness premia A and B. When repo specialness premia are large, the government bonds appreciate to reflect potential opportunities to make extra revenue from the repo market.

Equation (30) shows that the corporate bond price in my model can be decomposed into three parts. The first term is the present value of future coupon payments promised. The second term is a discount due to the credit risk of the corporate bond. The third term captures the portfolio rebalancing effect. The size of this term is increasing in the repo specialness premium B. If government bonds are more special in the repo market, the portfolio rebalancing of non-banks causes corporate bonds to become more expensive. The size of this term is also increasing in the search effort cost $K(\lambda_o)$. Larger search frictions in the repo market imply that it is costlier for non-banks to monetize repo specialness premium. Thus, owning a government bond is less attractive for non-banks.

4.1. The Impact of the PSPP

Koijen, Koulischer, Nguyen and Yogo (2018) shows that most transactions under the PSPP took place between the Eurosystem and foreign institutions that did not have access to the central bank deposit facility. When foreign financial institutions sell government bonds to the Eurosystem, they are credited with unsecured deposits at European banks (Cœuré, 2017). However, they do not like having large quantities of cash parked as unsecured deposits due to their risk management practices. Thus, they subsequently invest their cash in short-term European government bonds or the GC repo market (Mersch, 2017). Foreign financial institutions investing their cash get government bonds in return. Thus, they can be mapped into cash investors in my model.

To understand how the PSPP impacts asset prices, I see how the solution to my model changes as m_c (the cash investor mass) increases and m_n (the non-bank mass) decreases.

Theorem 2. ***The Impact of the PSPP on Asset Prices.*** *Let* $\theta = \frac{m_n}{m_c}$*. Then* $\frac{\partial}{\partial\theta}P < 0$ *and* $\frac{\partial}{\partial\theta}P_c < 0$*.*

Proof. See section B.2. □

The marginal impact of the PSPP on the repo market is to create excess demand for collateral. The equilibrium specialness premia A and B increase so as to equate demand and supply again. While the OW non-banks search for dealers more intensely, cash investors search for dealers less intensely. The PSPP can affect the revenue that dealers can make in the repo market in two ways. First, larger specialness premia imply that dealers can earn more revenue for each repo transaction with cash investors. Second, because cash investors search for dealers less intensely, dealers enter repo transactions less frequently. This effect reduces each dealer's revenue. Nevertheless, for an arbitrary increasing and convex search cost function $K(\cdot)$, I show that dealers that own government bonds are expected to earn more revenue in the repo market. As a result, dealers may be willing to pay more in the cash market to own government bonds.

Since dealers charge non-banks more for government bonds, without any change in the corporate bond price, non-banks substitute for government bonds with corporate bonds. Corporate bonds appreciate to clear the market.

4.2. The Impact of the Securities Lending Facility

Now I consider a hypothetical scenario in which the Eurosystem operates a securities lending facility akin to the reverse repo facility of the Federal Reserve. That is, the Eurosystem repos government bonds to cash investors. In my model, the operation of this facility will reduce the value of m_c - the mass of cash investors that try to obtain government bonds in the private market. Consequently, by theorem 2, both government bonds and corporate bonds appreciate.

4.3. Welfare Analysis of the Securities Lending Facility

I do not consider the cash market transaction of the PSPP. Instead, I take as given the cash market transactions under the PSPP. I study how the welfare changes with respect to the quantity of government bonds that the Eurosystem directly repos to cash investors. Suppose cash investors of mass x obtain government bonds directly from the facility without incurring any search cost. The remaining cash investors of mass $m_c - x$ obtain government bonds from the private repo market.

Because all investors in my model are risk-neutral, any monetary transfer between investors (e.g., repo specialness premium) is irrelevant for welfare analysis. I also assume that the central bank is risk-neutral. Hence, the specialness premium at which cash investors reverse-in government bonds from the securities lending facility is also irrelevant for welfare analysis.

For welfare analysis, allocative efficiency and search cost are relevant. Equation (31) shows my measure of social welfare as a function of x. The first term is allocative efficiency. The more cash investors obtain government bonds, the more they get the flow of benefit b per unit of time. The second term is the search effort cost of OW non-banks per unit of time. The third term is the search effort cost of CW cash investors per unit of time. Agent masses and search intensities vary with respect to x.

$$W(x) = b\left\{x + \mu^{C}(x)\right\} - \mu^{OW}(x)\, K(\lambda_o(x)) - \mu^{CW}(x)\, K(\lambda_c(x)) \tag{31}$$

Theorem 3. *Suppose the mass of government bond owners is larger than the mass of cash investors.* $\mu^{OW} + \mu^{O} > \mu^{CW} + \mu^{C}$. *The social welfare* $W(x)$ *is increasing in* x.

Proof. See section B.3. □

The security lending facility improves allocative efficiency. Since fewer cash investors try to obtain government bonds from the private repo market, repo specialness premia A and B decline. Consequently, each OW non-bank reduces its search effort while each CW cash investor exerts more search effort. With arbitrary convex and increasing search cost function $K(\cdot)$, reduction in search cost of non-banks and improvement in allocative efficiency dominate increase in search cost of cash investors.

5. Numerical Example

I calibrate the parameters of my theoretical model using the data as of June 1, 2016. I set unit of time equal to a year. One unit of government bond is equal to bonds with face value of 1 billion euros. For numerical exercises, I add another free parameter μ_D. If the OW non-bank chooses the search intensity parameter λ_O, it is matched to dealers at the rate $\mu_D \lambda_O$. Similarly, the CW cash investor that chooses the search intensity λ_c is matched to dealers at the rate $\mu_D \lambda_c$.

For numerical calibration purposes, government bonds in my model can be mapped to long-term German federal government bonds. I match the risk-free rate r to the overnight index swap (OIS) rate with 30 years tenor. Since I have perpetual bonds in my model, I choose the OIS rate with the longest tenor possible. I match the mass of non-banks holding government bonds $\mu^O + \mu^{OW}$ to the Securities Holdings Statistics data. Entities other than the central bank held 740.6 billion euros of German general government debt whose remaining maturity is no shorter than 1 year.

I set the non-pecuniary benefit of obtaining government bond b so that repo specialness premium in my model can be at least as large as the maximum specialness premium observed in the data. Large specialness premia observed in data imply that the opportunity cost of not obtaining German government bonds is very large, at least for a subset of investors.

I allow non-banks and cash investors to have different cost functions. I parameterize the non-bank search cost function as $K(\lambda) = k_o \cdot \lambda^{\alpha}$. I parameterize the cash-investor search cost function as $K(\lambda) = k_c \cdot \lambda^{\alpha}$. All the other parameters are set to match the trading volume in BrokerTec, bid-ask spread in European repo[9], 30-year German federal government bond yield and 30-year European investment grade corporate yield. Because perpetual bonds do not exist, I use the Bloomberg yield curves at the longest remaining maturity possible. Table 4 summarizes the parameter values used for the numerical calibration.

I see how solutions to my model change as the PSPP withdraws government bonds from circulation in the market. If the Eurosystem purchases x units of government bonds, m_n in

[9] According to Hill (2015a), the bid-ask spread for government bond term repo in Europe is around 7 basis points.

[10] This value is in line with Heynderickx, Cariboni, Schoutens and Smits (2016).

Parameter	Value	Parameter	Value	Parameter	Value
r	0.962%	c	0.03	α	2.5
z	0.1	b	4.5%	Λ[10]	0.4
μ_D	10	m_c	450	η	0.065
k_o	$10^{-0.3}$	k_c	$5 \times 10^{-2.3}$		

Table 1: Parameter Values for the Numerical Calibration

my model decreases by x. At the same time, m_c increases by x. Figure 7 shows the result, confirming theorem 2.

The government bond price and the corporate bond price in my model deviate from the expected value of future coupon payments entirely because of search frictions in the repo market. To check this intuition, I see how solutions to my model change with respect to the constant k_o that enters the search cost function of a non-bank. I fix the ratio of the search cost parameter k_c of a cash investor to that of a non-bank. Figure 8 shows the result.

Figure 8 shows that as the search cost asymptotically diminishes to zero, the specialness premium B converges to zero. Both the government bond price and the corporate bond price converge to the simple expected value of future coupon payments.

This result also implies that any regulation that impacts the collateral velocity and search frictions in the repo market can impact bond prices and, hence, potentially corporate financing costs.

6. Data

My sample period is from March 1, 2014 to February 29, 2016. I stop my analysis on February 29, 2016 because the Eurosystem announced its plan to purchase corporate bonds under the Corporate Sector Purchase Program (CSPP) in March 2016. I do not want the CSPP to affect my empirical analysis.

For government bonds, I consider debt instruments issued by the German federal government. During the PSPP period, the specialness premium of German government bonds was larger than that of any other European sovereign bond. I exclude inflation-linked bonds and Bund-Laender-Anleihe that is jointly issued by the German federal government and state governments. [11] I remove bonds if their remaining maturity is shorter than 15 days: bond prices tend to become very volatile if their remaining maturity gets closer to zero. I also remove bonds that are issued less than eight days before for a similar reason. I remove data on days that are close to quarter ends.[12] On quarter ends, banks tend to scale back their repo intermediation business to window-dress their balance sheets for regulatory purposes. Consequently, repo rates fluctuate very widely around reporting days.

[11] See the website of the German finance agency for more information.

[12] I remove data on days between March 25 and April 5, between June 25 and July 5, between September 25 and October 5, and between December 25 and January 5.

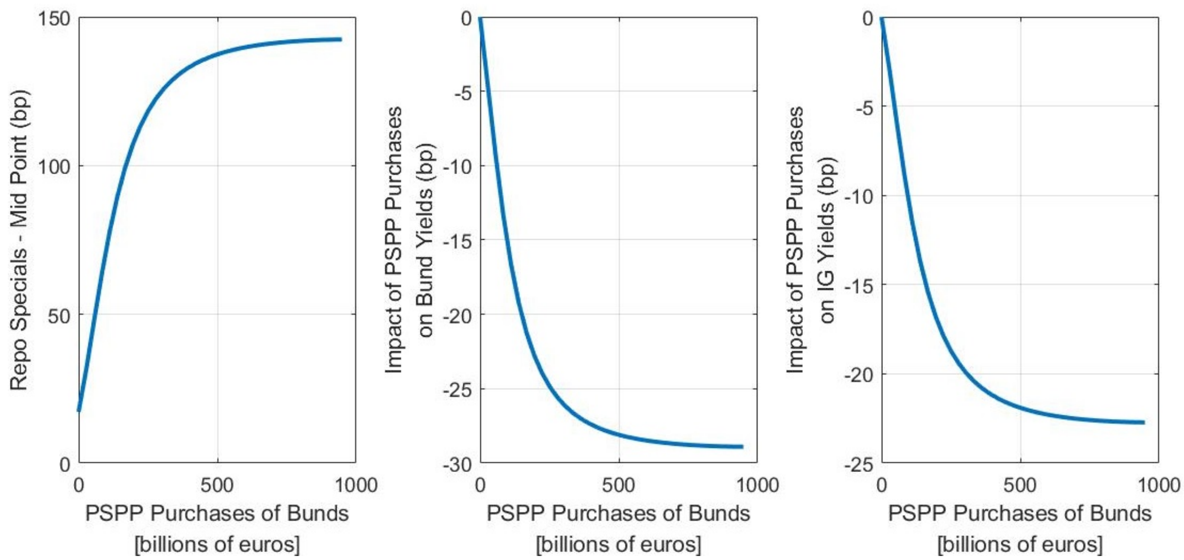

Figure 7: **The Impact of PSPP on Asset Prices.** The graph shows how the solutions to my model change as the PSPP withdraws government bonds from circulation in the private market. The midpoint on the left-most panel is the average of two repo specialness premia A and B. According to the ECB website, total 191 billion euros of German securities have been purchased by May 31, 2016 under the PSPP. This number includes not only the debt instruments of the German federal government but also those of regional governments and recognized agencies. Until December 2016, debt instruments were eligible for purchase only if their remaining maturities were no shorter than two years. The Eurosystem does not disclose how this number is broken down into different maturity buckets. Nevertheless, before December 2016, the Eurosystem was not allowed to purchase bonds with yields to maturity lower than the deposit facility rate. Thus, the majority of bonds held under the PSPP as of June 1, 2016 is likely to have been long-term bonds.

For corporate bonds, I consider all euro-denominated corporate bonds that appear on the Bloomberg Terminal during my sample period. I only consider bonds with fixed coupon payments. I only consider bullet bonds, as opposed to amortizing bonds. I download yields to maturity data and OIS data from Bloomberg.

Repo transaction data is from BrokerTec, an electronic platform on which the majority of interdealer German SC repo is traded (EMMI, 2017). I only use the transactions with tenor Spot-Next (SN), the most commonly observed tenor on BrokerTec. SN tenor implies that collateral and cash are exchanged two business days after both parties agree on the terms of the transaction. The collateral and cash are returned after another business day.

Specialness premium is defined as the Euro Overnight Index Average (EONIA) rate minus repo rate. Table 2 summarizes data.

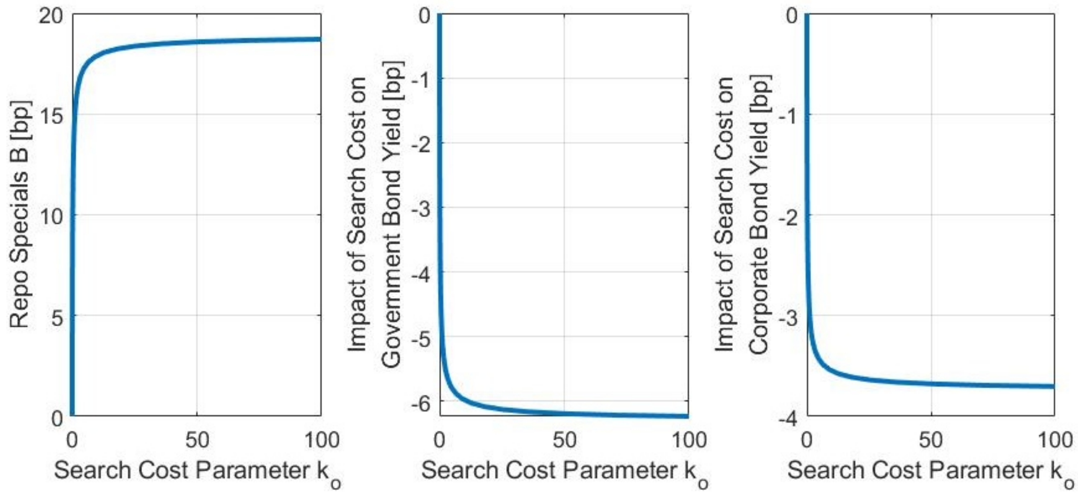

Figure 8: **The Impact of Search Cost on Asset Prices.** The graphs show how the solutions to my model change with respect to the search cost parameters k_o and k_c. The search cost functions of the non-bank and the cash investor are $k_o \lambda_o^\alpha$ and $k_c \lambda_c^\alpha$, respectively. The ratio of k_o to k_c is kept constant at 20. The panel furthest left is the specialness premium B at which the non-bank repos government bonds to bank dealers. The remaining two panels show the impact of search cost on the bond yields. The benchmark yields are what would be observed in a model without any search frictions. The benchmark government bond yield is the risk-free rate r. The benchmark corporate bond price is $\frac{c}{r} - (1 - \Lambda) \frac{c \cdot \eta}{r(r+\eta)}$, the first two terms in (30). The vertical axis in the middle panel and the right panel show the model-implied yields minus the benchmark bond yields.

