Corporate Debt Structure and the Financial Crisis*

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Abstract

We present a DSGE model where firms optimally choose among alternative instruments of external finance. The model is used to explain the evolving composition of corporate debt during the financial crisis of 2007-09, namely the observed shift from bank finance to bond finance despite the increasing cost of debt securities relative to bank loans. We show that substitutability among instruments of external finance is important to shield the economy from the adverse effects of a financial crisis on investment and output.

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1 Introduction

During the financial crisis of 2007-09, European banks experienced major difficulties to finance themselves in money markets. Starting in August 2007, concerns about their exposure to the US sub-prime market enhanced the perception of counterparty risk in the interbank market and triggered a drying-up of liquidity. Banks refrained from lending to each other and began to hoard liquidity. Their funding difficulties were soon passed on to the corporate sector. Euro area non-financial corporations - traditionally heavily dependent on bank-finance - faced progressively tightening lending standards.

Early in 2008, non-financial corporations started shifting the composition of their debt from bank loans towards debt securities (figure 1). At the same time, the cost of market debt raised above the cost of bank loans, where it remained throughout the crisis (figure 2). Despite the increase in the cost of external finance, aggregate debt to equity kept rising and only stabilized in 2009, while the default rate of non-financial corporations increased sharply. The turmoil on financial markets implied an aggregate drop in investment and output that was unprecedented since the introduction of the euro.

In this paper, we propose a model that can account for the stylized facts observed during the crisis both on the composition of corporate debt and on aggregate economic activity. We use the model to evaluate the role played by the composition of corporate debt in determining the response of investment and output during the crisis. In particular, we investigate the endogenously evolving debt structure, and the possibilities for companies to switch between bank financing and bond financing, and argue that it is important to account for this margin of adjustment when analyzing the effects of financial shocks on aggregate economic activity.

The framework we consider is a stochastic dynamic general equilibrium model where lenders and borrowers face agency costs, and where heterogeneous firms can choose among alternative instruments of external finance. The model is a version of the model analyzed in De Fiore and Uhlig (2011). There, we focussed on the steady state analysis, while the emphasis here is on the analysis of the dynamics and on the propagation of specific shocks, possibly accounting for the financial crisis. To do so, we enrich the model, allowing for nominal contracts and using a quarterly calibration.

The model generates an endogenous corporate debt structure as a result of two key features. The first is the existence of two types of financial intermediaries, where banks (which

intermediate loan finance) are willing to spend resources to acquire information about an unobserved productivity factor, while "capital mutual funds" (which intermediate bond finance) are not. Because information acquisition is costly, bond issuance is a cheaper - although riskier - instrument of external finance.

We view banks as financial intermediaries that build a closer relationship with entrepreneurs than dispersed investors. They assess and monitor information about firms' uncertain productive prospects and are ready to adapt the terms of the loans accordingly. Our modelling of banks builds on theories of financial intermediation that stress the higher flexibility provided by banks relative to the market (Chemmanur and Fulghieri (1994) and Boot, Greenbaum and Thakor (1993)). It is also consistent with the recent role taken by banks as originators of asset-backed securities, which requires screening of applicants' projects.

Entrepreneurs (or firms) in our model choose between obtaining bond finance, bank finance or abstaining from production, based on information available at that time. When they choose bank finance, a further, but costly investigation of the proposed production reveals additional information, and provides the entrepreneur with the option of not proceeding with the loan, if the expected gains then turn out to be lower than those from abstaining from production and saving the available net worth.

In equilibrium, firms experiencing high risk of default choose to abstain from production and not to raise external finance. This choice enables them to retain their net worth, which would otherwise get sized by financial intermediaries in case of bankruptcy. Firms with relatively low risk of default choose to issue debt securities because this is the cheapest form of external finance. Firms with intermediate risk of default decide to approach banks, as they highly value the option of getting further information before deciding whether or not to produce. The model delivers a distribution of firms among financing choices (whether or not to raise external finance) and among debt instruments (bank loans or debt securities) that reacts to aggregate conditions and evolves endogenously over the cycle.

We investigate the dynamic shift of these boundaries in response to key three financial shocks: an increase in the "iceberg" cost of obtaining bank financing (or a deterioration in bank efficiency), a decrease in capital quality similar to the capital quality shock in Gertler and Karadi (2011), and an increase in uncertainty, as in, say, Bloom (2009) or in Christiano, Motto and Rostagno (2010). We use these shocks to "build up" a quantitative interpretation of the 2007-09 financial crisis.

We obtain three sets of results.

First, we show that the model can qualitatively replicate the observed changes both in the composition of corporate debt and in aggregate variables, in response to a shock that increases information acquisition costs and reduces the efficiency of banks as financial intermediaries. This shock induces a fall in the ratio of bank loans to debt securities, as a larger share of firms with high ex-ante risk of default now finds the cost of external finance too high, and choose to abstain from production. Similarly, a larger share of firms experiencing intermediate realizations of the first productivity shock find the flexibility provided by banks too costly, and decides to issue bonds instead.

The shift in the composition of debt in turn affects the cost of external finance. Bond finance becomes more costly as the average risk of default for the new pool of market-financed firms is higher. The cost of bank finance also rises because the share of firms with low risk of default that move from bank-finance to bond-finance more than compensates the share of firms with high risk of default that move out of banking and decides not to produce. Overall, the increase in bond yields is higher than the increase in lending rates. The higher cost of external finance increases the average default rate. The shock further exerts contractionary effects on real activity as a consequence of the reduction in the fraction of producing firms. More firms decide not to approach a financial intermediary and a larger share of bank-financed firms decides not to produce, conditional on obtaining information on the uncertain productivity factor. The aggregate level of credit and investment fall, together with output.

Our second result relates to the ability of the model to match quantitatively the responses observed during the financial crisis. We show that the peak effects (relative to post-EMU averages) can be broadly replicated when all three shocks are combined.

Our third finding is that firms' ability to shift among alternative instruments of external finance has important implications for the effects of shocks on aggregate activity. We compare the real effects of a shock to bank costs when the corporate debt structure is endogenous to the effects obtained when it is kept unchanged. Consistent with recent empirical evidence documented in Becker and Ivashina (2011), we find that the effects on the cost of external finance, investment and output are greatly amplified when the debt structure is exogenous relative to the case when it reacts to aggregate conditions.

Our paper relates to recent work by Adrian, Colla and Shin (2011). As we do, they document and explain the fall in bank finance during the 2007-09 crisis, the compensating

increase in bond finance, and the rising price of both instruments. Different from us, in order to account for this evidence, they present a model that builds around a procyclical behaviour of leverage for commercial banks. In a recession, banks sharply contract lending through deleveraging. Risk-averse bond investors need to increase their credit supply to fill the gap in demand. In order to induce this outcome, risk premiums need to rise. In their model, as in ours, a contraction in economic activity arises because of the rising premiums, rather than because of a contraction in total credit.

Our paper is also related to an older literature that models the endogenous choice between bank finance and market finance. Holstrom and Tirole (1997) and Repullo and Suarez (1999) analyse this choice for firms that are heterogeneous in the amount of available net worth. In those models, moral hazard arises because firms can divert resources from the project to their private use. In Holstrom and Tirole (1997), moral hazard applies to both firms and banks, while it applies only to firms in Repullo and Suarez (1999). In both cases, it is assumed that monitoring is more intense under bank finance. The papers find that, in equilibrium, firms with large net worth choose to raise market finance, firms with intermediate levels of net worth prefer to raise bank finance, and firms with little net worth do not obtain credit. One implication of their model is that a contraction in net worth, as observed during the crisis, leads to a reduction of bond finance, at odds with the evidence observed during the recent financial crisis. In our model, firms financing choices depend on their risk of default. Hence, a fall in net worth needs not produce a reduction in the share of bond-financed firms. A second main difference relative to this literature is that we cast the analysis of corporate finance into a fully general equilibrium model. This enables us to relate the equilibrium choice of the instrument of external finance to the behaviour of real aggregate variables in the economy.

The paper proceeds as follows. Following a summary of the key facts of the 2007-09 financial crisis in the EMU in section 2, we describe the model in section 3. In section 4, we present the analysis and describe the equilibrium of the model. We refer to the appendix for a description of the methodology we use to log-linearize the equilibrium conditions. An additional and interesting challenge arises because of the need to aggregate across heterogeneous firms and because of the presence of endogenously changing regions of integration. Section 5 provides our results. We first document the response of financial and real variables under a temporary shock to bank information acquisition costs. Then, we document the ability of the model to match the peak effects observed during the crisis. Finally, we evaluate the importance of considering

firms' endogenous debt structure for assessing the investment and output effects of shocks. In section 6, we conclude. In the appendix, we provide details of the aggregation across firms; we define the financial variables used in the numerical analysis; we collect the conditions that characterize a competitive equilibrium in the model; we characterize the stochastic steady state and describe the numerical procedure used to compute it; and we illustrate how to obtain the coefficients of the log-linearized equilibrium conditions.

2 The key facts

We regard the following as key facts for the 2007-09 financial crisis for the European Monetary Union (EMU). We report the "peak effects" observed during the crisis, which we define as the maximum deviation over the period 2007-2009 of each series relative to the post-EMU average (over the period 1999-2011). The data are from the database of the Financial Stability Review of the European Central Bank. A detailed description of the data is planned for a future version of this paper, in appendix F.

1. The ratio of bank loans to debt securities (outstanding amounts) fell by 16 percent.
2. The cost of market finance (based on an average yield of corporate bonds with investment grade ratings and maturity of more than one year, and on a euro-currency high-yield index) rose by 70 percent.
3. The cost of bank finance (based on long-term lending rates to non-financial corporations of euro area banks) rose by 29 percent.
4. The default rate (for all grades) rose from 0.7 percent to 2.5 percent on an annual basis, or, 241 percent in relative terms.
5. The debt to equity ratio (ratio of loans, debt securities and pension fund reserves to financial assets of the non-financial corporations) rose by 15 percent.
6. The investment-to-gdp ratio fell by 3.6 percent.
7. GDP fell by 6.8 percent.

Our aim is to provide a model which qualitatively as well as quantitatively can match these facts or, at least, can come reasonably close. We will focus on three shocks in particular.

We investigate an increase in the "iceberg" cost (denoted by τ) of obtaining bank financing, motivated by the observed 40 percent increase in the item "commissions and fees" of pre-provisioning profits of euro area banks. We also investigate a decrease in capital quality, similar to the capital quality shock in Gertler and Karadi (2011). Finally, we investigate an increase in uncertainty, as in, say, Bloom (2009) or in Christiano, Motto and Rostagno (2010), which reflects the increase in stock markets volatility observed at the end of 2008 and beginning of 2009.

3 The model

We extend the model presented in De Fiore and Uhlig (2011). There, we focussed on the steady state properties, and used our results to shed light on the differences in the financial structure between the US and the EMU. Here, our focus is on the dynamic impact of key financial shocks to analyze the 2007-09 financial crisis. To do so, we need a somewhat richer structure.

Before describing the details, it is useful to provide an overview of the model. Time is discrete, counting to infinity. There are entrepreneurs, regular households, capital market funds, banks and a central bank. Households enter the period, holding cash as well as securities, and owning capital. They receive payments on their securities and may receive a cash injection from the central bank. Then aggregate shocks are realized. Households deposit cash at banks, buy shares of capital mutual funds and keep some cash for transactions purposes. They rent capital to firms as well as supply labor, earning a wage. After receiving wages and capital rental payments, they purchase consumption goods and investments, subject to a cash-in-advance constraint. The deposits and capital market fund securities pay off at the end of the period: the household receives these payments at the beginning of the next period.

Entrepreneurs enter the period, holding capital. The (end-of-period) market value of the capital is their net worth. They can operate a production technology, employing capital and labor, but to do so, they need to have cash at hand to pay workers and capital rental rates up front. Entrepreneurs can borrow a fixed multiple of their net worth to do so. The productivity of entrepreneurs is heterogeneous, and only part of that information is public information ex ante. The final amount produced is observable to the entrepreneur, but not completely observable to lenders, unless they undertake costly verification. The interest rate

at which entrepreneurs can borrow will therefore be endogenously determined, taking into account repayment probabilities and verification costs.

Capital market funds provide break-even costly state verification lending contracts to entrepreneurs based on the ex-ante publicly available productivity information. Banks are assumed to have closer relationships with entrepreneurs. At an iceberg cost to net worth, they can obtain some additional information about the productivity. Based on that additional information, the banks offer break-even costly state verification contracts covering the remaining uncertainty. Given the initial publicly available information, entrepreneurs choose whether to approach capital market funds or banks for a loan, or abstain. If they approach a bank, they can still abstain, after the banks have obtained the additional productivity information. If an entrepreneur obtained a loan, he proceeds to produce, learns the remaining uncertainty regarding his project, and then either repays the loan or defaults. In case of a default, there will be costly monitoring. The entrepreneur then splits end-of-period resources into consumption and capital held to the next period, as net worth.