7. Empirical Strategy

The goal of my empirical analysis is to measure the impact of bund repo specialness premia on euro-denominated corporate bond yields. See Table C for the list of definitions of symbols in this section.

7.1. Baseline Specification

The reduced-form empirical specification for my analysis is (32). Each corporate bond is indexed by i. y_{it} is the yield-to-maturity of corporate bond i. I include the corporate bond-level fixed effect term α_i to capture bond-specific credit risk premium. I include the linear time trend term $\gamma \cdot t$ in case variables are persistent. OIS_{it} is the overnight index swap (OIS) rate with the tenor identical to the remaining maturity of corporate bond i. This term captures the sum of the expectation hypothesis component and the term premium component.

Q_{it} is the aggregate outstanding amount of all euro-denominated corporate bonds with the same credit rating as corporate bond i. I include this term to control for the scarcity effect. Investors may have non-satiated demand for AAA corporate bonds, for example, due to regulations. Then yields on all AAA corporate bonds are positively related to the quantity of

Variable	Mean	Standard Deviation	Max	Min	Observations
Amount Issued of Bunds (Billions of Euros)	17.179	4.564	26	0.750	27,851
Spot-Next Repo Rate of Bunds (%)	-0.147	0.167	0.501	-1.487	27,847
Residual Maturity of Bunds (years)	7.043	7.704	32.455	0.041	27,851
Amount Issued of Corporate Bonds (Billions of Euros)	0.727	0.935	8	0.0012	908,046
Yield-to-Maturity of Corporate Bonds (%)	1.350	1.040	17.070	-1.651	926,552
Residual Maturity of Corporate Bonds (years)	7.649	4.164	29.997	0.085	926,552

Table 2: **Summary Statistics**

AAA corporate bonds. Krishnamurthy and Vissing-Jorgensen (2012a) show the existence of this effect for the case of the U.S. Treasury market.

$$y_{it} = \alpha_i + \gamma \cdot t + \beta_o \cdot OIS_{it} + \beta_Q \cdot Q_{it} + \beta_s \cdot s_{it} + \epsilon_{it} \tag{32}$$

The main explanatory variable is s_{it}, the repo specialness premium of bunds with the same remaining maturity. I compare a bund and a corporate bond with the same remaining maturity because the portfolio rebalancing effect is likely to be the strongest in such cases. Some investors have demand for debt instruments with specific maturities. For example, insurance companies want to match the duration of their assets and liabilities. Thus, if bunds with specific maturities become expensive, those investors are likely to look for alternative bonds with similar maturities.

Simple Ordinary Least Squares (OLS) estimate of (32) cannot identify the main coefficient of interest β_s due to simultaneity bias: the bund repo specialness premium s_{it} and the corporate bond yields y_{it} are jointly determined in equilibrium. Thus, I need to instrument bund repo specialness premia.

7.2. PSPP Implementation Rule

Because I use the implementation rule of the PSPP to construct my instrumental variable, I summarize the details of the PSPP in this section. The PSPP has a very clear rule on how to allocate aggregate purchase quantity across individual securities that are eligible for purchase. This rule was articulated from the start when the PSPP was first introduced in 2015. Although the rule has been updated several times over the course of the PSPP, most parts of the rules remained largely unchanged. I briefly summarize the rule as of March 4, 2015 below. See section E for a full summary.[13]

[13] Decision (EU) 2015/774 of the European Central Bank of 4 March 2015 on a secondary market public sector asset purchase programme (ECB/2015/10).

On March 4, 2015, the Eurosystem announced that it would expand the monthly net purchase under its asset purchase program from 10 billion euros to 60 billion euros. Prior to this announcement, the Eurosystem already had been running the Asset-Backed Securities Purchase Program (ABSPP) and the Covered Bond Purchase Program 3 (CBPP3). An extra net purchase of 50 billion euros was expected to come from the creation of the PSPP under the umbrella of an "expanded" Asset Purchase Program (APP).

Debt instruments issued by central governments in the euro area were eligible for purchase. In addition, debt instruments of recognized agencies, international organizations, and multilateral development banks were also eligible. 88% of the aggregate purchase - 44 billion euros per month - was to be made for debt instruments of central governments and recognized agencies.

44 billion euros were, in turn, distributed among member countries according to their capital keys. The capital key of a member country is the average of its GDP share and population share in the eurozone. For example, since the capital key of Germany was 25.7% in March 2015, the allocation for debts of the German central government and German agencies is 44 billion euros $\times$ 25.7% = 11.308 billion euros per month. 11.308 billion euros are, accordingly, distributed across eligible individual debt instruments in proportion to their outstanding amounts.[14] This rule is called the market neutrality rule.

There are other rules that govern the purchases by the Eurosystem.

1. **Limit on yield-to-maturity (YTM):** The Eurosystem is not allowed to purchase bonds with yields below the deposit facility rate at the time the purchase is made.

2. **Issuer limit:** The Eurosystem is not allowed to purchase more than 33% of the total outstanding debt instruments issued by one entity (e.g., the central government of Germany).

3. **Issue (ISIN) limit:** The Eurosystem cannot purchase more than 25% of outstanding amounts of any individual security.

4. **Currency denomination:** The security must be denominated in euros.

5. **Remaining maturity:** The remaining maturity of a bond should be between 2 years and 31 years.

6. **Eligibility for monetary operations of the Eurosystem:** The security must comply with the rules outlined in Guideline of the European Central Bank of 20 September 2011 on Monetary Policy Instruments and Procedures of the Eurosystem. For example, the security must have a credit rating above a certain threshold, which is not an issue for German federal government bonds.

7. **Blackout period:** The Eurosystem can purchase bonds from the secondary market only a certain number of days after the date of issuance. In addition, bonds with maturities

[14] See the ECB website https://www.ecb.europa.eu/mopo/implement/omt/html/pspp-qa.en.html.

close to the newly issued ones are also temporarily ineligible for purchase during the blackout period.

8. **Flexibility (Cœuré, 2015):** The Eurosystem reserves the right to deviate from the market neutrality rule if necessary. For example, to avoid exacerbating severe frictions in the market, the Eurosystem may avoid purchasing bonds that are extremely special in the repo market. Nevertheless, Arrata, Nguyen, Rahmouni-Rousseau and Vari (2018) documents that the Eurosystem largely followed the market neutrality rule initially proposed.

7.3. Instrumental Variable

Theory suggests that repo specialness premia should increase if there is a negative exogenous shock to the supply of collateral - government bonds. Hence, any exogenous shock to supply of collateral can be used to instrument repo specialness premia. I use the PSPP to construct a time series of negative supply shocks.

Let $Q_{o,j,t}$ denote the cumulative quantity of government bond j that the German finance agency auctioned to the market as of day t. Let $Q_{PSPP,j,t}$ denote the cumulative quantity of government bond j that the Eurosystem purchased under the PSPP by day t. Then $F_{j,t} = Q_{o,j,t} - Q_{PSPP,j,t}$ is the free float, the quantity of government bond j that is still held by investors other than the Eurosystem. Unlike Cœuré (2018), I do not subtract the quantity held by the foreign official sector. Since the Eurosystem does not actively supply the purchased bonds to the repo market, the free float $F_{j,t}$ is a rough measure of the supply of government bond j to the repo market. If $Q_{PSPP,j,t}$ is exogenous, I can use $F_{j,t}$ to instrument repo specialness premia $s_{j,t}$.

Nevertheless, the YTM restriction may cause the quantities purchased by the Eurosystem $Q_{PSPP,j,t}$ to become potentially endogenous. If an omitted variable causes government bond yields and corporate bond yields to move together, the quantity of government bonds that the Eurosystem purchased can be correlated with the omitted variable too. Joint movement in corporate bond yields and bund repo specialness premia can induce endogeneity in a similar manner.

To mitigate endogeneity concern, I simulate counter-factual purchasing decisions of the PSPP had the Eurosystem operated with only exogenous rules. By exogenous rules, I mean rules that do not rely on yields and repo specialness premia. I also ignore the blackout period rule and the flexibility rule because I do not have information on their details. Let $\hat{Q}_{PSPP,j,t}$ denote the cumulative quantity of bond j that the Eurosystem would have purchased by day t in this counter-factual scenario.

As the Eurosystem accumulates holdings of government bonds over time, a subset of bonds inevitably becomes ineligible for purchase, for example, due to the issue limit constraint. Then following Boermans and Keshkov (2018), I assume that the Eurosystem spreads the aggregate purchase quantity over the remaining eligible bonds in proportion to their outstanding amounts.

On the supply side, $Q_{o,j,t}$ can also be endogenous. First, the German finance agency[15] might be determining issuance schedule strategically. The German finance agency auctions each bond to the market through multiple auctions that spans several months. The agency might be tailoring its auctions to movements in repo specialness premia. Second, investors might be bidding in the auction strategically, too. For example, investors might bid more or less aggressively based on their observation of repo rates. Then the final volume of bonds auctioned off to the market may be endogenous.

To mitigate endogeneity, I assume that the agency sells to the private market 100% of its intended issuance volume at the first auction date. For example, suppose the agency initially planned to issue 10 billion euros of bond X. It planned to auction 6 billion euros of this bond on day 1 and 4 billion euros on day 2. Suppose bidders were not aggressive, so only 5.7 billion euros and 2.7 billion euros were allotted on day 1 and day 2, respectively. I assume that the entire 10 billion euros of this bond X were sold on day 1. Let $\hat{Q}_{o,j,t}$ denote the outstanding quantity of bond j on day t in this counter-factual scenario.

I compute the counter-factual free float of bond j as $\hat{F}_{j,t} = \hat{Q}_{o,j,t} - \hat{Q}_{PSPP,j,t}$. The time series $\hat{F}_{j,t}$ is based on information such as total outstanding quantities, original maturities and initial issuance date of government bonds. $\hat{F}_{j,t}$ can be constructed based on the debt issuance strategy of the German federal government. Compared to other eurozone countries, the German federal government is less likely to be strategically responding to day-to-day fluctuations in yield curves. In December of each year, the German finance agency announces issuance schedules of all federal government bonds in the next year.[16] The schedule outlines how much of each individual security will be auctioned on each day of the year.

I use $\hat{F}_{j,t}$ to construct a new variable $L_{i,t}$ that instruments repo specialness premia $s_{i,t}$. The definition (33) is based on the local supply measure in Cahill, DAmico, Li and Sears (2013). The local supply measure takes into account the fact that bonds with similar remaining maturities are close substitutes for each other. For example, suppose German government bond with 9.9 years of remaining maturity is particularly scarce, much more so than bonds with 10.1 or 9.8 years of remaining maturity. Then hedge funds seeking to short long-term bonds might as well search for a 10.1-year or 9.8-year bond instead of a 9.9-year bond. Consequently, the circulation quantity of 9.9-year bond alone is not sufficient to explain the repo rate of the 9.9-year bond. I need to consider the circulation quantity of all bonds with maturities close enough to 9.9 years.

$$L_{i,t} = \sum_j \hat{F}_{j,t} \max \left\{ 1, \frac{1}{\theta} \left(1 - \frac{\left| m^g_{j,t} - m^c_{i,t} \right|}{m^c_{i,t}} \right) \right\} \tag{33}$$

In (33), $m^g_{j,t}$ denotes the remaining maturity of government bond j on day t. $m^c_{i,t}$ denotes the remaining maturity of corporate bond i on day t. $L_{i,t}$ is the weighted sum of free floats of all German federal government bonds outstanding on day t. The weight is decreasing in the

[15] https://www.deutsche-finanzagentur.de.
[16] See the website for example.

absolute difference between the remaining maturity of government bond j and that of corporate bond i. The weight is parameterized as a triangular function, following Cahill, DAmico, Li and Sears (2013). I set $\theta = 0.7$.[17]

7.4. 2SLS Specification

The first-stage and the second-stage specifications are (34) and (35), respectively. $\hat{s}_{it}$ is the fitted value of s_{it} from the first-stage regression.

$$s_{it} = \alpha_{1i} + \gamma_1 \cdot t + \beta_{1o} \cdot OIS_{it} + \beta_{1Q} \cdot Q_{it} + \beta_{1L} \cdot L_{it} + \epsilon_{1it} \tag{34}$$

$$y_{it} = \alpha_{2i} + \gamma_2 \cdot t + \beta_{2o} \cdot OIS_{it} + \beta_{2Q} \cdot Q_{it} + \beta_{2s} \cdot \hat{s}_{it} + \epsilon_{2it} \tag{35}$$

8. Empirical Results

Table 3 shows two-stage least squares (2SLS) estimation of (32). I run regressions separately for each category of credit rating. For example, the first column is based on all euro-denominated corporate bonds with S&P credit rating of AAA.

The instrument L_{it} is strongly correlated with the main explanatory variable s_{it} in all of the five specifications. The F-statistic for the test of significance of β_{1L} in the first-stage regression is always larger than 10.

Nevertheless, the main coefficient of interest β_{2s} is estimated to be significant only for credit ratings of AAA, AA and A. This result is in line with theoretical prediction. The repo specialness effect is likely to show up most strongly for corporate bonds with high credit ratings. Non-banks such as insurance companies are likely to consider AAA-rated corporate bonds and German federal government bonds fairly substitutable for each other. On the other hand, speculative-grade corporate bonds and German federal government bonds are less likely to be considered substitutable for each other.

For A or better-rated corporate bonds, the coefficient β_{2s} is estimated to be negative. As bund becomes more special in the repo market, the portfolio rebalancing causes corporate bond yields to decline.

Taking parameter values from Table 3, I estimate how corporate bond yields would have moved if there were no bund repo specialness premia throughout the sample period. My 2SLS estimation implies that an increase in bund repo specialness premia by 1 basis point is associated with a decline in 9-year A-rated corporate bond yields by 1.731 basis points. Let $y_{9Yr,t}$ denote the 9-year A-rated corporate bond yield on day t. Let $s_{9Yr,t}$ denote the repo specialness premium of a 9-year bund on day t. Then $y_{9Yr,t} + 1.731 \cdot s_{9Yr,t}$ is my estimate of 9-year A-rated corporate bond yield in a hypothetical world without bund repo specialness premia.

[17] Cahill, DAmico, Li and Sears (2013) set $\theta = 0.5$.

Dependent Variable: Corporate Bond Yield y_{it}					
	AAA	AA	A	BBB	Speculative
s_{it}	-2.479^{***}	-2.967^{***}	-1.731^{***}	-2.160	-1.073
	(0.880)	(0.726)	(0.521)	(1.944)	(10.23)
OIS_{it}	1.199^{***}	1.300^{***}	1.360^{***}	1.204^{***}	0.170
	(0.042)	(0.043)	(0.034)	(0.099)	(0.639)
Q_{it}	4.627^{***}	1.890	-8.427^{***}	-18.37^{***}	-182.5^{***}
	(1.036)	(2.049)	(1.301)	(2.156)	(42.59)
t	-0.0013^{**}	0.0005	0.0043^{***}	0.0074^{***}	0.0104^{***}
	(0.0007)	(0.0005)	(0.0004)	(0.0005)	(0.0024)
Error Cluster	Bond Time	Bond Time	Bond Time	Bond Time	Bond Time
R^2	0.789	0.661	0.699	0.243	0.124
Observations	209,903	107,920	257,225	266,153	40,367
First-stage F-statistic of excluded instruments	17.84	44.45	53.21	29.17	20.42
Frequency	Daily	Daily	Daily	Daily	Daily

Table 3: **The Effect of Bund Repo Specialness Premia on Corporate Bond Yields.** The table shows two-stage least squares (2SLS) estimation of the effect of bund repo specialness premia on corporate bond yields. The first-stage model is $s_{it} = \alpha_{2i} + \gamma_2 \cdot t + \beta_{2o} \cdot OIS_{it} + \beta_{2Q} \cdot Q_{it} + \beta_{2L} \cdot L_{it} + \epsilon_{2it}$. The second-stage model is $y_{it} = \alpha_{1i} + \gamma_1 \cdot t + \beta_{1o} \cdot OIS_{it} + \beta_{1Q} \cdot Q_{it} + \beta_{1s} \cdot s_{it} + \epsilon_{1it}$. y_{it} is the yield-to-maturity of corporate bond i on day t. OIS_{it} is the overnight index swap (OIS) rate at the tenor equal to the remaining maturity of corporate bond i. s_{it} is the repo specialness premium of German federal government bond (bund) with the remaining maturity identical to that of corporate bond i. If no such OIS rate or German federal government bond exist, a nonparametric regression or an interpolation is used to impute the value. Bund repo specialness premium s_{it} is instrumented with L_{it}, the weighted sum of the free floats of bunds. The weight is decreasing in the absolute difference in the remaining maturities of the bund and the corporate bond i. See equation (33) for the definition of L_{it}. Q_{it} is the total outstanding quantity of euro-denominated corporate bonds with S&P credit rating identical to that of corporate bond i. The unit of Q_{it} is trillions of euros. Repo specialness premium is defined as the Euro OverNight Index Average (EONIA) rate minus the repo rate at Spot-Next tenor. Standard errors are in parentheses and are clustered two-way by corporate bonds and time. *, **, and *** indicate that the coefficient is statistically significant at 1%, 5%, and 10%, respectively. The sample period is from March 1, 2014 to February 29, 2016. The first-stage F statistic of the excluded instrument is the F test of the significance of β_{1L} in the first-stage regression (34).

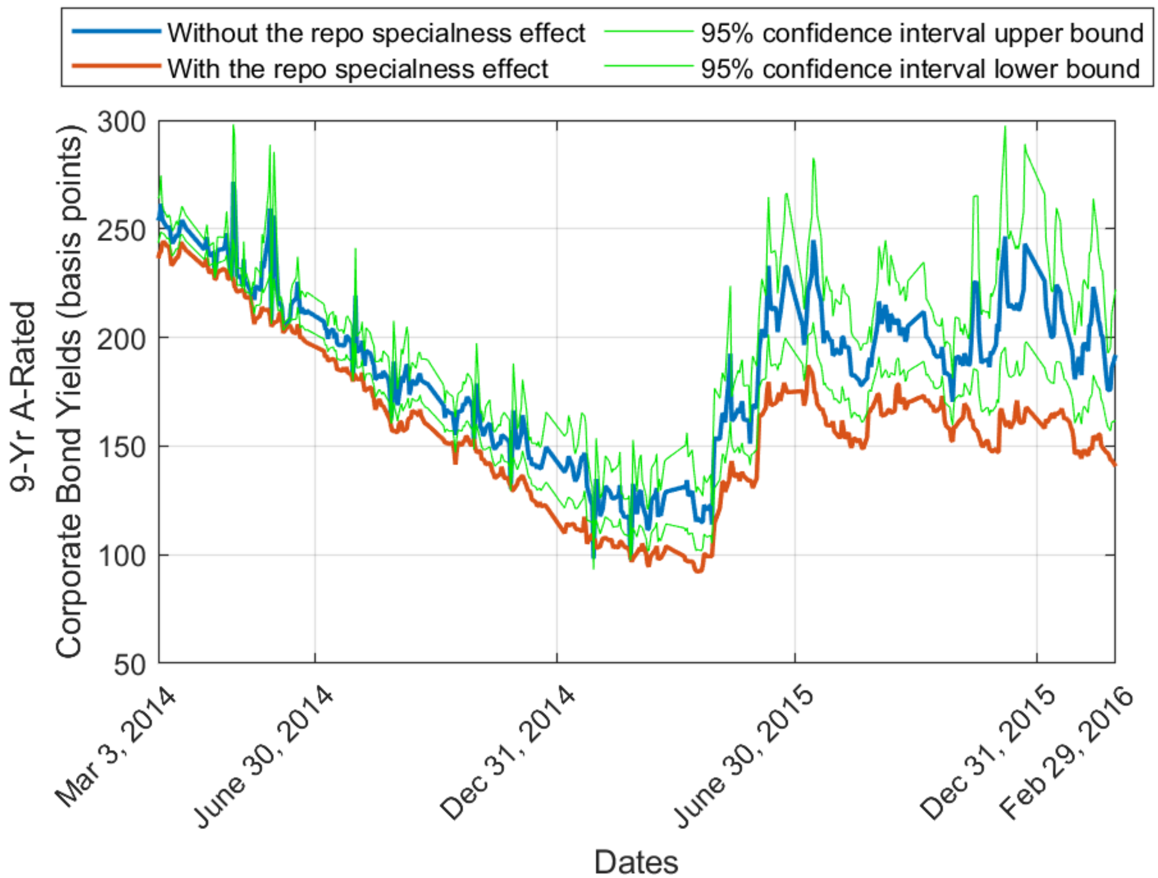

Figure 9: **Counter-factual Time-Series Evolution of 9-Year A-Rated Corporate Bond Yields.** The figure plots the time series evolution of A-rated euro-denominated corporate bond yields with residual maturity of 9 years. On each day during my sample period, I start with the universe of all euro-denominated A-rated corporate bond yields outstanding on that day. I run a Nadaraya-Watson kernel regression of yields-to-maturity on remaining maturity to estimate yields on a hypothetical A-rated bond with remaining maturity exactly equal to 9 years. The orange line is the plot of this estimation. Using the estimate from Table 3 and the observed time series of bund repo specialness premium, I estimate what the 9-year A-rated corporate bond yields would have been if the bund repo specialness premia were zero. The blue line shows the time-series of 9-year A-rated corporate bond yield in this hypothetical scenario. The two green lines show the 95% confidence interval around the orange line.

The orange line in Figure 9 shows that the actual 9-year A corporate bond yield declined by 95.7 basis points from March 3, 2014 to February 29, 2016. The blue line shows how the yield would have looked like if there were no bund repo specialness premia. I estimate that 31.9 basis points of the 95.7 basis points decline in 9-year A-rated corporate bond yield are related to the repo specialness effect. In section G of the appendix, I complete similar exercises for AAA-rated and AA-rated corporate bonds.

9. Discussion

9.1. The Repo Specialness Channel in Other Settings

I only present evidence of the transmission of the repo specialness premia of government bonds to investment grade corporate bond yields in this paper. Nevertheless, there is anecdotal evidence that in response to recent depression in euro-area government bond yields, European insurance companies increased the purchase of sub-sovereign and agency (SSA) bonds (AXA, 2016). Some insurance companies and pension funds have started investing in risky properties (Canepa, 2017). Such portfolio rebalancing behavior would cause those assets (e.g., SSA bonds) to appreciate.

The transmission can happen not only through insurance companies and pension funds but also through any financial institution that faces frictions in their use of the repo market. For example, new European regulation[18] on money market funds does not allow funds to repo more than 10% of their assets.

Another example is small banks. Small banks that are not members of CCPs participate in the repo market indirectly through large bank dealers. Small banks do not have a large flow of repo transactions that justify the fixed cost of becoming members of CCPs (ICMA, 2015). Hence, just like non-banks in my model, they incur extra costs such as search costs or relationship costs. As repo specialness premia cause government bond yields to become even lower, small banks are likely to substitute for government bonds with risky assets such as loans to firms. In fact, inducing the banking sector to substitute for government bonds with riskier corporate loans is an important channel that the Eurosystem had in mind in the context of the PSPP (Cœuré, 2015). Repo specialness premia on government bonds amplify this channel.

Once yields on investment-grade corporate bonds decline, there can be knock-on effects on firms that do not have access to the capital market. For example, as yields on investment-grade corporate bonds decline, firms with access to the capital market can substitute bank loans with corporate bonds. As a result, banks have more lending capacity available. Banks can subsequently use that capacity to make loans to firms that do not have access to the bond market. Grosse-Rueschkamp, Steffen and Streitz (2017) identified this channel for the case of the Corporate Sector Purchase Program (CSPP).

[18] See Regulation (EU) 2017/1131 of the European Parliament and of the Council of 14 June 2017 on Money Market Funds.

9.2. Investment Behavior of Insurance Companies and Pension Funds

Insurance companies and pension funds are buy-and-hold investors. When they write new policies, they receive cash from policy subscribers. To match the long-duration of their liabilities, they purchase long-term fixed-income assets, such as government bonds or corporate bonds. Once purchased, those bonds are often held until maturity.

The Securities Holdings Statistics data shows that the holdings of euro-denominated investment grade corporate bonds by the insurance sector declined recently. Nevertheless, much of that decline is due to the redemption of bonds that insurance companies purchased a long time ago (Davies and Hetland, 2017). If one looks at how insurance companies invest cash that they get from new underwritten policies, there has been a shift from government bonds to investment-grade corporate bonds (Davies and Hetland, 2017).

9.3. Fiscal Implication

An increase in the repo specialness of government bonds can help the government to issue its bonds more cheaply in the primary market. This effect is likely to have been particularly relevant to the governments of the two largest euro-area economies - France and Germany - during the PSPP period. Of the government bonds of the seven largest eurozone economies, the French and the German bonds were most special in the repo market (Arrata, Nguyen, Rahmouni-Rousseau and Vari, 2018). At the same time, both the French and the German governments rely on primary dealers to issue their bonds.[19] Primary dealers are banks that can actively repo their government bonds once acquired. Therefore, primary dealers may be willing to bid higher prices for purchases of government bonds that are on special in the repo market. Their more aggressive bids could have helped to lower the financing costs of the French and the German governments.

9.4. The Allocative Efficiency

An increase in the repo specialness of government bonds through the scarcity of available bonds can have an adverse impact on the allocative efficiency of both the bond market and the repo market. Specialness reflects heightened search frictions in the repo market. That is, collateral providers and cash investors are not matched to each other efficiently. The inefficiency arises because the bonds stay with the less efficient users of these bonds (collateral provider) instead of the more efficient users (cash investors).

Besides, dealers in the bond market often deliver bonds to their customers by reversing-in the requested bonds from the repo market (ICMA, 2015). Larger search frictions in the repo market imply that dealers may not be able to obtain the bonds promptly. Thus, search frictions

[19] According to the website of Agence France Trésor, "The principal method of issuing French government securities since 1985 has been" bid price auctions in which primary dealers participate. According to the website of the German Finance Agency, more than 90% of the bunds are auctioned to primary dealers.

in the repo market, signaled by large specialness, can also interfere with the matching of buyers and sellers of the bond.

10. Conclusion

The repo specialness of government bonds can magnify the impact on corporate bond prices of a central bank's government bond purchases. The specific case of the Public Sector Purchase Program (PSPP) of Europe motivated this paper. Unlike the Bank of Canada or the Federal Reserve System of the United States, the Eurosystem did not supply back most of the bonds that it purchased under the PSPP. The repo specialness premium cause government bonds to become more expensive in the cash market. As a result, non-banks substitute for expensive government bonds with investment-grade corporate bonds, thereby allowing firms to finance themselves more cheaply. Yields on 9-year A-rated euro-denominated corporate bonds dropped by 95.7 basis points from March 3, 2014 to February 29, 2016. I estimate that 31.9 basis points of this drop are associated with the repo specialness effect.

References

Albertazzi, U., Becker, B., Boucinha, M., 2018. Portfolio rebalancing and the transmission of largescale asset programmes: evidence from the euro area. ECB Working Paper Series URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp2125.en.pdf?e2aee48fbbfeccd4a608f2bc1d2083ee.

Altavilla, C., Carboni, G., Motto, R., 2015. Asset Purchase Programmes and Financial Markets: Lessons from the Euro Area. European Central Bank Working Paper Series URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1864.en.pdf, doi:10.1111/j.1467-629X.1980.tb00220.x.

Andrade, P., Breckenfelder, J., De Fiore, F., Karadi, P., Tristani, O., 2016. The ecb's asset purchase programme: an early assessment. ECB Working Paper Series URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1956.en.pdf.