3.1 Households

At the beginning of period t, aggregate shocks are realized and financial markets open. We use P_t to denote the nominal price level in period t. Households receive the nominal payoffs on assets acquired at time $t-1$ and the monetary transfer $P_t\theta_t$ distributed by the central bank, where θ_t denotes the real value of the transfer. These payments plus their cash balances $\tilde{M}_{t-1}$ carried over from the previous period are their nominal wealth. The households choose to allocate their nominal wealth among four types of nominal assets, namely cash for transactions M_t, nominal state-contingent bonds B_{t+1} paying a unit of currency in a particular state in period $t+1$, one-period deposits at banks D_t^B and one-period deposits at capital mutual funds D_t^C. The deposits earn a nominal uncontingent return. In order for the households to be indifferent between these two deposits, the returns must be the same, a condition that we henceforth impose. Write $D_t = D_t^B + D_t^C$ for total deposits, and R_t^d for the gross return to be earned per unit of deposit between period t and $t+1$. We can then write the budget constraint as

$$M_t + D_t + E_t\left[Q_{t,t+1}B_{t+1}\right] \leq W_t, \tag{1}$$

where nominal wealth at the beginning of period t is given by

$$W_t = B_t + R_{t-1}^d D_{t-1} + P_t \theta_t + \widetilde{M}_{t-1}. \tag{2}$$

Households own capital k_t, which they rent to entrepreneurs at a real rental rate r_t. They also supply labor h_t ("hours worked") to entrepreneurs for a real wage w_t. After receiving rental payments and wage payments in cash, the goods market open, where the household purchases consumption goods c_t and new capital, using total available cash and the cash value of their existing capital, but not more. They thus face a cash-in-advance constraint, given by

$$\widetilde{M}_t \equiv M_t - P_t \left[c_t + k_{t+1} - (1-\delta) k_t\right] + P_t \left(w_t h_t + r_t k_t\right) \geq 0. \tag{3}$$

The household's problem is to maximize utility, given by

$$U = E_o \left\{ \sum_0^\infty \beta^t \left[u\left(c_t\right) - v\left(h_t\right)\right] \right\}, \tag{4}$$

subject to the constraints (1,2,3), where β is the households' discount rate and $u\left(\cdot\right)$ and $v\left(\cdot\right)$ are felicity functions in consumption and hours worked.

3.2 Entrepreneurs, banks and capital market funds

There is a continuum $i \in [0,1]$ of entrepreneurs. They enter the period with capital z_{it}, which will earn a rental rate r_t and depreciate at rate δ. Entrepreneurs can post this capital as collateral, and therefore have net worth n_{it} given by the market value of z_{it},

$$n_{it} = \left(1 - \delta + r_t\right) z_{it}. \tag{5}$$

Each entrepreneur i operates a CRS technology described by

$$y_{it} = \varepsilon_{1,it} \varepsilon_{2,it} \varepsilon_{3,it} H_{it}^{\alpha} K_{it}^{1-\alpha}, \tag{6}$$

where K_{it} and H_{it} denote the capital and labor hired by the entrepreneur .

The shocks $\varepsilon_{1,it}, \varepsilon_{2,it}$ and $\varepsilon_{3,it}$ are random, strictly positive and mutually independent entrepreneur-specific disturbances with aggregate distribution functions denoted by Φ_1, Φ_2 and Φ_3, respectively. While we need to assume this for $\varepsilon_{2,it}$ and $\varepsilon_{3,it}$, and wish to assume this for $\varepsilon_{1,it}$ for simplicity, we can more generally allow serial correlation in $\varepsilon_{1,it}$. In that case, the distribution Φ_1 will depend on $\varepsilon_{1,it-1}$, with little influence on the subsequent analysis, but

perhaps with more palatable implications concerning the time series behavior of individual entrepreneurs[1].

The shocks are realized sequentially during the period, creating three stages of decision. In the first stage, $\varepsilon_{1,it}$ is publicly observed and realized at the time when the aggregate shocks occur, before the entrepreneur takes financial and production decisions. Conditional on its realization, the entrepreneur chooses between three alternatives. He can borrow fund from a capital mutual fund (henceforth: CMF) and produce. He can approach a bank and possibly receive bank loans to produce. He can abstain from production.

If the entrepreneur borrows funds from a CMF, he will obtain total funds in fixed proportion to his net worth

$$x_{it} = \xi n_{it}$$

and learns about $\varepsilon_{2,it}$ and $\varepsilon_{3,it}$ once production has taken place. In De Fiore and Uhlig (2011), we discuss and defend in greater detail the assumption of a fixed proportion as well as ruling out actuarily fair gambles. If the entrepreneur approaches a bank, the bank will investigate the quality of the project of the entrepreneur further, revealing $\varepsilon_{2,it}$ as public information. This investigation is costly to the entrepreur: his net worth shrinks from n_{it} to

$$\hat{n}_{it} = (1 - \tau_t)\, n_{it}$$

Given the additional information as well as the new net worth, the entrepreneur then decides whether to proceed with borrowing or with abstaining. If the entrepreneur borrows, he obtains total funds

$$x_{it} = \xi \hat{n}_{it}$$

from the bank (or a competing bank, as they now all have access to the same information). If the entrepreneur abstains either in the first or the second stage, the entrepeneur takes his (remaining) net worth to the end of the period, and splits it into a part to be consumed and into a part to be carried over as capital into the next period.

[1] Under the assumption that $\varepsilon_{1,it}$ is iid, firms could experience high volatility in ex-ante productivity and could frequently move from one instrument of external finance to the other. Assuming an AR1 process for $\varepsilon_{1,it}$ generates persistance both in firms' productivity and in the choice of the instrument of external finance. This, however, has no implications for the equilibrium allocations in the aggregate.

If the entrepreneur has obtained a loan, he proceeds with production, using the total funds obtained in order pay the factors of production

$$x_{it} = w_t H_{it} + r_t K_{it}. \tag{7}$$

Upon producing, the entrepreneur then learns about the remaining pieces of uncertainty, i.e. about $\varepsilon_{2,it}$ and $\varepsilon_{3,it}$, in case the loan came from a CMF, or $\varepsilon_{3,it}$, in case the loan came from a bank. These outcomes are not observable to the lender, however, unless the lender monitors the entrepreneur, destroying a fraction μ of the output in the process of doing so.

We assume that lending contracts are optimal and rely on revelation. As Townsend (1979) has shown, as is now well known and as we discuss in De Fiore and Uhlig (2011), the solution is a costly state verification contract, in which entrepreneurs promise to repay the loan $x_{it}(\xi - 1)/\xi$ with a prior-information dependent interest rate. They default if and only if they cannot repay the loan, in which case the lender monitors the project. If the entrepreneur did not default, he will repay the loan, and split the reminder between current consumption and capital to be held to the next period, as net worth.

Similar to Gertler and Karradi (2011), we assume that entrepreneurs face difficulties in transforming end-of-period resources into capital next period. If to-be-saved resources at the end are given by f_{it}, then capital next period is given by

$$z_{it+1} = \varkappa_t f_{it} \tag{8}$$

In this way, we can investigate aggregate disturbances to entrepreneurial activity and entrepreneurial net worth. We assume that the logarithm of $\varkappa_t$ follows an AR(1) process.

Entrepreneurs have linear preferences over consumption with rate of time preference β^e, and they die with probability γ. We assume β^e sufficiently high so that the return on internal funds is always higher than the preference discount, $\frac{1}{\beta^e} - 1$. It is thus optimal for entrepreneurs to postpone consumption until the time of death. When they die or default on the debt, entrepreneurs receive an arbitrarily small transfer from the government to restart productive activity.

3.3 Monetary policy and equilibrium

Monetary policy occurs through central banks' liquidity injections, carried out with nominal transfers $P_t\theta_t$ to households. The total amount of liquidity injections in the economy is

$$P_t\theta_t = M_t^s - M_{t-1}^s, \tag{9}$$

where M_t^s denotes money supply. We assume that the latter grows at the exogenous rate ν, $M_t^s = \nu M_{t-1}^s$.

An equilibrium is defined in the usual manner as sequences so that all markets clear and so that all entrepreneurs, households and financial intermediaries take the optimal decisions, given the prices they are facing.

4 Analysis

The analysis here builds on and extends the analysis in De Fiore and Uhlig (2011).

4.1 Households

Define real balances as $m_t \equiv M_t/P_t$ and the inflation rate as $\pi_t \equiv P_t/P_{t-1}$. The safe nominal rate satisfies $R_t = \left(E_t[Q_{t,t+1}]\right)^{-1}$. A comparison with the equation for the interest rate on deposits shows that $R_t = R_t^d$. Since we concentrate on equilibria with $R_t > 1$, we obtain the usual first-order conditions of the household,

$$\begin{aligned} \frac{v'(h_t)}{u'(c_t)} &= w_t \\ u'(c_t) &= \beta R_t E_t\left[\frac{u'(c_{t+1})}{\pi_{t+1}}\right] \\ u'(c_t) &= \beta E_t\left[(1-\delta+r_{t+1})\,u'(c_{t+1})\right]. \end{aligned}$$

4.2 Entrepreneurs: production

We solve the decision problem of the entrepreneur "backwards", starting from the last stage: production. If the entrepreneur obtained a loan and commences production, he maximizes expected profits

$$\varepsilon_{it}^e H_{it}^\alpha K_{it}^{1-\alpha} - w_t H_{it} - r_t K_{it}$$

subject to the financing constraint (7), where

$$\varepsilon_{it}^e \equiv \begin{cases} \varepsilon_{1,it} = E\left[\varepsilon_{1,it}\varepsilon_{2,it}\varepsilon_{3,it}|\varepsilon_{1,it}\right] & \text{if CMF finance} \\ \varepsilon_{1,it}\varepsilon_{2,it} = E\left[\varepsilon_{1,it}\varepsilon_{2,it}\varepsilon_{3,it}|\varepsilon_{1,it},\varepsilon_{2,it}\right] & \text{if bank finance} \end{cases} \tag{10}$$

is the expected part of the entrepreneur-idiosynchratic productivity piece by the time the loan is obtained. A straightforward calculation shows that

$$\begin{aligned} K_{it} &= (1-\alpha)\frac{x_{it}}{r_t} \\ H_{it} &= \alpha\frac{x_{it}}{w_t} \end{aligned}$$

Expected output at the time of loan contracting is given by

$$y_{it}^e \equiv \varepsilon_{it}^e q_t x_t \tag{11}$$

where

$$q_t \equiv \left(\frac{\alpha}{w_t}\right)^{\alpha}\left(\frac{1-\alpha}{r_t}\right)^{1-\alpha}. \tag{12}$$

can be understood as the aggregate entrepreneurial markup over input costs or as the aggregate finance wedge, while actual output is given by

$$y_{it} \equiv \omega_{it} y_{it}^e \tag{13}$$

where

$$\omega_{it} \equiv \begin{cases} \varepsilon_{2,it}\varepsilon_{3,it} & \text{if CMF finance} \\ \varepsilon_{3,it} & \text{if bank finance} \end{cases} \tag{14}$$

is the remaining uncertain part of entrepreneur-specific productivity.

4.3 Entrepreneurs: financial intermediaries and lending decisions

The optimal contract sets a threshold $\overline{\omega}_{it}$ corresponding to a fixed repayment of $P_t\varepsilon_{it}^j\overline{\omega}_{it}q_t x_{it}$ units of currency. If the entrepreneur announces a realization of the uncertain productivity factor $\omega_{it}^j \geq \overline{\omega}_{it}^j$, no monitoring occurs. If $\omega_{it}^j < \overline{\omega}_{it}^j$, the intermediary monitors the entrepreneur, at the cost of destroying a proportion $0 \leq \mu \leq 1$ of the firm output. Let Φ and φ be respectively the distribution and density function of ω_{it}, implied by our distributional assumptions for $\varepsilon_{2,it}$ and $\varepsilon_{3,it}$ as well as the lending decision of the entrepreneur. The residual uncertain factor $\omega = \omega_{it}$ of production in (13) needs to be split across the entrepreneur, the lender and the monitoring costs. Given the treshold $\overline{\omega} = \overline{\omega}_{it}$, define

$$f_j(\overline{\omega}) = \int_{\overline{\omega}}^{\infty}(\omega - \overline{\omega})\,\varphi(\omega)\,d\omega \tag{15}$$

as the expected share of final output acruing to the entrepreneur and

$$g(\overline{\omega}) = \int_0^{\overline{\omega}}(1-\mu)\,\omega\varphi(\omega)\,d\omega + \overline{\omega}\left[1-\Phi(\overline{\omega})\right] \tag{16}$$

as the expected share of final output accruing to the lender, with $\overline{\omega}\Phi(\overline{\omega})$ the share of final output lost due to monitoring. In De Fiore and Uhlig (2011), we provide the details for this contracting problem. Competition between banks results in the break-even condition

$$g_j(\overline{\omega}_{it}) = \frac{R_t}{\varepsilon^e_{it} q_t}\left(1 - \frac{1}{\xi}\right). \tag{17}$$

with $\overline{\omega}_{it}$ minimal among all solutions to this equations. We write this minimal solution as

$$\overline{\omega}_{it} \equiv \begin{cases} \overline{\omega}^c(\varepsilon^e_{it}; q_t, R_t) & \text{if CMF finance} \\ \overline{\omega}^b(\varepsilon^e_{it}; q_t, R_t) & \text{if bank finance} \end{cases} \tag{18}$$

to emphasize that the distribution of ω is either the distribution of $\varepsilon_{3,it}$ for bank finance or of $\varepsilon_{2,it}$ $\varepsilon_{3,it}$ for capital mutual fund finance. It is easy to see that $\overline{\omega}_{it}$ is increasing in R_t and decreasing in ε^e_{it} and q_t.

If the entrepreneur has approached a bank for a loan, he has learned the second-phase value $\varepsilon_{2,it}$ and needs to decide whether to proceed with a loan or abstaining, by comparing his expected share of output when proceeding with a loan to the opportunity cost of holding the remaining net worth to the end of the period. The former is given by $F^d(\varepsilon_{1,it}, \varepsilon_{2,it}; q_t, R_t)n_{it}$, where

$$F^d(\varepsilon_1, \varepsilon_2; q, R) = \varepsilon_1 \varepsilon_2 q f(\overline{\omega}^b(\varepsilon_1 \varepsilon_2; q, R))\xi \tag{19}$$

The entrepreneur will therefore proceed with the loan, if that second-phase value $\varepsilon_{2,it}$ exceeds a threshold $\varepsilon_{2,it} \geq \overline{\varepsilon}^d_{it} = \overline{\varepsilon}_d(\varepsilon_{1,it}; q_t, R_t)$, which satisfies

$$1 = F^d(\varepsilon_{1,it}, \overline{\varepsilon}^d_{it}; q_t, R_t). \tag{20}$$

In stage I and in light of $\varepsilon_{1,it}$ as well as aggregate information, the entrepreneur chooses whether or not to obtain a loan, and if so, whether to obtain it from a bank or from a capital market fund. The expected payoff for an entrepreneur, who proceeds with bank finance conditional on the realization of $\varepsilon_{1,it}$, is $F^b(\varepsilon_{1,it}; q_t, R_t, \tau_t)n_{it}$, where

$$F^b(\varepsilon_1; q, R, \tau) \equiv (1-\tau)\left(\int_{\overline{\varepsilon}_d(\varepsilon_1; q, R)} F^d(\varepsilon_1, \varepsilon_2; q, R)\Phi_2(d\varepsilon_2) + \Phi_2(\overline{\varepsilon}_d(\varepsilon_1; q, R))\right) \tag{21}$$

is the expected entrepreneurial payoff for each unit of net worth from either proceeding with a bank loan or abstaining, after learning ε_2. The expected payoff for an entrepreneur, who proceeds with CMF finance conditional on the realization of $\varepsilon_{1,it}$, is $F^c(\varepsilon_{1,it}; q_t, R_t)n_{it}$, where

$$F^c(\varepsilon_1; q, R) \equiv \varepsilon_1 q f(\overline{\omega}^c(\varepsilon_1; q, R))\xi. \tag{22}$$

Finally, the expected payoff for an entrepreneur, who abstains from production, is n_{it}. Knowing $\varepsilon_{1,it}$, each entrepreneur chooses his or her best option, leading to the overall payoff $F(\varepsilon_{1,it}; q_t, R_t, \tau_t)n_{it}$, where

$$F(\varepsilon_1; q, R, \tau) \equiv \max\{1; F^b(\varepsilon_1; q, R, \tau); F^c(\varepsilon_1; q, R)\}. \tag{23}$$

We assume that $(A1)$ $\frac{\partial F^b(\cdot)}{\partial \varepsilon_1} \geq 0$ and $(A2)$ $\frac{\partial F^b(\cdot)}{\partial \varepsilon_1} < \frac{\partial F^c(\cdot)}{\partial \varepsilon_1}$, for all ε_1. Under (A1), a threshold for ε_1, below which the entrepreneur decides not to raise external finance, exists and is unique. We denote it as $\overline{\varepsilon}_{bt}$. It is implicitly defined by the condition

$$F^b(\overline{\varepsilon}_{bt}; q_t, R_t, \tau_t) = 1. \tag{24}$$

The unique cutoff point is a function of aggregate variables only, $\overline{\varepsilon}_{bt} = \overline{\varepsilon}^b(q_t, R_t, \tau_t)$, and hence is identical for all firms. Under A1) and A2), a threshold for $\varepsilon_{1,it}$ above which entrepreneurs sign a contract with the CMF, also exists and is unique. We denote it as $\overline{\varepsilon}_{ct}$. It is implicitly defined by the condition

$$F^b(\overline{\varepsilon}_{ct}; q_t, R_t, \tau_t) = F^c(\overline{\varepsilon}_{ct}; q_t, R_t) \tag{25}$$

and it is thus identical across firms, $\overline{\varepsilon}_{ct} = \overline{\varepsilon}^c(q_t, R_t, \tau_t)$.

Conditional on q_t, R_t and τ_t, entrepreneurs split into three sets that are intervals in terms of the first idiosyncratic productivity shock $\varepsilon_{1,it}$. Denote the firm's decision on whether to produce with a dummy variable Θ_{it}:

$$\Theta_{it} = \begin{cases} 1 \text{ if } \varepsilon_{1,it} > \overline{\varepsilon}_{ct} \text{ or if } \overline{\varepsilon}_{bt} \leq \varepsilon_{1,it} \leq \overline{\varepsilon}_{ct} \text{ and } \varepsilon_{2,it} > \overline{\varepsilon}^d_{it} \\ 0 \quad \text{else} \end{cases}.$$

The functions $s^a(\cdot), s^b(\cdot), s^c(\cdot)$ and $s^{bp}(\cdot)$ measure respectively the shares of firms that abstain from producing, approach a bank, raise CMF finance, and produce conditional on having approached a bank,

$$s^a(q, R, \tau) = \Phi_1\left(\overline{\varepsilon}^b(q, R, \tau)\right) \tag{26}$$

$$s^b(q, R, \tau) = \Phi_1\left(\overline{\varepsilon}^c(q, R, \tau)\right) - \Phi_1\left(\overline{\varepsilon}^b(q, R, \tau)\right) \tag{27}$$

$$s^c(q, R, \tau) = 1 - \Phi_1\left(\overline{\varepsilon}^c(q, R, \tau)\right) \tag{28}$$

$$s^{bp}(q, R, \tau) = \int_{\overline{\varepsilon}^b(q,R,\tau)}^{\overline{\varepsilon}^c(q,R,\tau)} \int_{\overline{\varepsilon}^d(\varepsilon_1;q,R)} \Phi_2(d\varepsilon_2)\Phi_1(d\varepsilon_1). \tag{29}$$

Because the return on internal funds is always higher than the rate of time preference, entrepreneurs accumulate wealth and only consume before dying. It follows that in the aggregate,

entrepreneurs consume each period a fraction γ of their accumulated wealth. Entrepreneurial consumption and accumulation of capital are then given by

$$e_t = (1-\gamma)\,\psi^f\left(q_t, R_t, \tau_t\right) n_t, \tag{30}$$

$$z_{t+1} = \gamma \varkappa_t \psi^f\left(q_t, R_t, \tau_t\right) n_t, \tag{31}$$

where $\psi^f\left(q_t, R_t, \tau_t\right) n_t$ are aggregate profits of the entrepreneurial sector, and $\psi^f\left(q_t, R_t, \tau_t\right)$ is defined in appendix A. As in (8), $\varkappa_t$ is an aggregate shock to net worth accumulation. We assume assume that it follows an AR(1) process. It affects the ability of entrepreneurs to transform period t profits into period $t+1$ capital, and can be thought of as a shock to the quality of the existing capital (as in Gertler and Karadi (2011)).

For comparison to the data, the following calculations are useful. The loan rate R_{it}^j, defined as the nominal interest rate that is charged for the use of external finance, is implicitly given by the condition

$$R_{it}^j = \varepsilon_{it}^e q_t \overline{\omega}_{it} \frac{\xi}{\xi - 1}. \tag{32}$$

It follows that the risk premium on the external finance of a firm i, which has chosen to use instrument j, is given by

$$rp_{it} = \frac{R_{it}^j}{R_t} - 1. \tag{33}$$

4.4 Aggregation and market clearing

Aggregate demand for funds, x_t, output y_t, and output lost to agency costs y_t^a are given by:

$$x_t = \left[(1-\tau_t) s^{bp}\left(q_t, R_t, \tau_t\right) + s^c\left(q_t, R_t, \tau_t\right)\right] \xi n_t \tag{34}$$

$$y_t = \psi^y\left(q_t, R_t, \tau_t\right) \xi q_t n_t \tag{35}$$

$$y_t^a = \left[\tau_t s^b\left(q_t, R_t, \tau_t\right) + \psi^m\left(q_t, R_t, \tau_t\right) \mu \xi q_t\right] n_t \tag{36}$$

where the functions $s^b(\cdot)$, $s^c(\cdot)$ and $s^{bp}(\cdot)$ are given by (27)-(29). The function $\psi^y(\cdot)$ aggregates the realized productivity factors across all producing firms. The terms $\tau_t s^b\left(q_t, R_t, \tau_t\right)$ and $\psi^m(\cdot)\,\mu\xi q_t$ measure the loss of resources due respectively to bank information acquisition and to monitoring costs, per unit of net worth. All these functions are defined in Appendix A.

Aggregate factor demands are given by

$$w_t H_t = \alpha x_t \tag{37}$$

$$r_t K_t = (1-\alpha)\, x_t. \tag{38}$$

Market clearing for money, assets, labor and capital requires that $M_t^s = M_t + D_t$, $B_t = 0$, $K_t = k_t + z_t$ and $H_t = l_t$, respectively. Market clearing conditions for loans and output are, respectively,

$$D_t = P_t \left[(1-\tau_t) s^{bp}(q_t, R_t, \tau_t) + s^c(q_t, R_t, \tau_t) \right] (\xi - 1) n_t, \tag{39}$$

$$y_t^a = y_t - c_t - e_t - K_{t+1} + (1-\delta) K_t. \tag{40}$$

In appendix B, we provide analytical expressions for the aggregate financial variables that we use in our numerical analysis, namely the ratio of bank finance to bond finance, ϑ_t, the average risk premium for bank-financed firms, rp_t^b, and for CMF-financed firms, rp_t^c, the aggregate debt to equity ratio, χ_t, the default rate on corporate bonds, ϱ_t^c, the average default across firms, ϱ_t, and the net expected return to entrepreneurial capital, r_t^z. We collect the equations that characterize a competitive equilibrium in appendix C. In appendix D, we characterize the steady state and describe the procedure we use to compute it. In appendix E, we show how to log-linearize the equilibrium conditions around a stochastic steady state. This latter is a steady state where firms are hit by idiosyncratic shocks but aggregate shocks are set to their long-run values. A particular challenge arises from the heterogeneity of firms, and the need to log-linearize with respect to the boundaries of integrals, that is, by the need to aggregate across firms and by the presence of endogenously evolving regions of integration.

5 Results

We seek to investigate the ability of the model to qualitatively and quantitatively replicate the key facts observed during the crisis on corporate debt and macroeconomic activity. We then use the model to evaluate the importance of firms' ability to shift among alternative instruments of external finance for aggregate activity. The model is calibrated in line with the long-run evidence for the euro area documented in De Fiore and Uhlig (2011). The dynamics of the system is solved, using log-linearization and Uhlig (1999)'s toolkit.

5.1 Calibration

We assume the functional form $u(c_t) - v(h_t) = \log(c_t) - \eta h_t$ for some parameter η. We calibrate the model quarterly in order to match in steady state the financial facts documented for the euro area in De Fiore and Uhlig (2011). Since the model here is quarterly, while the

model there is annual, we use slightly different parameters. To that end, we briefly review our procedure for calibration. We set $\beta = .99$ and the inflation rate to 0.5 percent per quarter, corresponding to the annual average over the period 1999-2007 in the euro area. The corresponding nominal risk-free rate is $R = 1.015$. The depreciation rate is set at $\delta = .02$ and the discount factor at $\beta = .99$, implying a rental rate for capital of 3 percent. We choose $\alpha = .64$ in the production function and a coefficient in preferences η so that labor equal .3 in steady state. We set $\mu = .15$, a value commonly assumed in related literature.

The iid productivity shocks $v = \varepsilon_2, \varepsilon_3$ are lognormally distributed. $\log(v)$ is normally distributed with mean $-\sigma_v^2/2$ and variance σ_v^2, so that $E(v) = 1$.

The shock ε_1 is autocorrelated and such that $log(\varepsilon_{1,it}) = \rho_{\varepsilon_1} log(\varepsilon_{1,it-1}) + \left(1 - \rho_{\varepsilon_1}\right) \log(\kappa_{it})$, where $log(\kappa_{it})$ is normally distributed with mean $-\sigma_\kappa^2/2$ and variance σ_κ^2. It follows that $E(\kappa_{it}) = 1$ and $E(\varepsilon_{1,it}) = 1$.

We set the remaining six parameters, $\xi, \tau, \gamma, \sigma_{\varepsilon_1}, \sigma_{\varepsilon_2}$ and σ_{ε_3} to values that jointly minimize the squared log-deviation of the model-based predictions from their empirical counterparts for the following six financial facts : i) the ratio of aggregate bank loans to debt securities for non-financial corporations, ϑ, is 5.48; ii) the ratio of aggregate debt to equity, χ, is .64; iii) the average risk premium on debt securities, rp^c, is 143 bps; iv) the average risk premium on bank loans, rp^b, is 119 bps; v) the average default rate on debt securities, ϱ^c, is 4.96 percent; vi) and the expected return to entrepreneurial capital, r_t^z, is 9.3 percent.[2] The parameter values selected from our calibration procedure are $\tau = .017$, $\gamma = .977$, $\xi = 2.28$, $\sigma_{\varepsilon_1} = .007$, $\sigma_{\varepsilon_2} = .03$, $\sigma_{\varepsilon_3} = .237$.

The stochastic processes for τ_t and $\varkappa_t$ are assumed to have a persistence parameter of 0.9. The standard deviations are calibrated as to replicate, respectively, the maximum deviation observed during the 2007-2009 crisis of the ratio of bank loans to debt securities and of investment from their average over the post-EMU period.

5.2 Steady state

In order to understand the response of the composition of corporate debt to a shock to bank fees, it is useful to consider how a permanent reduction in τ affects firms' financing choices and risk premia in the steady state of our economy.

[2] These are annual averages observed over the period 1999-2007. See De Fiore and Uhlig (2011) for a description of the data.

In the model, an increase in bank fees τ induces a change in the expected profit function $F^b(\varepsilon_{1,it}; q_t, R_t, \tau_t)$. The higher the τ, the lower the advantage of approaching a bank and obtaining additional information on $\varepsilon_{2,it}$, before deciding whether or not to produce and raise external finance. From equations (24) and (25), it follows that an increase in τ shifts the thresholds $\overline{\varepsilon}_{bt}$ and $\overline{\varepsilon}_{ct}$, thus modifying the share of firms approaching banks and the share of firms raising external finance from CMFs. On the contrary, equation (20) shows that the level of τ does not affect firms' choice of proceeding with production, conditional on having approached a bank. The share of bank-financed firms that decide to drop out after observing the shock $\varepsilon_{2,it}$ remains unaffected.

Figure 1 plots the effect of a 40 percent permanent increase in τ on the share of firms choosing to abstain, to approach a bank and wait, and to raise CMF finance and produce.

The black solid line shows the density function $\varphi(\varepsilon_1)$. The red and purple dashed lines show respectively the threshold for bank-finance, $\overline{\varepsilon}_{bt}$, and the threshold for CMF finance, $\overline{\varepsilon}_{ct}$, when τ equals its benchmark value of .016. The green and pink dashed-dotted lines show the same thresholds when τ is increased to .023.