Arrata, W., Nguyen, B., 2017. Price Impact of Bond Supply Shocks: Evidence from the Eurosystem's Asset Purchase Program. Banque de France Working Paper URL: https://www.banque-france.fr/sites/default/files/medias/documents/document-de-travail-623_2017-03.pdf.

Arrata, W., Nguyen, B., Rahmouni-Rousseau, I., Vari, M., 2018. The Scarcity Effect of Quantitative Easing on Repo Rates: Evidence from the Euro Area. IMF Working Paper URL: https://www.imf.org/en/Publications/WP/Issues/2018/12/07/The-Scarcity-Effect-of-Quantitative-Easing-on-Repo-Rates-Evidence-from-the-Euro-Area-46404.

AXA, 2016. Solvency 2 Impact on European Bond Market. Presentation at the Bond Market Contact Group of the European Central Bank. URL: https://www.ecb.europa.eu/paym/groups/pdf/bmcg/160621/2016-06-21_-_Item%203.2._-_Solvency_2_impact_on_European_bond_markets.pdf?ab42efd51bf3522e5bacfbc02495b12a.

Bergant, K., Fiodra, M., Schmitz, M., 2018. International capital flows at the security level - evidence from the ecb's asset purchase programme. CEPS Working Paper URL: https://www.ceps.eu/publications/international-capital-flows-security-level-%E2%80%93-evidence-ecb%E2%80%99s-asset-purchase-programme.

Bernanke, B., 2011. Statement before the Committee on Financial Services of the U.S. House of Representatives. Speech. URL: https://www.federalreserve.gov/newsevents/testimony/bernanke20110713a1.pdf.

Blattner, T., Joyce, M., 2016. Net debt supply shocks in the euro area and the implications for qe. European Central Bank Working Paper URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1957.en.pdf?81ec0cf7f6e24f242d1f3ba083cd93cd.

Boermans, M., Keshkov, V., 2018. The Impact of the ECB Asset Purchases on the European Bond Market Structure: Granular Evidence on Ownership Concentration. DeNederlandscheBank Working Paper URL: https://www.dnb.nl/binaries/Working%20Paper%20No.%20590_tcm46-375317.pdf.

Boissel, C., Derrien, F., Ors, E., Thesmar, D., 2017. Systemic risk in clearing houses: evidence from the european repo market. Journal of Financial Economics 125, 511–536. URL: https://www-sciencedirect-com.stanford.idm.oclc.org/science/article/pii/S0304405X17301277.

Boysen-Hogrefe, J., Fieldler, S., Jannsen, N., Kooths, S., Reitz, S., 2016. Limits in terms of Eligible Collateral and Policy Risks of an Extension of the ECB's Quantitative Easing Programme. Technical Report. Directorate General for Internal Policies, the European Parliament. URL: http://www.europarl.europa.eu/cmsdata/105470/IPOL_IDA(2016)569994_EN.pdf.

Brand, C., Ferrante, L., Hubert, A., 2019. From cash-to securities-driven euro area repo markets: the role of financial stress and safe asset scarcity. ECB Working Paper Series URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp2232~c3399f4b01.en.pdf?4a5b7f70c587493dc299cc2d1768aa1e.

Brunnermeier, M., James, H., Landau, J.P., 2016. The Euro and the Battle of Ideas. Princeton University Press.

Bua, G., Dunne, P., 2017. The portfolio rebalancing effects of the ecb's asset purchase programme. Central Bank of Ireland URL: https://www.

centralbank.ie/docs/default-source/publications/research-technical-papers/07rt17---the-portfolio-rebalancing-effects-of-the-ecb-asset-purchase-programme.pdf?sfvrsn=2.

Buraschi, A., Menini, D., 2002. Liquidity Risk and Specialness. Journal of Financial Economics 64, 243–284. URL: https://www.sciencedirect.com/science/article/pii/S0304405X02000776.

Cahill, M., DAmico, S., Li, C., Sears, J., 2013. Duration Risk versus Local Supply Channel in Treasury Yields: Evidence from the Federal Reserve's Asset Purchase Announcements. Finance and Economics Discussion Series URL: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2976836.

Canepa, F., 2017. ECB unlikely to boost bond lending to ease end-March squeeze-sources. News. URL: https://www.reuters.com/article/ecb-policy-bonds-idUSL5N1GF5YT.

Carpenter, S., Demiralp, S., Ihrig, J., Klee, E., 2013. Analyzing federal reserve asset purchases: from whom does the fed buy? Finance and Economics Discussion Series of the Federal Reserve Board .

Chadha, J., Hantzsche, A., 2018. The impact of the ecb's qe programme: core versus periphery. Working Paper URL: http://www.euroframe.org/files/user_upload/euroframe/docs/2018/Conference/Session%208/EUROF18_Chadha_Hantzsche.pdf.

Corradin, S., Maddaloni, A., 2017. The Importance of Being Special: Repo Markets During the Crisis. European Central Bank Working Paper Series URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp2065.en.pdf?f63dfd95702fd923704fe00418aee009.

Cœuré, B., 2015. Embarking on Public Sector Asset Purchases. Speech at the Second International Conference on Sovereign Bond Markets. URL: https://www.ecb.europa.eu/press/key/date/2015/html/sp150310_1.en.html.

Cœuré, B., 2017. Bond Scarcity and the ECB's Asset Purchase Programme. Speech. URL: https://www.ecb.europa.eu/press/key/date/2017/html/sp170403_1.en.html.

Cœuré, B., 2018. The Persistence and Signalling Power of Central Bank Asset Purchase Programmes. Speech. URL: https://www.ecb.europa.eu/press/key/date/2018/html/ecb.sp180223.en.html.

D'Amico, S., Fan, R., Kitsul, Y., 2018. The Scarcity Value of Treasury Collateral: Repo Market Effects of Security-Specific Supply and Demand Factors. Journal of Financial and Quantitative Analysis 53, 2103–2129. URL: https://www.cambridge.org/core/journals/journal-of-financial-and-quantitative-analysis/article/scarcity-value-of-treasury-collateral-repomarket-effects-of-securityspecific-supply-and-dema
FA32EDBEDFB4381BBA47AB55EA9D1340.

DAmico, S., Pancost, A., 2017. Special Repo Rates and the Cross-Section of Bond Prices. Working Paper URL: http://www.greta.it/sovereign/sovereign5/papers/02_Pancost.pdf.

Davies, Z., Hetland, A., 2017. Low Yields Shape Demand. Technical Report. Barclays Credit Research.

De Santis, R., 2016. Impact of the asset purchase programme on euro area government bond yields using market news. ECB Working Paper Series URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1939.en.pdf?712abb4a54132af89260d47385ade9ef.

De Santis, R., Holm-Hadulla, F., 2017. Flow effects of central bank asset purchases on euro area sovereign bond yields: evidence from a natural experiment. ECB Working Paper Series URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp2052.en.pdf.

Duffie, D., 1996. Special Repo Rates. The Journal of Finance LI, 493–526. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.1996.tb02692.x.

Duffie, D., 2017. Dynamic Asset Pricing Theory. Unpublished edition ed., Princeton University Press.

Duffie, D., 2018. Post-Crisis Bank Regulations and Financial Market Liquidity, Working Paper.

Duffie, D., Garleanu, N., Pedersen, L.H., 2002. Securities Lending, Shorting, and Pricing. Journal of Financial Economics 66, 307–339. URL: https://www.sciencedirect.com/science/article/pii/S0304405X0200226X, doi:10.2139/ssrn.302015.

Duffie, D., Garleanu, N., Pedersen, L.H., 2005. Over-the-Counter Markets. Econometrica 73, 1815–1847. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1468-0262.2005.00639.x.

Duffie, D., Garleanu, N., Pedersen, L.H., 2007. Valuation in Over-the-Counter Markets. The Review of Financial Studies 20, 1865–1900. URL: https://academic.oup.com/rfs/article/20/6/1865/1575887.

Duffie, D., Malamud, S., Manso, G., 2009. Information Percolation with Equilibrium Search Dynamics. Econometrica 77, 1513–1574. URL: https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA8160.

Dunne, P., Hau, H., Moore, M., 2014. Dealer Intermediation between Markets. Journal of the European Economic Association 13, 770–804. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/jeea.12118.

Ejsing, J., Sihvonen, J., 2009. Liquidity Premia in German Government Bonds. European Central Bank Working Paper URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1081.pdf?ae81b6fa5af3f3e276eaa7308c665dad.

EMMI, 2017. Consultation Paper on a New Reference Index for the Euro Repo Market. Technical Report. European Money Market Institute. URL: https://www.emmi-benchmarks.eu/assets/files/D0307H-2017-Repo%20Consultation%20Paper.pdf.

ESRB, 2016. Macroprudential Policy Issues Arising from Low Interest Rates and Structural Changes in the EU Financial System. URL: https://www.esrb.europa.eu/pub/pdf/reports/161128_low_interest_rate_report.en.pdf.

Falagiarda, M., McQuade, P., Tirpak, M., 2015. Spillovers from the ecb's nonstandard monetary policies on non-euro area eu countries: evidence from an event-study analysis. ECB Working Paper Series URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1869.en.pdf?e537966259dd1344b3de8635d9591b64.

Ferdinandusse, M., Freier, M., Ristiniemi, A., 2017. Quantitative Easing and the Price-Liquidity Trade-Off. Sveriges Riksbank Research Paper Series URL: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2917294.

Ferrari, M., Guagliano, C., Mazzacurati, J., 2017. Collateral Scarcity Premia in Euro Area Repo Markets. European Systemic Risk Board Working Paper Series URL: https://www.esrb.europa.eu/pub/pdf/wp/esrb.wp55.en.pdf?878239b868a5d574391c5e9c2656c45e.

Ferrero, G., Loberto, M., Miccoli, M., 2017. The Collateral Channel of Unconventional Monetary Policy. Bank of Italy Working Paper Series URL: https://www.bancaditalia.it/pubblicazioni/temi-discussione/2017/2017-1119/index.html?com.dotmarketing.htmlpage.language=1.

Fleming, M., Keane, F., Schurmeier, J., Weiss, E., 2016. A Closer Look at the Federal Reserve's Securities Lending Program, Working Paper.

Gambetti, L., Musso, A., 2017. The Macroeconomic Impact of the ECB's Expanded Asset Purchase Programme. European Central Bank Working Paper URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp.2075.en.pdf.

Goldstein, I., Witmer, J., Yang, J., 2018. Following the money: evidence for the portfolio balance channel of quantitative easing. Bank of Canada Staff Working Paper URL: https://www.bankofcanada.ca/wp-content/uploads/2018/07/swp2018-33.pdf.

Gros, D., 2016. QE Infinity: What Risks for the ECB?, in: Limits in terms of Eligible Collateral and Policy Risks of an Extension of the ECB's Quantitative Easing Programme. The Directorate General for Internal Policies, the European Parliament. URL: http://www.europarl.europa.eu/cmsdata/105470/IPOL_IDA(2016)569994_EN.pdf.

Grosse-Rueschkamp, B., Steffen, S., Streitz, D., 2017. Cutting Out the Middleman – The ECB as Corporate Bond Investor, Working Paper.

He, Z., Milbradt, K., 2014. Endogenous Liquidity and Defaultable Bonds. Econometrica 82, 1443–1508. URL: https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA11039.

He, Z., Xiong, W., 2012. Dynamic Debt Run. The Review of Financial Studies 25. URL: https://academic.oup.com/rfs/article/25/6/1799/1594008.

Heynderickx, W., Cariboni, J., Schoutens, W., Smits, B., 2016. The Relationship between Risk-Neutral and Actual Default Probabilities: the Credit Risk Premium. Applied Economics 48, 4066–4081. URL: https://www.tandfonline.com/doi/pdf/10.1080/00036846.2016.1150953?needAccess=true.

Hill, A., 2015a. ICMA Impact Study for Mandatory Buy-Ins: the Repo Market. Technical Report. International Capital Market Association. URL: https://www.icmagroup.org/Regulatory-Policy-and-Market-Practice/Regulatory-Policy-Newsletter/Previous-versions/.

Hill, A., 2015b. Perspectives from the Eye of the Storm: The Current State and Future Evolution of the European Repo Market. An initiative of the ICMA European Repo Council URL: https://www.icmagroup.org/assets/documents/Regulatory/Repo/The-current-state-and-future-evolution-of-the-European-repo-market-181115.pdf.

Hill, A., 2017. Closed for Business. A Post-Mortem of the European Repo Market Break-down over the 2016 Year-End. Technical Report. International Capital Market Association. URL: https://www.icmagroup.org/assets/documents/Regulatory/Repo/ICMA-ERCC-year-end-report-2016-AndyHill-020317.pdf.

Huszar, Z., Simon, Z., 2017. The Liquidity and Welfare Implications of the Securities Lending Market for European Treasuries. Working Paper URL: https://www.netspar.nl/assets/uploads/E20181901_Paper_Zorka.pdf.

IAIS, 2016. 2015 Global Insurance Market Report. White Paper.

ICMA, 2015. Frequently Asked Questions on Repo. White Paper. URL: https://www.icmagroup.org/assets/documents/Regulatory/Repo/Repo-FAQs-4-December-2015.pdf.

Jank, S., Mönch, E., 2018. The impact of eurosystem bond purchases on the repo market. Bundesbank Research Brief, 21st edition URL: https://www.bundesbank.de/en/publications/research/research-brief/2018-21-bond-purchases-repo-market-761744.

Jank, S., Mönch, E., 2019. Safe asset shortage and collateral reuse. Deutsche Bundesbank internal memo .

Jordan, B., Jordan, S., 1997. Special Repo Rates: an Empirical Analysis. The Journal of Finance LII. URL: https://www.jstor.org/stable/2329473?seq=1#metadata_info_tab_contents.

Joyce, M., Liu, Z., Tonks, I., 2014. Institutional investor portfolio allocation, quantitative easing and the global financial crisis. Bank of England Working Paper URL: https://www.bankofengland.co.uk/-/media/boe/files/working-paper/2014/institutional-investor-portfolio-allocation-quantitative-easing-and-the-global-financial-cri pdf?la=en&hash=DB7AE58E928E244CBE48B320DF6498A6A9A73816.

Karadi, P., 2017. The ecb's announcements of non-standard measures and longer-term inflation expectations. ECB Research Bulletin URL: https://www.ecb.europa.eu/pub/economic-research/resbull/2017/html/ecb.rb170418.en.html.