At $\tau = .016$, firms experiencing a value of ε_1 at the left of the red dashed line find it optimal to abstain from production and to retain their net worth n_{it}. Their risk of default at the end of the period in case of production is too high. Firms experiencing a value of ε_1 between $\overline{\varepsilon}_{bt}$ and $\overline{\varepsilon}_{ct}$ rather find it optimal to raise external finance from banks. Their risk of default is sufficiently high that the "wait and see" option provided by banks compensate the extra-fee being charged. Only firms at the right of $\overline{\varepsilon}_{ct}$ are sufficiently safe to choose CMF finance.

Under the larger fee, $\tau = .023$, the thresholds $\overline{\varepsilon}_{bt}$ and $\overline{\varepsilon}_{ct}$ shift inwards. Firms facing a realization of ε_1 between the red dashed and the green dash-dotted lines now find the flexibility of banks too costly relative to the benefit. At the prevailing price of bank finance, their risk of default is sufficiently high to make it optimal for them to abstain from production. Similarly, the share of firms that experience a shock between the purple dashed line and the pink dashed-dotted line now find it optimal to shift from bank finance to bond finance. The higher τ induces them to face the higher risk of default associated with CMF finance.

Because the average creditworthiness (as measured by the realization of the first shock, $\varepsilon_{1,it}$) of CMF-financed firms falls, the average risk premium on bonds rises. The average risk premium on bank finance increases but not as much. The reduction in average creditworthiness due to some firms with high $\varepsilon_{1,it}$ moving to CMF-finance just more than compensate the

improved risk prospects due to firms with low $\varepsilon_{1,it}$ moving out of banking. Overall, the increase in the average risk premium is larger for bonds than for loans.

5.3 The response to a decrease in bank efficiency

In order to capture the evidence observed during the financial crisis, we need to account for the observed fall in bank loans relative to debt securities and the simultaneous rise in the cost of market finance relative to bank finance. We conjecture that the shift was induced by a negative shock to bank profitability as well as a decrease in the efficiency with which banks evaluates projects, having perhaps lost some of their confidence in standard procedures used up to that point. We explore this explanation through the lenses of our model.

We model this as a shock that increases bank information acquisition costs, τ_t, thus reducing the efficiency of banks as financial intermediaries. The shock can be seen as capturing the difficulties in raising liquidity faced by euro area banks in 2007-2009.[3] It is calibrated as to generate a fall on impact of the ratio of loans to bonds of 16 percent, in line with the peak effect observed during the crisis.

Figure 4 shows that the response of the economy is qualitatively consistent with the evidence. As the cost of information acquisition increases, firms move away from bank finance. A larger share of firms facing low realizations of ε_1 find the cost of external finance too high, and choose to abstain from production. A larger share of firms experiencing high realizations of ε_1 find the flexibility provided by banks too costly, and decides to issue bonds instead. The ratio of bank loans to corporate bonds falls.

As in the data, the cost of both bank finance and bond finance rise, and the latter increases to a greater extent than the former. The risk premium on bond finance unambiguously increases because the pool of CMF-financed firms now presents a higher average risk of default. The risk premium on loans also increases on impact (although to a lower extent than bond finance) because the share of firms with low risk of default that move from bank-finance to CMF-finance more than compensates the share of firms with high risk of default that move out of banking and decides not to produce.

The shock increases the aggregate default rate and the debt to equity ratio, as observed during the crisis. More frequent bankruptcies result from the larger cost of external finance,

[3] The shock is also consistent with the sharp increase observed in the item "commissions and fees" of pre-provisioning profits of euro area monetary and financial institutions. See Financial Stability Review (2011).

which increases due to higher banking fees and risk premia. The aggregate debt to equity ratio rises because the reduction in aggregate net worth, due to lower available net worth of bank-financed firms, is larger that the reduction in aggregate debt due to the shrinking share of producing firms.

The real effects of the shock to bank costs arise as a consequence of the reduction in the fraction of producing firms. As more firms decide not to approach a financial intermediary (the share of *abstain* increases) and a larger share of bank-financed firms decide to drop out after obtaining information on the second productivity shock, the aggregate level of credit and investment fall, together with output.

It is instructive to compare the quantitative strength of the responses with observed observed magnitudes. Under the shock to information costs τ, the model generates too large volatility in the ratio of bank loans to corporate bonds, relative to other variables. Aggregate default increases in the order of 0.4 percentage point, while bankruptcies have more than doubled during the crisis, relative to their long run average value. The increase in the risk premium on bonds is ten times larger than the one on bank loans, but both are tiny (the former rises by 0.5 percent vs 70 percent in the data). The debt to equity ratio increases by around 0.3 percent, well below the observed 15 percent. The investment to output ratio and output fall respectively by .05 and .02 percent in the model (vs 3.6 and 6.8 percent in the data).

5.4 The response to a shock to capital quality.

Figure 5 shows the impulse responses to a reduction in capital quality, $\varkappa_t$, which is normalized to produce the observed peak fall in the ratio of aggregate investment to GDP of 3.6 percent.

Like the bank efficiency shock to τ, this shock generates responses which are qualitatively in line with the evidence. The shock reduces entrepreneurs' capital and net worth in period $t+1$. It also reduces output, but not as much because a large fraction of the capital stock is owned by households and it is unaffected by the shock. Because leverage is constant for each producing firm, an equilibrium requires inducing a larger share of firms to borrow and produce. The share of producing firms indeed raises because the diminished net worth increases the average financial distortion, as measured by the markup q, contributing to raise expected profits from production. The higher profitability also explains why some of the firms which would otherwise be borrowing from banks now shift to bond finance. For those firms, improved production prospects reduce default risk and the value of the "wait-and-see " option offered by

banks. The average risk premium rises both on bonds and on loans, reflecting the inclusion of new firms with high default risk in the share of both bank-financed firms and CMF-financed firms. As a consequence, the economy faces a higher average risk of default.

Relative to a shock to banking fees, a reduction in capital quality generates more sizeable effects on real and aggregate financial variables. A shock normalized to replicate the peak effect observed on investment generates an increase in the ratio of aggregate debt to equity of 27 percent (vs 15 percent in the data). The fall in GDP and the increase in the spreads and aggregate default rate are larger, but still far from the levels observed during the crisis. Also, the shift from bank finance to bond finance is too mild (0.8 percent).

A combination of τ_t and $\varkappa_t$ better captures the magnitude of the responses observed during the crisis. The experiment is illustrated in figure 6, where the shock to bank efficiency is calibrated as to replicate the 16 percent drop in the ratio of bank loans to debt securities, while the shock to capital quality is set to generate an impact reduction in investment of 3.6 percent. The combined shock produces a sizeable increase in the debt to equity ratio and an output contraction that is closer to (although milder than) the data, relative to the case where τ_t is the only shock. Nonetheless, it generates too little movements in the average risk premia and in the aggregate default rate.

5.5 The response to an increase in uncertainty

To provide a fuller account for the key facts, we shall appeal to three shocks. Aside from the shock to bank efficiency and capital quality investigated above, we add a shock to the level of risk faced by firms, i.e. a general increase in uncertainty. Specifically, we consider a shock which increases the standard deviation of ε_{3t}. By affecting the default risk faced by all producing firms, this shock can produce large effects on risk premia and default rates. For pragmatic reasons, we focus on a permanent change in that standard deviation, as it allows us to calculate the response as the transition between steady states. In a future version of the paper, we plan to add a temporary change in bank efficiency τ and capital quality $\varkappa$ to this. Here, we show the effects of a combined *permanent* shock to τ, $\varkappa$ and σ_{ε_3}. The experiment is conducted by assuming that the economy starts from the calibrated steady state and converges to a new steady state where the three parameters τ, $\varkappa$ and σ_{ε_3} take up their "post-crisis" level.

Figure 7 shows the responses of the economy. The shock to σ_{ε_3} is normalized to replicate the observed increase in the cost of bond issuance (70 percent). The response is computed in percentage deviations from the old steady state.

A combination of these three shocks replicates the responses observed during the crisis reasonably well from a quantitative point of view. The increase in σ_{ε_3} produces large effects on both risk premia and the aggregate default rate, which almost double, and a deeper contraction of output (by 2.7 percent), although still milder than observed. The main shortcoming is that the debt to equity ratio falls rather than to increase. The reason is that a higher σ_{ε_3} reduces expected profits and the share of firms that decide to produce. As a consequence, total debt as a share of net worth is reduced. Also, the shock to σ_{ε_3} exerts equally large effects on the risk premium on loans and on the risk premium on bonds. They both increase by around 70 percent. In the data, they increase by 29 and 70 percent, respectively.

5.6 Exogenous thresholds

We evaluate the importance for the aggregate economy of firms' ability to shift among alternative instruments of external finance. We do so by comparing the impulse responses to a τ shock when thresholds $\overline{\varepsilon}_{bt}, \overline{\varepsilon}_{ct}$ and $\overline{\varepsilon}_{it}^{d}$ are endogenous to the case when they are fixed at their steady state level.

Figure 8 shows the results for the case of exogenous thresholds. The shares of firms that abstain, approach a bank, raise bank-finance and produce, and raise bond-finance and produce, remain constant. Nonetheless, the ratio of total bank loans to corporate bonds fall, because the available net worth for bank-financed firms is reduced, together with the amount of finance these firms can raise from banks. For the same reason, the overall debt to equity ratio falls. The reduction in available net worth and total credit, together with the fall in the markup q_t, is also responsible for the fall in investment and output. Risk premia on loans and on bonds rise because the overall share of producing firms is larger than what would be optimal at this higher level of bank fees. The average risk of producing firms increases together with the risk premia.

Interestingly, the effects of the shock on risk premia, investment and output are amplified relative to the case when the thresholds are endogenous (reported in figure 4). The risk premium on loans and the risk premium on bonds increase by 13 and 11 percent, relative to .06 and .5 percent, respectively, in the case of endogenous thresholds. Output and investment to

GDP fall by 0.5 and 2.2 percent, relative to .02 and .05 percent when thresholds are endogenous. The contractionary effect of the shock is much larger when firms are unable to substitute instruments of external finance.

Our results are consistent with recent empirical evidence documented in Becker and Ivashina (2011). Using firm-level data on US firms over the period 1990Q2:2010Q4, the authors show that the effect of a reduction in loan supply on investment is positive and significant for firms that raise debt finance and have access to both bond and loan markets. For firms that are excluded from bond markets, the contractionary effect is even larger.

6 Conclusions

We propose a dynamic stochastic general equilibrium model that enables to assess the macroeconomic consequences of firms' financial choices and of the evolving composition of corporate debt.

In response to a shock that increases banking costs and reduces bank efficiency in financial intermediation, the model replicates qualitatively the main facts observed during the crisis, namely the shift in corporate debt from bank finance to bond finance together with an increasing cost of debt securities relative to bank loans, and a contraction in investment and output.

The model points to an important role played by the composition of corporate debt in determining the response of real activity during the crisis. When firms have no access to the bond market, the negative effects on investment and output of a shock that reduces bank profitability are amplified. These findings suggest that abstracting from an endogenous corporate debt structure - as generally done in models that assess the impact of financial market imperfections - may overstate the negative consequences of adverse shocks on real activity.

These results also suggest that the post-crisis policy debate in Europe needs to be broadened beyond banks and financial intermediaries, and needs to include considerations of shifts in firm financing from banks to capital markets. Notwithstanding the central role of banks for ensuring financial stability, policy measures aimed at achieving easier substitutability of bank loans for other instruments of external finance may be equally important, as they reduce the adverse consequences on economic activity of periods of financial distress.

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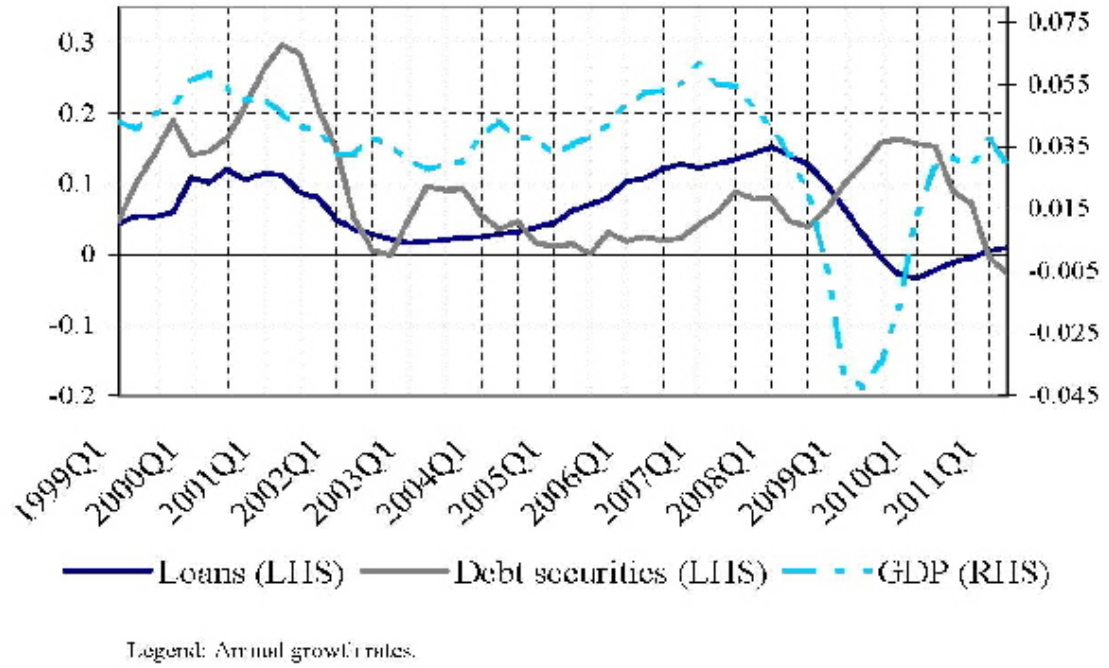

Figure 1: Bank loans and debt securities of non-financial corporations in the euro area.

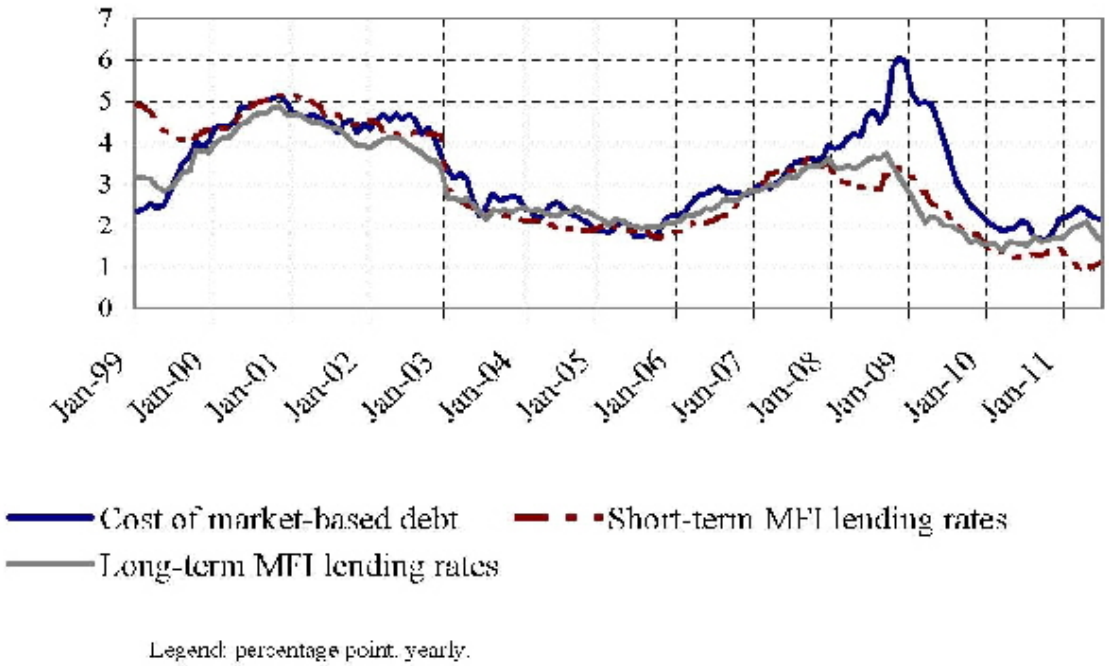

Figure 2: Cost of bank financing and bond financing in the euro area.

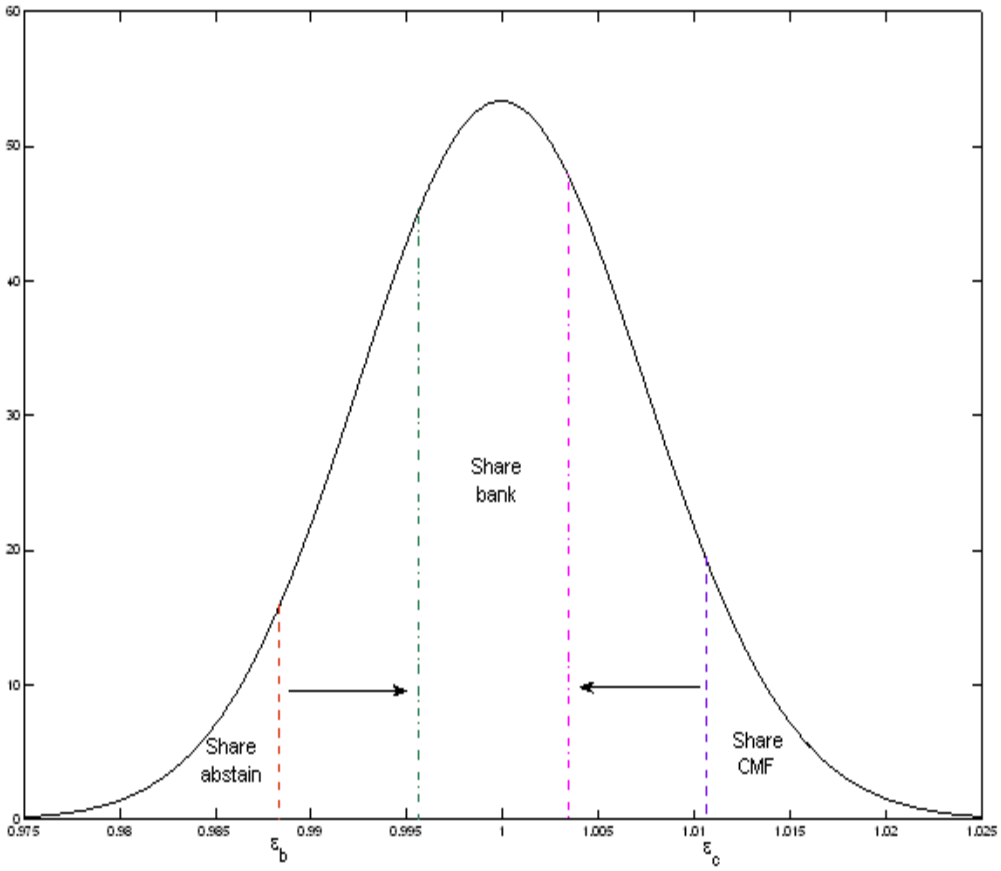

Figure 3: Impact on the steady state distribution of firms of an increase in τ

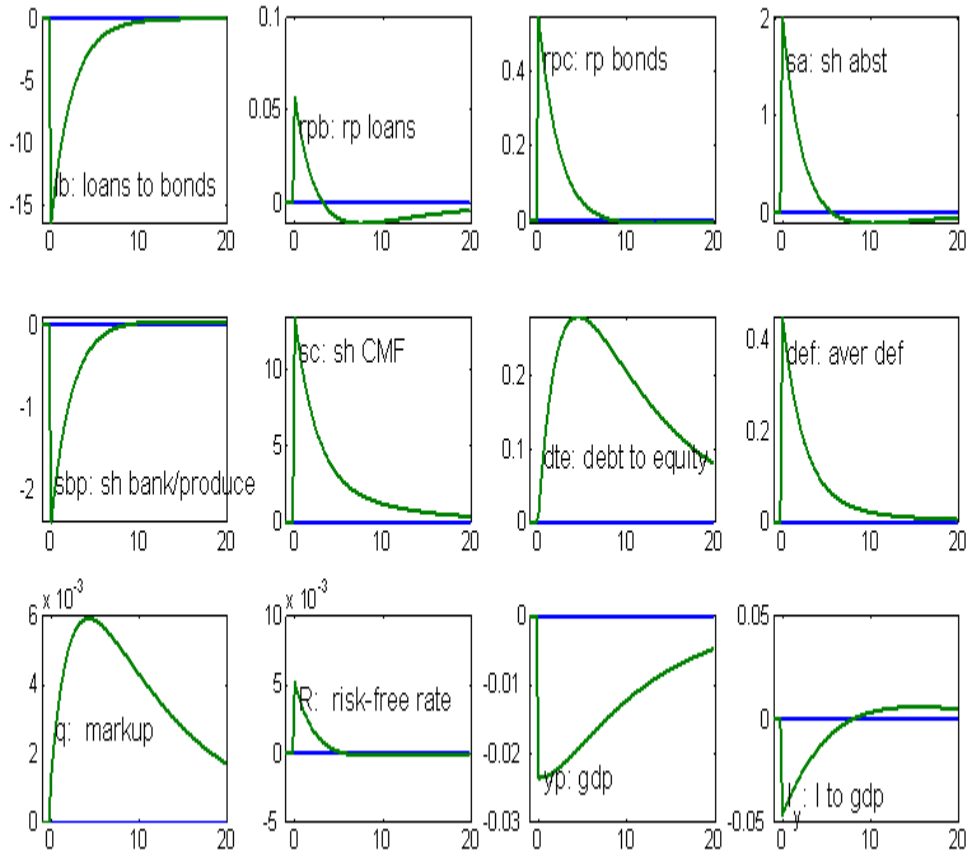

Figure 4: Impulse responses to an increase in bank costs, τ.

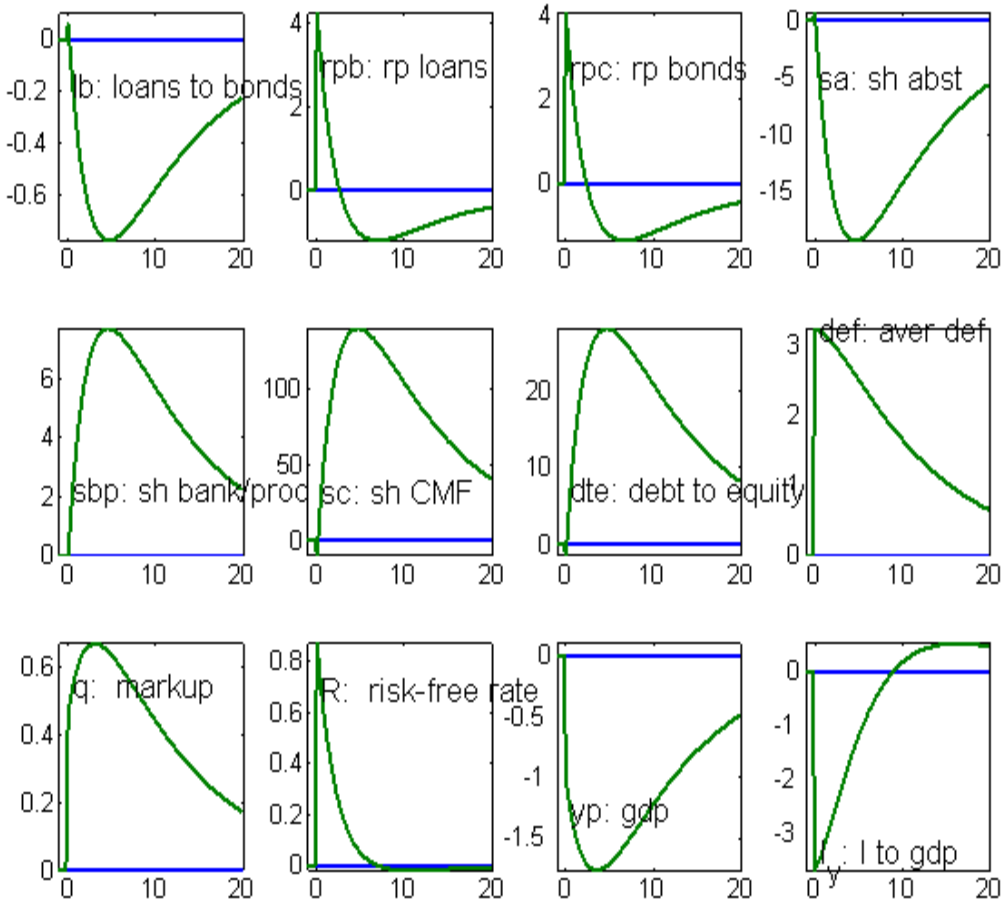

Figure 5: Impulse responses to a negative shock to capital quality, $\varkappa$.

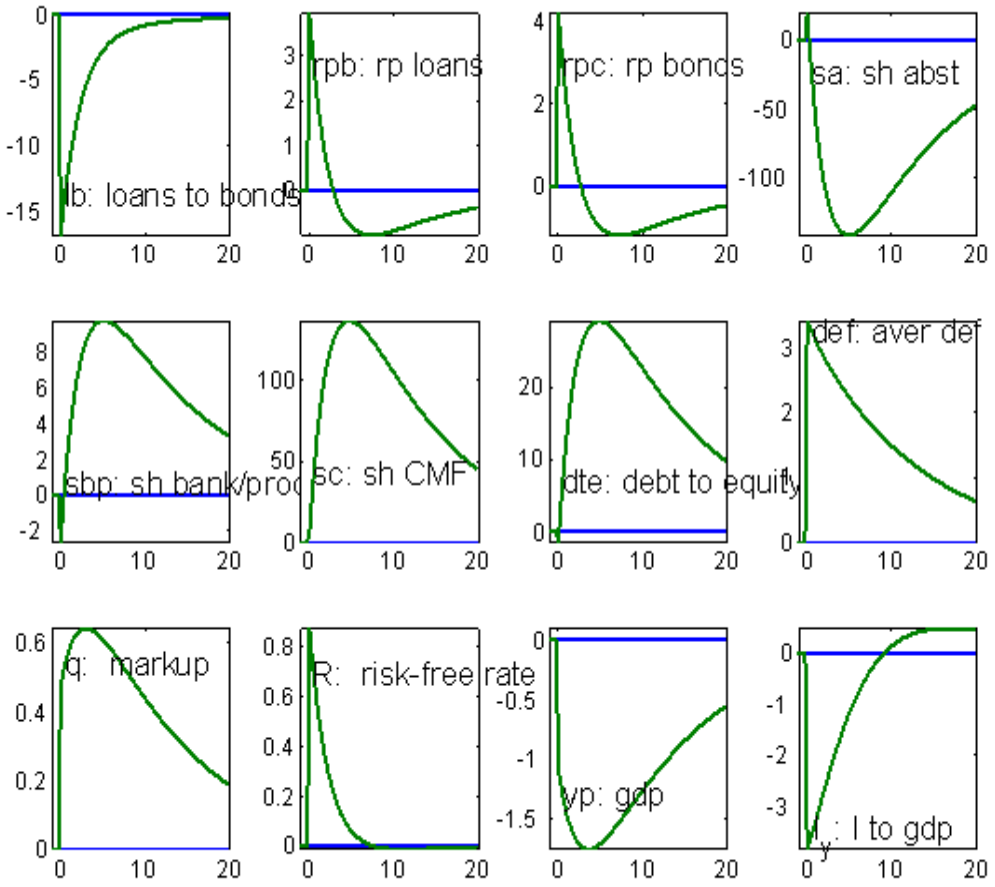

Figure 6: Impulse responses to a combined shock to τ and $\varkappa$.