Koijen, R., Koulischer, F., Nguyen, B., Yogo, M., 2018. Inspecting the Mechanism of Quantitative Easing in the Euro Area. Banque de France working paper URL: https://publications.banque-france.fr/sites/default/files/medias/documents/wp601.pdf.

Krishnamurthy, A., 2002. The Bond/Old-Bond Spread. Journal of Financial Economics 66. URL: https://people.stanford.edu/akris/sites/default/files/krishnamurthyjfe2002.pdf.

Krishnamurthy, A., Vissing-Jorgensen, A., 2012a. The Aggregate Demand for Treasury Debt. Journal of Political Economy 120, 233–267. URL: http://www.journals.uchicago.edu/doi/10.1086/666526, doi:10.1086/666526.

Krishnamurthy, A., Vissing-Jorgensen, A., 2012b. The Ins and Outs of LSAPs, in: Kansas City Federal Reserve Symposium on Global Dimensions of Unconventional Monetary Policy, Kansas City. URL: https://people.stanford.edu/akris/sites/default/files/ins_and_outs_of_lsaps.pdf, doi:10.1016/j.tcb.2004.07.007.

Lemke, W., Werner, T., 2017. Dissecting long-term bund yields in the run-up to the ecb's public sector purchase programme. ECB Working Paper Series URL: https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp2106.en.pdf?265269477f1f0e65555e6827aae3aab6.

Mancini, L., Ranaldo, A., Wrampelmeyer, J., 2015. The euro interbank repo market. The Review of Financial Studies 29, 1747–1779. URL: https://academic.oup.com/rfs/article-abstract/29/7/1747/2606985.

Maraffino, G., 2016. PSPP Securities Lending: Accepting Cash for Collateral. Dec 8 2016. Technical Report. Barclays.

Marraffino, G., 2017. Euro Money Markets Weekly: Collateral Squeeze to Continue. Jan 9 2017. Technical Report. Barclays.

McGuire, R., Graham-Taylor, L., Cairns, M., 2017. Looking at Quarterly Squeezes. Report. URL: https://services.rabobank.com/publicationservice/download/publication/token/ddJZydFvh3GEV0TJmWcJ.

Mersch, Y., 2016. Monetary Policy in the Euro Area: Scope, Principles and Limits. Keynote speech at the Natixis Meeting of Chief Economists. URL: https://www.ecb.europa.eu/press/key/date/2016/html/sp160623.en.html.

Mersch, Y., 2017. Ructions in the Repo Market -Monetary Easing or Regulatory Squeezing? Speech. URL: https://www.ecb.europa.eu/press/key/date/2017/html/sp170126.en.html.

Nyborg, K., 2017. Collateral Frameworks, the Open Secret of Central Banks. Cambridge University Press.

Nyborg, K., 2019. Repo rates and the collateral spread puzzle. Swiss Finance Institute Research Paper Series URL: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3335203#.

Paludkiewicz, K., 2018. Unconventional Monetary Policy, Bank Lending, and Security Holdings: the Yield-Induced Portfolio Rebalancing Channel. Deutsche Bundesbank Discussion Paper URL: https://www.bundesbank.de/Redaktion/EN/Downloads/Publications/Discussion_Paper_1/2018/2018_07_16_dkp_22.pdf?__blob=publicationFile.

Patterson, L., 2015. Fine-Tuning the Framework for the Bank's Market Operations. Speech at the CFA Society Vancouver. URL: https://www.bankofcanada.ca/2015/05/fine-tuning-framework-bank-market-operations/.

Pelizzon, L., Subrahmanyam, M., Tomio, D., Uno, J., 2018. Central Bank Driven Mispricing. Working Paper URL: http://people.stern.nyu.edu/msubrahm/papers/APPArbitrage.pdf.

Piquard, T., Salakhova, D., 2018. Substitution between secured and unsecured interbank markets, counterparty risk and the opportunity cost of collateral. Banque de France URL: https://adres2019.sciencesconf.org/236945/document.

Praet, P., 2015. The APP Impact on the Economy and Bond Markets. Intervention by Peter Praet, Member of the Executive Board of the ECB, at the annual dinner of the ECB's Bond Market Contact Group,Frankfurt am Main. URL: https://www.ecb.europa.eu/press/key/date/2015/html/sp150630.en.html.

Protter, P., 1990. Stochastic Integration and Differential Equations. Springer-Verlag, New York, New York.

Protter, S., 2018. The Supply of Money-Like Assets. Remarks for the American Economic Association Panel Session "The Balance Sheets of Central Banks and the Shortage of Safe Assets". URL: https://www.bis.org/review/r180115e.htm.

Sangiorgi, I., 2017. Essays on the Repo Market. Ph.D. thesis. University of Reading. URL: http://centaur.reading.ac.uk/76521/.

Schlepper, K., Hofer, H., Riordan, R., Schrimpf, A., 2017. Scarcity Effects of QE: A Transaction-Level Analysis in the Bund Market. BIS Working Papers URL: https://www.bis.org/publ/work625.htm.

Sun, Y., 2006. The Exact Law of Large Numbers via Fubini Extension and Characterization of Insurable Risks. Journal of Economic Theory 126, 31–69. URL: https://www.sciencedirect.com/science/article/pii/S0022053104002224.

Tischer, J., 2018. Quantitative Easing, Portfolio Rebalancing and Credit Growth: Micro Evidence from Germany. Deutsche Bundesbank Discussion Paper URL: https://www.bundesbank.de/Redaktion/EN/Downloads/Publications/Discussion_Paper_1/2018/2018_07_13_dkp_20.pdf?__blob=publicationFile.

UniCredit, 2016. Handbook of German States. White Paper. URL: https://www.research.unicredit.eu/DocsKey/credit_docs_2016_157740.ashx?M=D&R=41841879.

Vayanos, D., Vila, J.L., 2009. A preferred- habitat model of the term structure of interest rates. NBER Working Paper URL: http://www.nber.org/papers/w15487.pdf.

Wallace, N., 1982. A Modigliani-Miller Theorem for Open-Market Operations. The American Economic Review 71, 267–274. URL: https://www.jstor.org/stable/pdf/1802777.pdf?refreqid=search%3Afaa32bcfd95caeae3e04d7d3f85b1fdf.

Woodford, M., 2012. Methods of Policy Accommodation at the Interest-Rate Lower Bound, in: the Federal Reserve Bank of Kansas City (FRBKC) Economic Policy Symposium on the Changing Policy Landscape, Jackson Hole, Wyoming. URL: https://www.kansascityfed.org/publicat/sympos/2012/mw.pdf, doi:10.7916/D8Z899CJ.

A. List of Parameters and Variables in the Theoretical Model

Table 4: The list of parameters used in the model

Parameters	Meaning	Exogenous?
δ	The rate at which repo matures	Y
η	The rate at which a unit of corporate bond expires	Y
r	The risk-free rate	Y
λ_o	Search intensity of OW non-bank	N
λ_c	Search intensity of CW cash investor	N
A	The specialness premium at which cash investors reverse repo government bonds	N
B	The specialness premium at which OW non-banks repo government bonds	N
m_c	The mass of cash investors	Y
m_n	The mass of non-banks	Y
b	The benefit of obtaining government bond for cash investors per unit of time	Y
c	The coupon rate of a government bond and a corporate bond	Y
P_{ig}	The corporate bond price	N
P	The government bond price	N
z	Dealer's bargaining power	Y
N	The number of dealers	Y
S_g	The outstanding quantity of government bonds	Y
n_c	The quantity of new corporate bonds issued per unit of time	Y
f_{IG}	The mass rate of N non-banks that buy corporate bonds	N
f_{OW}	The mass rate of N non-banks that buy government bonds	N

Table 5: The list of random variables used in this model

Variable	Definition
τ_δ	Stopping time at which repo matures
τ_D	Stopping time at which a customer meets a dealer
τ_η	Stopping time at which a corporate bond defaults

Table 5: The list of random variables used in this model

μ_t^σ	The mass of investors of type $\sigma \in \{OW, O, CW, C, IG\}$ at time t.
θ_t^{RR}	Indicator variable that is 1 if and only if a dealer reverse-repos a government bond from a non-bank customer
θ_t^R	Indicator variable that is 1 if and only if a dealer repos a government bond to a cash investor
x_t	The number of units of government bonds that dealer owns at time t
Q_t^{RR}	The number of units of government bonds that dealer reverse repos at time t
Q_t^R	The number of units of government bonds that dealer repos at time t

B. Proofs

B.1. Theorem 1

Lemma 1. *Dealer's value function is given as (36).*

$$V_D(W_t, Q_t^{RR}, Q_t^R) = W_t + Q_t^R \frac{A + c - rP}{r + \delta} + Q_t^{RR} \frac{-c - B + rP}{r + \delta} + (A + c - rP) \frac{\lambda_c \mu_t^{CW}}{Nr(r + \delta)} + (-c - B + rP) \frac{\lambda_o \mu_t^{OW}}{Nr(r + \delta)} \quad (36)$$

Proof. I assume that dealers have an incentive to intermediate both sides of the repo market. I check this incentive compatibility later. Then the dealer's problem is isomorphic to (37). Cash collateral exchanged as part of repo and reverse repo transactions is completely irrelevant. Dealer can always undo that exchange of collateral by borrowing or lending at the rate r.

$$V_D\left(W_t, Q_t^{RR}, Q_t^R\right) = \sup_{x, C} \mathbb{E}_t \int_{s=0}^{\infty} e^{-rs} dC_{t+s} \quad (37)$$

s.t.

$$dW_t = rW_t dt - dC_t + AQ_t^R dt - BQ_t^{RR} dt + x_t c dt - P dx_t$$

$$x_t + Q_t^{RR} \geq Q_t^R$$

$$dQ_t^{RR} = -\delta Q_t^{RR} dt + \frac{1}{N} \lambda_o \mu_t^{OW} dt$$

$$dQ_t^R = -\delta Q_t^R dt + \frac{1}{N} \lambda_c \mu_t^{CW} dt$$

Because the dealer's time discount rate is equal to the risk free rate r, it is without loss of generality to assume that $W_t = 0$ for $\forall t$. Then the value function is linearly separable in W_t. Define a new value function $J_D(Q_t^{RR}, Q_t^R)$ such that $V_D(W_t, Q_t^{RR}, Q_t^R) = W_t + J_D(Q_t^{RR}, Q_t^R)$.

Duffie, Garleanu and Pedersen (2005) also made this assumption. Then the dealer's problem can be reformulated as (38).

$$J_D\left(Q_t^{RR}, Q_t^R\right) = \sup_x \mathbb{E}_t \int_{s=0}^{\infty} e^{-rs} \left\{AQ_{t+s}^R dt - BQ_{t+s}^{RR} dt + x_{t+s} c dt - P dx_t\right\} \tag{38}$$

$$s.t.$$

$$x_t + Q_t^{RR} \geq Q_t^R$$

$$dQ_t^{RR} = -\delta Q_t^{RR} dt + \frac{1}{N}\lambda_o \mu_t^{OW} dt$$

$$dQ_t^R = -\delta Q_t^R dt + \frac{1}{N}\lambda_c \mu_t^{CW} dt$$

If the government bond price P is below $\frac{c}{r}$, dealers would purchase government bonds in the inter-dealer cash market as much as possible. Thus, the government bond inter-dealer cash market cannot clear.

Suppose $P = \frac{c}{r}$. Suppose a dealer obtains the government bond from the inter-dealer cash market and subsequently repo the same bond to cash investors. The opportunity cost of using cash to purchase government bonds is zero. As long as b is positive and the dealer has non-zero bargaining power vis-a-vis cash investors, dealers can profit from this transaction. Hence, the dealer will not refuse to transact with cash investors. $\theta^R = 1$.

The feasibility constraint implies that to obtain a government bond for repo position vis-a-vis cash investors, the dealer can purchase it or reverse-in. If B is zero, then non-banks would exert zero search effort, and there would not be any flow of bonds to repo dealers. In equilibrium, dealers would have to go to the cash market to obtain government bonds to be delivered to cash investors. If B is positive, then any dealer would choose to go to the cash market to obtain a government bond instead of reverse repoing it. Dealer would not want to incur the cost of B per unit of time. In either case, dealers would go to the cash market to obtain government bonds. Therefore, in equilibrium, $x_t = Q_t^R > 0$ for all dealers. Nevertheless, I already mentioned that for the government bond cash market to clear, dealers as a whole must demand zero government bonds. Hence, the market cannot clear.

Thus, the government bond cash market can clear only when $P > \frac{c}{r} = \int_0^\infty e^{-rt} c dt$. Because government bonds are inflated, dealers would want to minimize their position in the cash market x_t. Thus, it must be the case that $x_t + Q_t^{RR} = Q_t^R$. Thus, $dx_t = dQ_t^R - dQ_t^{RR} = -\delta Q_t^R dt + \frac{1}{N}\lambda_c \mu_t^{CW} dt + \delta Q_t^{RR} dt - \frac{1}{N}\lambda_o \mu_t^{OW} dt$.

The Hamilton-Jacobi-Bellman equation can be formulated as (39). $\mathcal{D}$ is a symbol for generator.

$$\mathcal{D}J_D\left(Q^{RR}, Q^R\right) - r \cdot J\left(Q^{RR}, Q^R\right) + AQ^R - BQ^{RR} + c\left(Q^{RR} - Q^R\right) - P\left(-\delta Q_t^R + \frac{1}{N}\lambda_c \mu_t^{CW} + \delta Q_t^{RR} - \frac{1}{N}\lambda_o \mu_t^{OW}\right) = 0 \tag{39}$$

Conjecture that the value function is linear in state variables.

$$J_D\left(Q^{RR}, Q^R\right) = H_0 + H^{RR}Q^{RR} + H^RQ^R \tag{40}$$

Substitute the conjecture (40) into the HJB equation (39). Solve for the values of H_0, H^{RR}, and H^R such that the equation holds for any value of Q^{RR} and Q^R. □

Value functions of all the other investors are also linear in W_t. Thus, define value functions J_σ such that $V_\sigma(W_t) = W_t + J_\sigma$ for $\sigma \in \{OW, O, CW, W, N, IG\}$.