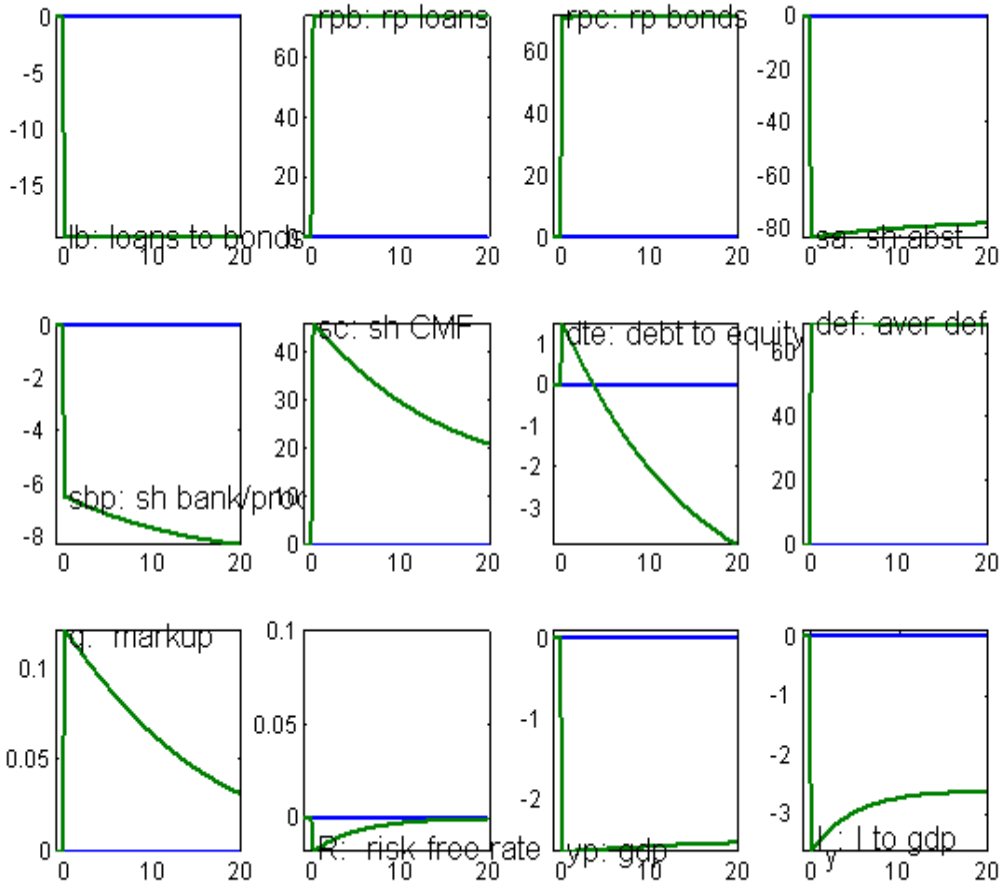

Figure 7: Impulse responses to a permanent combined shock to τ, $\varkappa$ and σ_{ε_3}.

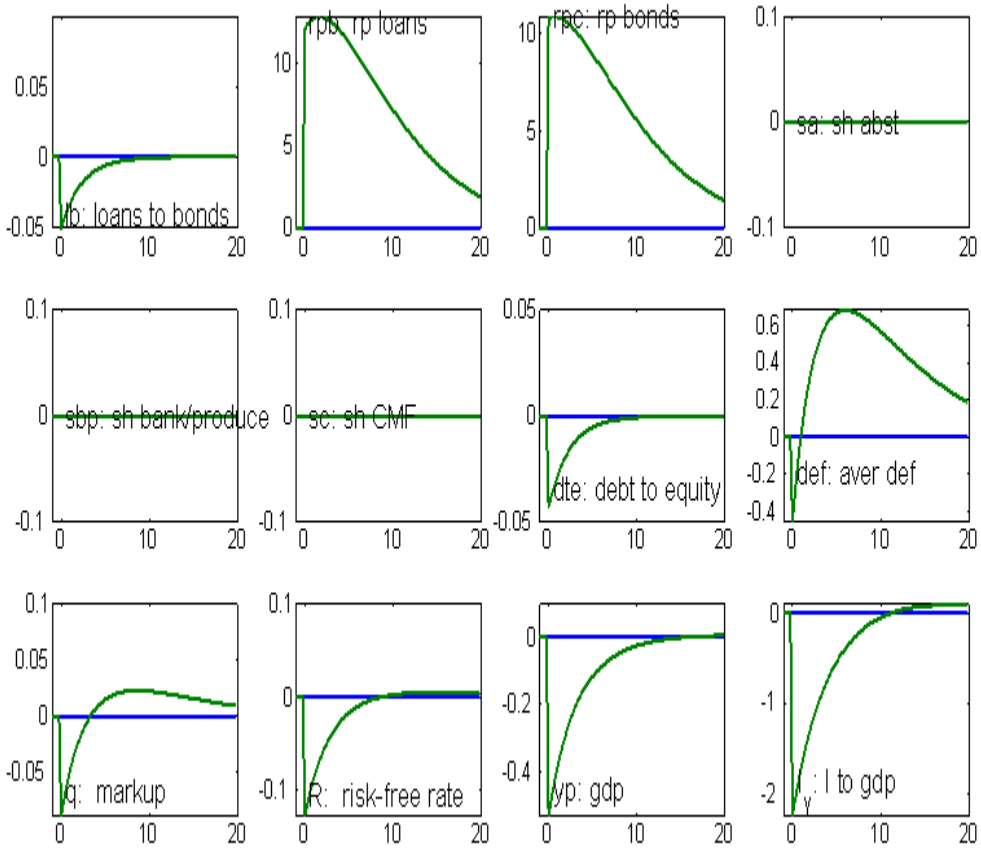

Figure 8: Impulse responses to an increase in bank fees, τ: exogenous thresholds.

APPENDIX

A Aggregating across firms

Aggregate profits of the entrepreneurial sector are given by $\psi^f(q_t, R_t, \tau_t)n_t$, where

$$\psi^f(q_t, R_t, \tau_t) \equiv \int F(\varepsilon_1; q_t, R_t, \tau_t)\Phi_1(d\varepsilon_1),$$

or, equivalently, by

$$\begin{aligned}\psi^f(q_t, R_t, \tau_t) &= s^a(q_t, R_t, \tau_t) + \int_{\overline{\varepsilon}_b(q_t,R_t,\tau_t)}^{\overline{\varepsilon}_c(q_t,R_t,\tau_t)} F^b(\varepsilon_1; q_t, R_t, \tau_t)\Phi_1(d\varepsilon_1) \\ &\quad + \int_{\overline{\varepsilon}_c(q_t,R_t,\tau_t)} F^c(\varepsilon_1; q_t, R_t)\,\Phi_1(d\varepsilon_1).\end{aligned}$$

Entrepreneurial consumption and accumulation of capital can then be written as equations (30) and (31) in the text.

Define

$$\psi^y(q_t, R_t, \tau_t) = (1-\tau_t)\int_{\overline{\varepsilon}_b(q_t,R_t,\tau_t)}^{\overline{\varepsilon}_c(q_t,R_t,\tau_t)} \varepsilon_1 \int_{\overline{\varepsilon}_d(\varepsilon_1;q_t,R_t)} \varepsilon_2\Phi_2(d\varepsilon_2)\Phi_1(d\varepsilon_1) + \int_{\overline{\varepsilon}_{c(q_t,R_t,\tau_t)}} \varepsilon_1\Phi_1(d\varepsilon_1) \tag{41}$$

and

$$\psi^m(q_t, R_t, \tau_t) = (1-\tau_t)\psi^{mb}(q_t, R_t, \tau_t) + \psi^{mc}(q_t, R_t, \tau_t),$$

where

$$\psi^{mb}(q_t, R_t, \tau_t) = \int_{\overline{\varepsilon}_b(q_t,R_t,\tau_t)}^{\overline{\varepsilon}_c(q_t,R_t,\tau_t)} \int_{\overline{\varepsilon}_d(\varepsilon_1;q_t,R_t)} \Phi_3\left(\overline{\omega}^b(\varepsilon_1\varepsilon_2; q_t, R_t)\right)\Phi_2(d\varepsilon_2)\Phi_1(d\varepsilon_1),$$

$$\psi^{mc}(q_t, R_t, \tau_t) = \int_{\overline{\varepsilon}_c(q_t,R_t,\tau_t)} \Phi_{2*3}\left(\overline{\omega}^c(\varepsilon_1; q_t, R_t)\right)\Phi_1(d\varepsilon_1),$$

and Φ_{2*3} is the distribution function for the product $\omega^c = \varepsilon_2\varepsilon_3$. Then, total output, y_t, and total output lost to monitoring costs, y_t^a, are given by equations (35) to (36) in the text.

B Financial variables

We provide analytical expressions for financial variables used in the numerical analysis.

The ratio of bank finance to bond finance, ϑ_t, is defined as the ratio of the funds raised by bank-financed firms to the funds raised by CMF-financed firms, and is given by

$$\vartheta_t = \frac{(1-\tau_t)\,s^{bp}(q_t, R_t, \tau_t)}{s^c(q_t, R_t, \tau_t)}. \tag{42}$$

Recall that the risk premium for a firm i, which has chosen to use instrument j, is given by (33). Let $\psi^{rb}(q_t, R_t, \tau_t)$ and $\psi^{rc}(q_t, R_t, \tau_t)$ be

$$\psi^{rb}(q_t, R_t, \tau_t) = \int_{\overline{\varepsilon}_b(q_t,R_t,\tau_t)}^{\overline{\varepsilon}_c(q_t,R_t,\tau_t)} \int_{\overline{\varepsilon}_d(\varepsilon_1;q_t,R_t)} \left[\frac{\left(\frac{\xi}{\xi-1}\right) q_t \varepsilon_1 \varepsilon_2 \overline{\omega}^b(\varepsilon_1\varepsilon_2; q_t, R_t)}{R_t} - 1 \right] \Phi_2(d\varepsilon_2)\Phi_1(d\varepsilon_1),$$

$$\psi^{rc}(q_t, R_t, \tau_t) \equiv \int_{\overline{\varepsilon}_c(q_t,R_t,\tau_t)} \left[\frac{\left(\frac{\xi}{\xi-1}\right) q_t \varepsilon_1 \overline{\omega}^c(\varepsilon_1; q_t, R_t)}{R_t} - 1 \right] \Phi_1(d\varepsilon_1).$$

The average risk premia for bank-financed firms, rp_t^b, and for CMF-financed firms, rp_t^c, are then given by

$$rp_t^b \equiv \frac{\psi^{rb}(q_t, R_t, \tau_t)}{s^{bp}(q_t, R_t, \tau_t)}, \tag{43}$$

$$rp_t^c \equiv \frac{\psi^{rc}(q_t, R_t, \tau_t)}{s^{c}(q_t, R_t, \tau_t)}. \tag{44}$$

Although the debt to equity ratio (leverage) is fixed at the firm level and given by $\frac{\xi-1}{\xi}$, the aggregate debt to equity ratio for the corporate sector, χ_t, is endogenous and depends on the share of firms that decide to produce. It is defined as the ratio of all debt instruments used by producing firms to the aggregate net worth of all firms,

$$\chi_t = (\xi - 1)\left[(1-\tau_t)\, s^{bp}(q_t, R_t, \tau_t) + s^{c}(q_t, R_t, \tau_t)\right]. \tag{45}$$

The default rate on bonds, ϱ_t^c, is given by the share of firms which borrow from CMFs but cannot repay the debt,

$$\varrho_t^c = \frac{\psi^{mc}(q_t, R_t, \tau_t)}{s^{c}(q_t, R_t, \tau_t)}. \tag{46}$$

The average default amounts to the share of firms which sign a contract with either a bank or a CMF but cannot repay the debt,

$$\varrho_t = \frac{\psi^{mb}(q_t, R_t, \tau_t) + \psi^{mc}(q_t, R_t, \tau_t)}{s^{bp}(q_t, R_t, \tau_t) + s^{c}(q_t, R_t, \tau_t)}. \tag{47}$$

Finally, we define the net expected return to entrepreneurial capital as

$$r_t^z = \psi^f(q_t, R_t, \tau_t)(1 - \delta + r_t) - 1 \tag{48}$$

C Competitive equilibrium

For the convenience of further analysis, we collect the relevant equations here.