Lemma 2. *Value functions of non-banks that own government bonds are given as (41) and (42). The optimal search intensity λ_o maximizes J_{OW}. The first-order condition for λ_o is (43).*

$$J_{OW} = \frac{c}{r} + \frac{-(r+\delta)K(\lambda_o) + \lambda_o B}{r(r+\lambda_o+\delta)} \tag{41}$$

$$J_O = -P + \frac{c}{r} + \frac{-\delta K(\lambda_o) + (r+\lambda_o)B}{r(r+\lambda_o+\delta)} \tag{42}$$

$$B + K(\lambda_o) = K'(\lambda_o)(r+\lambda_o+\delta) \tag{43}$$

Proof. Without loss of generality, assume zero bank-account process. Then non-bank's problem (2), (3), (4) and (5) can be simplified.

$$J_{OW} = \sup_{\lambda_o} \mathbb{E}\left[\int_0^{\tau_D} e^{-rt}\left\{-K\left(\lambda_o\right) + c\right\}dt + e^{-r\cdot\tau_D}\left(J_o + P\right)\right] = \sup_{\lambda_o} \frac{c - K(\lambda_o) + \lambda_o\left(J_o + P\right)}{r+\lambda_o} \tag{44}$$

$$J_o = \mathbb{E}\left[\int_0^{\tau_\delta} e^{-rt}(c+B)dt + e^{-r\cdot\tau_\delta}\left(J_{OW} - P\cdot e^{r\cdot\tau_\delta}\right)\right]$$

$$J_o + P = \frac{J_{OW}\delta + c + B}{r+\delta} \tag{45}$$

For each given value of λ_o, (44) and (45) can be solved jointly to get (41) and (42). Differentiate the right-hand side of (41) with respect to λ_o to obtain the first-order condition (43). A non-bank gets to choose the search intensity λ_o only in the OW state. A non-bank in the OW state faces the exact same stationary problem at every point in time. Hence, it is without loss of generality to assume that the non-bank chooses one value of λ_o that it will stick to forever. □

Lemma 3. *Value functions of cash-investors J_C and J_{CW} are given as (46) and (47) respectively. The optimal search intensity λ_c satisfies the first-order condition (48).*

$$J_{CW} = \frac{-(r+\delta)K(\lambda_c) + \lambda_c(b-c-A)}{r(r+\lambda_c+\delta)} \tag{46}$$

$$J_C = P + \frac{-\delta K(\lambda_c) + (\lambda_c + r)(b-c-A)}{r(r+\lambda_c+\delta)} \tag{47}$$

$$b - c - A + K(\lambda_c) = K'(\lambda_c)(r + \lambda_c + \delta) \tag{48}$$

Proof.

$$J_{CW} = \sup_{\lambda_C} \mathbb{E}\left[-\int_0^{\tau_D} e^{-rt} K(\lambda_C) dt + e^{-r \cdot \tau_D} (J_C - P)\right]$$

$$J_C = \mathbb{E}\left[\int_0^{\tau_\delta} e^{-rt}(b - c - A) dt + e^{-r \cdot \tau_\delta} (J_{CW} + P \cdot e^{r \cdot \tau_\delta})\right]$$

The remaining proof is similar to lemma 2. □

Lemma 4. *Nash bargaining between dealers and customers leads to specialness premia A and B as (49) and (50).*

$$A = z \cdot \{K(\lambda_c) + b\} + (1 - z) \cdot rP - c \tag{49}$$

$$B = -z \cdot K(\lambda_o) + (1 - z) \cdot (-c + rP) \tag{50}$$

Proof. Suppose a dealer repos a government bond to a cash investor. Dealer's Q_t^R increases by 1. Lemma 1 implies that the dealer's value function increases by $\frac{A + c - rP}{r + \delta}$. At the same time, the cash investor transitions from the CW state to the C state. Hence, the trading gain for the cash investor is (51).

$$J_C - J_{CW} - P = \frac{K(\lambda_c) + b - c - A}{r + \lambda_c + \delta} \tag{51}$$

According to the asymmetric Nash bargaining theory, A is a solution to (52).

$$\sup_A \left\{\frac{A + c - rP}{r + \delta}\right\}^z \left\{\frac{K(\lambda_c) + b - c - A}{r + \lambda_c + \delta}\right\}^{1-z} \tag{52}$$

Similarly, B is a solution to (53).

$$\sup_B \left\{\frac{-c - B + rP}{r + \delta}\right\}^z \left\{\frac{K(\lambda_o) + B}{r + \lambda_o + \delta}\right\}^{1-z} \tag{53}$$

□

In a steady-state equilibrium, the outstanding quantity of corporate bonds is $\frac{n_c}{\eta}$. Each non-bank holds either a government bond or a corporate bond. Hence, non-bank's demand for government bonds is $m_n - \frac{n_c}{\eta}$. Let x denote each dealer's demand for government bonds. Then the market-clearing condition for the government bond cash market is (54).

$$Nx + m_n - \frac{n_c}{\eta} = S_g \tag{54}$$

Equations (15) and (16) imply that the steady-state values of Q_t^R and Q_t^{RR} are $\frac{1}{N\delta}\lambda_o \mu_*^{OW}$ and $\frac{1}{N\delta}\lambda_c \mu_*^{CW}$, respectively. Throughout the remainder of this paper, the subscript $*$ indicates that the value is from the steady-state equilibrium. In the steady-state, dealer's demand for government bonds x_* is (55).

$$x_* = Q_*^R - Q_*^{RR} = \frac{1}{N\delta}\lambda_c\mu_*^{CW} - \frac{1}{N\delta}\lambda_o\mu_*^{OW} \tag{55}$$

To get steady-state investor masses, let all time-derivatives be zero for ordinary differential equations of agent mass dynamics in subsection 3.5.

$$\mu_*^{CW} = m_c\frac{\delta}{\delta+\lambda_c} \tag{56}$$

$$\mu_*^{OW} = \left(m_n - \frac{n_c}{\eta}\right)\frac{\delta}{\delta+\lambda_o} \tag{57}$$

Substitute (55), (56) and (57) into the market clearing condition (54). Also, use the relation $m_n = \frac{n_c}{\eta} + S_g$.

$$\frac{1}{\delta}\lambda_c m_c\frac{\delta}{\delta+\lambda_c} - \frac{1}{\delta}\lambda_o\left(m_n - \frac{n_c}{\eta}\right)\frac{\delta}{\delta+\lambda_o} + m_n - \frac{n_c}{\eta} = S_g$$

$$m_c\frac{\lambda_c}{\delta+\lambda_c} = \left(m_n - \frac{n_c}{\eta}\right)\frac{\lambda_o}{\delta+\lambda_o}$$

What remains is the corporate bond price. Non-banks are risk-neutral, and their mass is larger than the outstanding quantity of corporate bonds. Thus, the non-bank should be indifferent between owning a government bond and owning a corporate bond.

$$J_N = J_{IG} - P_c = J_{OW} - P \tag{58}$$

Solve for J_{IG} using (6) and (7).

$$J_{IG} = \mathbb{E}\left[\int_0^{\tau_\eta} e^{-rt}c dt + e^{-r\cdot\tau_\eta}\left\{J_{IG} - P_c + (1-\Lambda)\frac{c}{r}\right\}\right]$$

$$J_{IG} = \frac{c}{r}\frac{r+\eta}{r} - P_c\cdot\frac{\eta}{r} + (\mathit{\Lambda} - 1)\frac{c\cdot\eta}{r^2} \tag{59}$$

Substitute (41) and (59) into the indifference condition (58). Solve (50) for P. Then substitute the equation into (60). Solve for the corporate bond price P_c.

$$\frac{c}{r}\frac{r+\eta}{r} - P_c\cdot\frac{\eta}{r} + (\mathit{\Lambda}-1)\frac{c\cdot\eta}{r^2} - P_c = \frac{c}{r} + \frac{-(r+\delta)K(\lambda_o) + \lambda_o B}{r(r+\lambda_o+\delta)} - P \tag{60}$$

B.2. Theorem 2

Define a new parameter $m_o = m_n - \frac{n_c}{\eta}$. Solve (27) and (28) for A and B, respectively. Substitute resulting equations into (25) and (26), respectively. Combine the resulting two equations to substitute out P. Then I get (61). Equation (61) together with (62) characterize equilibrium.

$$K'(\lambda_c)(r+\lambda_c+\delta) + K'(\lambda_o)(r+\lambda_o+\delta) = (1-z)\left\{b + K(\lambda_c) + K(\lambda_o)\right\} \tag{61}$$

$$m_c \frac{\lambda_c}{\delta + \lambda_c} = m_o \frac{\lambda_o}{\delta + \lambda_o} \tag{62}$$

As I vary θ, the value of n_c or η does not change. Hence, to see how my solution changes with respect to θ, I can just vary m_o/m_c. Let $\hat{\theta} = m_o/m_c$. Then I can do implicit differentiation of (61) with respect to $\hat{\theta}$.

$$K''(\lambda_c)\frac{\partial \lambda_c}{\partial \hat{\theta}}(r+\lambda_c+\delta)+K'(\lambda_c)\frac{\partial \lambda_c}{\partial \hat{\theta}}+K''(\lambda_o)\frac{\partial \lambda_o}{\partial \hat{\theta}}(r+\lambda_o+\delta)+K'(\lambda_o)\frac{\partial \lambda_o}{\partial \hat{\theta}} = (1-z)\left\{K'(\lambda_c)\frac{\partial \lambda_c}{\partial \hat{\theta}} + K'(\lambda_o)\frac{\partial \lambda_o}{\partial \hat{\theta}}\right\}$$

$$\{K''(\lambda_c)(r+\lambda_c+\delta) + z \cdot K'(\lambda_c)\} \frac{\partial \lambda_c}{\partial \hat{\theta}} + \{K''(\lambda_o)(r+\lambda_o+\delta) + z \cdot K'(\lambda_o)\} \frac{\partial \lambda_o}{\partial \hat{\theta}} = 0 \tag{63}$$

Similarly, do implicit differentiation of (62) with respect to $\hat{\theta}$.

$$\lambda_c(\delta + \lambda_o) = \hat{\theta}\lambda_o(\delta + \lambda_c)$$

$$-\hat{\theta}\frac{\lambda_o \delta}{\lambda_c} \cdot \frac{\partial \lambda_c}{\partial \hat{\theta}} + \frac{\lambda_c \delta}{\lambda_o} \cdot \frac{\partial \lambda_o}{\partial \hat{\theta}} = -\lambda_o(\delta + \lambda_c) \tag{64}$$

Combine (63) and (64) in matrix form.

$$\begin{bmatrix} K''(\lambda_c)(r+\delta+\lambda_c) + z \cdot K'(\lambda_c) & K''(\lambda_o)(r+\delta+\lambda_o) + z \cdot K'(\lambda_o) \\ -\theta \frac{\lambda_o}{\lambda_c}\delta & \frac{\lambda_c}{\lambda_o}\delta \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda_c}{\partial \hat{\theta}} \\ \frac{\partial \lambda_o}{\partial \hat{\theta}} \end{bmatrix} = \begin{bmatrix} 0 \\ -\lambda_o(\delta + \lambda_c) \end{bmatrix}$$

Let $\mathbf{A} = \begin{bmatrix} K''(\lambda_c)(r+\delta+\lambda_c) + z \cdot K'(\lambda_c) & K''(\lambda_o)(r+\delta+\lambda_o) + z \cdot K'(\lambda_o) \\ -\theta \frac{\lambda_o}{\lambda_c}\delta & \frac{\lambda_c}{\lambda_o}\delta \end{bmatrix}$.

$$\begin{bmatrix} \frac{\partial \lambda_c}{\partial \hat{\theta}} \\ \frac{\partial \lambda_o}{\partial \hat{\theta}} \end{bmatrix} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} \lambda_o \{K''(\lambda_o)(r+\delta+\lambda_o) + z \cdot K'(\lambda_o)\}(\delta + \lambda_c) \\ -\lambda_o \{K''(\lambda_c)(r+\delta+\lambda_c) + z \cdot K'(\lambda_c)\}(\delta + \lambda_c) \end{bmatrix} \tag{65}$$

Because the search cost function is assumed to be convex and increasing, $K''(\lambda_c) > 0$, $K'(\lambda_c) > 0$, $K''(\lambda_o) > 0$ and $K'(\lambda_o) > 0$. Thus, $\det(\mathbf{A})$ is positive. Then inspection of the left-hand side of (65) allows me to conclude that $\frac{\partial \lambda_c}{\partial \hat{\theta}} > 0$ and $\frac{\partial \lambda_o}{\partial \hat{\theta}} < 0$.