1. (a) Households:

$$m_{t+1} + d_{t+1} = \frac{R_{t-1}}{\pi_t} d_t + \theta_t \tag{49}$$

$$0 = m_{t+1} + w_t h_t + r_t k_t - c_t - k_{t+1} + (1-\delta) k_t \tag{50}$$

(b) Entrepreneurs:

$$n_t = (1 - \delta + r_t) z_t \tag{51}$$

(c) Monetary authority:

$$\theta_t = (\nu - 1) \frac{m_{t-1}^s}{\pi_t} \tag{52}$$

$$m_t^s = \nu \frac{m_{t-1}^s}{\pi_t} \tag{53}$$

(d) Market clearing:

$$y_t^a = y_t - c_t - e_t - (k_{t+1} + z_{t+1}) + (1-\delta)(k_t + z_t) \tag{54}$$

$$m_t^s = m_t + d_t \tag{55}$$

$$d_t = \left[(1-\tau_t) s^{bp}(q_t, R_t, \tau_t) + s^c(q_t, R_t, \tau_t)\right](\xi - 1) n_t \tag{56}$$

(e) Production and aggregation:

$$x_t = \left[(1-\tau_t) s^{bp}(q_t, R_t, \tau_t) + s^c(q_t, R_t, \tau_t)\right] \xi n_t \tag{57}$$

$$y_t = \psi^y(q_t, R_t, \tau_t) q_t \xi n_t \tag{58}$$

$$y_t^a = \left[\tau_t s^b(q_t, R_t, \tau_t) + \psi^m(q_t, R_t, \tau_t) \mu \xi q_t\right] n_t \tag{59}$$

2. First-order conditions.

(a) Household:

$$\frac{\eta}{u_c(c_t)} = w_t \tag{60}$$

$$u_c(c_t) = \beta R_t E_t \left[\frac{u_c(c_{t+1})}{\pi_{t+1}}\right] \tag{61}$$

$$u_c(c_t) = \beta E_t \left[(1 - \delta + r_{t+1}) u_c(c_{t+1})\right]. \tag{62}$$

(b) Entrepreneurs:

$$
\begin{aligned}
q_t &= \left(\frac{\alpha}{w_t}\right)^{\alpha}\left(\frac{1-\alpha}{r_t}\right)^{1-\alpha} && (63)\\
r_t\left(k_t+z_t\right) &= (1-\alpha)x_t && (64)\\
w_t h_t &= \alpha x_t && (65)\\
e_t &= \gamma\psi^f\left(q_t,R_t,\tau_t\right)n_t && (66)\\
z_{t+1} &= \varkappa_t\left(1-\gamma\right)\psi^f\left(q_t,R_t,\tau_t\right)n_t && (67)\\
1 &= F^d(\varepsilon_{1t},\overline{\varepsilon}_t^d;q_t,R_t) && (68)\\
1 &= F^b(\overline{\varepsilon}_t^b;q_t,R_t,\tau_t) && (69)\\
F^b(\overline{\varepsilon}_{ct};q_t,R_t,\tau_t) &= F^c(\overline{\varepsilon}_t^c;q_t,R_t) && (70)
\end{aligned}
$$

where the functions F^b, F^c and F^d are defined in equations (21), (22) and (19). Note that these definitions require knowledge of the function $\bar{\omega}^b(\cdot)$ and $\bar{\omega}^c(\cdot)$, which are defined in equation (**??**) as solution to (17).

3. Financial structure:

$$\vartheta_t = \frac{\left(1-\tau_t\right)s^{bp}\left(q_t,R_t,\tau_t\right)}{s^c\left(q_t,R_t,\tau_t\right)}, \tag{71}$$

$$rp_t^b \equiv \frac{\psi^{rb}\left(q_t,R_t,\tau_t\right)}{s^{bp}\left(q_t,R_t,\tau_t\right)} \tag{72}$$

$$rp_t^c \equiv \frac{\psi^{rc}\left(q_t,R_t,\tau_t\right)}{s^c\left(q_t,R_t,\tau_t\right)}, \tag{73}$$

$$\chi_t = \left(\xi-1\right)\left[\left(1-\tau_t\right)s^{bp}\left(q_t,R_t,\tau_t\right)+s^c\left(q_t,R_t,\tau_t\right)\right], \tag{74}$$

$$\varrho_t^c = \frac{\psi^{mc}\left(q_t,R_t,\tau_t\right)}{s^c\left(q_t,R_t,\tau_t\right)}, \tag{75}$$

$$\varrho_t = \frac{\psi^{mb}\left(q_t,R_t,\tau_t\right)+\psi^{mc}\left(q_t,R_t,\tau_t\right)}{s^{bp}\left(q_t,R_t,\tau_t\right)+s^c\left(q_t,R_t,\tau_t\right)}. \tag{76}$$

4. Exogenous variables:

(a) Information acquisition costs

$$\log\tau_t-\log\tau=\rho_\tau\left(\log\tau_{t-1}-\log\tau\right)+\varepsilon_{\tau,t},\ \varepsilon_{\tau,t}\sim\mathcal{N}\left(0,\sigma_\tau^2\right),$$

(b) Net worth

$$\log\varkappa_t=\rho_\varkappa\log\varkappa_{t-1}+\varepsilon_{\varkappa,t},\ \varepsilon_{\varkappa,t}\sim\mathcal{N}\left(0,\sigma_\varkappa^2\right),$$

where we assume the shocks $(\tau_t, \varkappa_t)$ to be drawn at t and i.i.d. across time.

Given the exogenous variables τ_t and $\varkappa_t$, equations (49) to (76) need to be solved for the variables characterizing the households choices, $(m_t, d_t, c_t, k_t, h_t)$, the entrepreneurs choices $(e_t, z_t, n_t, \overline{\varepsilon}_t^b, \overline{\varepsilon}_t^c, \overline{\varepsilon}_t^d)$, the choices of the monetary authority (θ_t, m_t^s), aggregate quantities (y_t, y_t^a, x_t), financial variables $(\vartheta_t, rp_t^b, rp_t^c, \chi_t, \varrho_t^c, \varrho_t)$, and prices and returns $(\pi_t, R_t, r_t, q_t, w_t)$.

This is a system of 28 equations in 27 unknowns. Indeed, one equation is superfluous. By Walras' law, fulfillment of the budget constraints of the entrepreneurs and market clearing on all markets implies fulfillment of the budget constraints of the households as well.

D The stochastic steady state

We compute a steady state where we shut down the aggregate shocks, i.e. $\tau_t = \tau$ and $\varkappa_t = \varkappa$, for all t. We denote steady state variables by dropping the time subscript.

We find it convenient to specify one of the endogenous variables, q, as exogenous and to treat γ as endogenous.Under the assumed specification of the utility function, the unique steady state can be obtained as follows. For each value of q, we can compute π, r, w,and c by solving the equations

$$\begin{aligned}
\pi &= \beta R \\
r &= \frac{1}{\beta} - 1 + \delta \\
w &= \left(\frac{1}{q}\right)^{\frac{1}{\alpha}} \alpha \left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} \\
c &= \left(\frac{w}{\eta}\right)^{\frac{1}{\zeta}}.
\end{aligned}$$

To compute the overall expected profits $F(\varepsilon_1; q, R, \tau)$, given by the steady state version of (23), we use the following procedure. First, under our distributional assumptions about the productivity shocks $\varepsilon_1, \varepsilon_2$ and ε_3, we know that

$$\varphi\left(\overline{\omega}^j\right) = \varphi\left(x_j\right) \frac{1}{\overline{\omega}^j \sigma_j}$$

$$f_j(\overline{\omega}^j) = 1 - \Phi\left(x_j - \sigma_j\right) - \overline{\omega}^j \left[1 - \Phi\left(x_j\right)\right],$$

$$g_j(\overline{\omega}^j) = (1-\mu)\,\Phi\left(x_j - \sigma_j\right) + \overline{\omega}^j \left[1 - \Phi\left(x_j\right)\right].$$

where φ and Φ denote the standard normal, $x_j = \frac{\log \overline{\omega}^j + \frac{\sigma_j^2}{2}}{\sigma_j}$ and $j = b, c$. Second, we solve numerically the condition $\varepsilon^j q g_j(\overline{\omega}^j)\xi = R(\xi - 1)$ to obtain the function $\overline{\omega}^j(\varepsilon^j; q, R)$. The function $\overline{\omega}^b(\varepsilon_1\varepsilon_2; q, R)$ for bank-financed firms is derived by using the variance $\sigma^2_{\varepsilon_3}$ of the log-normal distribution. The function $\overline{\omega}^c(\varepsilon_1; q, R)$ for CMF-financed firms is derived by using the variance $\sigma^2_{\varepsilon_2} + \sigma^2_{\varepsilon_3}$. The cutoff value $\overline{\varepsilon}^d$ for proceeding with the bank loan is found by solving numerically the condition $F^d(\varepsilon_1, \overline{\varepsilon}^d; q, R, \tau) = 1$. Using $\overline{\varepsilon}^d$, it is then possible to compute the expected utility per unit of net worth for the bank-financed entrepreneur, $F^b(\varepsilon_1; q, R, \tau)$. The expected utility per unit of net worth for the CMF-financed entrepreneur can be computed as $F^c(\varepsilon_1; q, R) = \varepsilon_1 q f(\overline{\omega}^c(\varepsilon_1; q, R))\xi$. With this, it is possible to calculate the overall return $F(\varepsilon_1; q, R, \tau)$ to entrepreneurial investment, the thresholds $\overline{\varepsilon}^b$ and $\overline{\varepsilon}^c$, and the ratios $\frac{x}{z}$, $\frac{K}{x}$ and $\frac{l}{x}$, as given by

$$\frac{x}{z} = \left[(1-\tau)s^{bp} + s^c\right](\xi)(1 - \delta + r)$$

$$\frac{K}{x} = \frac{1-\alpha}{r}$$

$$\frac{l}{x} = \frac{\alpha}{w}.$$

Notice that in steady state,

$$m = \left(\frac{R}{\pi} - 1\right) d + \theta = c + \delta k - (wh + rk)$$

$$d = \left[(1-\tau)s^{bp} + s^c\right](\xi - 1)(1 - \delta + r)\varkappa z$$

$$\theta = (\nu - 1)\frac{m^s}{\pi} = \left(\frac{\pi - 1}{\pi}\right) m^s,$$

and

$$m^s = m + d = c - wh - (r - \delta)k + \left[(1-\tau)s^{bp} + s^c\right](\xi - 1)(1 - \delta + r)\varkappa z.$$

Now write the budget constraint of the household as

$$c = \left(\frac{R}{\pi} - 1\right) d + \theta + wh + (r - \delta)k$$

or as

$$\frac{c}{z} = (R-1)\left[(1-\tau)s^{bp} + s^c\right](\xi - 1)(1 - \delta + r)\varkappa + w\frac{l}{z} + (r - \delta)\frac{k}{z}.$$

Using the solution obtained, calculate z as $z = c/\frac{c}{z}$ and then compute the aggregate variables n, x, K, l and k. Then, use

$$z = \gamma \psi^f (q, R, \tau) n$$

to compute γ, the steady state version of equations (35) and (30) to compute y and e, and of the resource constraint (40) to compute y^a.

Finally, we use these results to compute the financial variables, given by (42)-(47), and the net expected return to entrepreneurial capital, given by (48), in steady state.

E Log-linearization

The equilibrium can be obtained by solving the system of equilibrium conditions, log-linearized around a stochastic steady state where $\pi = 1$ and the aggregate shocks are set to their steady state values. The log-linearized equations are standard and are therefore omitted here.

The difficulty arises in the computation of the coefficients multiplying the variables in the log-linearized equations. We illustrate here how they can be obtained. A detailed appendix with all the log-linearized equations and relative coefficients is available from the authors upon request.

Consider the log-linearized condition corresponding to equation (35),

$$\widehat{y}_t = \left[\frac{\psi_q^y(\cdot)q}{\psi^y(\cdot)} + 1\right]\widehat{q}_t + \frac{\psi_R^y(\cdot)R}{\psi^y(\cdot)}\widehat{R}_t + \frac{\psi_\tau^y(\cdot)\tau}{\psi^y(\cdot)}\widehat{\tau}_t + \widehat{n}_t.$$

Define $\Omega\left(\varepsilon_1; q, R\right) \equiv \varepsilon_1\varphi_1(\varepsilon_1)\int_{\overline{\varepsilon}_d(\varepsilon_1;q,R)}\varepsilon_2\Phi_2(d\varepsilon_2)$. From equation (41), evaluated at the stochastic steady state, we obtain

$$\begin{aligned}\psi_\upsilon^y\left(q,R,\tau\right) &= (1-\tau)\left[\begin{array}{c}\frac{\partial\overline{\varepsilon}_c(\cdot)}{\partial\upsilon}\Omega\left(\overline{\varepsilon}_c;q,R\right) - \frac{\partial\overline{\varepsilon}_b(\cdot)}{\partial\upsilon}\Omega\left(\overline{\varepsilon}_b;q,R\right)\\ -\int_{\overline{\varepsilon}_b}^{\overline{\varepsilon}_c}\left.\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial\upsilon}\right|_{(\varepsilon_1;q,R)}\varepsilon_1\overline{\varepsilon}_d\left(\varepsilon_1;q,R\right)\varphi_2(\overline{\varepsilon}_d\left(\varepsilon_1;q,R\right))\Phi_1(d\varepsilon_1)\end{array}\right]\\ &\quad -\frac{\partial\overline{\varepsilon}_c\left(q,R,\tau\right)}{\partial\upsilon}\overline{\varepsilon}_c\varphi_1(\overline{\varepsilon}_c),\end{aligned}$$

for $\upsilon = q, R$, and

$$\begin{aligned}\psi_\tau^y\left(q,R,\tau\right) &= -\int_{\overline{\varepsilon}_b(q,R,\tau)}^{\overline{\varepsilon}_c(q,R,\tau)}\int_{\overline{\varepsilon}_d(\varepsilon_1;q,R)}\varepsilon_1\varepsilon_2\Phi_2(d\varepsilon_2)\Phi_1(d\varepsilon_1) - \frac{\partial\overline{\varepsilon}_c\left(q,R,\tau\right)}{\partial\tau}\overline{\varepsilon}_c\varphi_1(\overline{\varepsilon}_c)\\ &\quad +(1-\tau)\left[\frac{\partial\overline{\varepsilon}_c\left(\cdot\right)}{\partial\tau}\Omega\left(\overline{\varepsilon}_c;q,R\right) - \frac{\partial\overline{\varepsilon}_b\left(\cdot\right)}{\partial\tau}\Omega\left(\overline{\varepsilon}_b;q,R\right)\right].\end{aligned}$$

To compute the value of $\psi_\upsilon^y\left(q,R,\tau\right)$ and $\psi_\tau^y\left(q,R,\tau\right)$, we now need to compute the derivatives of the thresholds $\overline{\varepsilon}_b, \overline{\varepsilon}_c, \overline{\varepsilon}_d$.

Consider first the threshold at stage II, $\overline{\varepsilon}_d\left(\varepsilon_1;q,R\right)$, which is implicitely defined by

$$F^d(\varepsilon_1, \overline{\varepsilon}_d; q, R) = 1.$$

Using the implicit function theorem, we have that

$$\left.\frac{\partial \overline{\varepsilon}_d(\cdot)}{\partial \varepsilon_1}\right|_{(\varepsilon_1;q,R)} = -\frac{F_1^d(\varepsilon_1,\overline{\varepsilon}_d;q,R)}{F_2^d(\varepsilon_1,\overline{\varepsilon}_d;q,R)} \tag{77}$$

$$\left.\frac{\partial \overline{\varepsilon}_d(\cdot)}{\partial \upsilon}\right|_{(\varepsilon_1;q,R)} = -\frac{F_\upsilon^d(\varepsilon_1,\overline{\varepsilon}_d;q,R)}{F_2^d(\varepsilon_1,\overline{\varepsilon}_d;q,R)}. \tag{78}$$

Using equation (17), we obtain

$$\begin{aligned}
F_1^d(\varepsilon_1,\varepsilon_2;q,R) &= \varepsilon_2 q\xi\left[f(\overline{\omega}^b(\varepsilon_1,\varepsilon_2;q,R)) + \varepsilon_1 f'(\overline{\omega}^b(\varepsilon_1,\varepsilon_2;q,R))\left.\frac{\partial \overline{\omega}^b(\cdot)}{\partial \varepsilon_1}\right|_{(\varepsilon_1,\varepsilon_2;q,R)}\right]\\
F_2^d(\varepsilon_1,\varepsilon_2;q,R) &= \varepsilon_1 q\xi\left[f(\overline{\omega}^b(\varepsilon_1,\varepsilon_2;q,R)) + \varepsilon_2 f'(\overline{\omega}^b(\varepsilon_1,\varepsilon_2;q,R))\left.\frac{\partial \overline{\omega}^b(\cdot)}{\partial \varepsilon_2}\right|_{(\varepsilon_1,\varepsilon_2;q,R)}\right]\\
F_q^d(\varepsilon_1,\varepsilon_2;q,R) &= \varepsilon_1\varepsilon_2\xi\left[f(\overline{\omega}^b(\varepsilon_1,\varepsilon_2;q,R)) + q f'(\overline{\omega}^b(\varepsilon_1,\varepsilon_2;q,R))\left.\frac{\partial \overline{\omega}^b(\cdot)}{\partial q}\right|_{(\varepsilon_1,\varepsilon_2;q,R)}\right]\\
F_R^d(\varepsilon_1,\varepsilon_2;q,R) &= \varepsilon_1\varepsilon_2 q\xi f'(\overline{\omega}^b(\varepsilon_1,\varepsilon_2;q,R))\left.\frac{\partial \overline{\omega}^b(\cdot)}{\partial R}\right|_{(\varepsilon_1,\varepsilon_2;q,R)}.
\end{aligned}$$

Computation of the derivatives of $F^d(\cdot)$ requires computing also the derivatives $\frac{\partial \overline{\omega}^b}{\partial \varepsilon_1}$, $\frac{\partial \overline{\omega}^b}{\partial \varepsilon_2}$, and $\frac{\partial \overline{\omega}^b}{\partial \upsilon}$, for $\upsilon = q, R$. Define $\widetilde{\omega}^b(\varepsilon_1,\varepsilon_2;q,R) \equiv \frac{g(\overline{\omega}^b(\varepsilon_1,\varepsilon_2;q,R))}{g'(\overline{\omega}^b(\varepsilon_1,\varepsilon_2;q,R))}$. From condition (17), we get

$$\left.\frac{\partial \overline{\omega}^b(\cdot)}{\partial \varepsilon_1}\right|_{(\varepsilon_1,\varepsilon_2;q,R)} = -\frac{\widetilde{\omega}^b(\varepsilon_1,\varepsilon_2;q,R)}{\varepsilon_1} \tag{79}$$

$$\left.\frac{\partial \overline{\omega}^b(\cdot)}{\partial \varepsilon_2}\right|_{(\varepsilon_1,\varepsilon_2;q,R)} = -\frac{\widetilde{\omega}^b(\varepsilon_1,\varepsilon_2;q,R)}{\varepsilon_2} \tag{80}$$

$$\left.\frac{\partial \overline{\omega}^b(\cdot)}{\partial q}\right|_{(\varepsilon_1,\varepsilon_2;q,R)} = -\frac{\widetilde{\omega}^b(\varepsilon_1,\varepsilon_2;q,R)}{q} \tag{81}$$

$$\left.\frac{\partial \overline{\omega}^b(\cdot)}{\partial R}\right|_{(\varepsilon_1,\varepsilon_2;q,R)} = \frac{\widetilde{\omega}^b(\varepsilon_1,\varepsilon_2;q,R)}{R}. \tag{82}$$

Define now $\Lambda^b(\varepsilon_1,\overline{\varepsilon}_d;q,R) = 1 - \frac{f'(\overline{\omega}^b(\varepsilon_1,\overline{\varepsilon}_d;q,R))}{f(\overline{\omega}^b(\varepsilon_1,\overline{\varepsilon}_d;q,R))}\widetilde{\omega}^b(\varepsilon_1,\overline{\varepsilon}_d;q,R)$. We can then write

$$\begin{aligned}
F_1^d(\varepsilon_1,\overline{\varepsilon}_d;q,R) &= \frac{F^d(\varepsilon_1,\overline{\varepsilon}_d;q,R)}{\varepsilon_1}\Lambda(\varepsilon_1,\overline{\varepsilon}_d;q,R)\\
F_2^d(\varepsilon_1,\overline{\varepsilon}_d;q,R) &= \frac{F^d(\varepsilon_1,\overline{\varepsilon}_d;q,R)}{\overline{\varepsilon}_d}\Lambda(\varepsilon_1,\overline{\varepsilon}_d;q,R)\\
F_q^d(\varepsilon_1,\overline{\varepsilon}_d;q,R) &= \frac{F^d(\varepsilon_1,\overline{\varepsilon}_d;q,R)}{q}\Lambda(\varepsilon_1,\overline{\varepsilon}_d;q,R)\\
F_R^d(\varepsilon_1,\overline{\varepsilon}_d;q,R) &= \frac{F^d(\varepsilon_1,\overline{\varepsilon}_d;q,R)}{R}\left[1-\Lambda(\varepsilon_1,\overline{\varepsilon}_d;q,R)\right].
\end{aligned}$$

and

$$\left.\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial\varepsilon_1}\right|_{(\varepsilon_1;q,R)} = -\frac{\overline{\varepsilon}_d}{\varepsilon_1} \tag{83}$$

$$\left.\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial q}\right|_{(\varepsilon_1;q,R)} = -\frac{\overline{\varepsilon}_d}{q} \tag{84}$$

$$\left.\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial R}\right|_{(\varepsilon_1;q,R)} = -\frac{\overline{\varepsilon}_d}{R}\left[\frac{1}{\Lambda(\varepsilon_1,\overline{\varepsilon}_d;q,R)}-1\right] \tag{85}$$

We now need to obtain derivatives of the threshold $\overline{\varepsilon}_b(q,R,\tau)$. This latter is implicitely defined by condition (24) evaluated at the steady state. Using the implicit function theorem, we have that

$$\frac{\partial\overline{\varepsilon}_b(\cdot)}{\partial\upsilon} = -\frac{F^b_\upsilon(\overline{\varepsilon}_b;q,R,\tau)}{F^b_1(\overline{\varepsilon}_b;q,R,\tau)}$$

$$\frac{\partial\overline{\varepsilon}_b(\cdot)}{\partial\tau} = -\frac{F^b_\tau(\overline{\varepsilon}_b;q,R,\tau)}{F^b_1(\overline{\varepsilon}_b;q,R,\tau)}$$

for $\upsilon = q,R$. Now, define $\Gamma(\varepsilon_1;q,R) = \varepsilon_1\overline{\varepsilon}_d(\cdot)\,qf(\overline{\omega}^b(\varepsilon_1\overline{\varepsilon}_d(\cdot);q,R))\xi\varphi_2(\overline{\varepsilon}_d(\cdot))$. Using condition (21), we get

$$F^b_1(\varepsilon_1;q,R,\tau) = (1-\tau)\left(\begin{array}{c} -\left.\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial\varepsilon_1}\right|_{(\varepsilon_1;q,R)}\Gamma(\varepsilon_1;q,R)+\int_{\overline{\varepsilon}_d(\varepsilon_1;q,R)}F^d_1(\varepsilon_1,\varepsilon_2;q,R)\Phi_2(d\varepsilon_2) \\ +\varphi_2(\overline{\varepsilon}_d)\left.\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial\varepsilon_1}\right|_{(\varepsilon_1;q,R)}\end{array}\right)$$

$$F^b_\upsilon(\varepsilon_1;q,R,\tau) = (1-\tau)\left(\begin{array}{c} -\left.\frac{\partial\overline{\varepsilon}_d}{\partial q}\right|_{(\varepsilon_1;q,R)}\Gamma(\varepsilon_1;q,R)+\int_{\overline{\varepsilon}_d(\varepsilon_1;q,R)}F^d_q(\varepsilon_1,\varepsilon_2;q,R)\Phi_2(d\varepsilon_2) \\ +\varphi_2(\overline{\varepsilon}_d(\cdot))\left.\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial q}\right|_{(\varepsilon_1;q,R)}\end{array}\right)$$

$$F^b_\tau(\varepsilon_1;q,R,\tau) = -\frac{F^b(\varepsilon_1;q,R,\tau)}{(1-\tau)}.$$

for $\upsilon = q,R$. Notice that $\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial\varepsilon_1}$ and $\frac{\partial\overline{\varepsilon}_d(\cdot)}{\partial\upsilon}$ are given by (77)-(78). Moreover, $\frac{\partial\overline{\omega}^b}{\partial\varepsilon_1}$, $\frac{\partial\overline{\omega}^b}{\partial q}$ and $\frac{\partial\overline{\omega}^b}{\partial R}$ are given by (79), (81) and (82).

Consider now the threshold for the first stage, $\overline{\varepsilon}_c(q,R,\tau)$. It is implicitely defined by condition (25), evaluated at the steady state. Using the implicit function theorem, we have that

$$\frac{\partial\overline{\varepsilon}_c(\cdot)}{\partial\upsilon} = -\left(\frac{F^b_\upsilon(\overline{\varepsilon}_c;q,R,\tau)-F^c_\upsilon(\overline{\varepsilon}_c;q,R)}{F^b_1(\overline{\varepsilon}_c;q,R,\tau)-F^c_1(\overline{\varepsilon}_c;q,R)}\right),$$

$$\frac{\partial\overline{\varepsilon}_c(\cdot)}{\partial\tau} = -\left(\frac{F^b_\tau(\overline{\varepsilon}_c;q,R,\tau)}{F^b_1(\overline{\varepsilon}_c;q,R,\tau)-F^c_1(\overline{\varepsilon}_c;q,R)}\right).$$

for $\upsilon = q, R$. Using condition (22), we get

$$\begin{aligned}
F_1^c(\varepsilon_1; q, R) &= \frac{F^c(\varepsilon_1; q, R)}{\varepsilon_1}\left[1 + \varepsilon_1 \frac{f'(\overline{\omega}^c(\varepsilon_1; q, R))}{f(\overline{\omega}^c(\varepsilon_1; q, R))} \left.\frac{\partial \overline{\omega}^c(\cdot)}{\partial \varepsilon_1}\right|_{(\varepsilon_1; q, R)}\right] \\
F_q^c(\varepsilon_1; q, R) &= \frac{F^c(\varepsilon_1; q, R)}{q}\left[1 + q \frac{f'(\overline{\omega}^c(\varepsilon_1; q, R))}{f(\overline{\omega}^c(\varepsilon_1; q, R))} \left.\frac{\partial \overline{\omega}^c(\cdot)}{\partial q}\right|_{(\varepsilon_1; q, R)}\right] \\
F_R^c(\varepsilon_1; q, R) &= F^c(\varepsilon_1; q, R) \frac{f'(\overline{\omega}^c(\varepsilon_1; q, R))}{f(\overline{\omega}^c(\varepsilon_1; q, R))} \left.\frac{\partial \overline{\omega}^c(\cdot)}{\partial R}\right|_{(\varepsilon_1; q, R)}.
\end{aligned}$$

Define $\widetilde{\omega}^c(\varepsilon_1; q, R) \equiv \frac{g(\overline{\omega}^c(\varepsilon_1; q, R))}{g'(\overline{\omega}^c(\varepsilon_1; q, R))}$ and $\Lambda^c(\overline{\varepsilon}_c; q, R) = 1 - \frac{f'(\overline{\omega}^c(\overline{\varepsilon}_c; q, R))}{f(\overline{\omega}^c(\overline{\varepsilon}_c; q, R))}\widetilde{\omega}^c(\overline{\varepsilon}_c; q, R)$. From condition (17), we get

$$\begin{aligned}
\frac{\partial \overline{\omega}^c}{\partial \varepsilon_1} &= -\frac{\widetilde{\omega}^c(\varepsilon_1; q, R)}{\varepsilon_1} \\
\frac{\partial \overline{\omega}^c}{\partial q} &= -\frac{\widetilde{\omega}^c(\varepsilon_1; q, R)}{q} \\
\frac{\partial \overline{\omega}^c}{\partial R} &= \frac{\widetilde{\omega}^c(\varepsilon_1; q, R)}{R}.
\end{aligned}$$

It follows that

$$\begin{aligned}
F_1^c(\overline{\varepsilon}_c; q, R) &= \frac{F^c(\overline{\varepsilon}_c; q, R)}{\overline{\varepsilon}_c}\left[\Lambda^c(\overline{\varepsilon}_c; q, R)\right] \\
F_q^c(\overline{\varepsilon}_c; q, R) &= \frac{F^c(\overline{\varepsilon}_c; q, R)}{q}\Lambda^c(\overline{\varepsilon}_c; q, R) \\
F_R^c(\overline{\varepsilon}_c; q, R) &= \frac{F^c(\overline{\varepsilon}_c; q, R)}{R}\left[1 - \Lambda^c(\overline{\varepsilon}_c; q, R)\right]
\end{aligned}$$

from which we can compute $\frac{\partial \overline{\varepsilon}_c(\cdot)}{\partial q}$, $\frac{\partial \overline{\varepsilon}_c(\cdot)}{\partial R}$ and $\frac{\partial \overline{\varepsilon}_c(\cdot)}{\partial \tau}$.