Now perform implicit differentiation of (26).

$$K'(\lambda_o)\frac{\partial \lambda_o}{\partial \hat{\theta}} + \frac{\partial B}{\partial \hat{\theta}} = K''(\lambda_o)\frac{\partial \lambda_o}{\partial \hat{\theta}}(r+\lambda_o+\delta) + K'(\lambda_o)\frac{\partial \lambda_o}{\partial \hat{\theta}}$$

$$\frac{\partial B}{\partial \hat{\theta}} = K''(\lambda_o)(r+\lambda_o+\delta)\frac{\partial \lambda_o}{\partial \hat{\theta}} < 0$$

Do implicit differentiation of (28).

$$(1-z) \cdot r \cdot \frac{\partial P}{\partial \hat{\theta}} = z \cdot K'(\lambda_o)\frac{\partial \lambda_o}{\partial \hat{\theta}} + \frac{\partial B}{\partial \hat{\theta}} < 0$$

Hence, $\frac{\partial P}{\partial \hat{\theta}}$ is negative. To see how the corporate bond price P_c varies with respect to $\hat{\theta}$, substitute (26) into (30).

$$P_c = \frac{c}{r} - (1-\Lambda)\frac{c\cdot\eta}{r(r+\eta)} + \frac{K'(\lambda_o)}{1-z}\frac{r+\delta+z\cdot\lambda_o}{r+\eta} \tag{66}$$

$$\frac{\partial}{\partial\hat{\theta}}P_c = \frac{K''(\lambda_o)(r+\delta+z\cdot\lambda_o)+K'(\lambda_o)z}{(1-z)(r+\eta)}\frac{\partial\lambda_o}{\partial\hat{\theta}} < 0$$

B.3. Theorem 3

Again, let $m_o = m_n - \frac{n_c}{\eta}$.

$$\mu^{CW}(x) = (m_c - x)\frac{\delta}{\delta+\lambda_c(x)} \tag{67}$$

$$\mu^{OW}(x) = m_o\frac{\delta}{\delta+\lambda_o(x)} \tag{68}$$

Substitute (67) and (68) into the equation for social welfare.

$$W(x) = xb - (m_c - x)\frac{\delta\cdot K(\lambda_c(x))}{\delta+\lambda_c(x)} + (m_c - x)\frac{\lambda_c(x)\cdot b}{\delta+\lambda_c(x)} - m_o\frac{\delta\cdot K(\lambda_o(x))}{\delta+\lambda_o(x)} \tag{69}$$

Lemma 5. *$\frac{\partial}{\partial x}\lambda_c(x) > 0$ and $\frac{\partial}{\partial x}\lambda_o(x) < 0$.*

Proof. For a given value of x, an equilibrium can be characterized with two equations (70) and (71).

$$K'(\lambda_c)(r+\lambda_c+\delta) + K'(\lambda_o)(r+\lambda_o+\delta) = (1-z)\left\{b + K(\lambda_c) + K(\lambda_o)\right\} \tag{70}$$

$$(m_c - x)\frac{\lambda_c}{\delta+\lambda_c} = m_o\frac{\lambda_o}{\delta+\lambda_o} \tag{71}$$

Do implicit differentiation of (70) and (71) with respect to x.

$$\begin{bmatrix} m_o\frac{\delta}{(\delta+\lambda_o)^2} & -(m_c-x)\frac{\delta}{(\delta+\lambda_c)^2} \\ K''(\lambda_o)(r+\lambda_o+\delta)+z\cdot K'(\lambda_o) & K''(\lambda_c)(r+\lambda_c+\delta)+z\cdot K'(\lambda_c) \end{bmatrix}\begin{bmatrix} \frac{\partial\lambda_o}{\partial\hat{\theta}} \\ \frac{\partial\lambda_c}{\partial\hat{\theta}} \end{bmatrix} = \begin{bmatrix} -\frac{\lambda_c}{\delta+\lambda_c} \\ 0 \end{bmatrix}$$

Let $\mathscr{A} = \begin{bmatrix} m_o\frac{\delta}{(\delta+\lambda_o)^2} & -(m_c-x)\frac{\delta}{(\delta+\lambda_c)^2} \\ K''(\lambda_o)(r+\lambda_o+\delta)+z\cdot K'(\lambda_o) & K''(\lambda_c)(r+\lambda_c+\delta)+z\cdot K'(\lambda_c) \end{bmatrix}$. Then $\det(\mathscr{A}) > 0$.

$$\begin{bmatrix} \frac{\partial\lambda_o}{\partial\hat{\theta}} \\ \frac{\partial\lambda_c}{\partial\hat{\theta}} \end{bmatrix} = \frac{1}{\det(\mathscr{A})}\begin{bmatrix} K''(\lambda_c)(r+\lambda_c+\delta)+z\cdot K'(\lambda_c) & (m_c-x)\frac{\delta}{(\delta+\lambda_c)^2} \\ -K''(\lambda_o)(r+\lambda_o+\delta)-z\cdot K'(\lambda_o) & m_o\frac{\delta}{(\delta+\lambda_o)^2} \end{bmatrix}\begin{bmatrix} -\frac{\lambda_c}{\delta+\lambda_c} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial\lambda_o}{\partial\hat{\theta}} \\ \frac{\partial\lambda_c}{\partial\hat{\theta}} \end{bmatrix} = \frac{1}{\det(\mathscr{A})} \begin{bmatrix} -\{K''(\lambda_c)(r+\lambda_c+\delta)+z\cdot K'(\lambda_c)\}\frac{\lambda_c}{\delta+\lambda_c} \\ \{K''(\lambda_o)(r+\lambda_o+\delta)+z\cdot K'(\lambda_o)\}\frac{\lambda_c}{\delta+\lambda_c} \end{bmatrix}$$

□

Differentiate $W(x)$ in (69) with respect to x.

$$\frac{dW(x)}{dx} = \delta\frac{b+K(\lambda_c)}{\delta+\lambda_c} + (m_c-x)\cdot\delta\cdot\frac{-(\delta+\lambda_c)K'(\lambda_c)+b+K(\lambda_c)}{(\delta+\lambda_c)^2}\frac{1}{\det\mathscr{A}}\frac{\lambda_c}{\delta+\lambda_c}\varphi_o \\ - m_o\delta\frac{(\delta+\lambda_o)K'(\lambda_o)-K(\lambda_o)}{(\delta+\lambda_o)^2}\frac{1}{\det\mathscr{A}}\frac{-\lambda_c}{\delta+\lambda_c}\varphi_c \quad (72)$$

φ_o and φ_c are defined as (73) and (74), respectively.

$$\varphi_o = K''(\lambda_o)(r+\lambda_o+\delta)+K'(\lambda_o)\cdot z \quad (73)$$

$$\varphi_c = K''(\lambda_c)(r+\lambda_c+\delta)+K'(\lambda_c)\cdot z \quad (74)$$

Because I assume $m_o > m_c - x$, (71) implies that $\lambda_o < \lambda_c$. Since there are more government bond owners than cash investors, each government bond owner searches for dealers less intensively than each cash investor.

Equation (72) implies that $dW(x)/dx > 0$ if and only if (75) holds.

$$b+K(\lambda_c)+\frac{\lambda_c}{\det\mathscr{A}}\left[(m_c-x)\,\varphi_o\frac{rK'(\lambda_c)+A}{(\delta+\lambda_c)^2}+m_o\varphi_c\frac{B-rK'(\lambda_o)}{(\delta+\lambda_o)^2}\right]>0 \quad (75)$$

Define a new function $f(\cdot)$ as (76).

$$f(\lambda) = K'(\lambda)(\lambda+\delta)-K(\lambda) \quad (76)$$

Due to the assumptions that I make about the search cost function, $f(0)=0$.

$$f'(\lambda) = K''(\lambda)(\lambda+\delta) > 0$$

Thus, for $\forall\lambda \geq 0$, $f(\lambda) \geq 0$. Because $\lambda_o \geq 0$, $K'(\lambda_o)(\lambda_o+\delta) \geq K(\lambda_o)$. Combine this inequality with (26) to get $B \geq rK'(\lambda_o)$. Hence, $m_o\varphi_c\frac{B-rK'(\lambda_o)}{(\delta+\lambda_o)^2}$ in (75) is positive.

I already showed that $\det(\mathscr{A}) > 0$. Hence, (75) holds.

C. Definitions of Symbols for Empirical Part

Symbol	Definition
y_{it}	The yield-to-maturity for corporate bond i on day t.
OIS_{it}	The overnight index swap (OIS) rate with tenor identical to the remaining maturity of corporate bond i as of day t.
Q_{it}	The aggregate outstanding amount of euro-denominated corporate bonds with credit ratings identical to that of bond i as of day t.
s_{it}	The repo specialness premium of bund with remaining maturity identical to that of corporate bond i on day t. If no such bund exists, I use a Nadaraya–Watson kernel regression.
$Q_{o,j}$	The outstanding amount of government bond j on day t
$Q_{PSPP,j,t}$	The cumulative quantity of government bond j that the Eurosystem purchased under the PSPP by day t.
$F_{j,t}$	The free float of government bond j on day t. $F_{j,t} = Q_{o,j} - Q_{PSPP,j,t}$
$\hat{Q}_{o,j}$	The outstanding amount of government bond j on day t in a hypothetical scenario where the agency issues all intended amount in the first auction.
$\hat{Q}_{PSPP,j,t}$	The cumulative quantity of government bond j that the Eurosystem would have purchased under the PSPP by day t in a hypothetical scenario where the PSPP only had fixed rules.
$\widehat{F}_{j,t}$	$\hat{Q}_{o,j} - \hat{Q}_{PSPP,j,t}$.
L_{it}	Instrumental variable. The weighted sum of free floats of bunds with remaining maturities close to that of corporate bond i.
m^c_{it}	The remaining maturity of corporate bond i on day t.
m^g_{jt}	The remaining maturity of government bond j on day t.
θ	Parameter that enters the definition of L_{it}.

D. The Supply of Bonds by the Central Banks in the United States and Canada

The Federal Reserve Bank of New York initiated the Fixed-Rate Overnight Reverse Repo (RRP) facility in September 2013. A wide range of financial institutions including non-banks can reverse repo Treasuries from the New York Fed. Each institution can borrow Treasuries up to 30 billion dollars on each day. Unlike its counterparts in Europe, the RRF facility does not have any limit on the overall size of operation, at least from December 2015. The New York Fed is willing to lend all bonds on its System Open Market Account (SOMA) portfolio to the extent possible. The impact on the effective size of collateral available in the private repo market can be limited to the extent that non-primary-dealer counter-parties cannot rehypotheticate

Treasuries from the RRP facility (Protter, 2018). Nevertheless, after reverse-repoing Treasuries from the New York Fed, these non-banks are less likely to be seeking extra collateral from the private market. Consequently, the net effect is likely to be increased availability of collateral in the private market (Protter, 2018).

The Fed RRP facility is much more accessible than the securities lending facility of the Eurosystem. For example, unlike the RRP facility, the Eurosystem cannot repo more than 50 billion euros of its PSPP-eligible bonds through its securities lending facility (Marraffino, 2017). Maraffino (2016) notes that the introduction of a facility akin to the RRP facility may considerably alleviate any problem associated with the reduced supply of collateral in the European repo market.

In addition, the Federal Reserve has been operating a securities lending program[20] since 1969. Through this program, primary dealers can borrow specific securities by positing any Treasury general collateral.[21] Investors used to borrow on-the-run securities by posting other Treasury general collateral.[22] Because on-the-run securities are far more special than off-the-run securities in the repo market, this program is likely to have reduced the repo specialness premium of on-the-run Treasuries (Arrata, Nguyen, Rahmouni-Rousseau and Vari, 2018).

From the market user's point of view, the securities lending program of the Federal Reserve is more attractive than the European counterparts (Arrata, Nguyen, Rahmouni-Rousseau and Vari, 2018). The minimum lending fee of the Fed facility was 5 basis points as of 2016 (Fleming, Keane, Schurmeier and Weiss, 2016). On the contrary, most national central banks of the Eurosystem have been repoing government bonds through their respective securities lending facilities at a repo rate at least 10 basis points lower than the prevailing market GC repo rate (Arrata, Nguyen, Rahmouni-Rousseau and Vari, 2018). Because the securities lending fee is economically equivalent to the repo specialness premium, Arrata, Nguyen, Rahmouni-Rousseau and Vari (2018) conclude that the Fed facility offered a pricing that is more favorable than the European facilities did.

Although the Bank of Canada did not explicitly implement QE, it has been consistently purchasing debt instruments of the federal government of Canada. As of 2015, the majority of the asset portfolio of the Bank of Canada, which is worth around 94 billion Canadian dollars, consists of bonds and bills of the government of Canada (Patterson, 2015). At the same time, the majority of repo transactions in Canada is collateralized by the federal government bonds. Consequently, the Bank of Canada is aware that its sizable purchase of federal government bonds can reduce the float size in the private market and negatively impact the repo market liquidity.

The Bank of Canada took several measures to address potential dry-up of liquidity in the repo

[20] See https://libertystreeteconomics.newyorkfed.org/2016/08/a-closer-look-at-the-federal-reserves-securities-lending-program.html for more details.

[21] According to the New York Fed website, a transaction needs "to be collateralized with Treasury bills, notes, bonds and inflation-indexed securities."

[22] See https://libertystreeteconomics.newyorkfed.org/2016/08/a-closer-look-at-the-federal-reserves-securities-lending-program.html.

market. For example, the Bank repos federal government bonds whenever the general collateral (GC) repo market rate is below the target rate of the Bank.[23] This measure contrasts with the policy of the ECB. The ECB did not take action when the German GC repo rate became substantially lower than the deposit facility rate.

In addition, the Bank of Canada has a securities lending facility through which primary dealers can borrow up to 50 percent of the bonds held by the Bank of Canada.[24] In return for borrowing federal government bonds, primary dealers can post a wide range of assets, including provincial government bond, municipal government bond, banker's acceptances, commercial papers, and covered bonds. Primary dealers can gain access to federal government bonds, the most widely used collateral in the Canadian repo market, in exchange for other less widely used assets. Therefore, this facility can mitigate any problem associated with the reduced supply of collateral in the repo market.

E. Implementation Details of the PSPP

I summarize implementation details only until March 31, 2017 because my data ends on that day. The information is from the following EU documents:

- Decision (EU) 2015/774 of the European Central Bank of 4 March 2015 on a secondary market public sector asset purchase programme (ECB/2015/10)
- Decision (EU) 2015/2101 of the European Central Bank of 5 Novemebr 2015 amending Decision (EU) 2015/774 on a secondary markets public sector asset purchase programme (ECB/2015/33)
- Decision (EU) 2015/2464 OF THE EUROPEAN CENTRAL BANK of 16 December 2015 amending Decision (EU) 2015/774 on a secondary markets public sector asset purchase programme (ECB/2015/48)
- DECISION (EU) 2016/702 OF THE EUROPEAN CENTRAL BANK of 18 April 2016 amending Decision (EU) 2015/774 on a secondary markets public sector asset purchase programme (ECB/2016/8)
- DECISION (EU) 2017/100 OF THE EUROPEAN CENTRAL BANK of 11 January 2017 amending Decision (EU) 2015/774 on a secondary markets public sector asset purchase programme (ECB/2017/1)

There have been four German agencies whose debt were eligible for the PSPP during this sample period: KfW, Landeskreditbank Baden-Wurttemberg Foerderbank, Landwirtschaftliche Rentenbank and NRW.Bank.

[23]https://www.bankofcanada.ca/markets/market-operations-liquidity-provision/framework-market-operations-liquidity-provision/

[24]https://www.bankofcanada.ca/2015/10/securities-lending-program/

Table 6: **Timeline of the implementation details of the PSPP**

Period	**APP monthly net purchase (billion euros)**	**The share of purchase quantities allocated to the debts of international organizations and multilateral banks**	**Minimum remaining maturity (years)**	**Maximum remaining maturity**
March 9, 2015 ~ April 18, 2016	60	12%	2	30 years and 364 days
April 19, 2016 ~ January 12, 2017	80	12%	2	30 years and 364 days
January 13, 2017 ~ March 31, 2017	60	12%	1	30 years and 364 days

Table 7: **Timeline of the implementation details of the PSPP**: eligibility of local and regional government debt, issuer limit, restriction on yield-to-maturity, and issue (ISIN) limit

Period	**Eligibility of debt instruments of regional and local governments**	**Issuer limit**	**Restriction on yield-to-maturity**	**Issue (ISIN) limit**
March 9, 2015 ~ November 9, 2015	N	33%	Yes, except inflation linked bonds[25]	25%
November 10, 2015 ~ December 31, 2015	N	33%	Yes, except inflation linked bonds	33%
January 1, 2016 ~ February 16, 2016	Yes technically. Nevertheless, anecdotal evidence suggest that the Eurosystem did not start purchasing during this period	33%	Yes, except inflation linked bonds	33%

[25] See Schlepper, Hofer, Riordan and Schrimpf (2017).

February 17, 2016 ~ April 18, 2016	Yes, but not municipal bonds[26]	33%	Yes, except inflation linked bonds	33%
April 19, 2016 ~ January 12, 2017	Yes, but not municipality bonds	33% but 50% for international organizations and multilateral development banks	Yes, except inflation linked bonds	33% but 50% for international organizations and multilateral development banks
January 13, 2017 ~ March 31, 2017	Yes, but not municipality bonds	33% but 50% for international organizations and multilateral banks	Bonds with yields-to-maturity below the deposit facility rate can be purchased to the extent necessary to meet capital key based allocation rule	30% but 50% for international organizations and multilateral banks

F. Other Institutional Details

F.1. The Political Cost of the PSPP

QE programs in Europe has been controversial legally because such program can be interpreted as monetary financing: the purchase of bonds of member states by the Eurosystem (Nyborg, 2017). The Treaty of the Functioning of the European Union (TFEU) explicitly prohibits monetary financing. In particular, Germany has been adamantly opposed to monetary financing because it dislikes the fiscal support of peripheral member countries (Brunnermeier, James and Landau, 2016). Hence, the PSPP could be carried out only after the European Court of

[26]It is because municipality bonds are not rated (UniCredit, 2016). To be eligible for the PSPP, securities must have credit ratings.

Justice confirmed the legality of such operations in 2015, subject to adherence to several conditions (Nyborg, 2017). For example, the PSPP never purchases bonds in the primary market to circumvent potential legal challenge (Nyborg, 2017). Nevertheless, right-wing politicians in Germany continued to challenge the legality of the PSPP. On August 15, 2017, the Federal Constitutional Court of Germany stated that the PSPP may be violating the mandate of the Eurosystem.

F.2. Why Insurance Companies and Pension Funds Cannot Easily Monetize Repo Specialness Premia

The European repo market is mostly intermediated by banks (Duffie, 2018). Since only banks can join electronic platforms such as BrokerTec, the only feasible way for insurers or pension funds to repo their bonds is through bilateral repo with banks. Nevertheless, recent regulatory drives have made banks increasingly reluctant to intermediate repo and reverse-repo transactions (Hill, 2017). One important factor is the Basel III Leverage Ratio requirement that forces banks to maintain leverage ratio - the ratio of Tier 1 capital to the total asset - above 3% (Hill, 2017). The Leverage Ratio requirement forces banks to economize the usage of their balance sheets (Hill, 2017). Consequently, banks are strongly discouraged from remaining in the repo intermediation business, which heavily expands the size of their balance sheet while bringing them a small profit (Hill, 2017).

There were signs that banks were beginning to retreat from the repo intermediation business in 2015 (Hill, 2015b). One exception is when banks can net repo and reverse repo transactions (Marraffino, 2017). By doing so, banks can avoid expanding their balance sheets. Banks often net repos and reverse repo by registering them at Central Counterparties (CCPs). Nevertheless, for any transaction to be registered at CCPs, both parties to the transaction need to be members of the CCP (Marraffino, 2017). Because membership is costly, not all financial institutions belong to CCP. Only financial institutions that have a constant flow of two way business (repos and reverse repos) have an incentive to become members of CCPs ICMA (2015). For this reason, CCP membership is mostly for banks (Hill, 2017). Insurers and pension funds without CCP membership is, thus, not favored clients of the repo desks of banks. To persuade the repo desk the profitability of transaction, insurers and pension funds need to accept unfavorable rates (Marraffino, 2017), or bring other profitable businesses on the side (Hill, 2015b).

In principle, insurers and pension funds can search for counterparties in the bilateral repo market. Nevertheless, doing so requires an ability to assess the credit risk of the counterparty, comply with legal requirements, and negotiate with a wide range of non-bank participants, which most buy-side firms do not possess (Hill, 2017). Thus, many buy-side firms are heavily reliant on bank-based intermediation (Hill, 2017).

F.3. The Restrictive Operation of the Securities Lending Facility

Although the European Central Bank did allow individuals to borrow government bonds through the securities lending facility, many structural factors prevented the facility from effectively addressing collateral scarcity in the repo market. The European Central Bank and National Central Banks in the Eurosystem operate their own securities lending facilities in a decentralized manner.

Before December 2016, in order to obtain specific collateral, firms had to bring another PSPP-eligible securities. Bringing cash was not acceptable. The securities lending facilities were said to have been operated in a cash-neutral manner. The operation of lending facilities in a cash-neutral manner does not change the overall quantity of collateral circulating in the market (Hill, 2017).

The Eurosystem allowed firms to reverse-repo bonds while posting cash collateral at the securities lending facility. Nevertheless, even this cash-collateral option could not fully mitigate the collateral scarcity problem due to the following features.

To obtain bunds, firms had to go to the security lending facility of the Bundesbank. Nevertheless, the Bundesbank facility had the following restrictive features as of the end of 2016.

- The firm needed to have "a credit line with the Bundesbank" (Hill, 2017).
- The lending operation was to be done with the European Master Agreement (EMA). Nevertheless, the norm in the private market is to use either the GMRA for repo and the GMSLA for securities lending (Hill, 2017).
- There was no option to automatically roll-over repo contract with the Bundesbank. The firm had to deliver the bond to the Bundesbank and then re-start the contract (Hill, 2017).
- The securities lending facilities in the Eurosystem were not allowed to lent more than 50 billion euros of securities with cash collateral option. This quote is distributed across facilities run by different national central banks. Thus, the Bundesbank facility was not allowed to lend more than 12 billion euros with cash collateral option (McGuire, Graham-Taylor and Cairns, 2017). This number is far smaller than the size of the PSPP. The Eurosystem purchased 304 billion euros of German securities under the PSPP by December 2016.
- Each transaction could not be larger than 200 million euros (Arrata, Nguyen, Rahmouni-Rousseau and Vari, 2018).
- Firms could reverse-in bunds only at a rate significantly less favorable than the market prevailing rates (Arrata, Nguyen, Rahmouni-Rousseau and Vari, 2018).

These restrictive features might have been the reason why the market did not utilize the cash collateral option to the fullest extent. The data on the ECB website shows that the total quantity of securities lent with cash collateral option has been consistently lower than the 50 billion euros cap each month.

F.4. The Portfolio Allocation Decisions of Life Insurance Companies

It is true that life insurers face strict regulation on their choice of investment strategies. Nevertheless, there is anecdotal evidence that European life insurers recently substituted for ultra low-yielding government bonds with alternative higher yielding assets. See Section C of ESRB (2016). Because a significant fraction of European insurers have sold products with guaranteed returns (IAIS, 2016), they need to make sure they generate sufficiently high returns from their investments to remain solvent.

G. Counter-factual Time-Series of AAA-Rated and AA-Rated Corporate Bond Yields

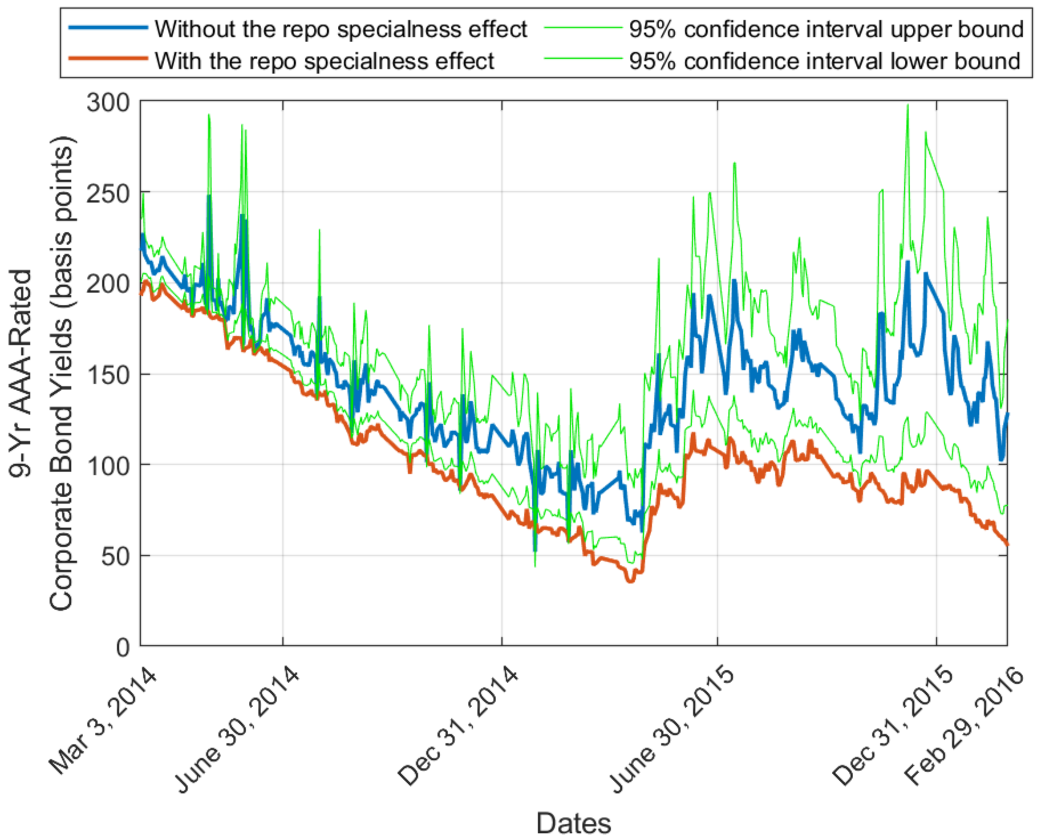

Figure 10: **Counter-factual Time-Series Evolution of 9-Year AAA-Rated Corporate Bond Yields** The figure plots the time series evolution of AAA-rated euro-denominated corporate bond yields with residual maturity of 9 years. On each day during my sample period, I start with the universe of all euro-denominated AAA-rated corporate bond yields outstanding on that day. I run a Nadaraya-Watson kernel regression of yields-to-maturity on remaining maturity. By doing so, I estimate yields on a hypothetical AAA-rated bond with remaining maturity exactly equal to 9 years. The orange line is the plot of this estimation. Using the estimate from Table 3 and the observed time series of bund repo specialness premia, I estimate what the 9-year AAA-rated corporate bond yields would have been if bund repo specialness premia were zero. The blue line shows the time-series of 9-year AAA-rated corporate bond yield in this hypothetical scenario. The two green lines show the 95% confidence interval around the orange line.

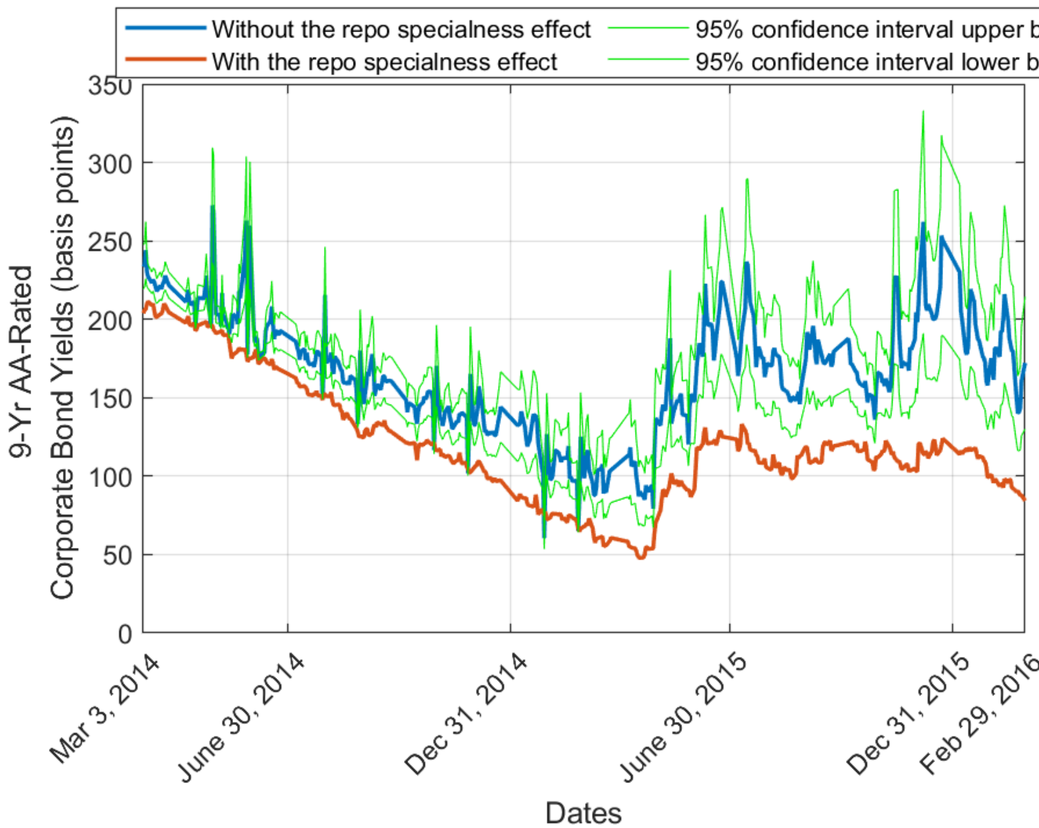

Figure 11: **Counter-factual Time-Series Evolution of 9-Year AA-Rated Corporate Bond Yields** The figure plots the time series evolution of AA-rated euro-denominated corporate bond yields with residual maturity of 9 years. On each day during my sample period, I start with the universe of all euro-denominated AA-rated corporate bond yields outstanding on that day. I run a Nadaraya-Watson kernel regression of yields-to-maturity on remaining maturity. By doing so, I estimate yields on a hypothetical AA-rated bond with remaining maturity exactly equal to 9 years. The orange line is the plot of this estimation. Using the estimate from Table 3 and the observed time series of bund repo specialness premia, I estimate what the 9-year AA-rated corporate bond yields would have been if bund repo specialness premia were zero. The blue line shows the time-series of 9-year AA-rated corporate bond yield in this hypothetical scenario. The two green lines show the 95% confidence interval around the orange line.